63-3-3

BSD-TDR-63-35

REPORT NO.
TDR-169(3153-10)TN-1

403 001

403001

TISIA

Stability and Response of Cylindrical Shells to Moving Loads

15 MARCH 1963

Prepared by JOHN P. JONES
Aerospace Corporation
and
PRAVIN G. BHUTA
Space Technology Laboratories, Inc.

Prepared for COMMANDER BALLISTIC SYSTEMS DIVISION
UNITED STATES AIR FORCE
Norton Air Force Base, California

LABORATORIES DIVISION • AEROSPACE CORPORATION
CONTRACT NO. AF 04(695)-169

MAY 8 1963
TISIA

BSD-TDR-63-35

Report No.
TDR-169(3153-10)TN-1

STABILITY AND RESPONSE OF CYLINDRICAL SHELLS TO MOVING LOADS

Prepared by

John P. Jones
Aerospace Corporation

and

Pravin G. Bhuta
Space Technology Laboratories, Inc.

AEROSPACE CORPORATION
El Segundo, California

Contract No. AF 04(695)-169

15 March 1963

Prepared for

COMMANDER BALLISTIC SYSTEMS DIVISION
UNITED STATES AIR FORCE
Norton Air Force Base, California

ABSTRACT

The stability of a circular cylindrical shell subjected to a moving ring load with a constant velocity has been examined in detail when both longitudinal and transverse inertia effects are included, using both the Timoshenko and the Flügge equations. A lower resonance speed has been found, which apparently has been concealed in a recent paper.

CONTENTS

ABSTRACT ii

NOMENCLATURE iv

I. INTRODUCTION 1

II. STATEMENT OF PROBLEM AND METHODS OF SOLUTION 3

III. ANALYSIS 5

Solution Using Timoshenko Equations 5

Solution Using Flügge Equations 15

Response in the Supercritical Range 21

IV. SUMMARY 23

REFERENCES 24

NOMENCLATURE

a	=	mean radius of shell
a, b	=	see Eq. (27)
a_1, b_1	=	see Eq. (28)
a_2, b_2	=	see Eqs. (61) and (62)
$\bar{a}_1, \bar{b}_1$	=	see Eqs. (39) and (40)
D.A.	=	see Eq. (34)
E	=	Young's modulus
h	=	shell thickness
p	=	$(1 - \nu^2)\bar{p}/\lambda E$
p_0	=	ring load per unit circumference
t	=	$\alpha\bar{t}/a$
$\bar{t}$	=	time
u, w	=	$\bar{u}/a, \bar{w}/a$
$\bar{u}, \bar{w}$	=	axial and radial displacements, respectively
V	=	velocity of ring load
V_{CR}	=	physical resonant speeds
v	=	V/α
v_{CR}	=	nondimensionalized resonant speeds
v_0, v_1	=	see Eq. (20)
x	=	$\bar{x}/a$
$\bar{x}$	=	axial coordinate
x_0, x_1	=	see Eq. (52)
z	=	$x - vt$

α^2	=	$E/\rho(1 - \nu^2)$
α_1^2, α_2^2	=	see Eqs. (73) and (74)
$\bar{\alpha}_1, \bar{\alpha}_2$	=	see Eq. (75)
β^2	=	$1 - v^2$
Δ	=	see Eq. (13)
Δ'	=	see Eq. (49)
$\delta(z)$	=	Dirac delta function
ϵ	=	$\lambda/\sqrt{3}$
ι	=	$+\sqrt{-1}$
λ	=	h/a
ν	=	Poisson's ratio
ξ	=	Fourier transform variable
ρ	=	shell density
ϕ	=	$v/\sqrt{\epsilon}$
$(\tilde{\ })$	=	Fourier transformed quantity

SECTION I
INTRODUCTION

In recent years, the problem of the dynamic response of circular cylindrical shells has received considerable interest because of their use in aerospace vehicles. In particular, many investigations have examined shells subjected to traveling loads: Nachbar [1][1] considered the dynamic response of an infinitely long cylindrical shell to moving discontinuous loads. Brogan [2] considered the problem of a uniform pressure front moving over a finite cylindrical shell which was taken to be internally pressurized, but the resonant modes were not considered and the solution obtained was not valid for all values of time. Bhuta [3] obtained the transient response of a finite shell which was not pressurized, but the resonant modes were studied in detail. It was also pointed out in [3] that a critical velocity exists for a semi-infinite shell for which the deflections become unbounded. The same critical velocity was obtained by Prieskin [4] for an infinitely long cylindrical shell.

In all of the forementioned investigations, only the axisymmetric loading was considered. However, in [1], viscous damping in the radial direction was assumed to be present. In [2], [3], and [4], the investigations concerned the response of undamped shells. Also, [1] differs from the rest in that the acceleration in the axial direction was considered, whereas in other prior investigations only radial inertia was taken into account. The critical velocity of [4] due to a bending resonance does not appear in the solution of [1], but rather a much greater critical velocity due to axial motion resonance appears. The resonance due to bending seems to have been concealed because of the assumed presence of damping. Unfortunately, however, as is shown in the present investigation, the resonance due to bending occurs at a much lower velocity than does the

[1] Numbers in brackets designate References at end of paper.

resonance due to axial motion and is important for aerospace vehicle design. The resonant character of the solution was brought out also in [3] by having let the shell length become successively larger and having noted that at the critical velocity the dynamic loading factor became infinite.

SECTION II

STATEMENT OF PROBLEM AND METHODS OF SOLUTION

It is the purpose of this paper to investigate the dynamic stability and solve the problem of the dynamic response of an infinitely long cylindrical shell subjected to a ring load moving with constant velocity, taking into account the accelerations in both the radial and the axial directions. It is shown that even when both accelerations are considered, the resonance due to bending occurs at a lower velocity than the one due to axial motion.

The problem is solved using Timoshenko's equations [5], as in [1] but without the presence of damping, and Flügge's equations [6]. The results of the analyses are compared, and it is found that Timoshenko's equations suffice for values of the load speed, up to the first critical speed. It is found that above that speed, there is really no proper or physically meaningful steady-state solution, as was concluded also in [4], and that an initial value problem should be solved for such speeds. Near the axial resonance, the effects of shear deformation and rotatory inertia should be included also. In addition, expressions for dynamic amplification in the deflections are derived.

The analysis is carried out using Fourier transforms. The resonances or instabilities are shown by the fact that for certain velocities the inversion integral fails to exist.

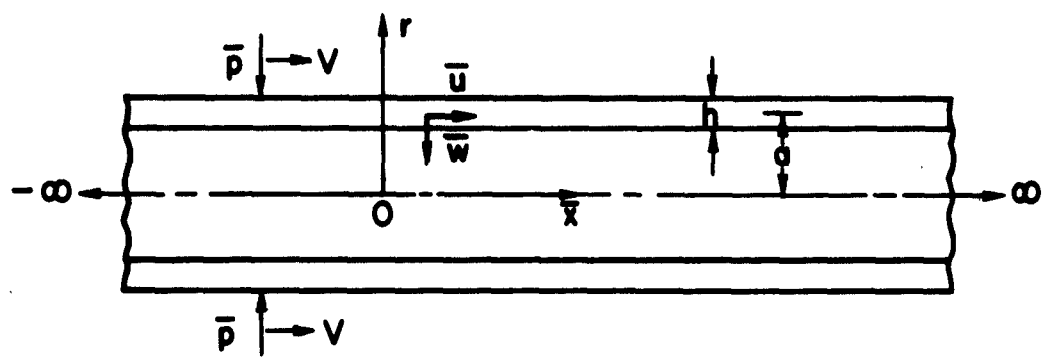

Fig. 1. Coordinate and load geometry.

SECTION III
ANALYSIS

Solution Using Timoshenko Equations

The shell--of thickness h and radius a, and of a material having a modulus of elasticity E, Poisson's ratio ν, and density ρ--is loaded by a ring load moving with a constant velocity V, as shown in Fig. 1, and the radial displacement $\overline{w}$ is assumed to be positive inward. The axial displacement and the pressure and axial coordinates are denoted by $\overline{u}$, $\overline{p}$, and $\overline{x}$, respectively. The equations of motion [5] for the shell are then

$$\frac{Eh}{1 - \nu^2}\left(\frac{\partial^2 \overline{u}}{\partial \overline{x}^2} - \frac{\nu \partial \overline{w}}{a \partial \overline{x}}\right) = \rho h \frac{\partial^2 \overline{u}}{\partial \overline{t}^2} \tag{1}$$

and

$$\frac{aEh^3}{12(1 - \nu^2)} \frac{\partial^4 \overline{w}}{\partial \overline{x}^4} + \frac{Eh}{1 - \nu^2}\left(\frac{\overline{w}}{a} - \nu \frac{\partial \overline{u}}{\partial \overline{x}}\right) = \overline{p}a - \rho h a \frac{\partial^2 \overline{w}}{\partial \overline{t}^2} \quad . \tag{2}$$

Next the following variables

$$\left.\begin{aligned} &\lambda = h/a \quad , \quad \alpha^2 = E/\rho(1 - \nu^2) \quad , \quad \overline{x} = ax \quad , \\ &\overline{t} = at/\alpha \quad , \quad \overline{u} = au \quad , \quad \overline{w} = aw \quad , \\ &\overline{p} = p\lambda E/(1 - \nu^2) \end{aligned}\right\} \tag{3}$$

are introduced into (1) and (2) to obtain the equations of motion in a dimensionless form:

$$\frac{\partial^2 u}{\partial x^2} - \nu \frac{\partial w}{\partial x} = \frac{\partial^2 u}{\partial t^2} \tag{4}$$

and

$$\lambda^2 \frac{\partial^4 w}{\partial x^4} + w - \nu \frac{\partial u}{\partial x} + \frac{\partial^2 w}{\partial t^2} = p(x,t) \quad . \tag{5}$$

For the moving ring load, the pressure is given by

$$p = p_0 \delta(x - vt) \tag{6}$$

where p_0 denotes the intensity of the load, δ the Dirac delta function, and v the nondimensional speed of the load. The actual speed V of the load is related to v by

$$V = \alpha v \quad . \tag{7}$$

To seek the steady-state solution, one introduces the transformation

$$z = x - vt \tag{8}$$

and sets all quantities to be functions of the new variable z. This results in

$$(1 - v^2)u'' - \nu w' = 0 \tag{9}$$

$$\frac{\lambda^2}{12} w^{iv} + v^2 w'' + w - \nu u' = p_0 \delta(z) \tag{10}$$

where $(\ )' \equiv d(\ )/dz$.

Operating with an exponential Fourier transform on (9) and (10) gives

$$\tilde{u} = \frac{\iota \nu p_0}{\xi(1 - \nu^2)\Delta} \tag{11}$$

and

$$\tilde{w} = \frac{p_0}{\Delta} \tag{12}$$

where

$$\Delta = \frac{\lambda^2}{12}\xi^4 - \nu^2\xi^2 + 1 - \frac{\nu^2}{1 - \nu^2} \quad . \tag{13}$$

Here ξ is the transform variable, and the transform of a quantity ( ) is denoted by $(\tilde{\ })$. Inverting (11) and (12), one has

$$u = \frac{\iota \nu p_0}{2\pi(1 - \nu^2)} \int_{-\infty}^{\infty} \frac{\exp(-\iota \xi z)}{\xi \Delta} d\xi \tag{14}$$

and

$$w = \frac{p_0}{2\pi} \int_{-\infty}^{\infty} \frac{\exp(-\iota \xi z)}{\Delta} d\xi \quad . \tag{15}$$

Determination of Resonant Speeds for Timoshenko Equations

Now the foregoing integrals are well defined except where the velocity is such that Δ has a double root on the real axis. The condition for this to be so is

$$v^4 - \frac{\lambda^2}{3}\left(1 - \frac{\nu^2}{1 - v^2}\right) = 0 \tag{16}$$

For these values of v, the integrands in (14) and (15) have second-order poles on the axis of integration, and for such a situation, the integrals involved do not exist even in the sense of a Cauchy principal value. It will be shown later in the analysis that such nonexistence of a solution to a physical problem implies a resonance or an instability in the sense that the displacements everywhere become unbounded. Values of v for which (16) is satisfied are the resonant speeds. It will be shown that there is one root near $v = (\lambda^2/3)^{1/4}$ and two near $v = 1$. Actually, there are other roots to (16) also, but since only positive real roots are of physical significance, the rest are discarded. The roots can be tabulated easily as functions of ν and λ. In most practical problems, λ is certainly less than 0.1 (i.e., a thin shell is assumed, since the statement $\lambda \leq 0.1$ implies that the thickness of the shell is less than 0.1 of the radius) for it is only in such regimes that the shell equations are valid. For values of $\lambda > 0.1$, it is doubtful if either Timoshenko's or Flügge's equations are valid. Thus, a perturbation solution in λ is a valid way of obtaining the approximate roots of (16). To obtain the root near $(\lambda^2/3)^{1/4}$, one sets $v = v_{CR1}$ to be given by

$$v_{CR1}^2 = \epsilon v_0 + \epsilon^2 v_1 + \cdots \tag{17}$$

where $\epsilon = \lambda/\sqrt{3}$. Substituting in (16) and equating the coefficients of the like powers of ϵ, one has

$$v_{CR1} = \sqrt{\epsilon}\,\sqrt[4]{1 - \nu^2}\left[1 - \frac{\nu^2\epsilon}{4(1 - \nu^2)^{1/2}}\right] + \cdots \quad . \tag{18}$$

Equation (18) is valid when $(1 - \nu^2)^{1/2} >> \epsilon\,\nu^2/4$. The maximum value of ν is 0.5, so that the above relation is always true. In terms of physical velocity V, (18) gives

$$V^2_{CR1} = \frac{Eh}{\rho a\,[3(1 - \nu^2)]^{1/2}} - \frac{Eh^2\nu^2}{6\rho a^2(1 - \nu^2)} + \cdots \quad . \tag{19}$$

The first term of (19) is the square of the bending resonant speed given in [4], and the additional terms in (19) arise from the consideration of longitudinal inertia in the present investigation. It is apparent from (19) that the contribution from the longitudinal inertia terms is indeed small.

To obtain the roots of (16) near v = 1.0, one sets $v = v_{CR2}$ to be given by

$$v^2_{CR2} = v_0 + \epsilon^2 v_1 + \cdots \quad . \tag{20}$$

Substituting from (20) into (16) yields

$$v^2_{CR2} = 1 + \nu^2\epsilon^2 + \cdots \quad . \tag{21}$$

In terms of physical speed

$$V^2_{CR2} = \frac{E}{\rho(1 - \nu^2)} + \frac{E\nu^2h^2}{3\rho a^2(1 - \nu^2)} + \cdots \quad . \tag{22}$$

Another resonant speed will occur when the constant terms in Δ[i.e., $1 - \nu^2/(1 - v^2)$] defined by (13) equal zero. This situation corresponds to a double root at $\xi = 0$. Setting the constant term to zero yields the third critical speed

$$V_{CR3}^2 = 1 - \nu^2 \quad . \tag{23}$$

In terms of physical speed, (23) gives

$$V_{CR3}^2 = \frac{E}{\rho} \quad . \tag{24}$$

It should be remarked that when $v^2 = 1$, i.e., $V^2 = E/\rho(1 - \nu^2)$, there is no meaningful solution to the differential equations (9) and (10). Thus, one can conclude that there are three resonant speeds, given by

1. $$V_{CR1}^2 = \frac{Eh}{\rho a[3(1 - \nu^2)]^{1/2}} - \frac{Eh^2\nu^2}{6\rho a^2(1 - \nu^2)} + \cdots$$

2. $$V_{CR2}^2 = \frac{E}{\rho(1 - \nu^2)} + \cdots$$

3. $$V_{CR3}^2 = \frac{E}{\rho} \quad .$$

The first resonance, and the one with the lowest speed, evidently corresponds to bending resonance. The second is a resonance which occurs when the load speed becomes equal to the plate-wave speed in the shell and is an axial vibration resonance. The third resonance occurs when the load speed becomes equal to the bar-wave speed and is also an axial vibration resonance. It is not clear physically as to why the bar-wave speed should enter at all. In fact, for load speeds of the order of magnitude of either the bar- or the

plate-wave speed, it is doubtful that the equations of motion without the effects of shear deformation and rotatory inertia are valid.

It is obvious from the manner in which the instabilities have been obtained here that these critical or resonant speeds depend only on the velocity of the pressure wave and not upon the manner in which the pressure may be changing with z above and behind the pressure wave front.

Evaluation of Displacements for Timoshenko Equations

The integrals (14) and (16) will be evaluated next. It is obvious that the integral in (14) can be obtained from (15) because

$$u = -\frac{\nu}{1 - v^2} w' \quad . \tag{25}$$

Hence, it suffices to evaluate (15), which in explicit terms is

$$w(z) = \frac{p_0}{2\pi} \int_{-\infty}^{\infty} \frac{\exp(-\iota \xi z)}{\epsilon^4 \xi^4/4 - v^2 \xi^2 + 1 - [\nu^2/(1 - v^2)]} d\xi \quad . \tag{26}$$

First, one considers the case when $v < v_{CR1}$. The roots of the denominator of the integrand are given by

$$\left.\begin{aligned} \xi^2 &= \frac{2v^2}{\epsilon^2} \pm \iota \frac{2}{\epsilon}\left(1 - \frac{\nu^2}{1 - v^2} - v^4\right)^{1/2} \\ &\equiv a \pm \iota b \quad . \end{aligned}\right\} \tag{27}$$

Since $v < v_{CR1}$, b is a real and positive number. Then the roots are located at

$$\xi = \pm\frac{a_1}{\sqrt{\epsilon}} \pm \iota \frac{b_1}{\sqrt{\epsilon}} \tag{28}$$

where

$$a_1 = \left[\left(1 - \frac{\nu^2}{1 - v^2}\right)^{1/2} + \frac{v^2}{\epsilon}\right]^{1/2} \tag{29}$$

and

$$b_1 = \left[\left(1 - \frac{\nu^2}{1 - v^2}\right)^{1/2} - \frac{v^2}{\epsilon}\right]^{1/2} \quad . \tag{30}$$

For $z > 0$, it suffices to integrate along the real axis and a semicircular contour in the lower-half plane, picking up the contributions from the poles at $\xi = \pm a_1/\sqrt{\epsilon} - \iota b_1/\sqrt{\epsilon}$. Performing the indicated integration results in

$$w = \frac{P_0}{2\sqrt{\epsilon}} \frac{\exp(-zb_1/\sqrt{\epsilon})}{[1 - \nu^2/(1 - v^2)]^{1/2}[1 - \nu^2/(1 - v^2) - v^4/\epsilon^2]^{1/2}}\left(a_1 \cos\frac{za_1}{\sqrt{\epsilon}} + b_1 \sin\frac{za_1}{\sqrt{\epsilon}}\right) \quad , \quad z \geq 0 \quad . \tag{31}$$

From symmetry considerations, w is an even function of z; hence

$$w = \frac{P_0}{2\sqrt{\epsilon}} \frac{\exp(zb_1/\sqrt{\epsilon})}{[1 - \nu^2/(1 - v^2)]^{1/2}[1 - \nu^2/(1 - v^2) - v^4/\epsilon^2]^{1/2}}\left(a_1 \cos\frac{za_1}{\sqrt{\epsilon}} - b_1 \sin\frac{za_1}{\sqrt{\epsilon}}\right) \quad , \quad z \leq 0 \quad . \tag{32}$$

The maximum value of w occurs at $z = 0$ and is given by

$$w_{max} = \frac{P_0}{2\sqrt{\epsilon}} \frac{\left|[1 - (\nu^2/1 - v^2)]^{1/2} + v^2/\epsilon\right|^{1/2}}{[1 - \nu^2/(1 - v^2)]^{1/2}[1 - \nu^2/(1 - v^2) - v^4/\epsilon^2]^{1/2}} \quad . \tag{33}$$

The dynamic amplification factor is defined here as the ratio of the maximum dynamic deflection to the maximum static deflection (i.e., the

deflection that would occur when v = 0). Hence, the dynamic deflection (D.A.) is given by

$$\text{D.A.} = \frac{\left\{[1 - \nu^2/(1 - v^2)]^{1/2} + v^2/\epsilon\right\}^{1/2}(1 - \nu^2)^{3/4}}{[1 - \nu^2/(1 - v^2)]^{1/2}[1 - \nu^2/(1 - v^2) - v^4/\epsilon^2]^{1/2}} \quad . \tag{34}$$

The dynamic amplification evidently becomes unbounded as v approaches one of the critical velocities.

To uncouple the effects of axial motion from the effects of radial motion, it suffices to neglect the term $\nu(\partial u/\partial x)$ in (5). The term $\nu(\partial w/\partial x)$ is retained in (4) so that the axial displacement u can be obtained. The results of this approximation can be analyzed from the theory previously considered in the present paper. The neglect of $\nu(\partial u/\partial x)$ removes the term $-\nu^2(1 - v^2)^{-1}$ in the equation for Δ, viz., Eq. (13). Hence, by this theory there are only two resonant speeds present, and they are given by

$$v_{CR1}^2 = \epsilon \qquad \text{or} \qquad V_{CR1}^2 = \frac{Eh}{a(1 - \nu^2)\sqrt{3}} \tag{35}$$

and

$$v_{CR2}^2 = 1 \qquad \text{or} \qquad V_{CR2}^2 = \frac{E}{\rho(1 - \nu^2)} \quad . \tag{36}$$

The first resonant speed given by (35) agrees with the one of [4] up to a factor of $(1 - \nu^2)^{1/2}$. The solution for the displacement now becomes

$$w = \frac{p_0}{2\sqrt{\epsilon}} \frac{\exp(-z\bar{b}_1/\sqrt{\epsilon})}{[1 - (v^4/\epsilon^2)]^{1/2}}\left(\bar{a}_1 \cos\frac{z\bar{a}_1}{\sqrt{\epsilon}} + \bar{b}_1 \sin\frac{z\bar{a}_1}{\sqrt{\epsilon}}\right) \quad , \quad z \geq 0 \tag{37}$$

and

$$w = \frac{p_0}{2\sqrt{\epsilon}} \frac{\exp(z\bar{b}_1/\sqrt{\epsilon})}{[1 - (v^4/\epsilon^2)]^{1/2}} \left(\bar{a}_1 \cos \frac{z\bar{a}_1}{\sqrt{\epsilon}} - \bar{b}_1 \sin \frac{z\bar{a}_1}{\sqrt{\epsilon}}\right) \quad , \quad z \leq 0 \tag{38}$$

where

$$\bar{a}_1 = \left(1 + \frac{v^4}{\epsilon^2}\right)^{1/2} \tag{39}$$

and

$$b_1 = \left(1 - \frac{v^4}{\epsilon^2}\right)^{1/2} \quad . \tag{40}$$

and $v < v_{CR1}$. The maximum deflection again occurs at $z = 0$, and is

$$w_{max} = \frac{p_0}{2\sqrt{\epsilon}} \frac{1}{[1 - (v^2/\epsilon)]^{1/2}} \quad . \tag{41}$$

The dynamic amplification factor is obtained from

$$D.A. = \frac{1}{[1 - (v^2/\epsilon)]^{1/2}} \quad . \tag{42}$$

The principal differences in the expression given above are in the neglect of the $\nu^2/(1 - v^2)$ terms. The first resonant speed v_{CR1}, corresponding to the case where the axial coupling is not neglected, differs from the case where the axial coupling is neglected by terms of order h/a. But because h/a is usually quite small, this approximation is certainly a valid one. Also, the resonance near the bar-wave speed disappears if the axial coupling is neglected. However, since the shell equations probably are not valid in this regime of the velocities, the difference is not of much importance.

Solution Using Flügge Equations

The two Timoshenko solutions will be compared now with the solution obtained from the Flugge equations ([6], pp. 209 - 219), viz.,

$$(1 - v^2)u'' + \nu w' - \frac{\epsilon^2}{4} w''' = 0 \tag{43}$$

$$\nu u' - \frac{\epsilon^2}{4} u''' + \frac{\epsilon^2}{4} w^{iv} + v^2 w'' + \left(1 + \frac{\epsilon^2}{4}\right) w = p \tag{44}$$

Here w is measured positive outward, p is positive when it is a suction rather than a compression, and u is measured positive in the positive x-direction. All quantities in (43) and (44) are taken to be functions of $z = x - vt$, and the notation of Flügge is changed to agree with that of Timoshenko.

Operating with Fourier transforms on (43) and (44) yields

$$-\xi^2(1 - v^2)\tilde{u} - \iota\xi\left(\nu + \frac{\epsilon^2}{4}\xi^2\right)\tilde{w} = 0 \tag{45}$$

and

$$-\iota\xi\left(\nu + \frac{\epsilon^2}{4}\xi^2\right)\tilde{u} + \left[\frac{\epsilon^2}{4}\xi^4 - v^2\xi^2 + \left(1 + \frac{\epsilon^2}{4}\right)\right]\tilde{w} = p_0 \tag{46}$$

where p, as before, has been taken to be $p_0\delta(z)$ for the traveling ring load.

Solving for $\tilde{u}$ and $\tilde{w}$, one has

$$\tilde{u} = \frac{\iota\xi[\nu + (\epsilon^2/4)\xi^2]p_0}{\Delta'(\xi)} \tag{47}$$

and

$$\tilde{w} = -\frac{\xi^2(1 - v^2)p_0}{\Delta'(\xi)} \tag{48}$$

where

$$\Delta'(\xi) = \xi^2\left\{\left(\nu + \frac{\epsilon^2}{4}\xi^2\right)^2 - (1 - v^2)\left[\frac{\epsilon^2}{4}\xi^4 - v^2\xi^2 + \left(1 + \frac{\epsilon^2}{4}\right)\right]\right\} \quad . \tag{49}$$

Again it suffices to study only w, since u can be obtained from w by using (43). Thus, one must consider the integral

$$w = -\frac{p_0(1 - v^2)}{2\pi}\int_{-\infty}^{\infty}\frac{\xi^2\exp(-\iota\xi z)}{\Delta'(\xi)}d\xi \quad . \tag{50}$$

Determination of Resonant Speeds for Flügge Equations

As before, the integral will not exist for values of v such that there are second-order poles on the real axis. The condition for this to be so is

$$\left[v^2(1 - v^2) + \frac{\nu\epsilon^2}{2}\right]^2 - \epsilon^2\left[1 - v^2 - \frac{\epsilon^2}{4}\right]\left[\left(1 + \frac{\epsilon^2}{4}\right)(1 - v^2) - \nu^2\right] = 0 \quad . \tag{51}$$

Equation (51) is a higher degree equation than (16). The extra root is located at a value of v greater than 1.0 and, as such, is not of much physical importance since it is unlikely that the shell equations without shear deformation and rotatory inertia are valid for values of v in this

range. As in the Timoshenko equations, there is a root near $v = \sqrt{\epsilon}$. Setting $v^2 = \epsilon x_0 + \epsilon^2 x_1$ gives $x_0 = (1 - \nu^2)^{1/2}$ and $x_1 = (-1/2)(\nu + \nu^2)$. Hence,

$$v^2 = \epsilon\left[\sqrt{1 - v^2} - \frac{1}{2}(\nu + \nu^2)\epsilon + \cdots\right] \quad . \tag{52}$$

This result compares well with the result from the use of the Timoshenko equations with the axial coupling, viz.,

$$v^2 = \epsilon\left[\sqrt{1 - v^2} - \frac{\nu^2}{2}\epsilon + \cdots\right] \tag{53}$$

obtained from (18). The critical physical speed is given by

$$V^2 = \frac{E}{\rho[3(1 - v^2)]^{1/2}}\frac{h}{a} - \frac{1}{6}\frac{(\nu + \nu^2)E}{\rho(1 - \nu^2)}\frac{h^2}{a^2} + \cdots \quad . \tag{54}$$

Again, the first term of (54) agrees with [4]. The two roots greater than unity are very close together, differing from unity by terms of order ϵ^2 and from each other by terms of order ϵ^4. They will not be investigated here, since their existence is doubtful, because when the equation of motion was derived, terms of order ϵ^2 were neglected. Hence, any term of order greater than ϵ^2 should be under suspicion. Dropping terms of order greater than ϵ^2 from (51) gives

$$v^4(1 - v^2)^2 + \nu\epsilon^2 v^2(1 - v^2) - \epsilon^2(1 - v^2)(1 - \nu^2 - v^2) = 0 \quad . \tag{55}$$

The meaning of the double root of $\Delta'(\xi)$ near $v = 1$ is now clear: It arises from the factor $(1 - v^2)$ common to all terms of (55). The differential equations themselves predict a resonance at this value of v in the axial mode.

Canceling the factor $(1 - v^2)$ yields

$$v^4(1 - v^2) - \epsilon^2[1 - \nu^2 - (1 + \nu)v^2] = 0 \quad . \tag{56}$$

The corresponding equation from the Timoshenko equations from (16) is

$$v^4(1 - v^2) - \epsilon^2(1 - \nu^2 - v^2) = 0 \quad . \tag{57}$$

The root near $v = \sqrt{\epsilon}$ is unaffected, but the root near $v = 1$ now becomes

$$v^2 = 1 + (1 + \nu)^2\epsilon^2 \tag{58}$$

which differs from the results of the Timoshenko equations by a term of the order of ϵ^2. The Timoshenko result is given by (21).

Finally, there is one resonance which will occur if the constant term in (49) disappears: There would be a second-order pole at the origin in the integrand for w. The condition that the constant term be zero is

$$\left.\begin{aligned} v^2 &= 1 - \frac{\nu^2}{(1 + \epsilon^2/4)} \\ &\approx 1 - \nu^2 + \frac{\nu^2\epsilon^2}{4} \end{aligned}\right\} \quad . \tag{59}$$

This value of v will always be slightly less than unity.

Evaluation of Displacements for Flügge Equations

The integral for w will now be evaluated. The procedure and results are similar to those for the Timoshenko theory. The terms of order greater

than ϵ^2 will be retained until the end of the analysis. For $v < v_{CR}$, there are four simple poles located at

$$\xi = \pm \frac{a_2}{\sqrt{\epsilon}} \pm \iota \frac{b_2}{\sqrt{\epsilon}} \tag{60}$$

where

$$a_2^2 = \frac{[(1 - \epsilon^2/4\beta^2)(1 - \nu^2/\beta^2 + \epsilon^2/4)]^{1/2} + (v^2/\epsilon + \nu\epsilon/2\beta^2)}{(1 - \epsilon^2/4\beta^2)} \tag{61}$$

$$b_2^2 = \frac{[(1 - \epsilon^2/4\beta^2)(1 - \nu^2/\beta^2 + \epsilon^2/4)]^{1/2} - (v^2/\epsilon + \nu\epsilon/2\beta^2)}{(1 - \epsilon^2/4\beta^2)} \tag{62}$$

and

$$\beta^2 = 1 - v^2 \quad . \tag{63}$$

As before, the integral for w is an even function of z and the integral for u is an odd function of z. For $z > 0$, the use of a semicircular contour in the lower-half plane is made in evaluating (50). The result of the integration is

$$w = \frac{p_0}{2\sqrt{\epsilon}} \frac{\exp(-zb_2/\sqrt{\epsilon})}{[(1 - \epsilon^2/4\beta^2)(1 - \nu^2/\beta^2 + \epsilon^2/4) - (v^2/\epsilon + \nu\epsilon/2\beta^2)^2]^{1/2}}$$
$$\times \frac{(1 - \epsilon^2/4\beta^2)^{1/2}}{(1 - \nu^2/\beta^2 + \epsilon^2/4)^{1/2}} \left[a_2 \cos\left(\frac{za_2}{\sqrt{\epsilon}}\right) + b_2 \sin\left(\frac{za_2}{\sqrt{\epsilon}}\right) \right] , \quad z > 0 \quad . \tag{64}$$

For $z < 0$, one substitutes $-z$ for z in (64). The maximum deflection again occurs at $z = 0$ and is given by

$$w_{max} = \frac{p_0}{2\sqrt{\epsilon}} \frac{1}{(1 - \nu^2/\beta^2 + \epsilon^2/4)^{1/2}} \frac{\left|[(1 - \epsilon^2/4\beta^2)(1 - \nu^2/\beta^2 + \epsilon^2/4)]^{1/2} + (\nu^2/\epsilon + \nu\epsilon/2\beta^2)\right|^{1/2}}{[(1 - \epsilon^2/4\beta^2)(1 - \nu^2/\beta^2 + \epsilon^2/4) - (\nu^2/\epsilon + \nu\epsilon/2\beta^2)^2]^{1/2}} \quad . \tag{65}$$

The maximum static deflection is obtained by putting $v = 0$ into (65) and is given by

$$w_{max\ static} = \frac{p_0}{2\sqrt{\epsilon}} \frac{1}{(1 - \nu^2 + \epsilon^2/4)^{1/2}} \frac{1}{\left|[(1 - \epsilon^2/4)(1 - \nu^2 + \epsilon^2/4)]^{1/2} - \nu\epsilon/2\right|^{1/2}} \quad . \tag{66}$$

The dynamic amplification factor, as defined earlier, is

$$D.A. = \frac{(1 - \nu^2 + \epsilon^2/4)^{1/2}}{(1 - \nu^2/\beta^2 + \epsilon^2/4)^{1/2}} \frac{\left|[(1 - \epsilon^2/4)(1 - \nu^2 + \epsilon^2/4)]^{1/2} - \nu\epsilon/2\right|^{1/2}}{\left|[(1 - \epsilon^2/4\beta^2)(1 - \nu^2/\beta^2 + \epsilon^2/4)]^{1/2} - (\nu^2/\epsilon + \nu\epsilon/2\beta^2)\right|^{1/2}} \quad . \tag{67}$$

Next, the powers of ϵ higher than first will be neglected. However, this must be done with caution since the order of magnitude of v is not known precisely. It is known that v is at least of the order of $\sqrt{\epsilon}$ but could be of higher order in ϵ. To carry out this procedure, one sets $v/\sqrt{\epsilon} = \phi$ and carries ϕ in all terms, assuming ϕ is of order one. The actual procedure then is to carry only first powers of ϵ, since a factor of ϵ^2 is already taken out by setting $\phi = v/\sqrt{\epsilon}$. The results are

$$a_2^2 = \frac{[(1 - \nu^2) - (2 - \nu^2)\epsilon\phi^2]^{1/2} + \phi^2(1 - \epsilon\phi^2) + \epsilon/2}{1 - \epsilon\phi^2} \tag{68}$$

$$b_2^2 = \frac{[(1-\nu^2)-(2-\nu^2)\epsilon\phi^2]^{1/2} - \phi^2(1-\epsilon\phi^2) - \epsilon/2}{1-\epsilon\phi^2} \tag{69}$$

$$w = \frac{p_0}{2\sqrt{\epsilon}} \frac{\exp(-zb_1/\sqrt{\epsilon})(1-\epsilon\phi^2)^{1/2}}{\left|(1-\nu^2)-\phi^4-\epsilon[(2+\nu-\nu^2)\phi^2-2\phi^6]\right|^{1/2}} [(1-\nu^2)-\epsilon\phi^2]^{1/2}$$

$$\times\left(a_2 \cos\frac{za_2}{\sqrt{\epsilon}} + b_2 \sin\frac{za_2}{\sqrt{\epsilon}}\right) \quad , \quad z > 0 \tag{70}$$

$$w_{max} = \frac{p_0}{2\sqrt{\epsilon}} \frac{\left|[(1-\nu^2)-\epsilon(2-\nu^2)\phi^2]^{1/2} + \phi^2 + [(\nu/2)-\phi^2]\epsilon\right|^{1/2}(1-\epsilon\phi^2)}{(1-\nu^2-\epsilon\phi^2)^{1/2}\left|(1-\nu^2)-\phi^4-\epsilon[(2+\nu+\nu^2)\phi^2-2\phi^6]\right|^{1/2}} \tag{71}$$

$$\text{D. A.} = \frac{(1-\nu^2)(1-\epsilon\phi^2)[(1-\nu^2)^{1/2}-\nu\epsilon/2]^{1/2}}{(1-\nu^2-\epsilon\phi^2)^{1/2}\left|[(1-\nu^2)-\epsilon(2-\nu^2)\phi^2]^{1/2}-\phi^2-[(\nu/2)-\phi^2]\epsilon\right|^{1/2}} \quad . \tag{72}$$

Response in the Supercritical Range

For values of v between the first and second resonances, given by (19) and (22), respectively, the integral (15) defining w can also be evaluated. In this case, there will be four simple poles, all on the real axis and located at $\xi = \pm a_1, \pm a_2$. Only the Timoshenko solution will be discussed for this case. The poles are given by

$$2a_1^2 = v^2 + \left[v^4 - \epsilon^2\left(1 - \frac{\nu^2}{\beta^2}\right)\right]^{1/2} \tag{73}$$

and

$$2a_2^2 = v^2 + \left[v^4 - \epsilon^2\left(1 - \frac{\nu^2}{\beta^2}\right)\right]^{1/2} \quad . \tag{74}$$

The integral (15) then is defined in the sense of a Cauchy principal value and is

$$w = \frac{P_0}{8} \frac{1}{[v^4 - \epsilon^2(1 - \nu^2/\beta^2)]^{1/2}} \left[\frac{\sin(\bar{a}_2 z)}{a_2} - \frac{\sin(\bar{a}_1 z)}{a_1} \right], \quad z > 0 \qquad (75)$$

where $\bar{a}_1 = (2/\epsilon)a_1$ and $\bar{a}_2 = (2/\epsilon)a_2$. The solution for $z < 0$ is obtained by setting $-z$ for z in (75).

The physical meaning of the solution (75) is not at all clear. The solution predicts that both the deflection and the slope are zero under the load and, thus, supplies an utterly improbable result; such a result does not appear to be physically realizable. It is felt that for values of v in this range, an initial value problem should be solved, since it is known [7] that the steady-state problem is an artificial one. Values of v should really be derived from a transient solution by allowing time to become unboundedly large. This problem is beyond the scope of the present investigation. For a related analysis, the reader is referred to [8].

SECTION IV
SUMMARY

The stability of a circular cylindrical shell subjected to a moving ring load with a constant velocity has been examined in detail when both longitudinal and transverse inertia effects are included, using both the Timoshenko and the Flügge equations. It has been found that the resonance due to bending occurs at a lower velocity than that reported in [1]. The resonance due to bending is concealed in the solution of [1] because of the assumed presence of viscous damping in the radial direction. However, a bending resonance occurs at a much lower velocity than that in [1] and is of importance for design of aerospace vehicles. The instabilities have been obtained in the present paper using Fourier transforms without obtaining the entire solution. The response of the shell is obtained in the subcritical range for Timoshenko and Flügge equations. Expressions for the dynamic amplification in the deflection have been given. The critical speeds obtained by using Flügge equations differ by only a small amount from those given by the Timoshenko equations.

A formal solution for the response in the supercritical range has been obtained for the Timoshenko equations. The formal solution does not appear to be physically meaningful. Therefore, it is felt that an initial value problem should be solved, including the effects of shear deformation and rotatory inertia, for values of velocity in the supercritical range.

REFERENCES

1 P. Mann-Nachbar, "On the Role of Bending in the Dynamic Response of Thin Shells to Moving Discontinuous Loads," Journal of Aerospace Sciences, June 1962, pp. 648-657.

2 W. L. Brogan, "Radial Vibrations of a Thin Cylindrical Shell," Journal of the Acoustical Society of America, vol. 33, no. 12, 1961, pp. 1778-1781.

3 P. G. Bhuta, "Transient Response of a Thin Elastic Cylindrical Shell to a Moving Shock Wave," Journal of the Acoustical Society of America. To appear in the January 1963 issue.

4 V. L. Prieskin, "The Stability of a Cylindrical Shell Subjected to a Moving Load," (in Russian) Izvestiya Akademii Nauk S. S. S. R. Otdelenie Tekhnicheskikh Nauk (Mekhanika i Mashinostroenie), no. 5., 1961, pp. 133-134.

5 S. P. Timoshenko, Theory of Plates and Shells, McGraw-Hill Book Company, Inc., New York, N. Y., 1940.

6 W. Flügge, Stresses in Shells, Springer-Verlag, Berlin, 1960.

7 J. J. Stoker, "Some Remarks on Radiation Conditions," Proceedings of the Fifth Symposium on Applied Mathematics of American Mathematical Society, 1954.

8 J. Dörr, "Der Unendlichen Federnd Gebetteten Balken Unter dem Einfluss eines Gleichformigen Bewegten Last," Ingenieur-Archiv, 1943.

UNCLASSIFIED

Aerospace Corporation, El Segundo, California.
STABILITY AND RESPONSE OF CYLINDRICAL SHELLS TO MOVING LOADS, prepared by John P. Jones and Pravin G. Bhuta. 15 March 1963. [29]p. incl. illus. (Report TDR-169(3153-10)TN-1; BSD-TDR-63-35)
(Contract AF 04(695)-169) Unclassified report

The stability of a circular cylindrical shell subjected to a moving ring load with a constant velocity has been examined in detail when both longitudinal and transverse inertia effects are included, using both the Timoshenko and the Flügge equations. A lower resonance speed has been found, which apparently has been concealed in a recent paper.

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California.
STABILITY AND RESPONSE OF CYLINDRICAL SHELLS TO MOVING LOADS, prepared by John P. Jones and Pravin G. Bhuta. 15 March 1963. [29]p. incl. illus. (Report TDR-169(3153-10)TN-1; BSD-TDR-63-35)
(Contract AF 04(695)-169) Unclassified report

The stability of a circular cylindrical shell subjected to a moving ring load with a constant velocity has been examined in detail when both longitudinal and transverse inertia effects are included, using both the Timoshenko and the Flügge equations. A lower resonance speed has been found, which apparently has been concealed in a recent paper.

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California.
STABILITY AND RESPONSE OF CYLINDRICAL SHELLS TO MOVING LOADS, prepared by John P. Jones and Pravin G. Bhuta. 15 March 1963. [29]p. incl. illus. (Report TDR-169(3153-10)TN-1; BSD-TDR-63-35)
(Contract AF 04(695)-169) Unclassified report

The stability of a circular cylindrical shell subjected to a moving ring load with a constant velocity has been examined in detail when both longitudinal and transverse inertia effects are included, using both the Timoshenko and the Flügge equations. A lower resonance speed has been found, which apparently has been concealed in a recent paper.

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California.
STABILITY AND RESPONSE OF CYLINDRICAL SHELLS TO MOVING LOADS, prepared by John P. Jones and Pravin G. Bhuta. 15 March 1963. [29]p. incl. illus. (Report TDR-169(3153-10)TN-1; BSD-TDR-63-35)
(Contract AF 04(695)-169) Unclassified report

The stability of a circular cylindrical shell subjected to a moving ring load with a constant velocity has been examined in detail when both longitudinal and transverse inertia effects are included, using both the Timoshenko and the Flügge equations. A lower resonance speed has been found, which apparently has been concealed in a recent paper.

UNCLASSIFIED