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Interim Research Memorandum
OPERATIONS EVALUATION GROUP
Center for Naval Analyses
WASHINGTON 25, D. C.
MODEL AND COMPUTER PROGRAM FOR CALCULATING THE KILL PROBABILITIES FOR CERTAIN ASW TACTICS

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IRM-25

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ABSTRACT

This memorandum describes a model and computer program designed to compute kill probabilities for certain firing tactics waged against an evading submarine. The model was developed assuming that the attack is imminent and that the weapon will be directed at the point where the submarine was last contacted. The model's design is centered around a determination of sets of points which represent the locus of the evading submarine. The locus is determined by variation of a simple evasion tactic. The aimpoint is considered to be circularly, normally distributed about the true position of the submarine at the time of last contact.
1.0 Discussion of the Model:

This model and program were developed to provide a means for computing kill probabilities for certain ASW tactics. The model and program, though predicated upon a particular tactical situation, are sufficiently flexible to handle a variety of cases and should permit design modifications without major revision.

1.1 Tactical Situation:

The tactical situation which this model is designed to treat is described as follows:

A helicopter, assisting a delivery aircraft in an attack, has a firmly established sonar contact with a submarine.

1.1.0 Condition 1:

If it is found that the assisting helicopter's distance from the sub is within a minimum safe distance, certain tasks must be performed before the assisting helicopter can retire, under emergency power, to the minimum safe distance.

The delivery aircraft must await communications and directions from the assisting helicopter, and then delay for at least some maximum time before maneuvering to the predicted position and dropping the weapon.

1.1.1 Condition 2:

If it is found that the assisting helicopter is outside the minimum safe distance from the submarine's position, the assisting helicopter tracks the submarine and continuously vectors the delivery aircraft until the weapon is dropped.

1.2 Analysis:

The key factor in these situations is the time that the submarine remains out of the sonar contact. That time, under condition 2, is, usually, negligibly small, even if a delay maneuver is incorporated. Not many escapes are possible here. The situation defined by condition 1 is such that a submarine's chance to escape is measurably enhanced. The probability of escape is a function of the weapon's lethal radius and the distance between the submarine and the weapon's burst point at detonation.

The locations attained by a submarine, for all evasion tactics executed within a given time interval, will be found at various points, more frequently in one spot, less frequently in another, at a greater or lesser distance from the origin, taken as the point of last sonar contact. The problem, then, is to find the values for the kill probability for distributions of points in the locus of the evading submarine.

2.0 Method of Solution:

The program computes sets of position which lie in a lobe of the locus of points of the evading submarine. The origin of the locus is the point of last contact, and the sub is assumed to be initially moving along the positive y-axis.
The terminal positions of the submarine are found from the initial speed of the submarine at the beginning of the blind time, the blind time interval, and variations of a simple evasion tactic. The blind time, TB, is the time between the severance of sonar contact and the weapon's detonation. The terminal positions so found comprise a representative sample of the distribution of points in the locus of an evading submarine. The evasion tactics are described as follows:

(a) During a multiple, t, of a fraction of the blind time, TB, the submarine cruises on course at speed, \( V_0 \). The value, (t), is reckoned from the beginning of the blind time, TB.

(b) After time, t, the submarine executes a turn with radius of turn, R, through a multiple, \( \Delta \), of a fraction of the turn angle, \( \theta \), at an average speed, \( V \), a function of \( V_0 \) and \( \Delta \).

(c) If there is any remaining blind time, the submarine cruises along its new course at speed, \( V_0 \), until all of the blind time is consumed.

(d) Then, for the given combination of TB, \( V_0 \), t and \( \Delta \), a terminal position is established for the submarine.

(e) The process is repeated for all combinations of a finite set of values in the range of t and a finite set of values in the range of \( \Delta \), where \( 0 \leq t \leq TB \) is the range of t and \( 0 \leq \Delta \leq \theta \) is the range of \( \Delta \).

2.1 The equations for calculating the coordinates, \( (X_{(i,j)}, Y_{(i,j)}) \), of the positions in the locus of the evading target are derived as follows: (see figure 1, appendix D)

**Given:**

\[ TB = \text{blind time} \]
\[ \theta = \text{turn angle} \]
\[ M = \text{a denominator} \]
\[ \Delta T = \frac{TB}{M}, \text{ a fraction of the blind time} \]
\[ N_i = \text{an integer which is allowed to take on successive values in the range}, \ 0 \leq N_i \leq M \]
\[ t = N_i \Delta T, \text{ a multiple of a fraction of the blind time} \]
\[ L = \text{a denominator} \]
\[ \Delta \theta = \frac{\theta}{L}, \text{ a fraction of the turn angle} \]
\[ P_j = \text{an integer which is allowed to take on successive values in the range}, \ 0 \leq P_j \leq L \]
\[ \Delta = P_j \Delta \theta, \text{ a multiple of a fraction of the turn angle} \]
\[ R = \text{turn radius} \]
\[ V_0 = \text{initial speed of submarine} \]
\[ \bar{V} = \text{average speed of submarine during turn} \]
\[ V_I = \text{the instantaneous speed of a submarine during a turn} \]
First, let us derive an expression for $V_I$. Assume that the instantaneous speed of a submarine during a turn is a continuous, linear function of the turn angle, $\theta$, and that the speed decreases to 55 percent of its initial value at $\theta = 90^\circ$. These assumptions are consistent with the results of studies made on submarine turning rates. Then,

$$(V_o, 0) \text{ and } (0.55V_o, \pi/2) \text{ are points on the line and}$$

$$(V_I - V_o)/\theta = -2V_o (1-0.55)/\pi = -0.90V_o/\pi$$

$$\pi V_I - \pi V_o = -0.90V_o \theta$$

$$V_I = (\pi V_o - 0.90V_o \theta)\pi = -V_o(1 - 0.90 \theta/\pi)$$

Then, the average value, $\overline{V}$, of the function, $V_I$, between $0$ and $\theta$ is,

$$\overline{V} = \frac{1}{\theta} \int_0^\theta V_o (1-0.90\theta/\pi) \, d\theta$$

$$\overline{V} = \frac{V_o}{\theta} \bigg|_{\theta}^{\theta - 0.90\theta^2/2\pi} = (V_o/\theta) \cdot \left( \theta - 0.90\theta^2/2\pi \right)$$

$$\overline{V} = V_o (1-0.90\theta/2\pi)$$

Now,

$$T_O = R(P_j\Delta\theta)/\overline{V} \quad \text{the time to turn through the angle, } \Delta = P_j\Delta\theta$$

$$T_I = T_B - N_1\Delta T \quad \text{the time remaining after the interval, } t = N_1\Delta T$$

$$T_2 = T_I - T_O \quad \text{time remaining to cruise on new course.}$$

Then,

$$X_{(i, j)} = R(1 - \cos(\overline{V}T_1/R)) \begin{cases} \quad T_2 < 0 \quad (1) \end{cases}$$

$$Y_{(i, j)} = V_o N_1 \Delta T + R \sin(\overline{V}T_1/R)$$

$$X_{(i, j)} = R(1 - \cos(P_j\Delta\theta)) \begin{cases} \quad T_2 = 0 \quad (2) \end{cases}$$

$$Y_{(i, j)} = V_o N_1 \Delta T + R \sin(P_j\Delta\theta)$$
\[ X_{(i, j)} = R \left( 1 - \cos \left( P_j \Delta \theta \right) \right) + V_o T^2 \sin \left( P_j \Delta \theta \right) \]
\[ Y_{(i, j)} = V_o N_t \Delta T + R \sin \left( P_j \Delta \theta \right) + V_o T^2 \cos \left( P_j \Delta \theta \right) \]

Equation (1) is used if the turn cannot be completed; equation (2) if it can but, in so doing, consumes all of the blind time; and equation (3) if all aspects can be accomplished within the blind time.

2.2 There now remains the problem of determining the point at which the attack on the target is aimed. That point is the point at which the target was last observed. The observations of that point are assumed to be circularly, normally distributed about the true position of the submarine at \( T = 0 \), the beginning of the blind time. Coordinates of the aimpoint can be established as follows:

Assume a polar, coordinate system with origin at the sighting ship, at \( T = 0 \), and the polar coordinates of the target are \( (R_t, \pi/2) \). (See figure 2), appendix D. Then,

- (a) \( (R_t, \pi/2) = \) polar coordinates of the target's true position.
- (b) \( (R_o, \alpha_o) = \) polar coordinates of the target's observed position
- (c) \( \Delta R = R_o - R_t = \) error in the range observation
- (d) \( \Delta \alpha = \alpha_o - \pi/2 = \) error in the azimuth observation

then, transforming to cartesian coordinates,

- (e) \( X_t = R_t \cos \pi/2 \) cartesian coordinates of the target's true position
- (f) \( Y_t = R_t \sin \pi/2 = R_t \)
- (g) \( X_o = R_o \cos \alpha_o \)
- \( Y_o = R_o \sin \alpha_o \) cartesian coordinates of the target's observed position

but, from item (d),

- (h) \( \alpha_o = \pi/2 + \Delta \alpha \), and therefore
- (i) \( X_o = R_o \cos \left( \pi/2 + \Delta \alpha \right) = R_o \sin \Delta \alpha \)
- (j) \( Y_o = R_o \sin \left( \pi/2 + \Delta \alpha \right) = R_o \cos \Delta \alpha \)

then, by translation of axes,

- (k) \( X'_o = X_o - X_t = R_o \sin \Delta \alpha \)
- (l) \( Y'_o = Y_o - Y_t = R_o \cos \Delta \alpha - R_t \)

then, \( (X'_o, Y'_o) = \) coordinates of the aimpoint.
The coordinates of the aimpoint with origin at the sub at \( T = 0 \), are found for each variation of the evasion; they are computed as follows:

Given,

\[ \sigma_R = \text{standard deviation in range error (2\% of range)} \]
\[ \sigma_\alpha = \text{standard deviation in azimuth error} \]

then, choose

\[ G_R \] a random number from a Gaussian distribution of random numbers with mean zero and standard deviation equal to 1.

and choose

\[ G_\alpha \] a random number from a Gaussian distribution of random numbers with mean zero and standard deviation equal to 1.

then,

\[ R_t = \sigma_R \cdot G_R / 0.02 = 50 \cdot \sigma_R \cdot G_R = \text{true range of aimpoint from origin at assisting ship at } T = 0. \]

\[ R_o = R_t = \sigma_R \cdot G_R = \text{an observed range from origin at assisting ship at } T = 0. \]

\[ \Delta \alpha = \sigma_\alpha \cdot G_\alpha = \text{the azimuth error} \]

and,

\[ (X'_o, Y'_o) = (-R_o \sin \Delta \alpha, R_o \cos \Delta \alpha - R_t). \]

Then, the distance between the target's position at \( T = T_B \) and the aimpoint is:

\[ D_{(i, j)} = \sqrt{(X_{(i, j)} - X'_o)^2 + (Y_{(i, j)} - Y'_o)^2} \]

2.3 This program uses the SHARE subroutine OCIP. OCIP uses the criterion of circular coverage to establish the probability that a weapon with a certain radius of effectiveness and a ballistic dispersion with standard deviation of \( \sigma_B \) will effectively dispose of a target randomly displaced from the aimpoint. The distribution of the target's displacement is characterized by a circular, normal
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IRM-25  In this application, target distribution has been generated by the
process already described; so \( \sigma_p \) should be set equal to zero.
The ballistic dispersion and the target's
displacement are centered at the aimpoint. For our purposes, \( D_{(i, j)} \) is the target's
displacement and the weapon's lethal radius, \( RKILL \), is the radius of effectiveness.

Input to OCIP must be given in the form:

\[
(D_{(i, j)}/\sigma, RKILL/\sigma, PK),
\]

where,

\[
\sigma = \sqrt{\sigma_B^2 + \sigma_p^2}
\]

and, the output parameter, \( PK \), in our usage, is the probability that the firing
tactic results in a kill.

2.4 Consequently, \( D_{(i, j)}/\sigma \) and \( RKILL/\sigma \) can be found for each position in the
sample of computed positions in the locus of the evading target and, using the
circular coverage function as programmed in OCIP, a kill probability for the
individuals in the sample can be calculated. The average of the kill probabilities
of the individuals in the sample is taken as the kill probability for the tactic
against an evading target. Kill probabilities are computed for several values of
lethal radius and submarine speed.

2.5 This model assumes that the time to turn is uniformly distributed throughout
the blind time and that the turn angle is, also, uniformly distributed through all
possible values of that angle. The model was constructed to calculate the kill
probability regardless of the evasion tactic. Consequently, the evasion tactic
contained in the model is designed only as a device for generating sets of points
which define a set of limiting distances attained by a target in a given time,
without giving special regard to favored evasions. The blind time is expected
to be of short duration. The blind time, thus, does limit the design of a more
sophisticated evasion which one could expect a submarine to accomplish under
imminent attack.

A study of the results, oriented toward studying evasion tactics, should
help to improve the model and may well suggest better countermeasures against
evasive maneuvers.

Consideration is currently being given to design modifications which will
enable specification of favored evasion schemes and provide for prediction of
aimpoints involving bearing and speed prediction errors, offset in range and azimuth.

3.0 Program Input:

The program will accept card input. At least two cards are required, a control
card and a test case card. A control card can be followed by any number of test
cases. Any number of test sets may be submitted.
3.1 The Control Card:

A set of test case cards must be preceded by a control card. The control card is used to indicate the number of sets and the number of test cases per set in the input deck and the output options. There are three output options. See paragraph 5.0, Program Output.

3.1.0 The Test Case Card:

There are eleven input parameters. The eleven parameters are described as follows:

3.1.1 Maximum Turn Angle:

This is the maximum angle, $\theta^\circ$, through which the evasive turn will be executed. A fraction of this angle is taken and successive multiples of that fraction are used to iterate the computation of the kill probability. Theta must be given in degrees.

3.1.2 Multiples of Turn Angle:

This parameter, L, is used by the program to determine the number of values of $P_j$ through which the submarine will be programmed to turn. $\Delta \theta = \theta^\circ / L$ and $0 \leq P_j \leq L$.

3.1.3 Blind Time:

Blind time, $T_B$, is the length of time between the bomb burst and the time when sonar contact is severed. A fraction of this value is determined, using $M$ (see paragraph 3.1.4), and successive multiples of that fraction are used to iterate the computation of the kill probability. Blind time is given in seconds.

3.1.4 Multiples of Time:

This parameter, $M$, is used by the program to determine the number of values of $N_j \Delta T$ so that the moment in time at which the evasive turn will be executed will be uniformly distributed. $\Delta T = T_B / M$ and $0 \leq N_j \leq M$.

3.1.5 The Turn Radius:

The turn radius, $R$, is the radius of the evasive turn. This value is given in yards.

3.1.6 Sub Speed:

This is the speed of the submarine. The speed, $V$, is given in knots. Five successive multiples of $V/5$ are used as the sub speed by the program.

3.1.7 The Damage Radius:

(RKILL) This specifies the distance from the point of the burst of the bomb within which a 100 percent probability of kill will be satisfied. RKILL is given in feet. Ten successive multiples of $RKILL/10$ are used as the lethal radius by the program.
3.1.8 **The Sonar Azimuth Error:**

$\sigma_a$ is given in degrees and is the standard deviation of the error in observation due to azimuth.

3.1.9 **The Sonar Range Error:**

$\sigma_R$ is given in feet. It is the standard deviation of the error in observation due to range.

3.1.10 **The Ballistic Error:**

$\sigma_B$ is given in feet and is the standard deviation of the ballistic error.

3.1.11 **Displacement Error:**

$\sigma_P$ is given in feet and is the standard deviation of the circular, normal distribution of the target's displacement from the aimpoint.

4.0 **Key Punching Instructions:**

4.1 **Control Card:**

There are four fields on a control card. The first three fields are six columns long, and all entries therein must be right-justified. The fourth field is fifty-four columns long and may be used for remarks.

- **Cols. 1-6:** This field is used to specify the number of test case cards under the purview of the given control card.

- **Cols. 7-12:** This field, if blank or punched zero, indicates that no other test sets follow the given set; if punched with a one, another test set is expected to follow. A test set consists of a control card followed by any number of test case cards.

- **Cols. 13-18:** This field is used to specify the output options; if blank or zero, the average, summary results of the computations from the $N_i \times P_j$ combinations of the multiples of blind time and turn angle will be printed; if punched with a one, the individual results from the computation on the $N_i \times P_j$ combinations of blind time and turn angle will be printed in addition to the average, summary results; if punched with a two, the zero-time combinations of blind time and turn angles will be printed in addition to the average, summary results.
4.2 Test Case Card:

There are eleven fields on a test case card. Each field is six columns long, and any entry in a field must be right-justified. The card should be punched as follows:

Cols. 1-6: Max Turn Angle ($\theta^\circ$), degrees.
Cols. 7-12: Multiples of $\theta^\circ$ (L).
Cols. 13-18: Blind Time (TB), seconds.
Cols. 19-24: Multiples of Blind Time (M).
Cols. 25-30: Turn Radius (R), yards.
Cols. 31-36: Sub Speed (V), knots.
Cols. 37-42: Damage Radius (RKILL), feet.
Cols. 43-48: Azimuth Error ($\sigma_\alpha$), degrees.
Cols. 49-54: Sonar Range Error ($\sigma_R$), feet.
Cols. 55-60: Ballistic Error ($\sigma_B$), feet.
Cols. 61-66: Displacement Error ($\sigma_P$), feet.

5.0 Program Output:

There are three output options. These options are set by key-punching the PBIT field (cols. 13-18) on the control card. The output is designed so that kill probabilities can be read out over ranges of speed and lethal radius.

5.1 Option A: (PBIT field is blank or zero)

The following are printed under option A.

(a) a value of the sub's speed in knots equal to $k \times V/5$, where $1 \leq k \leq 5$ and $V$ is the value of the speed given in the input.

(b) a value of lethal radius in feet equal to $1 \times RKILL/10$, where $1.0 \leq 1 \leq 10$ and RKILL is the damage radius given in the input.

(c) sigma, equal to $\sqrt{\frac{\sigma_B^2}{2} + \frac{\sigma_P^2}{2}}$.

(d) for each of the $k \times 1$ combinations of speed and RKILL, an average kill probability (PK), computed from the combinations of $N_1$ time intervals and $P_j$ angle intervals is printed.

(e) the total of the number of individual computations of PK involved in computing the average. Total = $(M+1.0) \times (L+1.0)$. 
5.2 **Option B: (PBIT field is set to one)**

For each of the \( k \times 1 \) combinations of speed and RKILL, the following will be printed under option B in addition to the output in option A:

(a) the time intervals; \((M + 1.0)\) values.
(b) the coordinates of the position of the submarine from the origin in feet; \((M + 1.0) \times (L + 1.0)\) values.
(c) the distance of the submarine from the origin in feet; \((M + 1.0) \times (L + 1.0)\) values.
(d) \(\sigma\), equal to \(\sqrt{\sigma_B^2 + \sigma_P^2}\).
(e) the fraction of the input turn angle; \((L + 1.0)\) values.
(f) the fraction of the input speed; \(k\) values.
(g) the fraction of the input lethal radius; \(l\) values.
(h) the kill probabilities, \((M + 1.0) \times (L + 1.0)\).

5.3 **Option C: (PBIT field is set to two)**

Option C is the same as option B but involves only those results concerning the zero time interval; that is item, a, under option B is always zero in option C.

6.0 **User's Instructions:**

A user must submit the following information for each test set:

(a) the number of cases in the test set.
(b) indication whether or not the given set is the last set in the input deck.
(c) indications of the output option desired.
(d) for each test case, values for: turn angle in degrees, multiples of turn angle, blind time in seconds, multiples of blind time, turn radius in yards, speed of sub in knots, lethal radius in feet, azimuth error in degrees, range error in feet, ballistic error in feet, and displacement error in feet.

This information can be conveniently given on an agendum sheet such as that shown in appendix B. The output shown in appendix C was derived from the sample input given in appendix B. In addition to the circular coverage subroutine, OCIP, the program uses the random number generator, GRNUMB. The binary decks for these subroutines must be incorporated with the binary deck for the program described in this document.
APPENDIX A

FORTRAN LISTING

AND

FLOW DIAGRAM
COMPUTE ASW KILL PROBABILITY

DIMENSION REMARK(9)

1 FORMAT(216,F6.0,9A6)

2 FORMAT(1H, 42H COMPUTATION OF EFFECTIVE KILL PROBABILITY//1H,
150X,19HMINPUT PARAMETER SET//1H 10MTURN ANGLE,1X,9HMULTIPLES,2X,5HBL
2INDX9HMULTIPLES2X4HTURNNTX3HSUBX6HDAMAGE5X7HAI MUTHX5HANGE6X9HB
3ALLISTIC2X6TARGET/1H 8H THETA ,5X,8HOF THETA,3X,4HTIME,5X,7HOF
4 TIME4X,6HRADIUS,9X,5HSPEED,4X,6HRADIUS,6X,SHERROR,6X,5HSERRO,8X,5
5SERROR,4X,5SERROR)

4 FORMAT(1IF6.0)

5 FORMAT(1H 10(F10.2,1X),F8.2)

6 FORMAT(1H,3X,4HTIME,6X,5HTHETA,4X,5HSUB1,5X,5HSUB1,5X,5HSUB1,
15X,5HSIGMA,5X,5HKILL,5X,5HSPEED,7X,2HPK)

7 FORMAT(1H0,30X,16HCOMPUTATION SETS)

21 FORMAT(1H lt2[6,F6.0,9A6)

P1=3.1415927

22 READ 1,NCASES,NSETS,PBIT,(REMARK_I),I=1,9)

PRINT 2,NCASES,NSETS,PBIT,(REMARK_I),I=1,9)

2 READ 4,ARC,ARCN,BT,BTM,ARCRAD,VSUB,DAMRAD,SA,SR,SB,SP

PRINT HEADING AND INPUT SET

PRINT 3

PRINT5,ARC,ARCN,BT,BTM,ARCRAD,VSUB,DAMRAD,SA,SR,SB,SP

PRINT 7

IF(PBIT=1.0)56,55,55

PRINT 6

55 PRINT 54

56 C=15.0

NARC=ARCN+1.0

MBT=BTM+1.0

VSUB1=1.689*VSUB/5.0

SA=PL*SA/180.0

TRAO=3.0*ARCRAD

ALPHA=PI*ARC/180.0*ARCN)

DT=BT/BTM

S=SQRT( (SB**2+SP**2)**2 )

Z=1.0

DO 155 M=1.10

FVSUB=Z*VSUB1

SPEED=FVSUB/1.689

Z=Z+1.0

R=1.0

DO 152 M=1,10

SUM=0.0

SUMPKR=0.0

TOTAL=0.0

ZPK=0.0

ZTIME=0.0

RKILL=R*DAMRAD/10.0

R=R+1.0

A=0.0

DO 151 J=1,NARC

THETA=A+ALPHA

ANGLE=THETA+180.0/PI

AVSPD=FVSUB*ABS( (1.0-0.9*THETA/(2.0*PI))

F0=TRAD*THETA/AVSPD

A=A+1.0

B=0.0

DO 150 K=1,MBT

T1=B+DT

T2=BT-T1

T3=T2-T0

A-3
CALL GRNUMB(A1)
DELTA=A1+50
CALL GRNUMB(A2)
TRUE=ABS(F(50,A2*SR))
ROBS=TRUE=A2+SR
XOBS=ROBS+SINF(DELTA)
YOBS=ROBS-COSF(DELTA)-TRUE
IF(IT3)=15,20
10 XSUBI=TR+A1-COSF(AVSPEED/T2/TRAD))
YSUBI=FXSUB*T1+TRADSINF(AVSPEED/T2/TRAD)
GO TO 25
15 XSUBI=TR+A1-COSF(AVSPEED/T2/TRAD)
YSUBI=FXSUB*T1+SINF(AVSPEED/T2/TRAD)
GO TO 25
20 XSUBI=TR+A1-COSF(AVSPEED/T2/TRAD)
YSUBI=FXSUB*T1+TRADSINF(AVSPEED/T2/TRAD)
YSUBI=SQRT((XSUBI-XOBS)**2+(YSUBI-YOBS)**2)
OPERS=OPERS+PS
DRPERS=RPK/S
CALL OCIP(DPERS,UPERS,PK)
SUM=SUM+PK
TOTAL=TOTAL+1.0
B=B+1.0
IF(BIT=1.0)26,28,26
28 IF(IT3)=C=15,35,30
30 PRINT 31,TI,ANGLE,XSUBI,YSUBI,OSUBI,S,RKILL,SPEED,PK
31 FORMAT(IN,8F10.2,10.3)
C=C+1.0
GO TO 150
35 PRINT 42
42 FORMAT(IN,1H1)
PRINT 6
C=5.0
GO TO 30
26 IF(IP=2.0)150,27,150
27 IF(IT1)150,37,150
37 ZPK=ZPK+PK
ZTIME=TOTAL+1.0
GO TO 28
150 CONTINUE
151 CONTINUE
152 IF(IP=2.0)162,160,162
160 ZPK=ZPK/ZTIME
IF(IF)63.0-C)=159,159,161
159 PRINT 42
PRINT 6
C=5.0
161 PRINT 43,SPEED,RKILL,S,ZPK,ZTIME
43 FORMAT(IN,6MSPEED=F6.2,2X,HMKILL=F10.2,2X,2HS=F10.2,2X,6HZPKAV=,
IF(F6.3,2X,6HTOTAL=F6.0)
C=C+1.0
162 AVPK=SUM/TOTAL
IF(IP=1.0)171,163,163
163 IF(IF)63.0-C)=170,170,171
170 PRINT 42
PRINT 6
C=5.0
171 PRINT 43,SPEED,RKILL,S,AVPK,ZTIME
13,2X,6HTOTAL=F8.0)
C=C+1.0
172 IF(IF)63.0-C)=-2.0)173,173,174
173 PRINT 42
PRINT 6
C=5.0
GO TO 152
174 PRINT 6
   C=C+1.0
152 CONTINUE
   IF(63.0-C)175,175,176
175 PRINT 42
   C=5.0
   GO TO 153
176 PRINT 41
   C=C+1.0
41 FORMAT(1HO,1H )
153 CONTINUE
   NCASES=NCASES-1
   IF(NCASES)154,154,2
154 IF(INSETS-1)50,22,50
50 CALL END JOB
END
### APPENDIX B

**TO:** OEG PROGRAMMING SECTION  
**FROM:**  
**PROGRAM NAME:**  
**DATE:**  
**CLASSIFICATION:**  
**ESTIMATED TIME:**

#### CONTROL

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(Reverse Blank)
### COMPUTATION OF EFFECTIVE KILL PROBABILITY

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**C-3**
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**Note:** The table above represents a sample of the data from the document. The full table includes additional rows and columns that were not shown here for brevity.
### COMPUTATION OF EFFECTIVE KILL PROBABILITY

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#### SPEED: 8.00 KILL* 1000.00*** 70.71 IPRK** 1000.00** TOTAL* 10.00

#### SPEED: 8.00 KILL* 2000.00*** 70.71 IPRK** 1000.00** TOTAL* 50.00

#### SPEED: 8.00 KILL* 3000.00*** 70.71 IPRK** 1000.00** TOTAL* 50.00
Tactical Situation:

TB = blind time
V = sub's speed
$\tau^o = N \Delta T =$ time sub begins to turn
R = turn radius
$\Delta = P \Delta \theta =$ turn angle

$(X_i, Y_i)$ represents the distributions of points in a lobe of the locus of an evading submarine at the end of the blind time.

FIGURE 1
$(X_t, Y_t) = (R_t \cos \pi/2) = (0, R_t) = \text{Sub's true position at } T = 0$

$(X_o, Y_o) = (R_o \cos \alpha_o, R_o \sin \alpha_o) = \text{Sub's observed position at } T = 0$

$(0, 0) = \text{origin at the assisting ship's location, at } T = 0$

$$d = \sqrt{(X_o - X_t)^2 + (Y_o - Y_t)^2} = \text{the observation error.}$$

**FIGURE 2**