A STUDY OF THE SLOT TRANSMISSION-LINE AND SLOT ANTENNA

Part 2. A Coaxial Amplitude-Insensitive Phase-Detection System

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Abstract

A coaxial amplitude-insensitive phase-detection system is discussed in detail. The system employs the hybrid junction in a balanced detector configuration to render a null reading independent of the relative magnitudes of both the reference and the unknown signals. A theoretical formulation, experimental results and evaluation of errors are presented.
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2.1 Introduction

The more widely used methods of phase measurement detect the phase of an unknown signal by comparing it with the phase of a known reference signal. If, for example, the phase of an unknown signal $U \cos(\omega t + \phi_U)$ is to be measured, a reference signal of known phase $R \cos(\omega t + \phi_R)$ is added to it. When $|\phi_R - \phi_U|$ is some odd multiple of $\pi$ a minimum signal is detected of magnitude $|R - U|$. In the case where $R = U$, a null is detected. As the magnitude of $U$ departs from that of $R$, the minimum becomes increasingly shallow until at amplitude ratios of 5 or more, no accurate reading can be made. As a consequence of this limitation, the amplitude of one or the other signals must be adjusted periodically to ensure null or near-null conditions for an acceptable degree of accuracy. In addition to being time consuming, this process introduces phase-shifting errors inherent in most calibrated attenuators.

The coaxial amplitude-insensitive phase-detection system presented in this study is not fundamentally new [1, 2, 3, 4]. However, the theory has not been made completely clear and the full capability has not been utilized.
It is the purpose of this effort to present the theoretical considerations supported by experimental evidence with a discussion of errors for this system.

2.2 Theory of Operation

The coaxial hybrid junction and balanced detectors are the cornerstones of the system presented in Fig. 2-1. The hybrid junction has four ports. A signal introduced into any port will be transmitted to the two opposite ports - to the far one with $0^\circ$ phase shift down 3 dB, to the near one with $90^\circ$ phase shift down 3 dB. For interest a typical isolation between an input and unused port is 20 dB.

In the case under consideration the two input signals are

$$e_1(t) = R \cos \omega t$$
$$e_2(t) = U(1 + m \cos \omega_m t) \cos (\omega t + \phi_d)$$

where the subscripts on $e(t)$ refer to their location on the schematic and $\phi_d$ is the difference in phase angle between the unknown and the reference signals. The hybrid junction combines the signals to give

$$e_3(t) = \frac{1}{2} R \cos \omega t + \frac{1}{2} U(1 + m \cos \omega_m t) \cos (\omega t + \phi_d' + \frac{\pi}{2})$$
$$e_4(t) = \frac{1}{2} R \cos (\omega t + \frac{\pi}{2}) + \frac{1}{2} U(1 + m \cos \omega_m t) \cos (\omega t + \phi_d')$$

By means of a technique similar to that suggested by Arguinbau [5], it is convenient to express the input-output characteristics of a diode about its operating point as follows

$$e_{\text{out}}(t) = a_0 + a_1 e_{\text{in}}(t) + a_2 e_{\text{in}}^2(t) + a_3 e_{\text{in}}^3(t) + \ldots$$
AMPLITUDE INSENSITIVE PHASE MEASURING SYSTEM

FIG. 2.1
Since from calibration the crystal is known to have a square law response, it is sufficient to consider only the \( a_2 e_{in}^2(t) \) term of the power expansion.

As a consequence, the output of the detectors are

\[
e_5(t) = \frac{1}{4} a_2 \left[ R \cos \omega t + U(1 + m \cos \omega_m t) \cos (\omega t + \phi'_d + \frac{\pi}{2}) \right]^2 \tag{2-6}
\]

\[
e_6(t) = \frac{1}{4} a_2' \left[ R \cos (\omega t + \frac{\pi}{2}) + U(1 + m \cos \omega_m t) \cos (\omega t + \phi'_d) \right]^2 \tag{2-7}
\]

With the trigonometric identity for the sum of two angles, Eqs. (6) and (7) yield

\[
e_5(t) = \frac{1}{4} a_2 \left[ R^2 \cos^2 \omega t - 2RU(1 + m \cos \omega_m t) \cos \omega t \sin (\omega t + \phi'_d) \right.
\]

\[+ U^2(1 + m \cos \omega_m t)^2 \sin^2(\omega t + \phi'_d) \right] \tag{2-8}
\]

\[
e_6(t) = \frac{1}{4} a_2' \left[ R^2 \sin^2 \omega t - 2RU(1 + m \cos \omega_m t) \sin \omega t \cos (\omega t + \phi'_d) \right.
\]

\[+ U^2(1 + m \cos \omega_m t)^2 \cos^2(\omega t + \phi'_d) \right] \tag{2-9}
\]

The current-shunting potentiometer provides the method of equating the \( a_2 \) and \( a_2' \) coefficients. With balanced coefficients and the trigonometric identity

\[
\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta) \tag{2-10}
\]

the output of the differencing IF transformer circuit is
\[ e(t) = \frac{1}{2} a_2 RU \sin \phi_d \cos \omega_mt. \]  
(2-11)

If \( \phi_d \) is zero or some integer multiple of \( \pi \), an \( a-f \) null is produced.

2.3 Experimental Results

A short-circuited section of transmission line with a 46dB VSWR was chosen to demonstrate the technique. The results are presented in Fig. 2.2. It is interesting to note that even in the sharp null region of the standing wave, readings were easily observed with no adjustments of relative amplitudes.

2.4 Discussion of Errors

There are three major sources of error in the amplitude-insensitive phase-detection system. The first and most obvious is a result of an improper equalization of the \( a_2 \) and \( a_2' \) coefficients. In this case, the system degenerates into the characteristics of the adding-type system previously mentioned. That is, with large variations in relative amplitude between reference and unknown signals, the null becomes a shallow minimum.

The second source of error is due to reflection from mismatched crystals. This reflection produces a feedback disturbing either the reference or unknown signal. Figure 2.3 demonstrates an example of this error. The third error is introduced by a non-unity standing-wave ratio on the reference line.
PHASE DISTRIBUTION

AMPLITUDE DISTRIBUTION

CURRENT PHASE ANGLE (DEGREES)

0°

90°

180°

270°

CURRENT AMPLITUDE (db)

-50db

-40db

-30db

-20db

-10db

0

POSITION ALONG SLOT TRANSMISSION LINE

λ/2

COMPARISON OF CURRENT AMPLITUDE AND PHASE DISTRIBUTION FOR OPEN-CIRCUITED SLOT TRANSMISSION LINE

FIG. 2.2
FIG. 2.3

PHASE DISTRIBUTION FOR OPEN-CIRCUITED SLOT TRANSMISSION-LINE WITH CRYSTALS MATCHED AND MISMATCHED TO HYBRID JUNCTION

VSWR = 1.2 BEFORE CRYSTALS

VSWR = 10 BEFORE CRYSTALS

\( \lambda/2 \)
PLOT OF $\Theta$ (PHASE ANGLE) VS. $\beta_z$ (DISTANCE MEASURED ALONG REFERENCE PHASE LINE) FOR VARIATIONS IN VSWR.

FIG. 2.4
This error is particularly disturbing when the phase function under study is continually varying. Figure 2.4 presents the effects of this error in its disturbance of the linear phase function required for simple operation of the reference phase line. A VSWR of 1.1, for example, introduces an error of magnitude equal to or less than 2.5° whereas a VSWR of 1.5 produces errors of magnitude less than 12°.
References


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