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Solid State Devices Applications For ASW

VLF FERRITE ANTENNAS

Technical Report

Contract No. 3358(00)
NR 373-804
Department of the Navy
Office of Naval Research
Washington 25, D.C.

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500 Washington Avenue
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DECEMBER 1963

November 1963
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Solid State Devices Applications For ASW

VLF Ferrite Antennas

Technical Report

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## INTRODUCTION

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VLF Ferrite Antennas

SUMMARY

The following report includes the results of the work performed at ITT Federal Laboratories on the design of VLF ferrite loaded antennas under Contract No. 3320 up to November 1963. The report consists of a general review of the properties of short electrical radiators, of a general theoretical analysis of the performance of ferrite loaded antennas, of the presentation and discussion of extensive experimental verifications of the theory, and, finally, of a preliminary design of an Omega type antenna system. The latter would operate at 100 kHz with a radiated power up to 20 kW, and with an efficiency of the order of 10%.

While this design is tentative, it constitutes a sound reference basis for the development of a final ferrite loaded antenna system.

The work discussed in the following indicates that the use of ferrite loading provides considerable wavelength reduction along the axis of the radiator and results in structures which have radiation resistance larger than that of metal-air antennas having the same height. The overall design may be achieved with satisfactory characteristics of efficiency, reliability, bandwidth and cost. The maximum radiated power is limited by the distortion which occurs when the magnetic material becomes saturated; however, in practice satisfactory levels of output power may be obtained for medium power applications.
1. **INTRODUCTION**

The design of transmitting antennas for the VLF to HF frequency range must satisfy in general a number of conditions whose relative importance varies from one case to another. These are:

1. Radiation efficiency
2. Power handling
3. Bandwidth
4. Cost of construction and of maintenance
5. Vulnerability with respect to enemy action
6. Frequency tuning ability
7. Radiation pattern and gain
8. Height ... etc.

The ideal case, which is taken as reference, is that of a quarter-wave monopole above infinitely conducting ground. For VLF to HF applications shorter antennas are needed; the basic approach consists of the use of top loaded monopoles. On the other hand, monopoles with inductive loading, where the inductance may be lumped at a suitable intermediate location or distributed along the radiator, may be used. In both cases the loading results in tuning the input impedance of the antenna. The current distribution of the loaded antenna depends on the particular type of loading and in certain cases tends to be uniform.

The approach followed in the present work is based on the use of distributed ferrite loading. In this case, the antenna structure and propagation characteristics are modified and, in particular, the axial propagation constant is increased, so that the axial wavelength is reduced with respect to the value in air. As a result, resonant monopoles possess a height lower than that of the corresponding air structures. If properly designed, these antennas provide high radiation efficiency and large relative bandwidth with respect to conventional metal-air antennas of the same height; on the other hand, the power handling is limited by the magnetization saturation of the ferrite material. Before proceeding to the discussion of the theory of ferrite loaded linear antennas, it is convenient to review briefly the theory of metal air antennas.

1. **CONFIDENTIAL**
2. THE LINEAR METAL-LINE ANTENNA

We consider first the case of thin wire antennas. With reference to a current element \( I dl \) located at the origin of a spherical coordinate system along the \( z \) axis, one obtains the radiation field by consideration of the corresponding vector potential:

\[
A = A_\phi = \frac{Idl}{4 \pi r} = -\frac{j \beta}{k} r_k
\]  

(1)

From Maxwell's equations there follow the well known field equations:

\[
\begin{align*}
H &= \frac{Idl \sin \phi}{4 \pi r} \left( j \beta + \frac{1}{r} \right) - j \beta r \\
E_\phi &= \eta \frac{Idl \sin \phi}{4 \pi r} \left( j \beta + \frac{1}{r} + \frac{1}{j \beta r^2} \right) - j \beta r \\
E_r &= \eta \frac{Idl \cos \phi}{4 \pi r} \left( \frac{2}{r} + \frac{2}{j \beta r^2} \right) - j \beta r
\end{align*}
\]

(2)

where \( \eta = \ldots \) and \( \beta = \omega \ldots \). The radiated power is computed from integration of the Poynting vector over the surface of a sphere of radius \( r \), having the current element at its center. The average power is contributed only by the far field components:

\[
\begin{align*}
H_0 &= j \beta \frac{Idl \sin \phi}{4 \pi r} - j \beta r \\
E_\phi &= j \beta \eta \frac{Idl \sin \phi}{4 \pi r} - j \beta r
\end{align*}
\]

(3)

and the corresponding time average Poynting vector is:

\[
\begin{align*}
S_{avg} &= \frac{1}{2} R \left( E \times H^* \right) = \frac{\beta}{2} \left( \frac{Idl \sin \phi}{4 \pi r} \right)^2
\end{align*}
\]

(4)
The overall radiated power is:
\[
P = \eta \frac{\beta^2 \Delta \omega^2}{12 \pi}
\]
where \( \beta \) is the maximum value of the current. The corresponding radiation resistance is:
\[
R_{rad} = \eta \frac{\beta^2 \Delta \omega^2}{6 \pi} = 80 \pi^2 \left( \frac{\Delta \omega}{\lambda} \right)^2
\]
For a short dipole of length with a triangular current distribution, having peak current \( I \) at its center and zero current at its ends, the radiated power is one quarter of that corresponding to a uniform current radiator of the same length and the radiation resistance is:
\[
R_{rad} = 20 \pi^2 \left( \frac{\Delta \omega}{\lambda} \right)^2
\]
For a short vertical monopole, above perfect ground, one has:
\[
R_{rad} = 10 \pi^2 \left( \frac{\Delta \omega}{\lambda} \right)^2
\]
More generally, if one considers a thin linear antenna of height 2H, having a sinusoidal current distribution:
\[
l = l_m \sin \beta (H-z) \quad \text{but, } z = 0
\]
\[
l = l_m \sin \beta (z - H) \quad \text{but, } z = 0
\]
where \( z \) is measured from the antenna mid-point, the resulting radiated electromagnetic field may be obtained by integration of suitable infinitesimal current elements. The far field components are:
\[
H = \frac{j \beta l_m}{\eta} \frac{\cos (2H \cos \theta) - \cos 3H \sin \theta}{\sin \theta}
\]
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\[ V = J \left( 60 \frac{L_a}{r} \right) e^{-jBr} \]

where \( r \) is the radial distance from the antennas mid-point. For a half-wave dipole (or quarter-wave monopole) one has \( \beta H = \pi/2 \); the radiation resistance computed through integration of the Poynting vector is:

\[ R_{rad} = \frac{30}{\sin^2 \beta H} \left\{ S_1 (2\beta H) - s_1 (4\beta H) - S_1 (2\beta H) \cos 2\beta H \right\} \]

\[ + \sin 2\beta H + (1 + \cos 2\beta H) S_1 (2\beta H) - \sin 2\beta H \sin S_1 (2\beta H) \]

for a dipole of half length \( H \). In the above expression:

\[ S_1 (x) = \frac{x}{2} \int_0^1 \frac{1 - \cos \nu}{\nu} d\nu, \quad S_1 (x) = \frac{x}{2} \int_0^1 \sin \nu d\nu \]

In the particular case of half-wave dipole and quarter-wave monopole above perfect ground, equation (71) provides respectively:

\[ R_{rad} = 73 \text{ ohms}, \quad R_{rad} = 36.5 \text{ ohms} \]

The radiation resistance can also be computed by the induced emf method. Consider, for example, a vertical monopole or helix in above perfect ground. Starting from the expression of the \( z \)-component of the electric field at the surface of the antennas, obtained assuming a given current distribution, one computes the power required to produce the assumed current against the induced voltage as follows:

4.

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where $\Phi$ is the phase angle between current and $E_z$. In the case of a sinusoidal distribution of current, one readily obtains the expression of $P_{\text{rad}}$ previously found.

The reactance of the ideal "thin wire" dipole antenna element is found to be infinitely large. This is due to the fact that the reactance depends strongly upon the "near field" components, and these cannot be defined accurately with the thin wire idealization. Therefore, in computations of antenna reactances, it is essential that the actual antenna cross-section be taken into account.

In general, considering a cylindrical antenna wire of radius $a$ carrying a sinusoidal current distribution, it is found that the reactance can be computed from the value of the reactive power required to produce the assumed current against the induced voltage. With reference to a vertical monopole of height $H$, one finds:

$$P_{\text{react.}} = \frac{1}{2} \int_{0}^{H} E_z \cdot I(z) \cdot \cos \Phi \cdot dz$$  \hspace{1cm} (13)

wherefrom the reactance of the antenna may be computed, letting $P = \chi_0^2/2$:

$$X = \frac{-15}{\sin^2 \Phi H} \cdot \sin 2\Phi H \cdot -0,5772 + \ln \frac{H}{a} \cdot 2 \cdot \text{Ci}(3H) - \text{Ci}(4\Phi H)$$

$$- \cos 2\Phi H \cdot \text{Si}(2\Phi H) - \text{Si}(4\Phi H)$$

$$+ 2 \cdot \text{Si}(2\Phi H)$$ \hspace{1cm} (14)

In particular, for a quarter-wave vertical monopole, the reactance is found to be $X = 21.25$ ohms; on the other hand, for shorter antennas, the reactance depends strongly on the value of the diameter of the antenna and decreases as the latter is increased.

3. **INFINITELY LONG RADIATOR**

A problem of particular interest for the study of ferrite loaded
anrennas is that of the radiation properties of an infinitely long conductor, carrying a uniform current $I = I_0 e^{j \omega t}$. Assuming that the conductor is surrounded by uniform lossless space, and that the conductor conductivity $\sigma$, the electromagnetic field distribution corresponding to the principal mode is found to be given by the following expressions: (1)

$$a, \quad E_z = a_0 J_0 (h_1) \cdot j(\omega t - \beta z)$$

$$E_z = a_0 \frac{h_1}{h} \cdot J_1 (h_1) \cdot j(\omega t - \beta z)$$

$$H_0 = \frac{k_1^2}{\mu_0} a_0 J_0' (h_1) \cdot j(\omega t - \beta z)$$

$$E_z = b_0 H_0 (2) (h_2) \cdot j(\omega t - \beta z)$$

$$E_z = j \frac{\beta}{h_2} b_0 H_0 (2)' (h_2) \cdot j(\omega t - \beta z)$$

$$H_0 = \frac{k_2^2}{\omega_0} b_0 H_0 (2)' (h_2) \cdot j(\omega t - \beta z)$$

In these relations $\beta$ is the axial propagation constant, $k_1^2 = -j\omega, \quad k_2^2 = \omega^2$.

and:

$$h_1^2 = k_1^2 - \beta^2, \quad h_2^2 = k_2^2 - \beta^2$$

The axial current is computed as follows:

$$I = 2 \pi \int_0^\infty \sigma E_z q \varphi dz = \frac{2\pi a_0 J_1 (h_1)}{h_1} \cdot j(\omega t - \beta z)$$

wherefrom the value of the coefficient $a_0$ is obtained as a function of the given current, i.e.

$$a_0 = \frac{h_1 I_0}{2\pi a \sigma_1 (h_2 a)} \quad (20)$$

Similarly, from the boundary condition at $a$ one obtains the value of the coefficient $b_0$ as follows:

$$b_0 = \frac{h_2}{2\pi a \sigma_1} \frac{i k_1^2}{k_2^2} \frac{I_0}{H_1(2)(h_2 a)} \quad (21)$$

It should be stated that, if the conductor is assumed to have infinite conductivity, the field vanishes for $a$ and one finds:

$$h_2 = 0, \beta = k_2 = \sqrt{2} \frac{\omega}{2} \quad (22)$$

i.e. the axial propagation occurs with speed equal to that of the external medium. The above expressions of the field cannot be applied in the present case, since $E_z(a) = 0$ and the argument of the Bessel functions vanishes.

Starting from Maxwell's equations and indicating with $\beta = \omega$ the axial propagation constant, one finds that the axial component $E_z$ is a solution of the wave equation:

$$\frac{\partial^2 E_z}{\partial \tau^2} + \frac{1}{\varepsilon} \frac{\partial E_z}{\partial \tau} + \beta^2 E_z = 0 \quad (23)$$

and

$$H_x = \frac{1}{\mu_0 \omega} \frac{\partial E_z}{\partial \tau} \quad (24)$$

There follows:
\[ E_z = \text{CH}_0(2) (\beta \gamma) \cdot j(\omega t - \beta z) \]  
(25)

\[ H_C = j \frac{\text{Re} \omega \mu_2}{\omega \mu_2} H_1(2) (\beta \gamma) \cdot j(\omega t - \beta z) \]  
(26)

From the boundary condition:

\[ I_o = 2a \frac{\partial}{\partial \rho} \left( \text{Ch}_0(2) (\rho) \right) \]  
(26)

there follows:

\[ C = \frac{I_o}{2 \pi a} \frac{\omega \mu_2}{\beta H_1(2) (\beta \gamma)} \]  
(27)

I.e.

\[ E_z = \frac{-j \omega \mu_2 I_o}{2 \pi a \beta H_1(2) (\beta \gamma)} H_0(2) (\beta \gamma) \cdot j(\omega t - \beta z) \]  
(28)

\[ H_C = \frac{I_o}{2 \pi a \beta H_1(2) (\beta \gamma)} H_1(2) (\beta \gamma) \cdot j(\omega t - \beta z) \]  
(29)

Since \( \beta a \ll 1 \) one has:

\[ \text{Ch}_1(2) (\beta a) \approx \frac{\beta a}{2} + \frac{21}{\pi \beta a} \]  
(30)

I.e.

\[ E_z = \frac{-j \omega \mu_2 I_o}{2} H_0(2) (\beta \gamma) \cdot j(\omega t - \beta z) \]  
(31)

\[ H_C = -j \frac{\beta I_o}{4} H_1(2) (\beta \gamma) \cdot j(\omega t - \beta z) \]  
(32)

8.

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Since $E_z$ must reduce to zero at $\varphi = a$, it is seen that there is present a field component

$$E_z = \frac{\partial}{\partial z} \left( \zeta (a) \right) = - \frac{\omega \mu_0 I}{\beta} H_0^2 (\beta a) \tag{31}$$

which represents the self-induced electric field produced by the current $I_0$.

One may now compute the external impedance of the wire per unit length; letting $I_0 = 2\pi a H_0^2 (a)$ one has

$$Z_{\text{ext}} = \frac{E_A}{I_0} = \frac{\omega \mu_0 I_0}{2\pi a} \frac{H_0^2 (\beta a)}{H_1^2 (\beta a)} \tag{32}$$

Letting for $\beta a \rightarrow 1$,

$$H_0^2 (\beta a) = 1 - \frac{2}{\pi} \ln \frac{1.781}{\beta a} \tag{33}$$

$$H_1^2 (\beta a) = \frac{\beta a}{2} + j \frac{2}{\pi \beta a}$$

one finds

$$Z_{\text{ext}} = \frac{\omega \mu_0 I_0}{2\pi a} + j \frac{\omega \mu_0 I_0}{2\pi} \ln \frac{2}{1.781 \beta a} \tag{34}$$

Thus the radiation resistance per unit length is $\omega \mu_0 I_0/2\pi$ and the corresponding reactance is $\omega \mu_0 I_0/2\pi \ln (2/1.781 \beta a)$. The preceding result may be generalized with consideration of the internal impedance of the cylindrical conductor obtained when the conductivity is finite and the frequency is low. In this case one finds:

$$Z_{\text{int}} = \frac{1}{\pi a^2 \gamma} + j \omega \frac{\gamma}{\beta_0} \tag{35}$$
per unit length. At high frequencies, skin effect phenomena occur and the internal impedance assumes the following expression:

\[ Z_{\text{int}} = \frac{1}{2\pi a} \left( \sqrt{\frac{\mu}{\epsilon}} + \frac{1}{2\pi z} \sqrt{\frac{\mu}{\epsilon}} \right) \]  \hspace{1cm} (36)

IV. BANDWIDTH AND EFFICIENCY CHARACTERISTICS OF METAL-AIR ANTENNAS

The input impedance of an antenna may be computed from the eqs. 11 and 15 and expressed in general as follows:

\[ Z_a = R_a + j X_a \]  \hspace{1cm} (37)

where \( X_a \) may be positive or negative depending upon the antenna length, and is zero at series resonance. The relative 3 db bandwidth \( \delta \omega \) of the antenna is computed solving for \( \delta \omega \) the equation:

\[ 2R_a (\omega_0) = R_a^2 (\omega_0 + j \omega) + X_a^2 (\omega_0 + j \omega)^{-1/2} \]  \hspace{1cm} (38)

Similarly the antenna \( Q \) is defined with the relationship

\[ Q = \omega_0 \delta \omega \]  \hspace{1cm} (39)

The general expressions 11 and 15 of the antenna impedance are too complicated to provide a simple expression for the bandwidth. If the antenna is electrically short \((BH < 0.5)\) an approximate expansion of the impedance may be used as follows \(^{(2)}\)

\[ Z_a = \frac{1}{\sqrt{z}} \frac{(BH)^2}{6\pi} - j \frac{1}{6\pi} \left( \ln \frac{H}{a} - 1 \right) \]  \hspace{1cm} (40)

Assuming that a series inductance is used for tuning, one has approximately

There follows from application of eq. 40

\[ 2 \frac{\delta_{\omega}}{\omega_o} \sim \frac{R_a}{X_a} = \frac{(BH)^3}{5 (\ln \frac{H}{a} - 1)} \]  

Recapitulating, the radiation resistance of an electrically short vertical dipole antenna varies proportionally to the square of the height \( H \) and the relative bandwidth varies proportionally to the third power of \( H \). Hence, with the decrease of \( H \) the radiation resistance and the relative bandwidth become very small. These characteristics represent basic limitations of short antennas since the efficiency depends upon the ratio \( R_{\text{rad}}/R_{\text{loss}} \) and \( R_{\text{loss}} \) is a function of the losses in the ground, in the tuning circuit, in the antenna conductors, in the antenna insulators, etc. On the other hand, the relative bandwidth is prescribed by operational conditions and typically may range from values of \( 2 \times 10^{-3} \) to \( 5 \times 10^{-2} \) at frequencies in the VLF to HF range.

In order to meet practical design requirements it is necessary to keep the antenna height sufficiently high, to use simple or multiple conductors with suitably large effective diameter, to make recourse to folded dipole or cage structures, etc.

It is of interest to summarize the characteristics of some VLF antenna designs for purpose of reference. We shall refer to the antennas installation of Forest-Port and of Cutler, Maine.

**FOREST-PORT ANTENNA**

The Forest-Port Antenna, part of an Air Force three-station VLF transmitting navigational system known as Project Omega, is located in Forest Fort, New York.
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The antenna consists of one 155 ft. bare insulated tower which uses its six top guys to provide the necessary capacitive top loading to make the antenna self resonant at 100 kc. By means of inductive base tuning the antenna operates at 10.2 kc with the following characteristics:

- Effective height: 155 ft.
- Radiation resistance: 0.071 ohms
- Maximum power radiated: 287 watts

CUTLER ANTENNA

The Cutler Antenna, part of the Navy VLF transmitting station located in Cutler, Maine, consists of two arrays whose centers are separated by one mile and which may be operated independently or in dual. Each array has a mean height of 845 ft. and a radius of 3070 ft.; it consists of one center tower of 955 ft. and 12 smaller towers ranging from 575 to 345 feet, which support its star shaped top hat.

The top hat of each array is made up of six independent diamond sections which are insulated from the towers and can be controlled separately, that is, they can be lowered or raised for repairs or for prevention of damage from heavy icing. A unique arrangement of the RF feed lines makes it possible to pass 60 cycle deicing currents through either array or remove ice under moderate icing conditions. Since the arrays may be operated independently, this arrangement allows one array to continue transmitting while the other is being deiced; hence the station may operate continuously.

Power is fed from the transmitter to a helix house at the base of each array through an underground coaxial transmission system. The helix house contains a tuning unit for the array and provides 6 feed points on its roof for feeding 6 vertical "up-leads" which are the radiating elements of the array. The "up-leads" are equally spaced on a circle of diameter 600 ft. providing a large effective diameter for improved bandwidth.

A large buried radial wire ground system helps to provide a low loss ground return for the antenna currents. The station is located on a peninsula and

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the ground system is terminated in the ocean by means of sea anchors which extend about 200 ft. into the ocean.

Below is a summary of the electrical characteristics of the antenna when operated as a dual array at its lowest frequency of operation 14 Ke.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance</td>
<td>0.122 uf each array</td>
</tr>
<tr>
<td>Resistance</td>
<td>0.22 ohms</td>
</tr>
<tr>
<td>Reactance</td>
<td>$-j \times 86.0$ ohms</td>
</tr>
<tr>
<td>Effective Height</td>
<td>407 feet</td>
</tr>
<tr>
<td>Efficiency</td>
<td>70%</td>
</tr>
<tr>
<td>Unloaded $Q$</td>
<td>484</td>
</tr>
<tr>
<td>Radiated Power</td>
<td>$10^5$ watts</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>60 cycles</td>
</tr>
</tbody>
</table>

5. GENERAL PROPERTIES OF LINEAR FERRITE LOADED ANTENNAS

The electrical properties of antennas are dependent upon the physical characteristics of the surrounding medium. Hence, if dipole antennas are embedded in a medium with dielectric permittivity $\varepsilon_1$ and magnetic permeability $\mu_1$ larger than the values of air, a corresponding increase of the value of the propagation constant $\beta' = \omega \sqrt{\mu_1 \varepsilon_1}$ is obtained and therefore, the axial wavelength distribution $\lambda' = c/\beta'$ becomes smaller than in air. If the medium extends to infinity, the same expressions described for the case of air apply, except that in general the losses and the saturation of the medium must be taken into account. From a practical point of view one might consider the effect resulting from adding the antenna in a medium of finite extent, such as for example a cylinder coaxial with the antenna.

The computation of the wavelength distribution, input impedance, and relative bandwidth for this type of antenna represents a difficult problem in general, since the boundary between medium and air is not a surface of constant phase or of constant amplitude for the radiated electromagnetic wave. From a general point of view, utilizing the principle of reciprocity, one may study the scatter of a plane wave incident on such an antenna structure and derive correspondingly the current distribution along the antenna as well as the input impedance and the
relative bandwidth. However, due to the fact that the resonant length of the antenna is not known in advance, the problem must be solved by approximation; in most cases, the solution is obtained in numerical form and gives little insight into the general properties of the antenna.

A simplification of the problem is obtained if the antenna is considered to consist of a cylindrical conductor of infinite length, radius $a$, embedded in a coaxial ferrite cylinder of infinite length and of radius $b$. In this case, one may compute in closed form the various permissible mode distributions and, in particular, the wavelength along the axis of the conductor for each mode. This result represents important design information, since it is found to be applicable with good approximation to the case of antennas of finite length. As far as the antennas input impedance and the relative bandwidth are concerned this approach does not provide convenient answers. In the following section the general theory of propagation of electromagnetic waves along infinite ferrite loaded conductors is discussed; this is followed by a description of various experimental results and by an outline of the design procedure of ferrite loaded antennas.

6. **Wave Propagation Along an Infinite Ferrite Loaded Conductor**

Consider an infinite straight conductor of infinite conductivity and radius $a$, surrounded by a uniform ferrite cylinder of inner radius $a$, and outer radius $b$, having permeability $\mu$ and permittivity $\varepsilon$ (Fig. 1). Using a cylindrical coordinate system $\rho, \phi, z$, one finds that the electromagnetic field distribution may be expressed in terms of two Hertz vector potentials $\mathbf{V}_1$ and $\mathbf{V}_2$, both parallel to the $z$ axis and such that their magnitudes satisfy the scalar wave equation

$$\left[ \nabla^2 - \frac{\mu}{\varepsilon} \frac{\partial^2}{\partial t^2} - \frac{\mu \varepsilon}{\partial \phi^2} \right] \varphi = 0$$

Assuming harmonic time and space dependence one may let in general

$$\varphi = f(\rho, \phi) \exp(\imath \omega t - \beta z)$$

\[14\]

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where \( f(\cdot) \) is a solution of the equation

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{2} \frac{\partial^2 f}{\partial z^2} + (\ell^2 - \mu^2) f = 0
\]

In the eq. 45 the coefficient \( K \) is expressed in terms of the characteristics of the medium, i.e.

\[
k^2 = c^2 - j \omega \mu
\]

and the coefficient \( \beta \) is the axial propagation constant, i.e.

\[
\beta = 2\pi / \lambda_z
\]

where \( \lambda_z \) is the wavelength along the z axis. The general expression of the electromagnetic field is derived from the vector potentials \( \vec{A}_1 \) and \( \vec{A}_2 \) using the following relations

\[
\vec{E} = \nabla \times \vec{A}_1 + j\omega \mu \nabla \times \vec{A}_2
\]

\[
\vec{H} = (c^2 - j\omega c) \vec{A}_1 + \nabla \times \nabla \times \vec{A}_2
\]

It is seen that the field derived from \( \vec{A}_1 \) has no axial magnetic field component, while the field derived from \( \vec{A}_2 \) has no axial electric field component; for this reason, the two field configurations are called respectively transverse magnetic and transverse electrical.

Clearly the problem of interest in the present investigation falls in the class of transverse magnetic modes. We can proceed accordingly neglecting \( \vec{A}_2 \) and solving eq. 45 separately in region I, i.e. \( a \leq z \leq b \) and in region \( \Omega \), i.e. \( b \leq z \leq \infty \).
In region 1 we may let in general

\[ E_{1z} = \sum_{n=0}^{\infty} (A_n J_n (h_{1z}) + B_n N_n (h_{1z})) J(n \omega - \omega t - \beta_1 z) \]

\[ H_{1z} = 0 \]

and in region 2 we may let

\[ E_{2z} = \sum_{n=0}^{\infty} C_n H_n (1) (h_{2z}) J(n \omega - \omega t - \beta_2 z) \]

In the above equations

\[ h_{1z}^2 = k_1^2 - \beta_1^2 \]

\[ h_{2z}^2 = k_2^2 - \beta_2^2 \]

Since we are interested in fields which are distributed uniformly with respect to the angle \( \gamma \), we shall limit our investigation to the zero order mode only, i.e. \( n = 0 \). Hence, applying eqs. 47 we find:

Region 1

\[ E_{1z} = \left[ A_0 J_0 (h_{1z}) + B_0 N_0 (h_{1z}) \right] J(\omega t - \beta_1 z) \]

\[ E_{1z} = \sqrt{\frac{k_1^2 - \beta_1^2}{\omega}} \left[ A_1 \frac{J_1 (h_{1z})}{h_{1z}} + B_1 \frac{N_1 (h_{1z})}{h_{1z}} \right] J(\omega t - \beta_1 z) \]

\[ H_{1z} = \frac{-j k_1^2}{\omega \mu_1 \sqrt{k_1^2 - \beta_1^2}} \left[ A_1 J_1 (h_{1z}) + B_1 N_1 (h_{1z}) \right] J(\omega t - \beta_1 z) \]
In order to find the eigenvalues of the problem and the values of the coefficients $A_1, B_1$ and $C_2$ we make recourse to the following boundary conditions

\[ \gamma = a, \quad E_{1z} = 0 \]  
\[ \gamma = b, \quad E_{1z} = E_{2z}, \quad H_{1y} = H_{2y} \]

From the first condition we obtain

\[ A_1 J_0 \left( \frac{h_1}{\gamma} \right) + B_1 N_0 \left( \frac{h_1}{\gamma} \right) = 0 \]  
(55)

i.e.

\[ B_1 = -A_1 \frac{J_0 \left( \frac{h_1}{\gamma} \right)}{N_0 \left( \frac{h_1}{\gamma} \right)} \]  
(56)

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Substituting in eqs. (52) we obtain:

\[
E_{1\alpha} = A_1 \left[ J_0 \left( h_1 \alpha \right) N_0 \left( h_1 \alpha \right) - J_0 \left( h_1 \beta \right) N_0 \left( h_1 \beta \right) \right] - j\omega \varepsilon_0 \frac{30A_1}{h_1}
\]

\[
E_{1\psi} = -jA_1 \left[ J_1 \left( h_1 \alpha \right) N_0 \left( h_1 \alpha \right) - J_0 \left( h_1 \alpha \right) N_1 \left( h_1 \beta \right) \right] - j\omega \varepsilon_0 \frac{30A_1}{h_1}
\]

\[
H_{1\phi} = \frac{j\mu_0^2}{\omega} A_1 \left[ J_1 \left( h_1 \alpha \right) N_0 \left( h_1 \alpha \right) - J_0 \left( h_1 \alpha \right) N_1 \left( h_1 \beta \right) \right] - j\omega \mu_0 \frac{30A_1}{h_1}
\]

Turning now to the boundary condition at \( \alpha = b \) we write:

\[
A_1 \left[ J_0 \left( h_1 b \right) N_0 \left( h_1 a \right) - J_0 \left( h_1 a \right) N_0 \left( h_1 b \right) \right] = C_2 H_0^{(1)} \left( h_2 b \right)
\]

\[
\frac{K_1^2}{\mu_1} A_1 \left[ J_1 \left( h_1 b \right) N_0 \left( h_1 a \right) - J_0 \left( h_1 a \right) N_1 \left( h_1 b \right) \right] = \frac{K_2^2}{\mu_2^2} H_1^{(1)} \left( h_2 b \right)
\]

In order to solve the above system of equations we introduce the following substitutions:

\[
h_1 a = u, \quad h_2 b = v
\]

\[
h_1 a = s u, \quad h_2 a = v s
\]

where \( s = a/b \). Hence eqs. (58) may be written as follows:
An equivalent expression of eq. 59 is written as follows:

\[
\begin{align*}
\frac{J_1(u) N_0(\text{us}) - J_0(\text{us}) N_1(u)}{J_1(u) N_0(\text{us}) - J_0(\text{us}) N_0(u)} = \frac{K_1^2}{\omega \mu_1 h_1} \frac{\mu_2 h_2}{K_2^2 + \omega_1 h_1} H_1'(v) = 0
\end{align*}
\]

(61)

If the quantities \( u \) and \( v \) are very small, as in the case of interest for the present investigations, eq. 62 can be simplified noting that:

\[
H_0(v) = \frac{21}{v} \ln \frac{\sqrt{2} \sqrt{v}}{2J}
\]

\[
H_1(v) = -\frac{23}{v} \cdot v
\]
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\[ J_0(u) \approx 1, \quad J_1(u) \approx 0 \quad (63) \]

\[ N_0(u) \approx \frac{2}{\pi} \ln \frac{2}{y u} \]

\[ N_1(u) \approx \frac{2}{\pi y u} \]

where \( y = 1.781 \)

Substituting into eq 62 one has finally

\[ \frac{\xi}{2} \cdot \frac{1}{2} \cdot \frac{1}{u^2 \ln \frac{1}{s}} + \frac{1}{v^2 \ln \frac{v}{2j}} = 0 \quad (64) \]

i.e. letting \( \frac{\xi}{2} = \xi \)

\[ v^2 \ln \frac{v}{2j} = u^2 \ln \left( \frac{1}{s} \right) \quad (65) \]

In order to remove the imaginary it is convenient to define a new set of variables, i.e.

\[ u = p = h_1 b \quad (66) \]

\[ v = t q = h_2 b \]

In terms of \( p \) and \( q \) equations 51 and 65 are written as follows:

\[ \frac{x_1^2 - b^2}{p^2} = \frac{x_2^2 - b^2}{q^2} = \frac{p^2}{b^2} = \frac{q^2}{b^2} \quad (67) \]

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\[ q^2 \ln \frac{2}{q} = p^2 \ln \left( \frac{b}{a} \right) \]

Up to this point, we have assumed that the conductivity of the ferrite is greater than zero. In practice, however, we may expect to use a low-loss ferrite material and correspondingly may let \( \mu = 0 \). There follows:

\[
\begin{align*}
\kappa_2^2 &= \frac{\omega^2 \mu_2}{\epsilon_2} = \left( \frac{2\pi}{\lambda_0} \right)^2 \\
\kappa_1^2 &= \frac{\omega^2 \mu_1}{\epsilon_1} = \left( \frac{2\pi}{\lambda_0} \right)^2 \epsilon
\end{align*}
\]

where \( \mu = \mu_1/\mu_2 \), \( \epsilon = \epsilon_1/\epsilon_2 \). Substituting in eqs. 67 and manipulating one finds:

\[
\begin{align*}
\left( \frac{2\pi}{\lambda_0} \right)^2 \epsilon &= \left( \frac{2\pi}{\lambda_z} \right)^2 = \frac{p^2}{b^2} \\
\left( \frac{2\pi}{\lambda_0} \right)^2 - \left( \frac{2\pi}{\lambda_z} \right)^2 &= -\frac{q^2}{b^2}
\end{align*}
\]

Finally adding and subtracting the first two equations of 69 one obtains:

\[
\begin{align*}
\kappa &= \frac{2\pi b}{\lambda_0} = \frac{\sqrt{p^2 + q^2}}{\sqrt{\epsilon \mu - 1}} \\
\lambda_z/\lambda_0 &= \frac{\sqrt{p^2 + q^2}}{\sqrt{2 \mu - 1}}
\end{align*}
\]

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The above system of equations provides the solution of the problem of the design of a resonant ferrite antenna. Given the values of \( \varepsilon \) and \( \varepsilon' \) for the ferrite material and of the radii \( a \) and \( b \), the ratio \( \lambda_2/\lambda_0 \) is computed. Alternatively, given \( \varepsilon \) and \( \lambda_2/\lambda_0 \) one can find the required values of \( a \) and \( b \) for which the desired wavelength reduction is obtained.

The information contained in equations (70) may be presented in graphical form in a number of ways. From the first of eqs. (70) it is seen that a family of circles for constant \( \lambda_2/\lambda_0 \) is described with the equation

\[
p^2 + q^2 = (\varepsilon \varepsilon' - 1) \left( \frac{2\varepsilon b}{\lambda_0} \right)^2 \tag{71}
\]

On the other hand the equation

\[
p^2 \ln \left( \frac{b}{a} \right) = \varepsilon^2 \ln \left( \frac{1.12}{q} \right) \tag{72}
\]

describes a family of curves, each characterized by a given ratio \( b/a \). The families corresponding eqs. (71) and (72) are plotted in fig. 2 for a given type of ferrite having relative average permeability \( \varepsilon = 100 \) and relative average permittivity \( \varepsilon' = 10 \). Using the latter diagram it is possible to find the values of the quantities \( p \) and \( q \) which correspond to a given set of values \( \lambda_2/\lambda_0 \) and \( b/a \). Substituting in the equation

\[
\frac{\lambda_2}{\lambda_0} = \frac{\sqrt{\frac{p^2 + q^2}{p^2 + q^2}}}{\sqrt{\frac{p^2 + q^2}{p^2 + q^2}}} \tag{73}
\]

the desired ratio \( \lambda_2/\lambda_0 \) is computed. It is of interest to note that in certain ranges of the variables \( \lambda_2/\lambda_0 \) and \( b/a \) two solutions are obtained; for instance letting \( \lambda_2/\lambda_0 = 100 \) and \( b/a = 5 \) one finds:
The maximum permissible value of q is 1.12; this corresponds to $\lambda_o/b = 177$ and to $p = 0$. In this case one finds $\lambda_e/\lambda_o = 1/\sqrt{\mu \xi}$ regardless of the ratio $b/a$. Associated with the same ratio $\lambda_e/b$ there is found also another solution for each value of $b/a$.

In experimental verifications the degeneracy above illustrated is not found and only the solutions corresponding to the smaller of the q values is obtained. The result may be attributed to the occurrence of additional boundary conditions not included in the above analysis.

A more compact representation of the design equations (70) is obtained eliminating the quantity p; in fact, solving the first equation in terms of p one has:

$$p = (\zeta \mu - 1) \left( \frac{2p b}{\lambda_o} \right)^2 - q^2 \quad (75)$$

Substituting in the remaining equations, these acquire the following form

$$\lambda_e/\lambda_o = \sqrt[2]{1 + \frac{\lambda_o q^2}{2 \pi b}} \quad (76)$$

$$(\zeta \mu - 1) \left( \frac{2p b}{\lambda_o} \right)^2 = q^2 \left[ 1 + \left( \ln 1.12 - \ln q \right) / \ln \frac{b}{a} \right]$$

From the latter expressions it is noted that, for a given value of the ratio $\lambda_o/b$, the ratio $\lambda_e/\lambda_o$ becomes smaller, the larger is the value of the parameter $q$. The parameter $q$ is obtained as a solution of the second equation of (76). Although
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the latter equation is of transcendental form, and, for this reason, cannot be
analyzed directly, some considerations of a qualitative nature may be derived.
Assume, for example, that $q$ is very small with respect to unity. In this case,
the said equation reduces approximately to the following form

$$(\xi \mu - 1) \left(2\pi b/\lambda_{g}\right)^2 = -q^2 \ln q/\ln \frac{b}{a}$$

(77)

The function $q^2 \ln q$ is zero at $q = 0$ and increases monotonically with the increase
of $q$; hence, $q$ increases with $\ln \frac{b}{a} \left(\xi \mu - 1\right)$ and $b/\lambda_{g}$. Quantitative values are
obtained plotting the variable $\lambda_{g}/\lambda_{g}$ as a function of $\lambda_{g}/b$ for given values of the
ratio $b/a$ and for a given type of ferrite material. An example is shown in fig. 3
computed for a ferrite material of characteristics $a = 10$, $\nu = 100$ and for $b/a$
values of 1.5, 3, 3.8, 10, 25 and 100.

A question of obvious interest is that of the optimization of the
ferrite characteristics. From the second of equations (76) it is seen that,
since $q$ must be less than 1.12, the quantity $\nu - 1$ must be less than a certain
limit value, depending upon the ratio $b/\lambda_{g}$. Physically, this result is interpreted
as a necessary condition for the existence of the principal mode $n = 0$ of trans-
verse magnetic type of eqs. (49) and (50). When the latter condition is not
satisfied, the electromagnetic field distribution must correspond to a different
mode distribution.

The above analysis has permitted the derivation of the axial wave-
length shortening obtained with a suitable ferrite loading of the conductor.
These results are directly applicable to resonant type antennas which have a
length equal to $\lambda_{g}/b$ or to a multiple of $\lambda_{g}/b$. It would be of interest to
complete the investigation, developing methods of computation of the input im-
pedance and of the relative bandwidth of the ferrite loaded antennas. These,
however, cannot be obtained following the previous reasoning, since they require
consideration of the finite length of the antenna and of the existence of modes
more complex than those corresponding to the infinite conductor.

In the case of very short ferrite loaded antennas an approximate
analysis of the input resistance may be developed assuming that the same equation valid for metal-air dipoles applies, except that the quantity $\alpha H$ is replaced with the value $\beta \cdot H$, where $\beta$ is the axial propagation constant of the infinite ferrite loaded conductor. With this reasoning one obtains the following expression for the radiation resistance of a ferrite loaded dipole of height $H$:

$$R_{rad} = \sqrt{\frac{\lambda_e}{\epsilon_2}} \left( \frac{\beta \cdot H}{\frac{\epsilon_2}{\epsilon_2}} \right)^2 = \sqrt{\frac{\lambda_e}{\epsilon_2}} \left( \frac{\beta \cdot H}{\frac{\epsilon_2}{\epsilon_2}} \right)^2 \left( \frac{\lambda_e}{\lambda} \right)^2$$

The radiation resistance is larger than that of a metal-air dipole having the same height $H$. This result is reasonably well verified by experimental data, as will be shown in the following section. On the other hand a similar reasoning cannot be developed for the computation of the reactance and of the relative bandwidth, because of the critical importance that the ferrite material represents with respect to the reactance of the antenna input impedance.

In conclusion, it has been shown that the wavelength reduction of a ferrite loaded dipole antenna may be computed with good accuracy from the values of the radii $a$ and $b$, and those of the permittivity and of the permeability of the ferrite material. An approximate expression can also be derived for the radiation resistance of very short antennas. From these results it is noted that ferrite loading gives a reduction of the axial wavelength to values as low as $1/\sqrt{\mu e}$ and an increase of the radiation resistance of short antennas with respect to that of metal-air monopoles of the same height as large as $\mu'$. However, the maximum permissible values of $\mu'$ and $\epsilon'$ for which the desired transverse magnetic mode may be sustained are limited; in the experience gained during the performance of this work it has been found that, using a ferrite with $\mu' = 10$ and $\epsilon' = 2000$ the above radiative mode could not be sustained at HF frequencies. Another consideration of importance is that of the ferrite losses; generally, the latter increase with the increase of the permeability, and, for this reason, they limit the advantage of the wavelength reduction obtained by ferrite loading.
7. EXPERIMENTAL VERIFICATION OF THE THEORY OF FERRITE-LOADED ANTENNAS

Experimental verifications of the theory of ferrite loaded antennas have been conducted on HF models, which have been resonated at frequencies in the range from 1 to 30 mc/s. An antenna testing ground has been utilized which consists of buried radial wires 75 meters long uniformly spaced at an angular separation of 3 degrees. This plane is considered good for tests down to about 1 mc/s.

Measurements of the input impedance and of the radiation efficiency have been made by using respectively an impedance bridge type C.R. 916A and an Empire Field Intensity Meter Type MF 105. The field intensity was measured at a distance of 800 feet, well within the far field range for all cases. The measurements of the efficiency were made with reference to a standard antenna consisting of a vertical monopole of height \( \lambda_n/4 \), sustained by means of a helium filled balloon. In this manner, errors due to uncertainty of the values of the loss resistances were eliminated.

Many measurements have been conducted with resonant ferrite loaded antennas; in this case it has been found that the radiation efficiency may be increased if folded type structures are used. The properties of these structures are well known; in particular if two monopoles of resonant height \( H \) are arranged in the configuration shown in fig. 4, the input impedance of the structure is approximately four times that of a single monopole, and the bandwidth corresponds to that of a single monopole of effective radius \( \sqrt{zd} \), where \( d \) is the separation of the wires and \( z \) is the radius of one of the conductors. In the case of a cage type structure, (fig. 5) similarly a radiative mode of operation exists, for which the input impedance is \( n^2 \) times that of a single monopole having the same height, if \( n \) is the number of members of the cage structure.

The experiments of resonance, radiation efficiency, bandwidth, etc. on ferrite loaded antennas have been conducted using various types of ferrite material; in Table 1 the electrical characteristics and the dimensions of the selected types are shown.

Measurements made on resonant single monopoles above perfect ground are summarized in Table 2 and in fig. 6. It is noted that in the case of type A ferrite loading the ratio \( H/\lambda_0 \) is 0.05 at 25.9 mc/s and 0.075 at 1.89 mc/s; the relative bandwidth is of the order of 3%; the input impedance is of the order of 12 ohms.

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and the radiation resistance is of the order of 5 ohms. Losses are produced within the ground plane and within the ferrite material.

The above experimental results may be evaluated in reference to the theoretical values obtained with the theory developed in the previous Section. For the case of ferrite of type A the values of the ratio \( \lambda_d / \lambda_0 \) have been computed in Table 3 and plotted in fig. 1 for the various cases. From the graphs of fig. 3 the theoretical results, corresponding to \( b/a = 1.67 \), have been derived and entered in the graph of fig. 7; it is seen that the check between experimental and theoretical values is satisfactory.

The variation of \( H/\lambda_0 \times d \) of the resonant height as a function of the frequency are also plotted in fig. 8 for the same type A of ferrite. These results indicate that the theory of the propagation of transverse magnetic modes of zero order in ferrite loaded antennas is well verified and that resonant ratio \( H/\lambda_0 \) of the order of 0.05 to 0.07 are obtained with ferrite structures of small dimensions.

The input resistance of resonant ferrite loaded antennas is less than that of resonant metal-air antennas; for example, a radiation resistance of the order of 5 ohms was obtained with monopoles of 8 or 16 feet height, and with values of \( a \) and \( b \) respectively \( a = 0.95 \) cm and \( b = 1.95 \) cm. Since there are inevitable losses due to the ferrite characteristics, the ground resistivity, etc. it is important that means of up-transforming the input impedance be utilized. One such possibility is obtained by recourse to folding techniques.

In the case of ferrite loaded structures two different types of folding may be used. (fig. 9) In one case the metal conductors may be folded within the ferrite envelope, and in the second case each ferrite loaded monopole separately can be folded, i.e. folding external to the ferrite is used. If the conductors are folded inside the ferrite, the dimensions of the latter cannot be made too large and the thickness of the ferrite is not utilized in full; to express this result in other words consider a ferrite cylinder of given ratio \( b/a \). The axial conductor may consist of a single wire of radius \( a \) or of a folded structure of equivalent radius \( a \). In either case the ferrite effect on wavelength shortening is of the same order of magnitude. The maximum total current which can be carried by the conductors is limited by the saturation of the ferrite material and is, say, \( I_{\text{sat}} \). If the conductors are
used in folding one has \( I_{\text{net}} = ni \), where \( I \) is the current in each conductor. The input resistance is \( R' = n^2 R \) where \( R \) is the resistance of a single monopole and corresponds to \( n = 1 \). The value of \( R \) may be considered approximately constant for a given ferrite geometry and for a resonant antenna height. There follows that, by folding \( n \) times, the input impedance is increased \( n^2 \) times and the input power is \( P_{\text{in}} = R' I^2 = R I^2 \). Since the increased value of resistance facilitates the realization of input matching, the recourse to folding is advantageous. There is a limit to the utilization of internal folding; for example, the mechanical resistance of the conductor is decreased and the construction becomes more complicated requiring the use of suitable isolators, etc. For this reason in general it is necessary to make recourse to external folding. This leads to structures with large effective radius and good mechanical resistance; however, the coupling between adjacent monopoles tends to decrease the resulting wavelength reduction ratio \( \lambda_e / \lambda_0 \) with respect to the value obtained with a single monopole. In order to illustrate this important result several experimental data have been taken. In Table 4 the data obtained with a resonant folded monopole of height 16 feet loaded with type A ferrite and consisting of two conductors of diameter 0.25", and axial separation 0.50" are shown. The resonant frequency is the same which applies for a single monopole, i.e. 4.29 mc/s, the input impedance is of the order of 50 ohms and the radiation efficiency is of the order of 55%. By comparing these data with the values shown in Table 2, the advantage of the folding operation becomes evident.

In Table 4 also examples of external folding are shown. The same monopoles of height 16 feet, loaded with type A ferrite were used. The axial separation was 0.5 feet, the resonant frequency was 4.69 mc/s, the input impedance was of the order of 50 ohms and the efficiency was of the order of 65%.

Multiple folding is illustrated in the data of Table 5. In this case, a number of identical ferrite loaded antennas was folded in a straight line. Strictly
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speaking this structure is not symmetrical and, for this reason, it does not support a completely radiative electromagnetic mode; as a result, with the increase of \( n \), the radiation efficiency reaches a maximum and then tends to decrease. The structure elements utilized in the experiments summarized in Table 5 were all four feet high and were placed at an axial separation of 6'. The \( h/a \) ratio was 5.85 and the wavelength reduction was of the order of 0.136; this low value of the ratio \( \lambda_4/\lambda_0 \) was obtained at the expense of the radiation efficiency which also was reduced; for a single monopole the efficiency was 4.9% and for a four-times folded structure it was 17%. The data of Table 5 are plotted in fig. 10.

If the same structural elements are connected in parallel rather than in folded mode the input impedance of the combination is found to decrease with the increase of \( n \); correspondingly, the radiation efficiency increases. This result is illustrated in Table 6 and fig. 11 which summarizes the experiments conducted on a number of 6 feet high monopoles, loaded with type A ferrite. The spacing was made again 6' and the number of elements was varied from one to four. All configurations were made resonant at 7.34 Mc/s in order to eliminate effects due to different frequencies of operation. The variation of the input impedance with the number \( n \) is due to the existence of a considerable loss component. While the radiation resistance remains unchanged when \( n \) is varied, the loss resistance is decreased.

An illustration of the operation in a folded mode is shown in Table 7 and fig. 12. Again 6' feet high monopoles with type A ferrite loading were used; axial spacing was 6' and frequency of operation was 7.34 Mc. The input impedance was found to vary with \( n^2 \) and the bandwidth and radiation efficiency were found to increase with \( n \).

Recapitulating, the previous data have shown that an improvement of the radiation efficiency may be obtained by parallel or by folded mode connection of the antennas. Since in either case the loss resistance of the ferrite material are decreased. On the other hand operation in the folded mode increases the input resistance, thus simplifying the problems of matching for most practical cases.

In the previous data it has been pointed out that operation of parallel...
monopole structures provides an interaction which worsens the effective wavelength reduction ratio. In order to investigate specifically this problem a series of measurements was made. In Table 8 and fig. 13 the results obtained using 4 foot high monopoles with ferrite A loading are shown. Two monopoles at distances varying between 6 feet and 1 foot were used; in each case the feeding was obtained by means of cables of fixed length, and with suitable base impedance match. It is seen that the wavelength ratio $\frac{\lambda_z}{\lambda_0}$ varies from 0.225 at 6 feet to 0.243 at one foot. Expressing this result in relative terms, the ratio $\frac{\lambda_z}{\lambda_0}$ is 0.225 at a relative axial distance of 0.062.

Another study of the proximity effect was made using three monopoles, rather than two and placing these either on a straight line or at the corners of an equilateral triangle. Again monopoles 4 feet high and with type A ferrite loading were used. The results are summarized in Table 9 and in fig. 14. The ratio $\frac{\lambda_z}{\lambda_0}$ varied from 0.228 at $d/\lambda_0 = 0.0425$ to 0.275 at $d/\lambda_0 = 0.004$. In the case of triangular arrangement the proximity effect was larger, for example at an axial separation $d/\lambda_0 = 0.016$ the ratio $\frac{\lambda_z}{\lambda_0}$ was 0.248 for straight line arrangement and 0.254 for the triangular arrangement.

In practical applications of ferrite loading to VLF antennas very short electrical structures must be used. In order to investigate the properties of electrically short antennas a number of experiments were conducted using short ferrite loaded monopoles which were resonated by means of top capacitive loading or by series inductive loading.

As an example, using 6" high monopoles with and without ferrite loading the following results were obtained. A straight arrangement of four monopoles with top capacitive load was resonated at 27 mc; the natural resonant frequency for 6" high monopoles in air is 500 mc/s. The top hat consisted of ten five feet long radials and the electrical height was $H/\lambda_0 = 0.01$. Adding ferrite loading of type A, the resonant frequency was lowered to 10.4 mc/s with a resultant electrical length $H/\lambda_0 = 0.0054$; the radiation efficiency was found to be 7%.

Further reduction of the resonant frequency was obtained by recourse to inductive loading.
Another example of very short antenna was obtained using two foot monopoles in folded structure with $n = 3$. Without ferrite loading and top loading the self resonant frequency is 125 mc/s; with type A ferrite loading and with top hat consisting of ten radials of length five feet, the resonant frequency was reduced to 6.16 mc/s and the ratio $H/\lambda$ was reduced to 0.012. The efficiency of this antenna was found to be of the order of 21% and the input impedance of the order of 28 ohms. Adding type A and type B ferrite loading the resonant frequency was further reduced to 5.05 mc/s and the electrical length was reduced to 0.010. However, the efficiency was also increased.

If top capacitive load and ferrite load are combined it is possible to obtain structures which have an inductive input impedance, thus requiring an input capacitance for tuning.

Another interesting observation is that top capacitive loading may be realized by means of direct capacitive connection to ground; in this case, a wire parallel to the radiator is used, which as an electrical moment opposite to that of the main dipole; however, since the electrical length of the ground return is much smaller than that of the main dipole, a net radiation is obtained.

One of the limitations of ferrite loading is the saturation effect of the magnetic permeability; because of this characteristic the maximum current and therefore the maximum power handling ability of the ferrite antenna is limited. Experimental verifications have been conducted to check quantitatively the importance of this characteristic.

If $H_{\text{sat}}$ is the value of magnetic field for which the ferrite material becomes saturated, the corresponding maximum antenna current is:

$$I_{\text{max}} = 2\pi a H_{\text{sat}}$$

(79)

where $a$ is the inner radius of the ferrite cylinder. The harmonic distortion which produced when the antenna current reaches the above value is also present in the radiated field, since the latter is proportional to the antenna current. Because
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of this phenomenon the maximum radiated power of a ferrite antenna is limited; however, it will be shown that these difficulties may be overcome at least in part by recourse to various design arrangements. One of these arrangements consists of the use of negative feedback: an error signal proportional to the distortion term may be obtained from the output and after suitable amplification and phase correction returned to the input feed point of the antenna. In certain cases it is possible to find a voltage waveform which produces a sinusoidal current waveform; since the radiated field is proportional to the antenna current rather than to the antenna voltage, the desired elimination of the distortion is achieved. Finally, the antenna power may be increased by recourse to multiple antenna structures in which each element is fed with a current less than that of saturation. Tests were conducted in order to determine experimentally the amount of the distortion components produced with large values of antenna currents. In addition, methods of compensation consisting of feeding the antenna with voltages at the harmonic frequencies were successfully devised.

A series of tests were performed using ferrite type C, in which the inner and outer radii are respectively:

\[ a = 0.218 \text{ cm}, \quad b = 1.27 \text{ cm} \]

The theoretical maximum values of magnetic field and of current are

\[ H_{\text{sat}} = 1550 \text{ A/m}, \quad I_{\text{max}} = 21.8 \text{ A} \]

A two foot conductor of radius 0.21 cm loaded uniformly with type C ferrite was used in the experiments. Passing currents of frequency 600 c/s and of intensities varying from 1 A to 12.5 A the corresponding distortion components were measured. These results are summarized in Table 10 and in fig. 15. It is seen that at 12.5 A the third harmonic component was 12.5 dB down (6%) and the fifth harmonic component was 17.5 dB down (1.8%). The even harmonic components were negligible.

Compensation of third or fifth harmonic distortion was achieved by simultaneously feeding a voltage at 1800 or 3500 cps, with proper amplitude and phase. The results are shown in Table 11 and fig. 16. It is seen that the...
The respective harmonic components were both decreased by more than 30 db.

In conclusion, in order to maintain the distortion in ferrite antennas below a prescribed limit, the antenna current must not exceed a critical value.

Further work was performed in studying the combination of inductive loading and ferrite loading. A helical structure consisting of a 16'' high monopole with 19 turns of diameter 1.25'' was constructed. The unit was loaded with type B ferrite and further provided with top capacitive loading consisting of ten five feet long radials.

The antenna was shown to resonate at 7.8 mc/s with an input impedance of the order of 1.5 ohms; its electrical length was $H/\lambda_o = 0.0105$ and its radiation efficiency was 21%.

Folding the above antenna inside the ferrite was accomplished using a second helix coaxial with the first one. In this case the resonant frequency was found to be 8.1 mc/s and the electrical length 0.011; the input impedance was of the order of 12 ohms and the radiation efficiency of the order of 23%.

Finally, tuning of the ferrite loaded antenna by means of short-circuits across the ferrite material was tested successfully. For this purpose, the inner conductor was provided with metal taps which were brought outside the ferrite sleeve. Shortcircuiting these taps removed the effect of the ferrite material and provided an increase of the resonant frequency. The results of this test are illustrated in fig. 17.

8. APPLICATION OF FERRITE LOADING TO THE DESIGN OF AN OMEGA ANTENNA SYSTEM

The results of the investigation developed in the previous sections have been utilized for a preliminary design of a 10.2 kc/s antenna system to be used in connection with the Omega Navigation System. The following design is tentative, because several data which have been assumed should be submitted to experimental verification. However, the design gives an overall idea of the practical possibilities of ferrite loading in connection with VLF antennas.
The ferrite material selected for this application has the following characteristics:

- Inner radius: \(a = 0.95\) cm
- Outer radius: \(b = 3.8\) cm
- Permeability: \(\mu = 100\)
- Permittivity: \(\varepsilon = 10\)
- Curie Point: \(T_c = 350^\circ C\)
- Loss Factor: \(2 \times 10^{-5}\)
- Density: \(4.5 \text{ g/cm}^3\)
- Saturation Flux: 3300 Gauss

The electrical design of the antenna is obtained using the equations derived in section 6. The frequency of operation of 10.2 kHz corresponds to a wavelength \(\lambda_0 = 29,400\) m. The ratios \(\lambda_o/b\) and \(b/a\) are respectively:

\[
\frac{\lambda_0}{b} = 7.9 \times 10^5 \quad \text{and} \quad \frac{b}{a} = 3.8
\]

From the graph of fig. 1 there follows that the wavelength reduction is

\[
\frac{\lambda_e}{\lambda_0} = 0.27
\]

i.e. \(\lambda_e = 7,940\) m, \(\lambda_{res} = 1980\) m. The actual antenna height is selected as

\[
H = 500^3 = 152^3
\]

This corresponds to a ratio \(H/\lambda_{res} = 0.077\). It is of interest to note that the values of similar ratios for the Forest-Fort and for the Cutler antennas are respectively:
Both are smaller than the values selected in the present design, although the corresponding actual heights of the antennas are respectively 1200' and 1000'.

The radiation resistance of this antenna is expected to be larger than that of a metal air radiator having the same height. In accordance with eq. 78 one has:

\[ R_{\text{rad}} = 0.146 \, \text{ohm} \]

In order to take into account other possible factors which reduce the radiation efficiency, a value of \( R = 0.10 \, \text{ohm} \) will be assumed. The actual input resistance will be larger because it will include the loss components.

Folding nine times the antenna gives an input impedance

\[ R_{\text{in}} = 8.1 \, \text{ohm} \]

exclusive of losses. These will depend upon the overall radiation efficiency; for example, if the efficiency is of the order of 10%, the input resistance will be of the order of 80 ohms and if the efficiency of the order of 50% the input impedance will be of the order of 16 ohms.

As far as the radiated power is concerned, this will be limited by the maximum antenna current allowable. Since the maximum saturation field is

\[ H_{\text{sat}} = 1580 \, \text{A/m} \]

the corresponding maximum value of antenna current is:

\[ I_{\text{max}} = 95 \, \text{A} \]
Assuming a safety factor of about 50%, a current of 50A may be considered as a practical maximum value. The corresponding antenna radiated power is of the order of:

\[ P_{\text{rad}} = 8.1 \times 2500 = 20.3 \text{ kW} \]

The distortion expected at this power level will be approximately of the order of 3%.

The relative bandwidth of the antenna system will depend upon the effective radius of the cage antenna structure; a value of 10^{-2} or better is expected.

An important consideration is that of the worsening of the wavelength reduction which is produced by the proximity effect. Assuming a separation of 200 m between the axes of contiguous antenna elements the relative separation is:

\[ d/\lambda_o = 0.006 \]

In accordance with the results of fig. 14 an increase of the wavelength reduction ratio by 20% is expected, i.e. the ratio \( \lambda_z/\lambda_o \) will be increased from 0.27 to 0.32. This will decrease the effective radiation resistance and the radiated power correspondingly. For example, assuming that the radiation resistance of a single monopole is 0.10 ohm, the modified value will be:

\[ 0.1 \times (0.27/0.32)^2 = 0.085 \text{ ohm} \]

and the corresponding radiative input resistance of the folded antenna will be 6.85 ohm. Finally, the radiated power for an antenna current of 50 A will be 17 kw. Of course, the proximity effect may be decreased by increasing the axial separation of the radiators; this may be done easily since the overall area covered by the antenna is rather small.
The capacitance of the top hat is a function of the ratio $H/H_{res}$ and of the dimensions of the radiator. Its value may be computed using the procedure of Schelkunoff\(^{(3)}\) for cylindrical antennas of radius $a$, height $H$, i.e.

$$Z_a = Z_0 \left( \frac{2}{a C_t} \right) = 60 \left[ \frac{1}{2} - \frac{1}{2} \cot \frac{H}{a} \right] +$$

$$Z_0 = 50 \left( \ln \frac{2nH}{C_t} - \frac{C_t}{2nH} - 1 - \cos \frac{2nH}{a} \right) \cot \frac{H}{a}.$$

In the above relation $Z_0$ is the average characteristic impedance of the antenna, $C_t$ is the top capacitance and $2nH \approx 2nH/a$. The value provided by the above relationship is approximate, and in practice a more accurate value should be obtained by experimental verifications. From the data of the Forest Port and the Cutler antennas, whose relative heights are smaller than the value assumed for the present design, an approximate value of top capacitance of the order of 0.1 uF should be expected. The top load capacitance may be realized by means of a horizontal system of wires or also by means of appropriate guy wires; it has been pointed out previously that the latter conductors have an electrical length which is smaller than that of the ferrite loaded antenna, so that, when said wires are returned to ground a net radiative moment is obtained.

In reference to the mechanical design of the antenna, two different approaches have been followed. These consist both of a system of nine steel towers arranged in correspondence of the vertices of an octagon; in one case the ferrite antennas are placed at the half-distance between the towers and in the second case they have been placed next to or within each tower.

The first arrangement is shown in fig. 18. In order to provide antennas of height 500 feet, the towers must be 660 feet high. Each antenna weights 7265 lbs. calculated as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ferrite material</td>
<td>6640 lbs</td>
</tr>
<tr>
<td>Steel rope 0.39 lb/ft</td>
<td>195</td>
</tr>
<tr>
<td>Steel and copper sleeves</td>
<td>50</td>
</tr>
<tr>
<td>Total</td>
<td>7265 lbs</td>
</tr>
</tbody>
</table>

The antenna is constructed with a steel rope of diameter 1/2"; the rope is surrounded with copper sleeves on which the ferrite cylinders are placed. The cylinders are supported in groups by means of suitable bushings which are pressed against the rope by a magnetoelectric process. This is realized simply by placing the rope and bushing within a coil through which a large surge of current is passed.

The selected rope is 6 x 37 special flexible hoisting type (monitor steel) having a breaking strength of 20,400 lbs. Therefore the safety factor is

$$\frac{20,400}{7285} = 2.8$$

The horizontal support is provided by another steel rope of diameter 5/8" having a breaking strength of 31,600 lbs. Simple pulley systems have been provided for the hoisting and support of the radiators.

The steel towers are base insulated and are supported by means of suitable guy wires. Structures suitable for this application are available on the market for the desired dimensions and load specifications.

The second type of design assumes that each ferrite antenna is supported by one tower. In this case the towers will be only 500 feet high and will be arranged as shown in fig. 19. The proximity of the insulated towers to the ferrite antenna may provide an interacting effect, but such effect may actually be equivalent to an increase of the top load capacitance. An experimental verification of the proximity effect in this case would be desirable.

Typical costs of the towers and of the ferrite material for the antenna system described are as follows:

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>660' heavy structure tower: steel</td>
<td>$17,500</td>
</tr>
<tr>
<td></td>
<td>erection work</td>
</tr>
<tr>
<td></td>
<td>base insulator</td>
</tr>
<tr>
<td></td>
<td>Total</td>
</tr>
</tbody>
</table>

38.

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### 660' Light Structure Tower:
- Steel: $12,500
- Erection Work: 5,500
- Base Insulator: 500
- Total: $18,500

### 500' Heavy Structure Tower:
- Steel: $13,000
- Erection Work: 5,500
- Base Insulator: 500
- Total: $19,000

### 500' Light Structure Tower:
- Steel: $9,000
- Erection Work: 5,500
- Base Insulator: 500
- Total: $15,000

The cost of the ferrite material is approximately $21,000 for a 500' radiator and is expected to be lower percentage-wise for larger quantities of ferrites. Assuming that the cost of the ferrites remains unchanged, the total ferrite cost for nine radiators would be of the order of 190,000 dollars. The cost of the ferrite material is of about the same order of magnitude as that of the steel towers. The above costs do not include consideration of the insulators, installation, connectors, ground plane or top hat.

### 9. CONCLUDING REMARKS

In this report the problem of the design of ferrite loaded antennas has been considered. In the past contradictory results have been published about the usefulness and feasibility of this type of design. The principal difficulty has been the absence of a practical generalized theory which might provide the possibility of design by modeling. Most results have been obtained either experimentally on specific radiators or theoretically on radiators embedded in spheres or other impractical geometries.

In this report a general theoretical analysis of cylindrical ferrite loaded antennas has been developed. The results of the analysis have been verified experimentally with numerous models in the RF range.
On the basis of these results, it is now possible to predict the wavelength reduction realizable with a given geometry and with a given ferrite type; the data required for the design are the ratio $b/a$ (outer radius to inner radius of the ferrite), and $\lambda_0/b$, as well as the values of $\mu$ and $\epsilon$ of the ferrite material. Graphs have been prepared for cases of practical interest.

The analysis has been applied to the design of a VLF antenna for a frequency of 10.2 kc/s. This type of design illustrates well the possibilities of the theory. One important advantage is that experimental results obtained at higher frequencies can be interpreted and applied to the design of VLF structures. The resulting design is, of course, tentative and should be considered as the center design of a more complete first realization, which would be improved with suitable experimental verifications.

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TABLE I

SUMMARY OF PROPERTIES OF FERRITE TOROIDS USED

<table>
<thead>
<tr>
<th>Property</th>
<th>Type A</th>
<th>Type B</th>
<th>Type C</th>
<th>Type D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_r )</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( \xi )</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Loss Factor = ( \frac{1}{\mu_0 \xi} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>at 1 Mc.</td>
<td>.00002</td>
<td>.00002</td>
<td>.00002</td>
<td>.00002</td>
</tr>
<tr>
<td>at 10 Mc.</td>
<td>.00016</td>
<td>.00016</td>
<td>.00016</td>
<td>.00016</td>
</tr>
<tr>
<td>Sat. Flux Density ( B_{max} ) (gauss)</td>
<td>3300</td>
<td>3300</td>
<td>3300</td>
<td>3300</td>
</tr>
<tr>
<td>Curie Point °C</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>350</td>
</tr>
<tr>
<td>Inner Radius &quot;a&quot; (cm)</td>
<td>0.95</td>
<td>1.75</td>
<td>0.216</td>
<td>0.316</td>
</tr>
<tr>
<td>Outer Radius &quot;b&quot; (cm)</td>
<td>1.59</td>
<td>2.38</td>
<td>1.27</td>
<td>7.62</td>
</tr>
</tbody>
</table>
### Table II

**Resonant Ferrite Monopoles**

<table>
<thead>
<tr>
<th>Height (ft.)</th>
<th>Ferrite Type</th>
<th>Reson. Freq. (Hz)</th>
<th>$\ell/\lambda_o$</th>
<th>$Z_{in}$ (Ohms)</th>
<th>Eff. (%)</th>
<th>BW (Kc)</th>
<th>BW (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>A</td>
<td>25.9</td>
<td>0.05</td>
<td>50</td>
<td>16</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>14.1</td>
<td>0.06</td>
<td>12</td>
<td>24</td>
<td>950</td>
<td>6.75</td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>7.34</td>
<td>0.06</td>
<td>12</td>
<td>47</td>
<td>250</td>
<td>3.3</td>
</tr>
<tr>
<td>16</td>
<td>A</td>
<td>3.72</td>
<td>0.06</td>
<td>12</td>
<td>37</td>
<td>100</td>
<td>2.7</td>
</tr>
<tr>
<td>40</td>
<td>A</td>
<td>1.89</td>
<td>0.075</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>8</td>
<td>C</td>
<td>8.50</td>
<td>0.034</td>
<td>28</td>
<td>4.6</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>11</td>
<td>A &amp; B</td>
<td>11.4</td>
<td>0.046</td>
<td>12</td>
<td>17</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
TABLE III

COMPARISON OF EXPERIMENTAL AND THEORETICAL $\lambda_e/\lambda_0$ VALUES

<table>
<thead>
<tr>
<th>Height (ft)</th>
<th>Freq. (Mc)</th>
<th>$\lambda_0^2$</th>
<th>$\lambda_e/\lambda_0$ (Exp.)</th>
<th>$\lambda_e/\lambda_0$ (Theor.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>25.9</td>
<td>11.6</td>
<td>1.24</td>
<td>0.26</td>
</tr>
<tr>
<td>4</td>
<td>14.1</td>
<td>21.2</td>
<td>1.35</td>
<td>0.24</td>
</tr>
<tr>
<td>5</td>
<td>7.5</td>
<td>40.0</td>
<td>2.28</td>
<td>0.24</td>
</tr>
<tr>
<td>16</td>
<td>4.3</td>
<td>70.0</td>
<td>4.40</td>
<td>0.26</td>
</tr>
<tr>
<td>24</td>
<td>3.0</td>
<td>100</td>
<td>6.30</td>
<td>0.27</td>
</tr>
<tr>
<td>32</td>
<td>2.3</td>
<td>130</td>
<td>8.17</td>
<td>0.32</td>
</tr>
<tr>
<td>40</td>
<td>1.9</td>
<td>156</td>
<td>9.90</td>
<td>0.36</td>
</tr>
</tbody>
</table>
## TABLE IV

### FOLDED FERRITE MONOPOLES

#### A. Folding inside Ferrite

<table>
<thead>
<tr>
<th>H (ft)</th>
<th>Ferrite Type</th>
<th>Freq. (Kc)</th>
<th>H/λ o</th>
<th>Z_in</th>
<th>Eff. %</th>
</tr>
</thead>
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<tr>
<td>16</td>
<td>A</td>
<td>4.29</td>
<td>.068</td>
<td>50</td>
<td>55</td>
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</table>

#### B. Folding outside Ferrite

<table>
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<th>Freq. (Kc)</th>
<th>H/λ o</th>
<th>Z_in</th>
<th>Eff.</th>
<th>BW (Kc)</th>
<th>d (ft)</th>
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<tr>
<td>6</td>
<td>A</td>
<td>7.34</td>
<td>.059</td>
<td>50</td>
<td>68</td>
<td>300</td>
<td>0.5</td>
</tr>
<tr>
<td>16</td>
<td>A</td>
<td>4.69</td>
<td>.075</td>
<td>50</td>
<td>65</td>
<td>300</td>
<td>0.5</td>
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<tr>
<td>4</td>
<td>C</td>
<td>8.5</td>
<td>.034</td>
<td>50</td>
<td>10</td>
<td>--</td>
<td>0.5</td>
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*d - separation between elements*
MULTIPLY CONNECTED ANTENNAS

In-Line Arrangement of Elements Using Type C Ferrite

<table>
<thead>
<tr>
<th>Number of elements n</th>
<th>d (in)</th>
<th>H (ft)</th>
<th>Freq. (Mc)</th>
<th>Z_in (Ohms)</th>
<th>Eff. (%)</th>
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</thead>
<tbody>
<tr>
<td>one element</td>
<td>1</td>
<td>4</td>
<td>8.5</td>
<td>.28</td>
<td>4.9</td>
</tr>
<tr>
<td>one driven one folded</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>8.5</td>
<td>.90</td>
</tr>
<tr>
<td>one driven two folded</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>8.5</td>
<td>.200</td>
</tr>
<tr>
<td>one driven three folded</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>8.5</td>
<td>.12</td>
</tr>
<tr>
<td>two driven two folded</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>8.5</td>
<td>.12</td>
</tr>
<tr>
<td>four in parallel</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>8.5</td>
<td>.12</td>
</tr>
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</table>

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TABLE VI

PARALLEL CONNECTED ANTENNAS

In-Line Arrangement of Elements Using Type A Ferrite

<table>
<thead>
<tr>
<th>n</th>
<th>H (ft)</th>
<th>Freq. (Mc)</th>
<th>ζ (°)</th>
<th>Eff. (%)</th>
<th>t/H (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>7.34</td>
<td>12</td>
<td>51</td>
<td>3.25</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>7.34</td>
<td>9</td>
<td>74</td>
<td>3.45</td>
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<tr>
<td>3</td>
<td>8</td>
<td>7.34</td>
<td>5</td>
<td>77</td>
<td>4.4</td>
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<tr>
<td>4</td>
<td>8</td>
<td>7.34</td>
<td>4</td>
<td>78</td>
<td>5.4</td>
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</tbody>
</table>
**MULTIPLY FOLDED ANTENNAS**

In-Line Arrangement of Elements Using Type A Ferrite

| 1  | 6  | 8  | 7.34 | ~12 | 51 | 3.25 |
| 2  | 6  | 8  | 7.34 | ~50 | 74 | 4.15 |
| 3  | 6  | 3  | 7.34 | ~90 | 68 | 5.5  |
TABLE VIII

PROXIMITY EFFECTS OF TWO ANTENNAS

Two 4 ft. Monopoles Using Type A Ferrite

<table>
<thead>
<tr>
<th>Frequency (Mc)</th>
<th>Separation (ft)</th>
<th>$Z_{11}$ ((\Omega))</th>
<th>Eff.</th>
<th>BW</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.1</td>
<td>--</td>
<td>-12</td>
<td>21</td>
<td>6.1</td>
</tr>
<tr>
<td>14.1</td>
<td>--</td>
<td>12</td>
<td>24</td>
<td>6.7</td>
</tr>
<tr>
<td>15.1</td>
<td>1</td>
<td>4.5</td>
<td>24</td>
<td>7.3</td>
</tr>
<tr>
<td>14.6</td>
<td>2</td>
<td>4.5</td>
<td>25</td>
<td>7.2</td>
</tr>
<tr>
<td>14.2</td>
<td>3</td>
<td>4.5</td>
<td>34</td>
<td>7.4</td>
</tr>
<tr>
<td>14.1</td>
<td>4</td>
<td>4.5</td>
<td>34</td>
<td>7.45</td>
</tr>
<tr>
<td>14.0</td>
<td>5</td>
<td>4.5</td>
<td>30</td>
<td>7.5</td>
</tr>
<tr>
<td>14.0</td>
<td>6</td>
<td>4.5</td>
<td>35</td>
<td>7.5</td>
</tr>
</tbody>
</table>


**TABLE IX**

**PROXIMITY EFFECTS OF THREE ANTENNAS**

Three 4 ft. Monopoles Using Type A Ferrite

Each Antenna Alone Resonated at 14.0 Mc.

<table>
<thead>
<tr>
<th>Separation (ft)</th>
<th>Resonant Freq. (Mc)</th>
<th>Resonant Freq. (Mc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>17.1</td>
<td>17.3</td>
</tr>
<tr>
<td>0.75</td>
<td>15.6</td>
<td>--</td>
</tr>
<tr>
<td>1.0</td>
<td>15.3</td>
<td>15.6</td>
</tr>
<tr>
<td>1.5</td>
<td>14.7</td>
<td>--</td>
</tr>
<tr>
<td>2.0</td>
<td>14.3</td>
<td>14.6</td>
</tr>
<tr>
<td>3.0</td>
<td>14.0</td>
<td>--</td>
</tr>
</tbody>
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### TABLE X

**HARMONIC DISTORTION IN FERRITE ANTENNA**

Harmonics Measured in dB below Fundamental

Type C Ferrite Used

\( f_0 = 600 \text{ Cps.} \)

<table>
<thead>
<tr>
<th>I (Amps)</th>
<th>3( f_0 ) (-db)</th>
<th>5( f_0 ) (-db)</th>
<th>7( f_0 ) (-db)</th>
<th>9( f_0 ) (-db)</th>
<th>11( f_0 ) (-db)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>--</td>
<td>21.0</td>
<td>28.0</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>5.0</td>
<td>37</td>
<td>19.5</td>
<td>23.0</td>
<td>27.0</td>
<td>32.0</td>
</tr>
<tr>
<td>6.0</td>
<td>24.5</td>
<td>19.5</td>
<td>23.0</td>
<td>24.0</td>
<td>30.0</td>
</tr>
<tr>
<td>7.0</td>
<td>20.5</td>
<td>19.0</td>
<td>23.0</td>
<td>24.0</td>
<td>29.0</td>
</tr>
<tr>
<td>8.0</td>
<td>16.0</td>
<td>16.5</td>
<td>23.5</td>
<td>24.0</td>
<td>27.0</td>
</tr>
<tr>
<td>9.0</td>
<td>14.5</td>
<td>17.0</td>
<td>25.0</td>
<td>25.0</td>
<td>30.0</td>
</tr>
<tr>
<td>10.0</td>
<td>14.5</td>
<td>19.0</td>
<td>27.5</td>
<td>27.5</td>
<td>30.5</td>
</tr>
<tr>
<td>12.5</td>
<td>12.5</td>
<td>17.5</td>
<td>26.0</td>
<td>27.0</td>
<td>32.5</td>
</tr>
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</table>
TABLE XI

HARMONIC COMPENSATION

Harmonic Reduced by at least 30 dB

<table>
<thead>
<tr>
<th>Antenna Current (Amps)</th>
<th>Fundamental Voltage</th>
<th>Harmonic Voltage</th>
<th>% Harmonic Injected</th>
</tr>
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<tbody>
<tr>
<td>Third Harmonic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compensation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>20</td>
<td>0.9</td>
<td>4.5</td>
</tr>
<tr>
<td>7.0</td>
<td>20</td>
<td>1.6</td>
<td>8.0</td>
</tr>
<tr>
<td>8.0</td>
<td>20</td>
<td>2.2</td>
<td>11.0</td>
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<tr>
<td>9.0</td>
<td>20</td>
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<td>13.5</td>
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<td>10.0</td>
<td>20</td>
<td>3.0</td>
<td>15.0</td>
</tr>
<tr>
<td>12.5</td>
<td>20</td>
<td>3.5</td>
<td>17.5</td>
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<tr>
<td>Fifth Harmonic</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Compensation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>22</td>
<td>1.85</td>
<td>8.4</td>
</tr>
<tr>
<td>7.0</td>
<td>22</td>
<td>1.85</td>
<td>8.4</td>
</tr>
<tr>
<td>8.0</td>
<td>22</td>
<td>1.85</td>
<td>8.4</td>
</tr>
<tr>
<td>9.0</td>
<td>22</td>
<td>1.90</td>
<td>8.6</td>
</tr>
<tr>
<td>10.0</td>
<td>22</td>
<td>1.90</td>
<td>8.6</td>
</tr>
<tr>
<td>12.5</td>
<td>22</td>
<td>1.95</td>
<td>8.9</td>
</tr>
</tbody>
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FIGURE 1
CYLINDRICAL COORDINATE SYSTEM

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Figure 2: Plots of Equations 71 and 72
Figure 3. Wavelength reduction for infinite ferrite loaded line.
FIGURE 4  FOLDED MONOPOLE

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FIGURE 5. CAGE ANTENNA

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Figure 6. Height vs. Resonant Frequency for Type A Ferrite Monopoles
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Figure 8
Heights and $H/\lambda_0$ versus frequency.
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A. FOLDING INSIDE FERRITE

B. FOLDING OUTSIDE FERRITE

FIGURE 9. FOLDED FERRITE MONOPOLES

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FIGURE 12. MULTIPLY FOLDED ANTENNAS

- Number of Elements $n$
- Efficiency (%) $E$
- Bandwidth $B$

$\times\text{ B.M.}$
$\triangle\text{ E.T.}$
$\circ\text{ LIF.}$
$\times\text{ L.N.}$

AX2352373

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Figure 13. Proximity Effect of Two Antennas
HARMONIC VOLTAGE ADDED TO REDUCE HARMONIC COMPONENT IN CURRENT BY 30 DB

![Diagram showing harmonic compensation](image)

**Figure 16. Harmonic Compensation**
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