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RHOMBIC ANTENNA THEORY
AS AN APPROACH TO DETERMINING THE R-F SUSCEPTIBILITY OF ELECTRO-EXPLOSIVE DEVICES

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MARCH 1962

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March 1962

ONS CODE NO. 4230.1.8839.701

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ACKNOWLEDGEMENTS

The author wishes to acknowledge the guidance given by his supervisor, Mr. Sidney Kravitz, and the assistance in programming and compiling the results of the computer by Pvt L. Schmele. Thanks are also given to Mr. Richard Aaron for his review of the work reported.
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ABSTRACT

This report presents a novel technique for determining the r-f susceptibility of electro-explosive devices. The technique is general enough to be of value in any case where the electro-explosive device may be considered to be part of a rhombic antenna circuit.

SUMMARY

An artificial index of the relative sensitivities of antennas, called the "Configuration Index", is defined. This quantity is easily calculable knowing the geometry of the antenna and the frequency of the impinging radiation. Using the Configuration Index, graphs may be drawn from empirical data of Configuration Index versus power density for a "no-fire" condition at any frequency. From these graphs the safe separation distance of electro-explosive devices from the r-f source may be determined for rhombic circuits.

CONCLUSIONS

The data obtained from the experiments outlined in this report will enable one to determine the safe separation distance of electro-explosive devices from r-f sources for the particular cases tested. If these cases fall into the pattern predicted in this study, then future tests will not
DISCUSSION

I. Problem:

We wish to determine whether or not a particular configuration of blasting caps is likely to explode when exposed to electromagnetic radiation from radars. "Configuration" is a term used to designate a particular arrangement of blasting caps and associated circuitry.

II. Assumptions:

It has been determined that blasting caps explode in r-f fields because the impinging radiation induces a current in the associated circuitry of the cap, which acts as an antenna, causing the bridgewire to heat, and initiate the explosive in the cap. From FM 5-25, "Explosives and Demolitions", it is noted that most configurations of blasting caps are used in series, parallel, or series - parallel circuits. These circuits
are, in general, rhomboidal in shape, or are composed of rhomboids. For this reason, it has been decided that profitable knowledge about blasting caps in r-f fields may be obtained from a study of the rhomboid antenna. In order to conduct this study, we must assume that blasting cap configurations are entirely analogous to rhombic antennas (i.e., have characteristic terminating impedances, sides of equal length, negligible self-coupling, etc.).

III. Overall Plan of Testing:

It is proposed that in order to evaluate the r-f hazard to blasting caps, the following plan be utilized:

1. Assign a theoretically calculable quantity to each possible antenna configuration. Such a quantity may be the gain, aperture, or any other measure of relative sensitivity of the antenna in question, providing that it be based on the geometry of the antenna and the frequency or wavelength of the radiation. For example, this quantity which will be called "Configuration Index" or "CI" will take L, H, φ, and λ or ν into account (see Figure 1). Let us assume that a suitable CI has been devised and that its magnitude increases with increasing sensitivity of antenna. (The question of how to set up the CI is discussed below).

*λ and ν are the wavelength and frequency (respectively) of the radiation transmitted or received. The other variables are defined on pages 7 and 8.
REPRESENTATION OF HORIZONTAL RHOMBIC ANTENNAS SHOWING PARAMETERS DISCUSSED IN TEXT.
2. Choose a suitable antenna taking into consideration the size and CI, and choose one frequency for the first test.

3. Immerse this antenna, with blasting caps in its circuit, in an r-f field of the chosen frequency. Determine the "no-fire" point with this configuration and measure the power density required to produce it. We now know, remembering that the magnitude of the CI increases with increasing sensitivity, that all configurations with a higher CI than the antenna under test should also fire with their probability of fire increasing with their CI. We also know that configurations of lower CI than the antenna under test should not fire.

4. Repeat the above procedure for a variety of CI's. The information thus accrued will enable us to then plot a graph of CI versus power density for "no-fire", at a particular frequency. Repeating the entire procedure for the various frequencies of interest will provide us with a family of curves which may in general look like Figure 2. It may be possible to derive an empirical equation for the CI versus power density curves at particular frequencies, and then find a relationship from these equations that will apply at all frequencies.

5. Having this family of curves and recalling that the CI may be calculated, knowing the geometrical configuration of the antenna, one can determine whether or not a particular configuration is hazardous. This will be done by finding the
SKETCH OF WHAT A FAMILY OF CONFIGURATION INDEX VERSUS POWER DENSITY CURVES MIGHT LOOK LIKE.
CI of the suspect antenna, measuring the power density at the location where the antenna will be used, and then examining the graph to see if the CI is greater or less than that value which produces a "no-fire" condition at the particular power density under consideration.

IV. **Criteria for Defining a CI:**

1. It must be conveniently calculable.
2. It must take the geometrical properties of the antenna, and the frequency of the radiation into account.
3. It must be a self-consistent indicator of the relative sensitivities of various antenna configurations.

V. **Definition of a CI:**

1. From Kraus (1), the expression for the relative field intensity of a radiating rhombic antenna, for a constant power input is:

   \[ E = \frac{\cos \phi \sin(Hr \sin \alpha) \sin(\mu L \lambda)}{\gamma} \]

   (See Figure 1)

   Where:
   \[ Hr = \frac{2\pi H}{\lambda} \quad Lr = \frac{2\pi L}{\lambda} \quad \mu = \frac{(1 - \sin \phi \cos \alpha)}{2} \]

   and \( \alpha \) = elevation angle at which an r-f beam impinges on or is transmitted from the antenna. It is measured with respect to the plane of the rhombic.

   \( H \) = height of the antenna above a perfectly conducting ground. (Bruce, Beck and Lowry(2) have shown that the real
physical ground very closely approximates a perfectly conducting
ground).

\[ L = \text{length of one side of the rhombic.} \]
\[ \phi = \text{half the included side angle of the antenna.} \]
\[ \lambda = \text{wavelength of the impinging (emitted) radiation.} \]

2. The power gain of an antenna is defined as (Kraus: (1) p. 26 footnote).
\[ G = \left( \frac{E}{E_0} \right)^2 \]

We will define CI as equal to \( E^2 \) for the reasons given below.

In the equation for \( G \), \( E \) is the maximum field intensity of
the antenna in question due to a specific power input, and \( E_0 \) is the maximum field intensity of an isotropic radiator with
that same power input. For our problem, \( E_0 \) is not conveniently
found, and is defined as 1. The justification for this is that
as the CI is perfectly arbitrary, it may be arbitrarily defined
as long as it satisfies the criteria set up for it.

The resultant equation is no longer an expression for gain,
but is still an indicator of the relative sensitivities of
various antennas. It must be borne in mind though, that a
scale based on \( E^2 \) is a relative one if and only if we restrict
our discussion to one particular frequency. If we choose
another frequency, the scale will have a different base level,
but will still maintain the same relationship among its elements.
Stating it in another way, the CI's of antennas operating at different frequencies may not be compared, even though they may be compared when operating at the same frequency. (one of the objects of this experiment will be to find a relationship among the various scales of CI's corresponding to different frequencies). The reason for making the above stipulation is that electro-explosive devices are thought to be more sensitive to some frequencies than to others.

It is seen then that CI = $E^2$ satisfies all the criteria for defining a CI.

VI. Discussion of CI:

1. At this stage of the investigation, it was possible to choose a suitable antenna for testing. Picking a few configurations at random, and finding their CI's, soon showed us that this was a naive approach because the CI's invariably were less than 1, while the nature of the function indicated that it could assume much larger values. Because of this knowledge it was decided to maximize the function by following the procedures in Kraus.

2. Accordingly, the function was first maximized with respect to $H$:

$$E = \cos \phi \left[ \sin(H - \sin \lambda) \right] \left[ \sin(\varphi' \lambda) \right]^2$$

$$\frac{\partial E}{\partial H} = \left[ \sin(\varphi' \lambda) \right]^2 \cos \phi \left[ \cos(H - \sin \lambda) \sin \lambda \right] \frac{2 \pi}{\varphi' \lambda}$$
Setting the derivative to zero;
\[
\cos(H_r \sin \phi) = 0
\]
\[
H_r \sin \phi = \frac{N \pi}{2}
\]
Where, by finding the second derivative, \( N = 1, 5, 9, 13 \ldots \)
but: \( H_r = \frac{2 \pi N}{\lambda} \)
therefore: \( H = \frac{N \lambda}{4 \sin \phi} \)

3. The function was then maximized with respect to \( L \):
\[
E = \cos(\sin(H_r \sin \phi)) \frac{1}{\sin(\psi L \phi)}
\]
\[
\frac{\partial E}{\partial L} = \cos(\sin(H_r \sin \phi)) \frac{2 \sin(\psi L \phi) \cos(\psi L \phi) 2 \pi}{\lambda}
\]
Setting the derivative equal to zero, \( \sin(\psi L \phi) \cos(\psi L \phi) = 0 \)
\[
\psi L \phi = \frac{M \pi}{2}
\]
(and by finding the second derivative it was found that the function is a maximum only if \( M \) is an odd integer).
but: \( \psi = \frac{(L \phi \cos \phi)}{2} \)
\[
L \phi = \frac{2 \pi L}{\lambda}
\]
Finally:
\[
L = \frac{M \lambda}{2 (1 - \sin \phi \cos \phi)}
\]

4. Lastly, the function was maximised with respect to \( \phi \).
The differentiation is simplified by first substituting the condition on \( L \), as found above, into the function. This is a permissible operation when seeking a maximum condition.
\[ E = \frac{\cos \phi \sin (\lambda - \sin \phi) \sin (\psi L \phi)}{\lambda} \]

Let \( 2 \sin (H \sin \phi) = K \)
and, since \( L = \frac{2 \pi L}{\lambda} \), where \( L = \frac{M \lambda}{2 (1 - \sin \phi \cos \phi)} \)
\( \psi L \phi = \frac{2 \pi M}{\lambda} \) remembering that
Then \( E = \frac{K \cos \phi (\sin \frac{2 \pi M}{\lambda})^2}{1 - \sin \phi \cos \phi} \)
\[ \psi = \frac{2 (1 - \sin \phi \cos \phi)}{1 - \sin \phi \cos \phi} \]

Setting the derivative equal to zero; \( \sin \phi = \cos \phi \)

which occurs for \( \phi = 90^\circ - \alpha \)

5. We have maximized the function with respect to
several of its variables separately. To maximize the function
with respect to all of the variables, we must simultaneously
apply each of the independent maximizing conditions on the
function. Summarizing the conditions on the variables:
\[ \phi = 90^\circ - \alpha \]
\[ H = \frac{N \lambda}{4 \sin \phi} \]
\[ L = \frac{M \lambda}{2 (1 - \sin \phi \cos \phi)} \]
From which is obtained:

\[ \psi = \frac{\sin^{2} \alpha}{2} \]

\[ L = \frac{M \alpha}{2 \sin^{2} \alpha} \]

\[ Hr = \frac{\pi N}{2 \sin \alpha} \]

substituting these expressions into the equation for \( E \):

\[ E_{\text{max}} = \frac{2 \sin \frac{\pi N}{2} \left[ \sin \frac{\pi M}{2} \right]^{2}}{\sin \alpha} \]

For the case when \( N = 1 \) and \( M = 1 \), \( E_{\text{max}} = 2 \csc \alpha \), and \( CI_{\text{max}} = 4 \csc \alpha \). This means that for any frequency, a graph of \( CI_{\text{max}} \) versus \( \alpha \) will look like Figure 3. Figure 3 clearly shows that theoretically there is no unique "maximum of all maximums".

For an expanded graph of \( CI_{\text{max}} \) versus \( \alpha \) see Figures 4A, 4B, 4C and 4D. Each of these graphs show different sections (with different scales) of Figure 3. The heights and lengths used to obtain any point on the graph are shown as well as a scale representing the angle \( 2\phi \). This graph effectively indicates the most dangerous configurations for the horizontal rhombic that are possible. Although the information displayed is predicated on a frequency of 1000 Mc, it is simple to gain information for any other frequency.
This graph is drawn for a 1000 MHz at any other frequency, for any CI MAX, 
alpha and phi will remain the same, while the new 
height and length may be gotten by dividing the heights 
and lengths shown, by the 
new frequency in kilo-mesacycles.

Optimum design parameters for the horizontal phonicc antenna.

Figure 3

Page 15
For a different frequency, at any point, the CI max, angle $2\theta$, and $\lambda$ will be the same. At 1000 Mc the length, equals the CI max multiplied by .1230, and the height equals the square root of CI max multiplied by .1230. To get the height and length at any frequency other than 1000 Mc, take the height and length at 1000 Mc, and divide it by the new frequency in kilo-megacycles, the result will be in feet.

Although the graph indicates that it is possible for CI max to approach infinity as $\lambda$ approaches zero, this is not truly the case; for as $\lambda$ approaches zero, $L$ gets larger and larger, and the radiation resistance of the antenna also increases. This increase of radiation resistance is not completely negligible at larger $\lambda$'s, and becomes more and more prominent as $\lambda$ gets smaller. This effect has not been thoroughly studied, but it is thought that even if carefully accounted for it will not cause the CI curve to have a unique peak. For further information on the radiation resistance of rhombic antennas consult Jasik(3).

At this time, it should be noted that the effect on test results of the inclusion of a terminating impedance in the blasting cap circuit for test purposes is open to speculation. If the impedance is included, the circuit more nearly realizes the conditions for which the equations were originally derived.
Without the impedance the circuit more nearly realizes the field conditions under which the caps will be used; but omitting the impedance in the circuit might cause the antenna to act more like a loop antenna than a rhombic. As a result of experimentation along the lines outlined in this report, it may prove necessary to analyze blasting cap configurations as analogous to loop antennas.

6. In order to obtain a suitable range of CI's for testing purposes (see III, 4 above), a convenient means of changing the CI of a particular antenna had to be found. A rectangular metal plate whose dimensions are at least twice the rhomboid's major axis by twice the rhomboid's minor axis, is an effective perfectly conducting artificial ground. By mounting the antenna above and parallel to this plane, and keeping the angle of the impinging radiation constant, but changing the angle that the metal plane makes with the horizontal, we can in effect change the angle $\alpha$. If the dimensions of the antenna are held constant while $\alpha$ changes, the CI will change as well.

Choosing particular antennas, known to give high CI's from Figures 4A, 4B, 4C and 4D, and using the IBM 709 Computer, plots were made of CI versus $\alpha$ for many different configurations. An example is shown in Figure 5. For this particular antenna
CONFIGURATION INDEX
VERSUS ALPHA FOR
A TYPICAL ANTENNA

$2\phi = 138.4^\circ$
$L = 3.0020 \text{ ft.}$
$H = 5330 \text{ ft.}$
$V = 1300 \text{ Mc}$

$36.6964 \text{ CI}_{\text{MAX}}$

$31.7206 \text{ PREDICTED CI}_{\text{MAX}}$
FOR HORIZONTAL ANTENNA

CONFIGURATION INDEX

ALPHA

FIGURE 5
DATA FROM FIGURE 5 PLOTTED IN POLAR COORDINATES. THIS IS A REPRESENTATION OF THE ACTUAL RECEIVING (TRANSMITTING) PATTERN OF THE ANTENNA EXCLUDING THE MINOR LOBES.

$2 \phi = 138.4^\circ$
$L = 3.0020 \text{ FT.}$
$H = 5330 \text{ FT.}$
$\nu = 1300 \text{ Mc}$

$20.8^\circ$ PREDICTED WAVE ANGLE FOR CÎMAX OF 31.7205 FOR A HORIZONTAL ANTENNA

$17.5^\circ$ FOR A CÎMAX OF 36.6954

FIGURE 6
the calculated CI max was 31.7205. It may be seen, however, that the graph does not peak at this value. The reason for this is that the equations used are for an antenna always parallel to the ground, for which the CI cannot exceed 31.7205 for the configuration chosen. Even if the antenna were at an angle with the ground, the CI could be no larger. However, if the antenna is held parallel to the artificial ground described above, and both are held at an angle to the horizontal, it is believed that the greater CI of 36.6954 can be achieved.* Beyond the range of the graph shown, there are many minor peaks out to $\alpha = 90^\circ$. If the data is plotted in polar coordinates, the graph will be an actual representation of what the transmitting (receiving) pattern of the antenna is like.

Figure 6 is a plot of Figure 5, in polar coordinates, where the antenna lies in the X-Z plane pointing towards the right. At $\alpha = 20.8^\circ$, the CI assumes the value of 31.7205. This is the maximum value that the CI can assume if the antenna is held parallel to the ground as in Figure 7a. Tilting the antenna with respect to the ground will keep the reflected wave from reinforcing the incident wave, and the CI will be less than 31.7205 (see Figure 7b). Using the

*This last statement requires both theoretical justification and experimental verification.
FIGURE 7A
REFLECTED WAVE REINFORCES INCIDENT WAVE AND MAXIMUM POSSIBLE CI IS 31.7208

FIGURE 7B
SHAPE OF LOBE IS NOT KNOWN REFLECTED WAVE DOESN'T REINFORCE INCIDENT WAVE AND CI = 31.7208

FIGURE 7C
THIS SETUP MAY ALLOW ACI OF 36.0964 TO BE REALIZED

$\phi = 138.4^\circ$
$L = 5.0020 \text{ FT.}$
$H = .5335 \text{ FT.}$
$V = 1300 \text{ MC}$

Figure 7
artificial ground though, and tilting it with respect to the physical ground should move the main lobe of the antenna to its most favorable position of $17.5^\circ$, with respect to the incident wave, resulting in a CI of 36.6954, Figure 7c. (See footnote on preceding page).

In general, on the basis of all the graphs plotted as above, the peaks of such graphs become narrower and higher for configurations used at higher frequencies, and appear closer to $\alpha = 0^\circ$. At higher frequencies then, the antennas are very sharply tuned. For example, at 20,000 Mc, where $2\theta = 172^\circ$, $L = 5.0570$ ft., and $H = .1764$ ft., the CI goes from 0 to 952.2090 as $\alpha$ goes from $0^\circ$ to $3.4^\circ$. This might explain in part why firings of blasting caps are less common at the higher frequencies.

VII. Comments:

1. It is recommended that a testing program be initiated to verify the results of this study. For such a program a suitable size antenna may be chosen by reference to Figure 4. Having chosen this particular antenna a graph such as Figure 5 may be plotted from which particular points of interest may be chosen for testing. The "no-fire" points should then be found for these configurations, as well as the power density required to produce them. Graphs similar to Figure 2 may then be plotted.
2. From this study, antennas can be designed which will have CI's of zero, or which are immune to r-f pickup. This point may prove worthy of future investigation, for by choosing lengths and heights which are not permissible multiples (see the maximizing criteria on L and H paragraph VI, 2 & 3), the CI becomes zero, producing antennas which are safe to use in r-f fields.

3. It is believed that following the recommended testing plan will produce valuable information about the behavior of blasting caps, or other electro-explosive devices, in rhomboidal configurations. In addition, whether successful or not, these tests will provide valuable information about how to conduct future investigations, and will also affirm or disaffirm the hypothesis that blasting cap configurations act like rhombic antennas. If the tests do not prove successful, then future analyses will have to look into the more complicated aspect of working with the real gain of the antenna, gotten from taking the radiation resistance and terminal impedance into account, and if this does not prove successful, then blasting cap configurations should be analyzed as being analogous to loop antennas.
REFERENCES


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