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SEAWATER DENSITY IN THE OCEAN AS A FUNCTION OF DEPTH, AND A METHOD FOR UTILIZING THIS INFORMATION IN DESIGN OF PRESSURE VESSELS WHICH WILL REMAIN IN A CONSTANT DEPTH RANGE BETWEEN THE SURFACE AND THE BOTTOM.

8 DECEMBER 1961

UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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Prepared by:
William F. Warren

ABSTRACT: Curves of maximum and minimum seawater density are given for the North Atlantic and for the World Ocean (Atlantic, Indian, and Pacific). These curves show that a sub-surface free floating pressure vessel is possible for all depths below 300 fathoms. If an expendable weight is used to sink the vessel to its design depth, then it might be possible to plant as close as 200 fathoms below the surface. The extent of vertical migration can be predicted from the curves, being on the order of several hundred fathoms for a reasonably stiff pressure vessel near the surface. For deeper depths, this migration zone decreases until it is something less than fifty fathoms for 3,000 fathoms depth and below. For a particular vessel design, the migration zone can be made smaller by increasing stiffness of the vessel; i.e., by increasing wall thickness or by adding ribs to the vessel. If a more limited area of the ocean is to be considered, then a minimum planting depth even less than that stated above should be possible, using the methods as described in this report.
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This report presents data and general procedures which are intended for design planning purposes and for preliminary design work on ordnance projects. The basic idea which generated this work; i.e., the idea of a neutrally buoyant weapon, was first proposed to the author by Mr. L. C. Fisher and Mr. C. C. Vogt of the Naval Ordnance Laboratory. Where opinions are expressed they represent the opinion of the author and of the Underwater Mechanical Engineering Department of the Naval Ordnance Laboratory. The work was done on Bureau of Weapons Task Number RUME 2E-000 PA#031.

W. D. COLEMAN
Captain, USN
Commander

L. C. FISHER
By direction
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1 Extreme Values of Temperature and Salinity at .... 5 Various Depths in the Atlantic, Pacific and Indian Oceans

REFERENCES

(a) "Properties of Water Substance" by Dorsey
(b) U. S. Coast & Geodetic Survey Special Publication No. 108 by N. H. Heck and J. H. Service
(c) "The Oceans" by Sverdrup, Johnson and Fleming
(d) "Die Zusammendrueckbarkeit des Meerwassers" by Ekman
SEAWATER DENSITY AS A FUNCTION OF DEPTH

1. Presented here are curves which show the maximum and minimum values of seawater density, expressed as specific volume, which are to be expected over the whole ocean area and from the surface down to the bottom; i.e., everywhere in the entire volume of ocean water. (See Appendix A for methods used in determining these density values). Each of the three sheets, figures 1, 2 and 3, therefore, must contain two curves, a maximum and a minimum; and all possible density values in the ocean must lie between the two curves. So, if we choose at random any longitude, latitude, water depth, and time, and then measure the density of the seawater there, we can be confident that this measured value will be somewhere between the maximum and minimum values for that depth as shown on the curves. The degree of our confidence need depend only on the temperature-salinity data which was provided by the Navy Hydrographic Office (Table 1) and on the Heck and Service experimental data as contained in reference (b).

A METHOD FOR ORDNANCE DESIGN

2. If it is desired to place a piece of ordnance at some depth in the ocean and have it remain there we might proceed as follows:

   a. Design and build a suitable pressure vessel which can withstand the expected pressures and maintain approximately neutral buoyancy.

   b. Measure the over-all density of the assembled unit over the expected pressure range; i.e., put it in a pressure chamber and measure the net buoyancy for the various pressures. Plot curve of net buoyancy versus pressure.

   c. Add trim weights to the unit so as to match the value from the maximum density curve (curve on the left) for the upper design depth.

3. If these things are done then the unit can be dropped anywhere in the ocean and it will sink below the "design depth" never to rise above it again. The deepest possible depth which it can then reach will be defined by the minimum density curve (curve on the right) where it intersects with the buoyancy-pressure curve, as determined above, for the particular unit involved. So the unit will remain always within a specific depth zone. It can, of course, move up and down in the zone.
as time changes and as the unit drifts into new areas (i.e., new temperature and salinity conditions) but it can never sink out of the zone or rise above it. The vertical dimension of this depth zone will be less for a stiff pressure vessel such as a thick walled sphere, than for a more compressible shape, such as a thin unsupported cylinder.

4. However, almost any vessel should work (i.e., always remain below the "design depth") provided it can withstand the hydrostatic pressures involved. Of course the minimum design depth and the zone of migration will both depend on the stiffness; the greater the stiffness of a vessel, the closer it can operate to the surface and the smaller will be its zone of migration. (Note: These facts might help to explain why oceanographic floats, which are long thin unsupported cylinders, have not been successful in depths shallower than 2000 meters).

EXAMPLE OF THE METHOD FOR AN ALUMINUM SPHERE

5. The method previously described will now be used to examine the performance of an aluminum sphere (any diameter, but wall thickness to be 1/10 of the mean spherical radius) as a pressure vessel for a constant depth range item of ordnance. Of course, this is just one configuration picked more or less at random to serve as an example; other configurations could be expected to perform better or worse depending on their relative compressibilities.

6. For this simple shape the compressibility can be accurately computed with specific volumes, V, as a cubic function of the depth, d, as follows:

\[ V = V_0 \left(1 - Ad + Bd^2 - Cd^3\right) \]

For this example the values of B and C were found to be negligibly small for all values of d from 0 to 35,000 feet, so V became a straight-line function of d for this case. This function produced a family of curves with \( V_0 \) as the parameter, where \( V_0 \) is the specific volume of the sphere at zero pressure. The seawater density curves have been modified by adding these "compressibility curves" (in this case a family of parallel straight lines) to show the depth ranges which are possible for each "design depth" as shown in figures 4, 5, and 6. It is believed that this general approach should be applicable to a wide variety of shapes, but only the simplest shapes can be computed; most shapes being experimentally measured as explained in paragraph 2 above.
COMMENTS FOR FUTURE WORK

7. If a weapon design can be restricted to a limited area of the ocean such as the upper latitudes of the Atlantic (above 45 degrees North), or perhaps the equatorial zone of the World Ocean (between 15 degrees North and 15 degrees South), a shallower minimum planting depth should be possible and the range of migration should also be less.

8. This should also hold true for separate areas such as the Arctic Ocean and the Baltic Sea, where even more uniform densities might be expected.

9. When a new project is to be started, the planting area should first be established, then a table for this area, similar to Table 1 of this report, should be obtained from the National Oceanographic Data Center, Washington 25, D. C. A set of curves could then be constructed and the project could proceed on the basis of these curves as explained above.
From Dorsey, reference (a), we have the formula:

\[ D_{s,t,d} = D_{35,0,d} - (6.48 + 0.00375d) t - 0.46t^2 \]
\[ + (f - 0.283t - 0.005t^2)(s-35) \]

where

- \( D_{s,t,d} \) = specific volume in \( \text{cm}^3/\text{gm} \)
- \( D_{35,0,d} \) = \( D_{35,0,d} \) at \( t = 0^\circ C \) and \( s = 35 \text{ gm/lkm} \)
- \( f \) = tabulated function of depth (in reference (a))
- \( s \) = salinity - total salts per kg of seawater
- \( t \) = temperature in degrees Centigrade
- \( d \) = depth below surface in meters

It is assumed that the maximum possible density at each depth will correspond to the minimum temperature and maximum salinity as given in table I of this report. Likewise, the minimum density is assumed to correspond with the maximum temperature and minimum salinity from table I. Maximum and minimum densities were obtained in this way for the various depths. This information was then used to produce the curves.

See also table I of reference (b) where this same information is available in tabular form. This table is usually more convenient than the above formula for ordinary computations. It was used for all computations in this report. The formula is included for future reference on work which might involve a digital computer, where it should prove to be more easily used than the tabular data.

The formula above has been verified as usually correct to within two units in the fifth significant digit with Heck and Service experimental data as contained in reference (b).
### TABLE I

Extreme values for temperature and salinity at various depths in the Atlantic, Indian, and Pacific Oceans as furnished by the Navy Hydrographic Office

<table>
<thead>
<tr>
<th>Depth (meters)</th>
<th>Temperature (°C)</th>
<th>Salinity (gm/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max.</td>
<td>Min.</td>
</tr>
<tr>
<td>North Atlantic (equator to Greenland)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>* surface</td>
<td>28.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>200</td>
<td>27.5</td>
<td>1.0</td>
</tr>
<tr>
<td>400</td>
<td>18.5</td>
<td>2.0</td>
</tr>
<tr>
<td>600</td>
<td>18.0</td>
<td>2.0</td>
</tr>
<tr>
<td>800</td>
<td>14.0</td>
<td>2.0</td>
</tr>
<tr>
<td>1000</td>
<td>11.5</td>
<td>2.0</td>
</tr>
<tr>
<td>1250</td>
<td>11.0</td>
<td>2.0</td>
</tr>
<tr>
<td>1500</td>
<td>9.5</td>
<td>3.0</td>
</tr>
<tr>
<td>2000</td>
<td>5.2</td>
<td>3.0</td>
</tr>
<tr>
<td>3000</td>
<td>3.9</td>
<td>2.5</td>
</tr>
<tr>
<td>5000</td>
<td>2.45</td>
<td>2.4</td>
</tr>
<tr>
<td>* 8500</td>
<td>3.0</td>
<td>2.9</td>
</tr>
<tr>
<td>beyond 5000</td>
<td>Temperature increases slightly with depth to bottom</td>
<td></td>
</tr>
</tbody>
</table>

South Atlantic (equator to Antarctica)

<table>
<thead>
<tr>
<th></th>
<th>Temperature (°C)</th>
<th>Salinity (gm/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max.</td>
<td>Min.</td>
</tr>
<tr>
<td>200</td>
<td>21.0</td>
<td>-1.2</td>
</tr>
<tr>
<td>400</td>
<td>15.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>600</td>
<td>11.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>800</td>
<td>6.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1000</td>
<td>4.5</td>
<td>0.0</td>
</tr>
<tr>
<td>1250</td>
<td>4.5</td>
<td>0.0</td>
</tr>
<tr>
<td>1500</td>
<td>4.5</td>
<td>0.0</td>
</tr>
<tr>
<td>2000</td>
<td>4.0</td>
<td>-0.25</td>
</tr>
<tr>
<td>5000</td>
<td>2.5</td>
<td>0.0</td>
</tr>
</tbody>
</table>

*These values were estimated from data in reference (c).*
### TABLE I (cont.)

<table>
<thead>
<tr>
<th>Depth (meters)</th>
<th>Temperature (°C)</th>
<th>Salinity (gm/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max.</td>
<td>Min.</td>
</tr>
<tr>
<td>Pacific</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>5.5</td>
<td>0.6</td>
</tr>
<tr>
<td>2000</td>
<td>3.5</td>
<td>0.6</td>
</tr>
<tr>
<td>3000</td>
<td>2.1</td>
<td>0.6</td>
</tr>
<tr>
<td>4000</td>
<td>2.1</td>
<td>0.6</td>
</tr>
<tr>
<td>5000</td>
<td>1.8</td>
<td>0.6</td>
</tr>
<tr>
<td>6000</td>
<td>1.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Beyond 6000  
Salinity extremes remain constant (no change) from 6000 meters to the bottom.

Indian Ocean

<table>
<thead>
<tr>
<th>Depth (meters)</th>
<th>Temperature (°C)</th>
<th>Salinity (gm/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max.</td>
<td>Min.</td>
</tr>
<tr>
<td>1000</td>
<td>8.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>2000</td>
<td>4.5</td>
<td>-1.0</td>
</tr>
<tr>
<td>3000</td>
<td>2.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>5000</td>
<td>1.2</td>
<td>-1.0</td>
</tr>
</tbody>
</table>
FIG. 3 SEAWATER DENSITY CURVES FOR WORLD OCEAN (ATLANTIC, INDIAN, PACIFIC)

SPECIFIC VOLUME (CM$^3$/GM X 10$^3$) vs. DEPTH (FATHOMS)
PARALLEL SLANT LINES REPRESENT THE SPECIFIC VOLUME VERSUS DEPTH FOR AN ALUMINUM ALLOY SPHERE WITH WALL THICKNESS = 1/10 MEAN RADIUS.

NUMBERS REPRESENT THE "PRESET" SPECIFIC VOLUME AT ZERO PRESSURE.

FIG. 4 - COMPRESSIBILITY CURVES FOR ALUMINUM ALLOY SPHERE IN NORTH ATLANTIC OCEAN (EQUATOR TO GREENLAND) IN FATHOMS.

SPECIFIC VOLUME (CM$^3$/GM X 10$^3$)

DEPTH (FATHOMS)
FIG. 5 COMPRESSIBILITY CURVES FOR ALUMINUM ALLOY SPHERE IN NORTH ATLANTIC OCEAN (EQUATOR TO GREENLAND) IN FEET

SPECIFIC VOLUME (cm$^3$/gm x 10$^3$)

DEPTH (FEET)

NOTE:
PARALLEL SLANT LINES REPRESENT THE SPECIFIC VOLUME VERSUS DEPTH FOR AN ALUMINUM ALLOY SPHERE (WITH WALL THICKNESS = 1/10 MEAN RADIUS)

NUMBERS REPRESENT THE "PRESET" SPECIFIC VOLUME: IE THE SPECIFIC VOLUME AT ZERO PRESSURE
Fig. 6: Compressibility curves for aluminum alloy sphere in world ocean (Atlantic, Indian, Pacific) in fathoms

Note: Parallel slant lines represent the specific volume versus depth for an aluminum alloy sphere with wall thickness = 1.0 mean specific volume at zero pressure.
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2. Oceans -
   Depth
3. Pressure
   vessels -
   Design
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