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"MICROWAVE DETERMINATION OF AFTERGLOW TEMPERATURES
AND ELECTRON COLLISION FREQUENCIES
IN NITROGEN AND OXYGEN"

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ABSTRACT

The results of microwave attenuation, phase shift and radiation temperature measurements in the late after-gloows of pulsed oxygen discharges of various types are discussed from the point of view of the collision frequency and G-factor determination.

A new general theory for the evaluation of the G-factor from the radiation temperature increases under microwave heating conditions is reported. The values computed with this theory in nitrogen are in very good agreement with the previous diffusion measurements by Crompton and Sutton, whereas in oxygen the agreement with the measurements by Healey and Kirkpatrick is much less satisfactory.

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MEASUREMENTS IN OXYGEN

The first step in the research program for the year May 1, 1961 - April 30, 1961 has been to perform a set of measurements in oxygen quite similar to the measurements previously done in nitrogen.

This set consisted in measuring the attenuation increase and the phase shift of a microwave signal at 9650 Mc/s and the microwave radiation temperatures due to an oxygen afterglow plasma. The set-up is described in details in the Technical (Scientific) Note N°1 (see also ref.2). Here we recall only that the plasma is generated in a cylindrical container, one meter long and with an internal diameter of 16 mm, placed in a circular waveguide with a 20 mm diameter. The preparation takes place in the dominant TE_{11} mode.

The measurements are performed at different pressures around few mm Hg, discharge currents and afterglow times. For the first measurements, like in nitrogen, we have used the positive plasma column of a conventional glow d.c. discharge, generated across two electrodes placed outside the waveguide, at the two ends of the container.

The results have been similar to those found in nitrogen, namely the electron temperatures are much higher than room temperature, being around 2-3 thousands degrees at post-discharge times between 50 and 200 μsec. If the experimental conditions are changed over the pressure, current and afterglow times ranges, at which the measurements may be performed with sufficient accuracy, only small variations in the radiation temperature are obtained. The resulting temperature range is then too small for a deter-
mination of the electron collision probability in oxygen.

To increase this range we have first tried to heat the plasma with two high-power microwave signals at 9000 Mo/s and 9200 Mo/s respectively, according to the scheme reported in the Technical Summary Report N° 1. With this experiment we hoped to determine also the fractional energy loss per collision (G-factor) from the measured temperature increase due to the applied microwave heating power, as it was previously accomplished in nitrogen.

However, no appreciable change in the afterglow temperatures was observed in our oxygen experiment, also when the heating klystron power was increased up to 2 W. This indicates a very large G-factor, which is in agreement with Healey and Kirkpatrick, who report a value 60 times larger than the elastic collision value.

Considering these results, the problem of decreasing the electron temperature down to the thermal values was again regarded as the basic step for a successful determination of the basic collision parameters in oxygen.

A hopeful approach to the problem to obtain a low temperature oxygen plasma is the use of the negative glow of a cold cathode discharge. With this type of discharge J.M. Anderson 4) found in Helium radiation temperatures as low as 350°K. For this purpose a new discharge tube and a new modulator suitable for this type of experiment were designed.

In this new design the discharge takes place between two 0.2 mm tungsten wires, which are stretched along the
inside wall of the standard 0.75" pyrex cylindrical container. The wires are located at the two ends of a diameter, so that their distance is 0.75"; at this distance the gas pressure being around 3mmHg, most of the discharge is formed by the negative glow. A T-300 Brown Boveri triode, driven by two 5687 tubes, provides for a 3.5 KV spike to start the discharge and for a 600 V pulse to maintain it with a current of 1 A for about 100 μsec.

Two sets of measurements were performed: the first one with the wire electrodes in a vertical plane and the second with these wires in a horizontal plane. In the second case the wires were located at the two positions of minimum microwave electric field, so that a better microwave match and lower attenuations can be achieved. In both cases measurements have been performed both during the discharge and in the afterglow; microwave heating has also been used.

The radiation temperatures we have thus measured range from 8000 K to 80000 K. The corresponding measured values of the parameter \( \frac{\omega}{\sigma_i \rho_e} \) (\( \sigma_r \) and \( \sigma_i \) being the real and the imaginary parts of the plasma conductivity, \( \omega \) the signal frequency and \( \rho_e \) the normalized pressure) plotted versus the radiation temperatures (fig.1) show, however, a large scattering. This scattering is probably due in part to some residual mismatches introduced in the waveguide by the discharge and in part to the different shapes of the electron distribution function, which may be far from the Maxwellian one, in the various experimental conditions.

The errors due to the non-contemporaneity of the phase-
attenuation and temperature measurements, which were regarded as another source of scattering at the time of the first measurements, were removed using a new set-up, where these data are taken at the same times.

Considering the large scattering of the above experimental data, no computation of the collision frequency has been made. From the measurements of radiation temperature increase, when the microwave heating is present, consistent determinations of the G-factor have been possible.

Two considerations may explain why it is possible to evaluate the G-factor and not the collision frequency. The collision frequency is directly dependent on the measured microwave attenuations, which are the less accurate data, whereas the attenuations enter the G-factor determinations only as corrective terms. Moreover, the shape of the distribution function does not appear explicitly into the formula for computing the G-factor, whereas it has a major role in the collision frequency formula.

In the Technical Summary Report № 1 the theory for the G-factor determination was given for the case of a constant electron collision frequency and of a Maxwellian velocity distribution. We shall present in the next Sections a more general theory and a review of the entire problem. The experimental data will be discussed at the end.
THE $G$-FACTOR PROBLEM

The problem to be discussed here is the determination of the fractional energy loss suffered by an electron, on the average, on each collision with the gas molecules. This quantity, which we shall call $G$, is exactly defined saying that $G(mv^2/2)$ is the average energy lost by an electron of velocity $v$, which collides with a gas molecule at rest. In general $G$ is a function of the velocity $v$.

When a microwave signal is propagating in a plasma, the electron mean energy is increased, because the electrons have their ordered oscillatory motion due to the field changed to random motion on each collision with the gas molecules. If we measure the microwave energy absorbed, on the average, by each electron, as well as the plasma microwave radiation temperature, both in the presence and in the absence of this microwave heating field, the fractional electron energy loss per collision may be determined. For this we need an energy balance equation written on an electron basis.

In fact if the collision frequency is independent of the electron velocity, the plasma microwave radiation temperature ($T_p$) is a direct measure of the electron mean energy ($\bar{U} = 3kT_p/2$). If $G$ is also velocity independent, then $G$ times the change in the electron mean energy must be equal to the energy absorbed from the field between collisions, which in this case is the well known $^1$: $e^2E_h^2/mw_h^2$, where $E_h$ is the r.m.s. applied microwave field and $w_h$ its radian frequency (low pressure case: $w_h^2 \gg \gamma_m^2$, $\gamma_m$ being the electron collision frequency for momentum transfer).
When the collision frequency is dependent on the velocity, the problem can still be solved easily, if the measurements are performed in the late afterglow of a decaying plasma in which electrons are in thermal equilibrium with the gas molecules. In this case their distribution function is Maxwellian and it remains such, when the microwave heating signal is applied, provided that $G$ is velocity independent.

Since the temperature of this Maxwellian distribution is equal to the measured radiation temperature, the electron distribution is completely known. If the $\nu_m(v)$ function is known, it is possible to compute the power absorbed by each electron from the field and the rate of energy losses due to collisions with the gas molecules. Equating these quantities we obtain a relation, where $G$ is the only unknown. If the $\nu_m(v)$ function is unknown, it can be determined during this same experiment according to a well-known method, measuring the plasma microwave complex conductivity as a function of the radiation temperature.

Also if $G$ is velocity dependent, the problem may in principle be solved, because the distribution function is here given by an expression containing $G(v)$ and a set of integral equations may be written, which is sufficient for the determination of the unknown $\nu_m(v)$ and $G(v)$. Thus the problem may be solved, at least by numerical methods.

However, in the case of nitrogen and oxygen, the collision frequency is not velocity independent and the isothermicity is not achieved in our late afterglows.
because special heating processes are active during the decays and keep the electrons at a mean energy higher than that of the molecules. This experimental result indicates that the electron distribution function has to be considered unknown during the afterglow; the problem of determining the G-factor seems thus hopeless for the lack of sufficient data from the experiment.

Fortunately, a more detailed analysis shows that the problem can be solved also in this case, if we may consider G as velocity independent, at least over the main part of the concerned electron distribution functions, and if the plasma microwave conductivities are measured, besides the microwave radiation temperatures. This analysis is reported here.

Let us assume a plasma in a steady-state and with uniform characteristics over the entire space region of interest. No external static magnetic field is applied to the plasma. A microwave signal is sent through the plasma and the changes in its amplitude and phase due to the plasma are measured. From these the real and imaginary parts of the plasma microwave conductivity \( \sigma = \sigma_r + j \sigma_i \) are determined. These parameters are, in the low pressure case, related to the plasma parameters by the well-known formulas:

\[
\sigma_i = \frac{n e^2}{m \omega^2} \quad (1)
\]

\[
\sigma_r = -\frac{n e^2}{3m \omega^2} \int_0^\infty \frac{d f}{d \nu} \cdot 4\pi \nu^2 d\nu \quad (2)
\]

Here \( n \) is the electron density, \( f \) is the spherically symmetrical part of the electron distribution function, normalized so that \( \int_0^\infty 4\pi \nu^2 d\nu = 1 \), and \( \omega \) is the radian
frequency of the measuring signal

The main quantity, which is measured in the present experiment, is the incoherent radiation emitted by the plasma at microwave frequencies. In particular the experiment is performed sending through the plasma the microwave radiation of a black body at a known temperature and finding the temperature at which the absorbed radiation equals the emitted one. This black body temperature is called the plasma radiation temperature \( T_r \).

Bekefy, Hirshfield and Brown \(^3\) have shown that this temperature is related to the plasma parameters by the formula:

\[
\frac{K T_r}{m} = - \int_0^\infty \nu m^2 \cdot \frac{4 \pi \nu^4}{d\nu} \int_0^\infty \nu m^2 \cdot \frac{4 \pi \nu^4}{d\nu} \]  

\( \) (3)

We shall now apply to the plasma a microwave heating field, uniform throughout space, with a radian frequency \( \omega_h \) and derive an energy balance equation, where the unknown \( G \) is present and the above defined plasma parameters may be substituted in.

We start by writing the equilibrium condition that the rate at which electrons would pass a velocity \( v \) in the upward direction, under the influence of the applied microwave heating field and in the absence of collision losses, must be equal to the rate at which electrons would pass the same velocity \( v \) in the downward direction, at the moment the microwave field is removed. The equation is:

\[
- \frac{4 \pi}{3} \cdot \frac{\varepsilon}{m^2 \omega_h^2} \cdot J_r (B_B - B_l) \cdot 4 \pi \nu^4 \]  

\( \) (4)
where one prime will indicate throughout the values of the quantities without the microwave heating field and double primes those in the presence of the heating field. In this equation $B$ is the spherically symmetrical part of the rate of change of the probability to find an electron in a velocity volume element and is given by the difference between the probability that the electron be scattered into that volume in the unit time and the probability to be scattered out.

According to Allis$^{1}$, when $G$ is velocity independent, and the scattering angle is distributed as for elastic collisions, the function $B(v)$ can be written as:

$$B = G \cdot \frac{i}{2} \nu \frac{d}{d\nu} \left[ \nu \nu^3 \left( f + \frac{KT_g}{mv} \right) \right]$$  \hspace{1cm} (5)

where $T_g$ is the temperature of the gas molecules.

Introducing this value of $B$, eq. (4) reduces to:

$$- \frac{4\pi}{3} \nu \nu^2 \frac{d}{d\nu} \left[ \nu \nu^3 \left( f + \frac{KT_g}{mv} \right) \right] = 2\pi G \nu \nu^3 \left[ f + \frac{KT_g}{mv} \right] - f - \frac{KT_g}{mv}$$  \hspace{1cm} (6)

Multiplying both sides by $mv$, integrating over the entire velocity space and rearranging, we obtain:

$$\left[ - \frac{e^2}{3mv^3} \int_0^\infty \nu \nu^2 \frac{d}{d\nu} \left( 4\pi \nu^3 \right) \right] \cdot E_h = C \int_0^\infty \nu \nu^2 \left( f + \frac{KT_g}{mv} \right) \cdot g \nu d\nu$$  \hspace{1cm} (7)

This is clearly an energy balance equation. The left hand side is $\int_{E_n}^E \left( E_n - E_p \right) \cdot \nu d\nu$, namely the mean power absorbed from the field by each electron; the right hand side is the difference between the mean power lost by an electron in collisions, when the heating field is present, and the mean power lost in the absence of this field. The terms containing $T_g$ come in because the gas molecules are not
at rest, as in the definition of $G$, but have a Maxwell distribution with temperature $T_g$.

Fortunately enough, all the integrals in eq.(7) are of the types given in eqs.(2) and (3); when these equations and eq.(1) are substituted into (7), this can be solved for $G$ to give:

$$G = \frac{2 e^2 E^2}{3 m w^2} K \Delta T$$

where:

$$\Delta T = T_r'' - T_g - \frac{\sigma_r''/\sigma_i''}{\sigma_r''/\sigma_i''} (T_r'' - T_g)$$

(9)

The quantity $\Delta T$, which has the meaning of a temperature difference, is easily computed from all the measured parameters. In particular, when the collision frequency is velocity independent, we have $\Delta T = T_r'' - T_r'$ and when the afterglow plasma is isothermal, we have $\Delta T = T_r'' - T_C$.

Eqs. (8) and (9) are thus the basic relations which solve our problem, within the discussed limitations. They have been used for computing $G$ in nitrogen and oxygen from our set of measurements of the microwave conductivity ratios $\sigma_r/\sigma_i$ and radiation temperatures with and without the microwave heating.

In the actual experiment, however, the plasma is generated, heated and measured in a waveguide so that the fields and the plasma characteristics are not constant in space, as assumed by the previous theory. In our experiment the geometry of the discharge and the microwave heating are such that all the quantities of interest are functions
only of the position in the cross-section of the cylindrical plasma column and not of the position along the axis.

Instead of the conductivity ratios, we thus measure the attenuation/phase shift ratios, \((A/\phi)\) where \(A\) is the attenuation in nepers, and \(\phi\) the phase shift in radians), given by:

\[
\frac{A}{\phi} = \frac{\int \sigma_r E^2_m dS}{\int \sigma_i E^2_m dS}
\]

where \(E_m\) is the measuring field configuration over the plasma cross-section \(S\).

The measured radiation temperatures are given by:

\[
<T_r> = \frac{\int \sigma_r E^2_m dS}{\int \sigma_i E^2_m dS}
\]

In the presence of a microwave heating field, not uniform over the cross-section, we distinguish between two limiting cases: 1) the case of infinite thermal conductivity of the plasma, so that the distribution function is independent of the position in the plasma, 2) the case of zero thermal conductivity, so that the distribution function is varying with the position, but has at each point the uniform case values under the local heating field.

In the first case the energy balance equation for electron (7) may still be considered valid, if we take for \(E^2\) a convenient average value over the plasma; a simple physical reasoning shows that this convenient value is:

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\[ \overline{E_h^2} = \int_S n E_m^1 dS / \int_S n dS \]  

If we now multiply both sides of (7) by \( n E_m^1 dS \) and integrate over the plasma cross-section, assuming the density ratio \( n'/n \) independent of position, we obtain the same eq. (8) for \( G \) with \( E_h^2 \) in place of \( E_h^2 \) and:

\[ \Delta T = \langle \frac{n'/n}{\mu''/\mu''} \rangle \left( <T_e>' - T_3 \right) \]  

In the zero thermal conductivity case eq. (7) is valid at each point. If we multiply both sides by \( n E_m^1 dS \) integrate over the plasma cross-section, assuming the density ratio \( n'/n \) independent of position, and finally divide by the factor \( \int_S n E_m^1 dS \), we obtain again for \( G \) eq. (8), where \( \Delta T \) is still given by (13), but \( E_h^2 \) has to be substituted by the average value:

\[ \overline{E_h^2} = \int_S \sigma_p E_h E_m^1 dS / \int_S \sigma_p E_m^1 dS \]  

A simple criterion may be given to determine whether the zero or the infinite thermal conductivity formula has to be preferred. The zero conductivity formula (14) will be chosen, when for the average electron the energy relaxation time \( \tau_e \approx 1/G \nu_m \) is shorter than the time required by the electron to diffuse over a distance of the order of the heating field disuniformities \( \tau_d \approx \nu_m \ell^2/2v_i^2 \) where \( \ell \) is the above said distance. The infinite conductivity formula (12) will be used in the opposite case.
G-FACTOR DETERMINATIONS FROM EXPERIMENTAL DATA

The results of the previous analysis will now be applied to compute again the G-factor in nitrogen, which was determined in the Technical Summary Report No. 1 under the simplified assumptions of a constant collision frequency and of a Maxwellian distribution. They will also be applied to the oxygen experimental data obtained in the measurements reported in Section 1 of the present Report.

In our experimental conditions the zero thermal conductivity formula (14) for $E_h$ has to be preferred. In fact, both in nitrogen and in oxygen using Phelps data and our results, the energy relaxation times are less than 1/us, or at the most equal, whereas the diffusion times over the distance of one waveguide radius are always longer than 1/us. The conductivity $\sigma$, which enters into Eq. (14), is proportional to the density $n_e$ and depends on the local electron energy; in our experiment we may, however, disregard this dependence on the basis that the energy increase is only a fraction of the initial electron energy. For the spatial variation of the electron density we have assumed a constant density in the nitrogen plasma (recombination controlled decay) and a radial Bessel $J_0$ variation in the oxygen plasma (attachment and diffusion controlled decay).

The results are as follows:

1) In nitrogen, at radiation temperatures between 3000 and 5500 K, $G = (3.95 \pm 0.25) \times 10^{-4}$ and no dependence of this value on the temperature is observed.
2) In oxygen, at radiation temperatures between 850 and 1900 K, $G = (4.8 \pm 1.0) \times 10^{-3}$ and the afterglow measurements seem to indicate a decrease in $G$, when the temperature increases.

From the data obtained by Crompton and Sutton 6) in nitrogen we have $G = 3.3 \times 10^{-4}$ at the electron temperatures of 4000 K and 4700 K, assuming a Maxwellian distribution for the electrons in their experiments, and $G = 4.1 \times 10^{-4}$ and $4.3 \times 10^{-4}$ at the same temperatures, assuming a Druyvesteyn distribution. These data are in very good agreement with our measurements.

From the data obtained by Healey and Kirkpatrick (7) in oxygen we have $G = 7.3 \times 10^{-3}$ at the electron temperature of 1600 K, a greater value than ours.

FUTURE WORK PROGRAM

On the basis of the above results and according to the Contract Extension Proposal, our future work program is as follows:

1) Using a transverse static magnetic field we increase both the real and the imaginary parts of the plasma microwave conductivity. We can thus improve the accuracy of our attenuation measurements and extend them later in the afterglow up to the time when the plasma has reached the isothermicity condition. From these data we expect to be able to derive reliable collision frequency values for the oxygen case.

2) Microwave heating is larger when its frequency is near
to the gyro-resonance. We expect to extend our G-factor determinations in oxygen up to larger electron energy values than here reported.

3) By the same microwave methods the electron collision frequency and the factor energy loss per collision will also be determined in nitrogen-oxygen mixtures.

4) We shall also investigate the possibility of performing similar measurements in atomic oxygen, produced by a discharge in oxygen under convenient conditions.
REFERENCES


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