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A Systematic Method for Character Recognition

by

David L. Fritzsche

Contract AF 33 (616) - 6137
Task No. 50682

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REPORT

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Task Number

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Investigation of

Guidance and Sensing Techniques for Advanced Vehicles.

Subject of Report

A Systematic Method for Character Recognition

Submitted by

David L. Fritzsche
Antenna Laboratory
Department of Electrical Engineering

Date

15 November 1961
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CHAPTER I
INTRODUCTION

The rapid expansion of digital computer usage has created the need for a means of transmitting written data directly to the computer without the manual translation such as that commonly performed by card punch operators. A satisfactory system which will accept written data and translate these data to machine language would have numerous applications in addition to those in the computer field. Automatic sorting of mail, office automation, and machine translation of written material from one language to other languages are some of the applications that come to mind. At the present time, most methods for automatic recognition of written data have been restricted to use of a carefully selected format for a group of written symbols.

A paper, "Pattern Detection and Recognition", by S.H. Unger presents a method which does not limit character size and style severely. The method suggested and tested requires the superimposing of line type patterns on a grid. All grid squares through which a line passes are tabulated. This information is processed by a computer. The computer answers a series of questions, such as, "Does the pattern consist of straight lines only?", "Does the pattern contain arcs opening to the

*All references are listed in the bibliography.
left?", "Does the pattern contain arcs opening to the right?", etc. By
the process of elimination, the pattern is identified providing the data
for the pattern are stored in the computer memory.

The report, "Identification of Shape," by R. L. Cosgriff demonstrates that shapes can be uniquely represented by a continuous periodic
function. The function can be represented by a number of the orthogonal
sets. This method of representation and a technique for recognition is
developed and extended in this paper.

A second method which uniquely describes a character is a curve
of R versus θ, Fig. 1-1. This method can be used only with the simple
characters since multiple values of the R(θ) function occur with
complex characters. Also, this representation has the disadvantage of
being dependent upon the origin location.

Fig. 1-1. Geometrical description of R vs. θ

representation method.

In the discussion of mathematical techniques for symbol recogni-
tion, it is necessary to remember that the nature of the selectivity
(ability to distinguish two dissimilar symbols) will be dependent upon the techniques involved. Furthermore, we should not expect an exact correspondence between selectivity of any mathematical technique and the selectivity of the human. Notice the characters in Fig. 1-2 and see the similarities as you progress from a to b to c. Yet these same characters, if rotated $90^\circ$, form the letters H, K, and X. H, K, and X each has a definite but different meaning. The human is conditioned to observe the differences rather than the very pronounced similarities of these three letters. We can not expect a mathematical comparison of characters to indicate to the same degree those differences or similarities which we observe.

Fig. 1.2. Simple similar characters with different meanings.
CHAPTER II

REPRESENTATION OF CHARACTERS

Characters or figures can be classified in many ways. In this paper, a character or figure will be considered to be simple if it can be defined by a single closed curve. If the figure can not be defined by a single closed curve, the figure will be considered complex. Complex figures require a finite number of closed curves or shade variation for satisfactory description. Many figures are composed of a group of lines with finite width. If the outside boundary of such figures can be used to define the figure the figure is said to be reducible to simple form. For example, all printed letters which can be represented by intersecting lines which do not enclose an area or areas are reducible to simple form. On the other hand, the letters B, P, etc. cannot be reduced to simple form without altering the figure. However, some such letters or figures may be easily recognized by the outside boundary alone. Techniques for representing or approximating complex figures can be developed but will not be considered. Only simple figures or characters will be considered.

Two basic characteristics of simple figures are employed in the development of a mathematical representation. The first variable is arc length.

\[ L = \int_{c} d\ell \]
where \( L \) is the total length and \( dl \) is the incremented arc length of the line which defines the figure. A suitable technique for figure recognition is not dependent upon the line length \( L \). Therefore, normalizing the total line length of all characters to \( 2\pi \) will eliminate possible restrictions on character size and will be convenient in further development of a mathematical representation. The normalized length \( l_1 \) is such that

\[
L_1 = \int_{c_1} dl_1 = \frac{2\pi}{L} \int_0^L dl = 2\pi
\]

Consider the second characteristic of a simple figure. If the figure is composed of straight lines only, then the sum of the exterior angles \( \phi_{en} \) is:

\[
(3) \quad \sum_{h=1}^{N} \phi_{en} = 2\pi
\]

providing the defining lines are traversed only once. If the lines are traversed in times then

\[
(4) \quad \sum_{h=1}^{N} m \phi_{en} = 2m\pi
\]

If the straight line segments of the figure are joined by curves, then Eq. (4) can be expressed in integral form.

\[
(5) \quad 2m\pi = \oint \frac{d\phi}{dl} dl
\]
where \( \frac{d\phi}{dl} \) at any given point is equal to \( 1/r_c \), with \( r_c \) the radius of curvature, \( m \) the number of times the figure is traversed, and \( \phi \) the angle the tangent to the curve makes relative to some arbitrary reference line in the plane of the figure.

The two fundamental characteristics of simple figures allows all such figures to be represented graphically. A plot of the tangent angle versus \( l \) is a unique representation. The tangent angle is measured with respect to some arbitrary reference in the plane of the figure. The figures are represented by

\[
\phi = F(l_1) \tag{6}
\]

after the normalization of \( l \). The second characteristic considered above indicates that \( F(l_1) \) is a curve of ever increasing value if the figure is traversed \( m \) times. However, \( F(l_1) \) is always equal to \( mL_1 \) after traversing the figure \( m \) times. The curve can be made periodic by subtracting the ramp function \( \phi = l_1 \) from Eq. (4) giving

\[
\phi - l_1 = G(l_1) \tag{7}
\]

A simple figure can be uniquely represented by the function \( G(l_1) \). \( G(l_1) \) is a piecewise continuous, bounded, and periodic function and can be represented by a Fourier series.

\[
G(l_1) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos n l_1 + b_n \sin n l_1 \right) \tag{8}
\]
The constant $a_0$ can be eliminated by choosing the reference angle properly and will be neglected.

Figure 2-1 illustrates the technique for representing any simple curve as some function $G(t)$. Figure 2-1a is a sketch of a simple character with an indicated starting point. (The line is traversed in the counterclockwise direction.) An arbitrary reference angle and the tangent to the curve at a given point are also shown. This figure has been chosen to test the performance of computer programming and accuracy because the Fourier series for $G(t)$, as shown in Fig. 2-1c, can be readily calculated. Appendix Table A-35 shows the computer results and calculated results for Fig. 2-1a.
a) Sketch of a simple character.  
b) The $\phi$ vs. $l$ curve for this character.  
c) The $\phi-l'$ curve for this character.

Fig. 2.1
CHAPTER III
COMPARISON OF CHARACTERS

A. Method of Comparison

In the last chapter it was demonstrated that any given simple figure can be represented by a Fourier series. This series can be reduced to:

\[ G(t_1) = \sum_{n=1}^{\infty} \frac{a_n}{n} \cos(n \cdot t_1 - \Theta_n) \]

where the constant \( a_0 \) has been neglected for reasons previously indicated.

In this chapter, a technique will be developed for specifying an arbitrary character as being identical or similar to one of a group of \( P \) standard characters. Each of the \( P \) characters can be represented as \( G_p(t_1) \) for \( 1 \leq p \leq P \). The arbitrary character is represented as \( G_q(t_1) \).

Because of the orthogonality of the Fourier series, it is convenient to express the degree of similarity between \( G_q(t_1) \) and \( G_p(t_1) \) as a mean square error \( \varepsilon_{pq} \).

\[ \varepsilon_{pq} = \frac{1}{2\pi} \int_{0}^{2\pi} |G_p(t_1) - G_q(t_1)|^2 \, dt_1 \]

\[ = \frac{1}{2\pi} \sum_{n=1}^{\infty} \int_{0}^{2\pi} |C_{np} \cos(n \cdot t_1 - \Theta_{np}) - C_{nq} \cos(n \cdot t_1 - \Theta_{nq})|^2 \, dt_1 \]
\[ (10) \quad = \frac{1}{2} \sum_{n=1}^{\infty} \left[ (C_{np})^2 + (C_{nq})^2 - 2C_{np}C_{nq} \cos \alpha_n \right] \]

where \( \alpha_n = \theta_{np} - \theta_{np} \). Practical considerations require limiting the summation to \( N \) terms such that:

\[ (11) \quad G(t_1) = \sum_{n=1}^{N} C_n \cos (nl_1 - \theta_n) \]

and

\[ (12) \quad E_{pq} = \frac{1}{2} \sum_{n=1}^{N} \left[ (C_{np})^2 + (C_{nq})^2 - 2C_{np}C_{nq} \cos \alpha_n \right] \]

Before continuing further along this line of thought, it clarifies the problem if each \( G_p(t_1) \) is represented in 2N dimensional vector space.

The unit vectors in 2N dimensional space are

\[ \vec{r}_n \text{ and } \vec{r}_n \quad n = 1, 2, 3, \ldots N. \]

Since

\[ G_p(t_1) = \sum_{n=1}^{N} C_{np} \cos (nl_1 - \theta_{np}) \]

\[ = \sum_{n=1}^{N} a_{np} \cos nl_1 + b_{np} \sin nl_1 \]

then \( G_p(t_1) \) can be represented as a unique point in 2N vector space. The
vector from the origin to the point $P$ is:

$$
\mathbf{OP} = \sum_{n=1}^{N} \left[ r_n \mathbf{a}_{np} + i_n \mathbf{b}_{np} \right]
$$

$$
= \sum_{n=1}^{N} \left[ \mathbf{a}_{np} + \mathbf{b}_{np} \right]
$$

where $|\mathbf{a}_{np}|$ and $|\mathbf{b}_{np}|$ are in the $r_n$ and $i_n$ directions, respectively.

The character $G_q(t_1)$ can be represented in like manner, so that:

$$
G_q(t_1) = \sum_{n=1}^{N} \left[ \mathbf{a}_{nq} + \mathbf{b}_{nq} \right]
$$

It follows that the vector $\mathbf{PQ}$ is a measure of the similarity of $G_p(t_1)$ and $G_q(t_1)$, because if, $G_p(t_1) = G_q(t_1)$ then $\mathbf{PQ} = 0$.

$$
\mathbf{PQ} = \sum_{n=1}^{N} \left[ \mathbf{a}_{nq} + \mathbf{b}_{nq} - \mathbf{a}_{np} - \mathbf{b}_{np} \right]
$$

$$
= \sum_{n=1}^{N} \left[ \mathbf{C}_{nq} - \mathbf{C}_{np} \right]
$$

where $|\mathbf{C}_n| = (a_n^2 + b_n^2)^{\frac{1}{2}}$.

The absolute magnitude squared of the vector $\mathbf{PQ}$ is identical to twice the mean squared error as determined by Eq. (12).

$$
|\mathbf{PQ}|^2 = E_{pq} = 2 E_{pq}
$$
Henceforth, the error $E_{pq}$ shall be used to designate the degree of similarity between the $p^{th}$ and $q^{th}$ figures.

B. Preliminary Adjustments

The function $G(t_1)$, Eq. (7), is dependent upon the origin of $F(t_1)$, (Eq. (6)). Also, $F(t_1)$ is dependent upon the starting point for traversing the defining curve of a simple figure. It logically follows that the vector $\overrightarrow{PQ}$ is dependent upon the origin of $F_p(t_1)$ and $F_q(t_1)$. In order to use the error $E_{pq}$ as a true indication of similarity, it must be minimized to its least possible value.

Allow the vector representation of $G_p(t_1)$, $\overrightarrow{OP}$, to remain fixed in vector space. Allow the vector representing $G_q(t_1)$ to describe some closed contour in $2N$ dimensional vector space. This closed contour is generated by shifting the origin of $G_q(t_1)$ from 0 to $2\pi$. This shifting operation necessitates rewriting the expression for $G_q(t_1)$, $\overrightarrow{OQ}$, as

$$\overrightarrow{OQ} = \sum_{n=1}^{N} C_{nq} \cos (\theta_{nq} + n\beta) + C_{nq} \sin (\theta_{nq} + n\beta)$$

Then the vector difference

$$\overrightarrow{PQ} = \sum_{n=1}^{N} \left[ a_{np} \overrightarrow{am} + b_{np} \overrightarrow{bm} - \left( C_{nq} \overrightarrow{cm} \cos (\theta_{nq} + n\beta) + C_{nq} \overrightarrow{cm} \sin (\theta_{nq} + n\beta) \right) \right]$$

$$= \sum_{n=1}^{N} \left[ \overrightarrow{c_{np}} - \overrightarrow{c_{nq}} \right]$$
where $|\vec{C}_{np}|$ and $|\vec{C}_{nq}|$ have the same definition as previously indicated. Then the error $E_{pq}$ is:

$$E_{pq} = |PQ|^2 = \sum_{n=1}^{N} \left[ (C_{np})^2 + (C_{nq})^2 - 2C_{np}C_{nq}\cos(\alpha_n + n\beta) \right]$$

where $\alpha_n = \theta_{nq} - \theta_{np}$. The maximization of the last term on the right side of Eq. (18) results in a minimization of $E_{pq}$. The minimum $E_{pq}$ results with a proper choice of $\beta$. This choice of $\beta$ is most readily determined with a digital computer if the $q^{th}$ figure is to be compared with $P$ standard figures. A graphical solution for $\beta$ is possible although it is a lengthy and tedious process.

It has been shown that the starting point on the line defining a simple figure is arbitrary for the determination of $G(l_1)$. However, the starting point must be adjusted mathematically if the minimum value of $E_{pq}$ is to be calculated. This corresponds to using a common starting point for identical figures. If two figures are identical, then a common starting point is determined by

$$\theta_{1q} - \theta_{1p} + \beta = 0$$

where the phase of the $p^{th}$ figure remains fixed.

Now consider the case where $C_{1q}$ of $G_q(l_1)$ is zero. $\theta_{1q}$ can assume any arbitrary value between 0 and $2\pi$. The solution of $\beta$ in Eq. (20) is not unique and a common starting point can not be found in this...
manner. In general, the same argument holds if \( C_{1q} \ll C_{nq} \), such that small errors in the calculation of \( C_{1q} \) result in large errors in \( \theta_{1q} \). The solution for \( \beta \) in Eq. (20) is possible for the general case but is not sufficiently accurate.

A second method for determining an approximate minimum value of \( E \) is to locate the largest product \( 2C_{np}C_{nq} \) in the summation

\[
\sum_{n=1}^{N} 2C_{np}C_{nq} \cos (\alpha + n\beta).
\]

Then \( \cos (\alpha + n\beta) \) for this term is set equal to one. This method assumes that the \( n^{th} \) term of the \( p^{th} \) and \( q^{th} \) figures are larger than all other terms. This assumption is valid if only figures with a high degree of similarity are of interest. In order to systematize this method, the starting points for each \( G_p(\ell_1) \) and \( G_q(\ell_1) \) are adjusted such that \( \theta_{np} \) and \( \theta_{jq} \) are zero, where \( \theta_{np} \) and \( \theta_{jq} \) are the phases of the largest term in each series. The error \( E_{pq} \) calculated in this manner will approach its minimum value for similar figures but will not necessarily approach the minimum value for dissimilar figures. One discrepancy may arise with the use of this technique if \( C_j \) and \( C_{j+k} \) are of nearly equal magnitudes. Which of these two terms should be phased to zero? This question can be reconciled by computing the error \( E_{pq} \) twice, once with \( \theta_j + \beta_j = 0 \) and once with \( \theta_{j+k} + \beta_{j+k} = 0 \). The lesser value of \( E_{pq} \)
will be the more accurate indication of similarity.

For this paper, the starting point on each describing curve is first chosen by visual inspection such that $E_{pq}$ approximates its minimum possible value. The approximation is improved in accuracy by adjusting the phase of the largest term of each $G(l_1)$ to zero. This corresponds to small adjustments of the starting points.

C. Absolute Upper Bound

The method to be used for determining the approximate minimum value of $E_{pq}$ can be checked for accuracy as follows. The absolute minimum value of $E_{pq}$ is:

\[
(E_{pq})_{\text{min}} = \sum_{n=1}^{N} |C_{np} - C_{nq}|^2
\]

which assumes all terms of $G_p(l_1)$ and $G_q(l_1)$ are in phase. For similar figures, this value of $(E_{pq})_{\text{min}}$ will be approached. If the calculated value of $E_{pq}$ does not approach $(E_{pq})_{\text{min}}$ then this indicates either that the starting points have been chosen improperly or that the figures are dissimilar. The calculation of $(E_{pq})_{\text{min}}$ can be used as a rapid and convenient indication of the degree of similarity. A large value of $(E_{pq})_{\text{min}}$ immediately shows dissimilarity. On the other hand, a small value of $(E_{pq})_{\text{min}}$ does not necessarily show a high degree of similarity. The upper bound $(E_{pq})_{\text{min}}$ establishes one level of similarity.
A second level is established by some upper allowable limit to $E_{pq}$. This second level is specified according to the purpose of the identification system when employed as in $(E_{pq})_{\min}$.

D. **Sample Characters**

The figures shown on Appendix I of this paper (Figs. A-1 through A-34) represent a sample of the figures which can be identified using the technique previously described. The series $G(i_1)$ which represents each character are tabulated in the Appendix I. The figures and corresponding tables have identical numbers. The amplitudes of each term of the harmonic series have been computed for the first thirty terms and the phase of each term has been adjusted so that $\theta_n$ for the largest term is zero.

Most of the characters are letters of the alphabet or are a modification of these characters. These characters have been chosen for their practical value. A second reason for choosing letters is because the series $G(i_1)$ exhibit convergence in the first thirty terms. The other characters (animals, faces, etc.) do not converge rapidly in the first thirty terms and are not entirely satisfactory for this initial study. These figures do show the limitations of such a technique.
E. Tabulated Error and Discussion of Results

There are over seven hundred possible comparisons which can be made using the characters referred to in the previous section. A visual check of the Fourier series coefficients and phase angles as tabulated in the Appendix I readily shows that \((E_{pq})_{\text{min}}\) for characters which appear dissimilar will be quite large \((i.e., \text{greater than } 0.5)\). For this reason the majority of the possible comparisons are not carried out.

Forty one comparison have been made and are tabulated in Table 3-1. The first column of the table designates the characters being compared. The second column labeled \(E_5\) is the error obtained by using only the first five terms of each series \(G(l_1)\). Columns 3, 4 and 5 have similar significance. It will be noted that the columns labeled \(E_{20}\) and \(E_{30}\) are not complete. A study of those comparisons which are complete will show that two figures which appear similar to the eye will have a small \(E_{pq}\) using only five or ten terms of \(G(l_1)\). Here again a small \(E_{pq}\) is assumed to be less than 0.5. This value of \(E_{pq}\) and \((E_{pq})_{\text{min}}\) has been arrived at after a study of Table 3-1 and the figures presented in Appendix I. A lesser or greater value may be used depending upon the degree or level of similarity desired.

Several of the comparisons in Table 3-1 bear special attention.

Note the error \(E_5\) for comparing the X, H, and K on Figs. A-1, A-6 and
### TABLE 3-1

**TABULATED ERROR**

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<th>Figure Compared</th>
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<th>$E_{10}$</th>
<th>$E_{20}$</th>
<th>$E_{30}$</th>
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<td>A-1 A-2</td>
<td>0.038</td>
<td>0.074</td>
<td>0.317</td>
<td>0.458</td>
</tr>
<tr>
<td>A-1 A-3</td>
<td>0.080</td>
<td>0.160</td>
<td>0.210</td>
<td>0.222</td>
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<tr>
<td>A-1 A-5</td>
<td>2.699</td>
<td>2.927</td>
<td>3.005</td>
<td>3.025</td>
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<tr>
<td>A-1 A-6</td>
<td>0.902</td>
<td>1.079</td>
<td>1.142</td>
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<td>A-1 A-7</td>
<td>0.741</td>
<td>0.886</td>
<td>0.958</td>
<td>0.973</td>
</tr>
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<td>A-1 A-8</td>
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<td>A-1 A-10</td>
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A-7 respectively. The error for these comparisons indicates a relatively high degree of similarity although the figures were not drawn for the purpose of showing this similarity. The degree of similarity between Fig. A-1 and A-2 and also Fig. A-3 and A-4 is quite high which is the desirable result.

The errors resulting from all possible comparisons of Fig. A-21, A-21a, A-21ab, and A-21abc require further explanation. These figures have been used as a basis for studying distortion. Distortions as used here refers to the modification of a standard figure in some small way such that the standard figure is still easily recognized visually. Furthermore, the modification effects only some small portion of the standard figure. Using only five terms in the calculation of $E_{pq}$ for the figures indicated results in a satisfactory indication of similarity although not what the human might expect. For example, Fig. A-21abc is expected to compare most favorably with Fig. A-21ab. However, Fig. A-21abc compares most favorably with Fig. A-21. If all thirty terms of $G(t_1)$ are used in the comparisons, this discrepancy between the eye and mathematical results is further enhanced.

Small distortions, i.e., the line length which describes the distortion is small compared to the line length of the total standard figure, can be treated as being a linear addition to $G_s(t_1)$. $G_s(t_1)$ is the function which describes a standard figure. A small distortion which adds a rectangular pulse to $G_s(t_1)$ contains frequency components
whose magnitude is proportional to \( \sin \pi t_2 f / \pi t_2 f \), where \( t_2 \) is the length of the distortion. Thus, a rectangular pulse adds a significant amount to \( G_s(t_1) \) over the frequency range up to \( f = 1/t_2 \). A distortion of length \( t_3 \), which adds the derivative of a triangular pulse to \( G_s(t_1) \), adds frequency components of increasing magnitude up to \( f = 1/t_3 \) and can be considered insignificant above \( f = 2/t_3 \). These two examples of possible distortions show the difficulties which arise if distorted figures are considered. Multiple distortions on a given standard figure or increased size of the distortions also modify \( G_s(t_1) \) to the extent where similarity can not be shown using the described technique. The results obtained from the comparison of Fig. A-33 and A-34 indicates the large error obtained in this case.

The error calculated for the comparison of Fig. A-1 and A-2 shows that the addition of small distortions placed symmetrically on a standard figure may possibly be treated as a linear addition to \( G_s(t_1) \). The distortions add components primarily to the higher harmonics. This is only one of many possible examples of symmetrical addition and is sufficient for showing general conclusions.

A plot of \((E_{pq})_{\text{min}}\) and \(E_{pq}\) is shown in Fig. 3-1. This figure uses only the first five terms of \( G(t_1) \) and demonstrates that starting points have been properly chosen for an approximation of the minimum possible \( E_{pq} \). Those points denoted by open dots are comparisons which
are considered to show a desired level of similarity both visually and mathematically. Those points denoted by closed dots show dissimilarity both mathematically and visually. Not all forty one comparisons have been entered in Fig. 3-1, but a sufficient number are shown to indicate the type of plot which is obtained.

Fig. 3-1. Representative sample of $(E_{pq})_{min}$ vs. $E_{pq}$ for first five terms of $G(L)$. 

21
CHAPTER IV

CONCLUSIONS

The technique which has been described for character recognition in this paper has been shown to possess several characteristics necessary for satisfactory recognition. Not all of these characteristics are present in many of the methods which have been used for character recognition. The desirable characteristics are:

1. Each character is representable by a unique mathematical expression.
2. The recognition technique is independent of character size.
3. The technique is independent of character orientation.
4. The gross features of a character are identified by using only a limited number of terms of the mathematical expression.

The principal disadvantage of the technique is that it is sensitive to irregularities or distortions of standard figures. This may be considered an advantage if it is known that such distortions represent the difference between two given figures.

The technique as presented is feasible for the recognition of typed letters or numbers, where the figures are distinct and are identical for each appearance. Hand written letters and words are not adaptive to this technique because of the immense number of standard characters which would be necessary.
Earlier methods which were considered for character recognition involved optical matching. The difficulties which arose were due primarily to orientation and size of characters. These problems have been overcome for the method shown by describing each character as a function $G(I_1)$. The optical matching of these functions may prove to be more satisfactory than the mathematical matching as described in the paper.
APPENDIX I

TABULATION OF G(\ell_1)

[Fourier Cosine Coefficients and Phase Angles]

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Fig. A-6

Fig. A-7

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Figure A-21 is used for the specific purpose of determining whether small random distortions can be added to a smooth character such that recognition is still possible. From this point, Fig. A-21 will signify the smooth character with no distortions. Fig. A-21a is the same as Fig. A-21 except for distortion a. Fig. A-21ab is the same as Fig. A-21a except for distortion b. Fig. A-21abc is the same Fig. A-21ab except for distortion c. The tables and all further references to the above character will have this same designation.
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Fig. A-34
APPENDIX II

TESTING OF DIGITAL COMPUTER RESULTS

The testing of digital computer programming and results was accomplished by using the character in Fig. 2-1. This character has a defining function $G(\lambda_1)$ for which the Fourier series is readily calculated. Table A-35 gives the computer and calculated results for the test character. The error in the calculation of coefficients exceeds 20 per cent for terms greater than the sixteenth harmonic. However, only 240 points were used to define the character in the x-y plane. These points were not uniformly spaced. The computation of the thirtieth harmonic coefficient results in a point on the defining curve every $\pi/4$ radians on the average. All subsequent characters have been represented by a minimum of 450 points and in most cases 550 or more points were used. Also, these points were more uniformly spaced than for the test character.

Two of the representative series calculated by the digital computer have been plotted using only the dominant terms. An analogue computer was used for this task. Seven terms from the series in Table A-3 are shown graphically in Fig. A-36. The calculated results are shown in Fig. A-35. Twenty one terms from the series in Table A-2 are shown as Fig. A-38. Fig. A-37 is the calculated result for this case.
These checks on digital computer performance indicate that acceptable accuracy is obtained in the calculations.

**TABLE A-35**

**FOURIER COSINE SERIES COEFFICIENTS AND PHASE ANGLES FOR FIGURE 2-1a**

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