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DESIGN PRINCIPLES FOR AN ACCURATE LARGE-APERTURE TRACKING ANTENNA

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ABSTRACT

The combination of a number of techniques, some of which are novel, appear to make possible the building of fully steerable antennas of large size which achieve high resolution and accurate performance at a lower cost than has heretofore prevailed. Also, these antennas can be made to track through the zenith with only two axes of rotation. The study presented here is preliminary in nature and describes principles rather than presenting detailed engineering solutions.
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1. **Introduction**

Radio astronomy and space communication demand very large and very accurate tracking antennas for missions requiring high resolution and high gathering power at short wavelengths. Satisfying these demands with current structural techniques leads to cumbersome and expensive structures and deterioration of performance.

For example, if the 600-foot diameter radio telescope now under construction is to be used to monitor 21 centimeter hydrogen radiation with high resolution, it will be desirable to maintain the surface shape of the reflector to tolerances of a small fraction of a wavelength. The shape of the parabolic reflector must be preserved as the dish is tilted in elevation (which imposes varying gravity loads) and despite wind loading and thermal changes due to ambient temperature and sunlight conditions.

The standard approach to preserving dish geometry under loading has been to keep the stress levels in the support structure low. This is achieved by using members of large cross section; consequently, as the dish grows in size, heavier members must be used and the structure tends to sag under its own weight when it is tilted in elevation. It also becomes progressively more unwieldy to move about. This is being offset in certain cases by servo control of individual panels of the paraboloid to make up for the deflections incurred under load and temperature changes.

However, it is possible to solve these problems by an alternate approach which greatly reduces the amount of structure
required to support the parabolic reflector. This approach involves the application of simple feedback techniques to preserve the geometry of the dish, and still permits the support structure to be stressed up to its yield point without deflection. By proper geometry and structural techniques it is also possible to simplify other problems, such as obtaining proper servo response from a large structure, making it survive hurricanes, or assembling the structure at the site. These techniques involve the following basic principles:

1. The "infinite stiffness" structural member.
2. A statically determinate dish support structure.
3. A "prescribed deformation" support structure.
4. Support of the tracking member with "infinite compliance" and three reference support points.
5. An attitude reference independent of support structure.
7. An offset reflector.

Not all of these features need be used simultaneously; a tracking antenna can be built with almost any combination of these elements.

Figure 1 is an artist's sketch of a 120-foot antenna incorporating all of the features listed above; individual discussions of each of them follow.

2. The "Infinite Stiffness" Structural Member

Solids change in dimension principally through elastic deformation under application of forces and through thermal changes. In the design of many large structures, rigidity has been achieved by keeping strain low; this involves using stress levels well below the yield point of the structural material employed, and has been the traditional method of preserving the shape of large antennas under gravity and wind load. The low stress requirement means cross-sections that are more bulky than necessary, and which in-
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PROPOSED 120 FT TRACKING ANTENNA

Figure 1
crease the load to be driven in moving the structure.

It is possible to compensate for strain changes, however, by means of appropriate thermal changes. Thus, if a member is placed under compressive load, its length may be kept constant by a suitable rise in temperature; likewise, a suitable temperature drop will offset a tension load. Hence, extremely accurate control of the dimensions of a member under varying load can be established by applying simple feedback techniques and thermal control.

In a test conducted at the Instrumentation Laboratory, temperature compensated strain gages were laid along a 48-inch long tube of 4130 chrome-moly steel with a 2-3/4 inch OD and a wall thickness of 0.065 inch, which was then surrounded with heater wiring. An amplifier fed by the bridge signals from the strain gages controlled the on-off flow of power to the heater wiring. Figure 2 is a functional schematic of the system, and Figure 3 is a photograph of the test set-up.

Under application of a 1000-pound load, the member deflected about 0.003 inch; the thermal stabilization system returned the member to within 0.0002 inch, or 4 parts per million, of the total length. Figure 4 is a plot of the performance of the member under sudden application of load. In general, the member was operated at a mean temperature sufficiently above ambient so that the time constants of heating after compression and cooling after tension were approximately equal. The results shown returned to null after 1-1/2 minutes; this can be speeded up by providing for greater rates of heat input or removal.

The threshold of observation which determined the short-term performance of the system and the size of the limit cycle, as shown in Figure 5, was about $4 \times 10^{-6}$ inches, or about 1 part in 10,000,000 of the total length. With further testing and with improvement of the experimental equipment, it is reasonable to expect that the long term drift can be brought down to a level not much above this, and well below the $2 \times 10^{-4}$ inches or 4 parts per
Figure 2  Functional schematic of dimensional stabilization system
Fig. 3 Laboratory test setup
Response with 1000 lb load applied

Figure 4
Long term stability

Figure 5
million obtained in the first tests.

The development and tests of the "infinite stiffness" structural member are described more in detail in Reference 1*. The work can be summarized by stating that long term stability of 4 parts per million and a threshold of 1 part in ten million have been achieved; and that with improved test equipment results better than 1 part per million should be obtainable. However, it should be noted that the servo system involved has the rather long time constant associated with heat flow. If applied to antenna structures, the system could compensate for gravity loads at modest tracking rates (such as are involved in tracking space vehicles) or for steady state wind, but not for sudden gust loading or violent slew rates.

3. The Statically Determinate Support Structure

The problem considered in this section is that of assembling an "infinitely rigid" structure of the members described in the preceding section. The thermal control techniques permit each member of a structure to be controlled in length with high accuracy. If a three-dimensional assembly, such as a tetrahedron, can be made in such a manner that no bending moments are applied to any individual member, it becomes possible to preserve the geometry of the entire assembly by controlling the length of its individual members.

If the individual structural members are connected, as shown in Figure 6, by spherical connectors with a pin-joint, or ball-and-socket arrangement, no bending moments are applied. The cylindrical structural members are butted up against lead gaskets on the connecting spheres by prestressing through a tension bar along the axis of the cylindrical member. The ball at

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TYPICAL PIN JOINT CONNECTION -
ANTENNA STRUCTURE

BALL END JOINT

PRE-STRESSING TENSION MEMBER

THERMALLY CONTROLLED COMPRESSION MEMBER

Figure 6
the end of the tension member rides in a conical socket in the con-
necting sphere. No bending moments are applied to the members
of a structure utilizing this arrangement; such a statically deter-
mine structure is proposed to support the reflecting dish of the
antenna system.

Among the advantages of this type of structure is the fact
that it is easily assembled at the site with simple tools. The in-
dividual members are small enough so that they can be fabricated
wherever desired and shipped with ease to the desired location.
There are no bulky weldments to handle in assembling the struc-
ture. The panels of the dish, which are then attached to an "in-
finitely rigid" truss support, need be restrained only in the di-
rection normal to the surface of the paraboloid, and can be free
to slide tangentially. The over-all structure should be considerably
lighter and, at the same time, more rigid than conventional truss
construction.

In the tests conducted on the "infinite stiffness" member,
the maximum input wattage to maintain the member above ambient
was 75 watts per linear foot. Actual average power was consid-
erably below this level; however, a demand for a quicker response
time might make it desirable to increase the thermal input. Per-
haps 200 kilowatts might be necessary to achieve thermal control
of the structure required for a 120-foot diameter dish, which was
roughly estimated at 4000 linear feet. Various alternatives are
possible to increase the rate of heat loss, thereby requiring less
heat by making it possible to operate the system at a temperature
closer to the ambient.

4. The "Prescribed Deformation" Support Structure

The preceeding sections have produced the concept of a
structural support for a reflector which is held to extreme rigidity
by the use of thermal control, but which has a rather slow response
to sudden changes in load. Sudden gust loading of the reflector
would cause a temporary distortion of the support structure.
In an effort to meet this problem, it was conjectured that by suitable choice of the mounting points which connect the reflector to its support structure, the support structure could be designed to deflect in such a manner that the linear motion of all of the reflector mounting points would keep these points on the surface of a paraboloid with a focus still at the proper location. Reference 2* contains a study of this problem. It does not appear possible to have the entire dish move with translation only, i.e. to have all support points and the focus deflect by the same amount and in the same direction. However, the study did show that it is possible to maintain the shape of a paraboloid when a gust load is applied in a particular direction by suitable choice of locations of support points and stiffnesses of members. For example, under application of a load along the direction of the axis of the parabolid, it is possible to have all support points move with a combination of axial and radial motion which places them on a new paraboloid of the same focal length. A specific set of design parameters can be obtained for each direction of load. Since gust loads are most commonly horizontal in direction, and since the reflector will be used at varying elevation angles, the design cannot compensate for all gust conditions. However, by estimating the distribution of gusts in various directions, and the expected distribution of tracking angles in use of the antenna, a solution can be obtained which is optimum in a statistical sense. That is, a mean square error can be defined in suitable manner, and the structural design can be chosen to minimize this error.

For further details on this phase of the work, the reader is referred to Reference 2.

5. An "Infinite Compliance" Support with Three Reference Points

The antenna, as so far developed, consists of a reflector
attached to an "infinite stiffness" support structure which pres-
serves the geometry of the reflecting paraboloid. However, since
the antenna is steerable, there is no need for the entire structure
to be designed for infinite stiffness. As long as the shape of the
reflector is preserved, the antenna drives can compensate for
deflections of that part of the support structure which holds the
"infinite stiffness" structure.

A rigid structure can be firmly fixed to a flexible structure
at only three points without imposing redundant restraint. These
three points determine a support plane which completely orients
the rigid structure. Conceivably then, the rigid reflector structure
could be supported by the flexible lower structure at three points
without impairing its geometry. However, a three point support
involves high loads at the three locations, and it would be desirable
to provide a more distributed support structure if possible. This
can be done if the three support points are made reference points
by using a load cell or force transducer to measure the support
forces at the reference points. A number of "infinite compliance"
support points can then be added in the plane of the three reference
points to distribute the load.

An "infinite compliance" support point, as used here, is
defined as a connection made through a pneumatic or hydraulic
member servoed to maintain whatever force level is desired.
Through proper computation, for example, the measurements
made by the three load cells at the reference points can be made
to call for suitable support forces at perhaps 12 points in the sup-
port plane, thus distributing the load. A deflection of the flexible
structure at one such support point would cause a change in the
force level at this point. The servo system could then extend or
retract the piston member at this particular point until the
force level is at the required value, thus creating what amounts
to an "infinitely compliant" or prescribed-force support point.

The principal value of this type of support system is the
distribution of load over a greater area and the increased symmetry of the structure. By spreading the load over more members, the load in each support point is reduced, and hence the effect of sudden load changes too rapid for the thermal servo to follow is reduced in magnitude. A pneumatic or hydraulic servo system could be made to operate with a time constant short enough to take care of any anticipated gust or gravity effects; however, this is done at the expense of introducing the added complexity of load cells, computation, and servos. Nevertheless, the value of this system in helping preserve rigidity is such that it seems worth serious consideration.

6. **Attitude Reference Independent of Support Structure**

The use of a flexible base requires that accurate measurements of the attitude of the reflector must be made independently of the supporting elements. In general, it is desirable to avoid making precise measurements through a structure under heavy and varying load. Independent attitude measurements are being used in the 600-foot antenna now under construction; a variety of stellar, optical, and inertial references are possible, depending on weather conditions and accuracy demands.

7. **Non-orthogonal Axis Geometry**

Various arrangements of axes of rotation are possible to permit the tracking of either a fixed star or a moving object from a base mounted on the earth. In optical astronomy, the customary arrangement is the equatorial mount, in which the first axis from the fixed base is parallel to the earth's axis. A declination axis normal to the polar axis permits acquisition of a fixed star by rotations of the two axes, after which a constant rate sidereal drive keeps the line of sight on the star. The line of sight is normal to the declination axis. This arrangement is also convenient for radio astronomy; however, it becomes extremely unwieldy and impractical when one attempts to apply it to very large dishes.
Consequently, the larger radio telescopes, such as those at Jodrell Bank or Sugar Grove, are constructed with the altazimuth arrangement of axes. The first axis of motion is vertical, i.e. an azimuth axis. The next axis is horizontal and is called the elevation axis, and the axis of symmetry of the reflector is normal to the elevation axis. Figure 7 shows the geometry of the equatorial and altazimuth arrangements. Although the altazimuth system is suitable for tracking moving objects, it requires a variable speed drive on two axes to follow fixed stars. In addition, altazimuth designs either have elevation limit stops, which, with the typical gun-mount type of support, prevent tracking through the zenith, or require an extensive support structure to permit the guidance of limit stops. Three-axis tracking systems, introducing the additional complexity of another degree of freedom, have been designed to cope with this problem in radar systems.

However, if the customary orthogonality of successive tracking axes and of the tracking line is discarded, a satisfactory solution can be obtained with only two axes. Figure 8 is a sketch of a slant axis mount with an azimuth axis, a slant elevation axis at 45° to the azimuth axis, and a reflector axis of symmetry at an angle of 45° to the slant elevation axis. This configuration can track from horizon to zenith without calling for infinite rates at the zenith, although it does call for large rates at the horizon. In practice, an object being tracked at the horizon is at considerable distance and hence requires only moderate angular velocities of the tracking system. Angular velocities and accelerations in typical tracking situations are similar for both the slant axis and altazimuth mounts, a somewhat lower azimuth acceleration for the slant axis mount being offset by a somewhat higher slant elevation acceleration.

Particular advantages of this configuration, as shown in Figure 8, are that the support of the dish can be distributed over a large area, and that the slant elevation drives can act at a large radius of action, without the use of additional support structure.
Figure 7a  Equatorial mount

Figure 7b  Altazimuth mount
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BASIC GEOMETRY OF TRACKING ANTENNA

Figure 8
The natural configuration of a support plane permits placing the slant axis drives at the perimeter of the support circle where their torque capabilities will be greatest. Another advantage is that the structure can withstand heavy environmental loads, such as those caused by hurricanes, without being controlled to some specified attitude. This eases the problems of survival and storage, which may present difficulties with extremely large structures such as the 600-foot dish now under construction.

8. **The Offset Reflector**

It is customary to use a section of the paraboloid which is centered on the axis of symmetry in both optical and radio-radar reflectors. However, this is not mandatory, since any section of the paraboloid performs the same reflecting function. If one uses the non-orthogonal axes described in the preceding section, the perimeter of a centered reflector lies in a plane making an angle of $45^\circ$ with the plane of the slant-elevation support, as shown in Figure 9a. Thus, one part of the reflector must be at a considerable distance from the support plane, implying extra structural members to connect the two surfaces, asymmetry about the slant-elevation axis, and requirement for a considerable amount of counterweighting to reduce the asymmetry.

However, if one offsets the reflector by using that part of the paraboloid which best fits the $45^\circ$ support plane, as shown in Figure 9b, one can keep most of the reflector at a modest distance from the support plane. This reduces the structure needed between the two surfaces and considerably increases the symmetry about the slant elevation axis. Thus, structure and counterweighting are reduced, and moments of inertia are drastically reduced, easing the requirements on the slant elevation drive system.
Figure 9a  Slant-axis mount with centered reflector

Figure 9b  Slant-axis mount with offset reflector.
9. Conclusion

The design principles outlined in the preceding sections appear to be feasible means of antenna construction. They seem particularly well suited to maintaining accurate geometry of a paraboloidal reflector with the tolerances desired as shorter wavelengths come into use. There is some doubt as to how well the "prescribed deflection" reflector support can be made to function, in terms of error, over a variety of conditions. However, in general, these techniques should permit drastic reductions in the amount of structural support required for extremely large dishes, primarily by permitting stress levels to reach values closer to the yield points of the materials involved. The application of these techniques and the ease of assembly of a complete structure should permit large radio telescopes to be built at a much more moderate cost than has prevailed thus far.

Although much work remains to be done before an antenna can be built on the principles described here, the authors know no reasons which would make the design impossible.