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PROPELLER STATIC THRUST

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PROPELLER STATIC THRUST

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2.0  PREFACE

This report has been prepared in support of work performed under ONR Contract NONR 1710 (00) and under Ryan Engineering Work Order No. 96-01-1075. This work has to do with studies being made in the field of direct ascent using the slipstream deflection principle.
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4.0 SUMMARY

An investigation of the static thrust characteristics of unducted propellers, using available experimental data, was made to determine means of possible improvement, methods for the rapid determination of static thrust for most conventional propellers, and to suggest areas for future experimentation. Possible improvement may lie in reducing blade twist and increasing activity factor. An expression for the useful determination of propeller static thrust \( T = 10.42 \left( \frac{F \cdot M \cdot D \cdot BHP}{\sigma^2 / 3} \right)^{\frac{2}{3}} \) was developed. Areas for future experimentation may lie in the systematic determination of the effect of varying the design parameters (solidity, activity factor, twist, etc.).
5.0 NOMENCLATURE

\[ C_T = \text{thrust coefficient} = \frac{T}{\rho n^2 D^4} \]

\[ C_P = \text{power coefficient} = \frac{P}{\rho n^2 D^5} \]

\[ T = \text{static thrust in lbs.} \]

\[ P = \text{power in ft. lbs./sec.} \]

\[ \rho = \text{mass density in slugs per ft}^3 \]

\[ n = \text{propeller speed in RPS} \]

\[ D = \text{propeller diameter in feet} \]

\[ F.M. = \text{figure of merit} = \frac{(0.798) C_T^{3/2}}{C_P} \]

\[ V = \text{velocity of airstream at the propeller disk \ fts} \]

\[ V_s = \text{final slipstream velocity \ fts} \]

\[ A = \text{propeller disk area \ ft}^2 \]

\[ A_s = \text{final slipstream area \ ft}^2 \]

\[ \sigma = \text{density ratio} = \frac{\text{ambient density}}{\text{standard sea level density}} \]

\[ C_Q = \text{helicopter torque coefficient} = \frac{Q}{\rho \Omega R^2 \Omega R^3} \]

\[ Q = \text{propeller torque in ft. lbs.} \]

\[ \Omega = \text{propeller rotational speed in radians per sec.} \]

\[ R = \text{propeller radius in ft.} \]

\[ M = \text{Mach No. (At propeller tip in this report)} \]

subscripts:

\[ P = \text{propeller} \]

\[ H = \text{helicopter} \]
6.0 INTRODUCTION

Only little attention by the aircraft industry and research agencies has been given the static thrust of propellers, because static thrust has, until recently, been significant only for the initial part of the take-off. Where a propeller is to be used as a means of providing the static thrust and thus the lift for a direct ascent type of aircraft, the subject of static thrust becomes all-important. Presented here will be a general discussion of the means of evaluating the static thrust, the available test data, fundamental propeller theory, correlation between propeller and helicopter data, and general conclusions.
7.0 METHOD OF APPROACH

Evaluation

The term, "figure of merit," has been in use for some time by the helicopter industry. It is simply the ratio of the ideal power input to a propeller as determined by the momentum theory to the actual power input obtained in test, for the same static thrust. Obviously, the higher the figure of merit, the least power will be required to develop a given thrust, or greater thrust will be developed for a given power.

The use of the propeller coefficients, $C_P$ and $C_T$, appear formidable, but are really simple expressions which are extremely useful in propeller analysis. Each is a dimensionless coefficient, like $C_D$ and $C_L$, and indicates the variation of one quantity as any other is changed. Thus, for a constant $C_T$ (which is simply $\frac{T}{\rho n^2 D^4}$), the thrust will vary as $D^4$, or $n^2$, or as $\rho$. Like $C_L$ and $C_D$ for airfoils, there will be a unique variation of $C_T$ with $C_P$ for each propeller configuration.

It is shown later in this paper that the figure of merit for an unshrouded propeller can be expressed in terms of $C_T$ and $C_P$ as simply

$$F.\ M. = (0.798) \frac{C_T^{3/2}}{C_P}$$

This is the expression in use by the helicopter industry for some time. If a given propeller is operated at the $C_P$ for the maximum figure of merit, it will develop the maximum static thrust for a given power.
Various propeller experimenters have used simply $C_T/C_p$ for the "figure of merit." Operating at the $C_p$ for a maximum $C_T/C_p$ is meaningless, since it will not give the highest static thrust for the available power.

**General**

A given propeller will develop a maximum static thrust for a given power when operating at a power coefficient where the figure of merit is a maximum. This point does not correspond to the power coefficient for maximum $C_T/C_p$. Notice that $C_p = \frac{P}{n^2D^5}$, but since $\frac{357}{\rho} = M$ for S. L.,

$$C_p = \frac{P}{\rho M^3D^3 \frac{357}{\rho}}$$

For S. L., $\rho = 0.002378$,

then $D = \frac{(0.0714) \left(\frac{\text{BHP}}{C_p}\right)^{\frac{1}{2}}}{M^{3/2}} \quad (1)$

Equation (1) is useful in finding an optimum propeller diameter. A plot of test values of figure of merit versus $C_p$ for the desired propeller solidity, etc., will yield the desired $C_p$. The tip Mach number can be taken as, say 0.8. Knowing BHP, the propeller diameter for maximum static thrust results.

The momentum theory indicates that, for an unshrouded propeller, half the final slipstream velocity is imparted ahead of the propeller and the remainder behind the propeller. This results in a contraction of the slipstream behind the propeller. Shrouding a propeller restricts this contraction and decreases the final slipstream velocity. Since the power required to develop a certain thrust is a function of the final slipstream velocity, the power required to develop a given thrust is considerably lower for a shrouded over an unshrouded propeller. (See appendix for derivation of various momentum expressions.
involving shrouding, etc.). It appears that there exists a field for development of means to expand the slipstream which would eliminate the use of a shroud.

The use of dual rotation propellers offers attractive possibilities for increasing static thrust for a given power by recovering the energy otherwise lost in rotation of the slipstream.
8.0 DISCUSSION

Review of Data

Static thrust data obtained in wind tunnel testing of propellers by extrapolating the data to a zero advance ratio is unsatisfactory from several standpoints. First, the effect of the tunnel walls is unknown. Second, the propeller slipstream is recirculated through the tunnel and results in an effective advance ratio.

Curves have been plotted of figure of merit versus $C_p$ at a tip Mach Number of 0.5 (where possible) using the data from the reports to obtain figure of merit. These curves are presented as Fig. 1 (data of ref. a), Fig. 2 (data of ref. b), Fig. 3 (data of ref. d), Fig. 4 (data of ref. c) and Fig. 5 (data of ref. f, h, i, and j).

Of the static thrust data available, the work of David Bierman in 1940 is probably the most reliable (ref. a). The test stand was mounted outdoors away from buildings, and high enough to minimize any ground interference. The data contained in ref. b is good, but is limited in scope because of the unique propeller configuration. The data of ref. c and d was performed to test the effect of using laminar flow airfoil sections for reduction in compressibility losses at the higher Mach numbers. There is rather poor agreement between the results of ref. c and d. The very low figure of merit values obtained with a symmetrical propeller section of ref. c are difficult to explain in the light of other data. The helicopter data obtained in the full-scale tunnel is useful for comparative purposes, but cannot be used for absolute values because of the severe ground effect in the tunnel. This is shown clearly
in Fig. 5 where the F.M. values obtained on the helicopter tower (where ground effect is negligible) drop normally with $C_p$ while the tunnel values increase with $C_p$. The effect of solidity is clearly shown, also, on Fig. 5.

**Effect of Solidity**

The effect of increasing solidity is to move the figure of merit curves to the right along the $C_p$ scale, and to expand the region of high figure of merit as shown.

![Graph showing the effect of increasing solidity on figure of merit curves](image)

This would be expected. As the solidity increases, the power absorbed increases approximately as the solidity for a given blade angle, etc. This is shown in Fig. 1 for the two and three blade data. It apparently makes little difference if the solidity is increased by changing the number of blades or the width of existing blades.

In order to reduce the diameter as much as possible, it appears that high solidity is desirable. High solidity permits the use of a high power coefficient, which will yield a smaller diameter for the same figure of merit (see eq. (1)).

Fundamentally, a high solidity permits the use of a small blade angle, which is necessary to yield a high figure of merit. The optimum blade angle
\( (\beta @ 0.75R) \) for the highest figure of merit, from all the data examined, is \( 8^\circ - 10^\circ \).

**Effect of Blade Airfoil Section**

The available data appears to indicate that, generally, there is little effect of blade airfoil section on the figure of merit for low tip Mach numbers. Fig. 1 indicates that, for the two-blade data, there is no consistent variation of figure of merit with airfoil section. The three-blade data indicates some shift with an increase in camber (changing from a Clark Y to an RAF6 section). The data of reference c and d indicate that, when using a laminar flow airfoil, there is a substantial effect due to camber, both on the magnitude of the maximum figure of merit and on its position along the \( C_p \) scale. The helicopter data (ref. h and i), however, shows very little effect of changing from a symmetrical section to the 23012.6 section. (This is shown on Fig. 5). At the higher tip Mach number, the use of a laminar flow airfoil is beneficial.

**Effect of Twist**

The twist is simply the total difference between the blade angle at the root \((0.2R)\) and the tip \((0.9R)\). The helicopter blades tested had either zero twist or only \(8^\circ\) twist. The propeller blades had from 20 to 36\(^\circ\) of twist. Since other configuration items were changed along with twist, it is impossible to obtain the single effect of twist. However, it is seen that the helicopter maximum figure of merit values are somewhat higher than the propeller values. The principal differences between the two are solidity and twist. Since solidity was seen by the data to have little effect on the maximum figure of merit, but merely changed its position along the \( C_p \) scale, it may be concluded that using
less twist than most conventional airplane propellers would be beneficial for static thrust. This beneficial effect of less twist might result from a change in the blade loading with radius in such a way as to increase the blade loading at the tip which might help to reduce the detrimental contraction of the slipstream.

Effect of Activity Factor

The activity factor is a function of the blade planform. The available data cannot be used to determine the effect of activity factor, because this was never varied independently of the other configuration variables. It may be surmised, however, that the effect of an increasing activity factor may be somewhat the same as a reduction in twist, causing an increase in the disk loading near the periphery, thus realizing an increase in the maximum figure of merit.

Effect of Tip Mach Number

Ref. 1 indicates that the effect of tip Mach number was to reduce the figure of merit by 7% in going from a tip Mach number of 0.5 to 0.8 using an RAF6 airfoil. The data of ref. 6, however, indicates that the maximum figure of merit increases slightly in going from a tip Mach number of 0.5 to 0.625. Ref. 6 indicates that the reduction of the maximum figure of merit with tip Mach number is considerably reduced using a laminar flow blade section. It appears that the use of tip Mach numbers in excess of 0.8 should be avoided to present excessive losses due to compressibility.
9.0 CONCLUSIONS

1. The principal factor governing the generation of large static thrust is solidity. A high solidity permits absorbing the high powers necessary for high static thrust while maintaining a high figure of merit.

2. Blade airfoil shape has only a minor effect on the propeller characteristics, except that using a laminar flow airfoil will reduce the compressibility losses at the higher tip Mach numbers.

3. The use of as little twist as possible and yet remain compatible with the high speed flight condition appears to be desirable.

4. The use of tip Mach numbers in excess of 0.8 should be avoided.

5. A convenient expression for the determination of static thrust is
   \[ T = (10.42) \left( \frac{\text{Fig. of merit}}{D \times \text{BHP}} \right)^{2/3} \sigma^{1/3} \]

6. Experimentation should be performed to determine means of increasing the figure of merit. The available data is poor from the standpoint of consistency, failure to systematically investigate the effect of varying the design parameters, and in trying to picture the physical nature of flow through a propeller developing static thrust. Systematic investigation should be made to determine the effect, independently and in combination, of the design parameters -- single or dual rotation, solidity, airfoil section, twist, activity factor, tip Mach No., and means of accomplishing the purpose of ducting.
10.0 REFERENCES


Figure of Merit Evaluation

Single Rotation

Twist = 22°

M = 0.5

Ref: NACA TR 684
FIGURE OF MERIT EVALUATION

SINGLE ROTATION

TWIST = 20°

M = 0.5

REF: NACA TN 5228

4 BLADE, NACA 16 SERIES LAMINAR FLOW, C_L = 0.4

FIG. 2
FIGURE OF MERIT EVALUATION
SINGLE ROTATION 2 BLADE
TWIST = 36°
M = 0.5

REF: NACA RM LBH250

FIG. 3
Figure of Merit Evaluation

Single Rotation 2 Blade
Twist = 30°
M = 0.5

Ref: NACA RM 651 L28

Effect of Camber:
Design lift coef. $-C_L = 0.5$

$C_L = 0.3$

$C_L = 0.5$

$C_L = 0.0$

Fig. 4
**Figure of Merit Evaluation**

**Helicopter Rotors**

\[ M = 0.3 - 0.5 \]

\[ C_T = (3.76) \text{mean} C_T = T/\rho n^2 D^4 \]

\[ C_p = (24.4) \text{mean} C_p = P/\rho n^2 D^2 \]

---

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<th>Test No.</th>
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<th>Mean Thrust</th>
<th>Mean RPM</th>
<th>Pitch</th>
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<tbody>
<tr>
<td>Tunnel 3</td>
<td>230/6</td>
<td>0</td>
<td>38</td>
<td>Small</td>
</tr>
<tr>
<td>4</td>
<td>2 Sym.</td>
<td>0</td>
<td>75</td>
<td>Wide</td>
</tr>
<tr>
<td>Tower 3</td>
<td>230/6</td>
<td>0</td>
<td>40</td>
<td>Single</td>
</tr>
<tr>
<td>Tunnel 5</td>
<td>230/6</td>
<td>0</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>230/6</td>
<td>0</td>
<td>80</td>
<td></td>
</tr>
</tbody>
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**Figure 5**
IF $T_0 =$ THRUST FOR DUCTED FAN
$T_u =$ THRUST FOR UNDUCTED FAN

$$T_0 = \frac{2^{1/3} \rho^{1/3} A^{1/3} \rho^{2/3}}{(A/\lambda)^{1/3}}$$
$$T_u = 2^{1/3} \rho^{1/3} A^{1/3} \rho^{2/3}$$

$$\frac{T_0}{T_u} = \frac{2^{1/3}}{(A/\lambda)^{1/3}} = 2^{1/3} \left(\frac{A_2}{A_1}\right)^{1/3} = 1.26 \left(\frac{A_2}{A_1}\right)^{1/3}$$

OTHER USEFUL EXPRESSIONS:

$T_u = (2 PA)^{1/3} \rho^{1/6}$

$\rho = 0.002375$  S.L. STD. COND.

$A = \pi D^2/4$

$P =$ BHP (550)

$$T_u = \left[\frac{(2 \times 0.002375) \pi^{1/6} D^{1/2} BHP^{3/4}}{4}\right]$$

$T_u = (10.42)(D \cdot BHP)^{3/4}$

$P^{3/8} = \frac{T}{(2 PA)^{1/3}} ; \quad P = \frac{T}{(2 PA)^{1/3}}$

FIG. MERIT: IDEAL POWER = $\frac{3^{1/6} \rho^{1/2} \frac{T}{PA}^{1/2}}{C_p (2 \pi m)^{1/2} \rho D^{1/2} \frac{D}{4}}$

$FIG. \text{ MERIT} = (0.795) \frac{C_T^{3/4}}{C_p}$

ALSO, $T_u = (10.42)(\text{FIG. MERIT} \cdot D \cdot \text{BHP})^{3/8}$

$\frac{T}{BHP} = (37.9) \frac{\text{FM} \cdot 0^{1/2}}{(T/A)^{1/2}}$

THrust of unDucted prop for any fig. of Merit & B.
**Relation Between Helicopter & Prop $C_T$ & $C_P(C_P)$**

**Prop.**

\[ C_T = \frac{T}{\rho n^2 D^4} \]

**Helicopter**

\[ C_T = \frac{T}{\rho (nR)^2 R^2} = \frac{T}{\frac{\pi}{16} n^2 D^4} \]

\[ T = C_T \rho n^2 D^4 = C_T \cdot \frac{\pi}{16} n^2 D^4 \]

\[ n^2 = \frac{N}{4 \pi^2} \]

\[ C_T \frac{4 \pi^2}{4 \pi^2} = C_T \frac{\pi}{16} \]

\[ C_T = C_T \frac{4 \pi^2}{16} = (7.76) C_T \]

\[ C_T = (7.76) C_T \]

\[ C_T = (0.129) C_T \]

\[ C_T = (0.129) C_T \]

**Prop.**

\[ C_P = \frac{P}{\rho n^3 D^5} \]

**Helicopter**

\[ C_P = \frac{P}{\rho (nR)^2 R^3} = \frac{2\pi n^3 \rho (nR)^2 R^3}{2\pi n^3 \rho (nR)^2 R^3} \]

\[ P = C_P \rho n^3 D^5 = 2\pi n^3 \rho (nR)^2 R^3 C_P = 2\pi n^3 \rho (nR)^2 R^3 C_P = 2\pi n^3 \rho (nR)^2 R^3 C_P = \frac{2}{3} \pi n^3 \rho (nR)^2 R^3 C_P = \frac{2}{3} \pi n^3 \rho (nR)^2 R^3 C_P \]

\[ C_P = \frac{n^3}{4} \rho C_P; C_P = C_P \frac{n^3}{4} = C_P (24.4) \]

\[ C_P = (24.4) C_P \]

\[ C_P = (0.041) C_P \]

\[ C_P = (0.041) C_P \]
Propeller Theory (Static)

\[ T = M \alpha = \rho V A V_s \quad ; \quad P = \text{A.F.E.} = \frac{1}{2} \rho V A V_s^2 \]

\[ P = \frac{T V_s^3}{2} \quad \frac{V_3}{V} = \frac{A}{A_s} \]

\[ V_s = \left( \frac{V_s}{V} \right) V \]

\[ T = \rho V A \left( \frac{V_s}{V} \right)^2 V - \rho A V \left( \frac{V_s}{V} \right) \]

\[ P = \frac{1}{2} \rho V A \left( \frac{V_s}{V} \right)^2 V^2 = \frac{1}{2} \rho V^3 \left( \frac{V_s}{V} \right)^2 A \quad \rightarrow \quad (1) \]

Solve (1) for \( V \), then \( V_3^3 \), and subst. in (2):

\[ V_3^3 \left[ \frac{T}{\rho A(V_3)} \right]^{1/2} \quad \text{from} \quad V_3^3 \quad \frac{T}{\rho A(V_3)} \quad ; \quad V = \left[ \frac{T}{\rho A(V_3)} \right]^{1/2} \]

\[ P = \frac{1}{2} \rho \left( \frac{V_s}{V} \right)^2 A \left[ \frac{T}{\rho A(V_3)} \right]^{3/2} = \frac{1}{2} \rho \left( \frac{A}{A_s} \right)^2 A_3 \frac{T^{3/2}}{A^{5/2} (A/A_s)^{3/2}} \]

\[ P = \frac{T^{3/2} \left( \frac{A}{A_s} \right)^{1/2}}{2 \rho^{1/2} A_3^{1/2} \left( \frac{A}{A_s} \right)^{1/2}} \quad V_3^3 = \frac{2 \rho^{1/2} A_3^{1/2} P^{3/2}}{\left( \frac{A}{A_s} \right)^{1/2}} \]

\[ T = \frac{(2)^{1/3} \rho^{1/3} A_3^{1/3} P^{3/2}}{\left( \frac{A}{A_s} \right)^{1/3}} = \frac{(2)^{1/3} \rho^{1/3} A_3^{1/3} P^{3/2}}{(2)^{1/3}} = (2 \rho A)^{1/3} P^{3/2} \]

For unducted, \( (A/A_s) = 2 \),

\[ T = \frac{(2)^{1/3} \rho^{1/3} A_3^{1/3} P^{3/2}}{(2)^{1/3}} = (2 \rho A)^{1/3} P^{3/2} \]