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SOME NOTES ON LOGICAL PRODUCTS AND ASSOCIATIONS

TECHNICAL REPORT NO. 9

PREPARED UNDER

CONTRACT NO. Nonr-1305(00)

for

THE OFFICE OF NAVAL RESEARCH

FC

APRIL
1955

DOCUMENTATION
Incorporated

WASHINGTON 8, D.C.

SOME NOTES ON LOGICAL PRODUCTS AND ASSOCIATIONS*

In the paper Class Definition and Code Construction[1], which is Paper Number VI of the series "Machine Literature Searching", the authors employ the algebra of classes and its symbols in a manner which is certainly questionable if not actually wrong. But since most of what is said is fairly clear in spite of the unfortunate use of symbols and the algebra of classes, we did not feel it necessary to bring this fact to the attention of the readers of American Documentation. But once again the dictum attributed to Leibniz, one of the great progenitors of modern symbolic logic, to the effect that a good symbolism accomplishes half the activity of thinking, is proven by this instance of its disregard. For the careless use of symbols in this paper has led to actual substantive error in Paper Number VII and in the paper by Tyler, Myers, and Kuipers, both[2] of which appeared in the January 1955 issue of American Documentation.

The passage in Machine Literature Searching VI, which contains the basic symbolic awkwardness to which subsequent error is traceable is the following:

"For convenience in applying class definition to the problems of

* Preprinted for official use from the July, 1955, issue of American Documentation.

"machine searching, we shall use capital letters to denote the individual characteristics used in defining classes. If X is a characteristic used in defining the class x in Fig. 1, and Z a characteristic used in defining class z, and if the elements of the intersect are all characterized by both X and Z, then the intersect $x \cap z$ is characterized by the logical product

$$X \cdot Z.$$

Similarly, the horizontally shaded region corresponds to the intersect of x and y, symbolized by $x \cap y$. This intersect may be characterized as the logical product $X \cdot Y$, if in the same way as before, all the elements in the intersect are characterized by both X and Y. The cross-hatched area is the intersect of y with $x \cap z$ and may be symbolized as

$$y \quad (x \cap z)$$

and this intersect may be characterized, in the same way as before, by the logical product

$$Y \cdot X \cdot Z$$

which is, obviously, the same as

$$X \cdot Y \cdot Z \quad \text{or} \quad Z \cdot X \cdot Y, \text{ etc.}$$

(in the algebra of classes with which we are concerned, the commutative law prevails)." (3)

The statement included in this passage that ". . . . . the intersect $x \cap z$ is characterized by the logical product $X \cdot Z$" doesn't really make sense because the intersect '$x \cap z$' <u>is</u> the logical product of of the class 'x' and the class 'z'. Further, the symbol ' • ' is usually

read as the sign of conjunction between propositions. In another part of the above quoted passage, the statement, "If X is a characteristic used in defining the class x and Z a characteristic used in defining the class z, and if the elements of the intersect are all characterized by both X and Z . . ." the second "if" clause is redundant. The elements or members of a class which is the intersect or logical product of two other classes, must by definition be characterized by the characteristics which define the members of the intersecting classes. Actually, classes are properties taken in extension and properties are classes taken in intension. According to Quine(4), "It matters little whether we read '$x \in y$' as 'x is a member of the class y', or 'x has the property y' . . . . Classes may be thought of as properties in abstraction from any differences which are not reflected in difference of instances. For mathematics certainly [including the algebra of classes] and perhaps for discourse in general there is no need of countenancing properties in any other sense". Apparently this failure to observe, at this point, the logical identity of classes and properties or characteristics leads in the passage from Machine Literature Searching VI to the use of two sets of symbols, "x" for the class x and "X" for the property X. Then, with two sets of symbols for classes and properties, there is generated an apparent need for two sets of relations: "$\cap$" for intersect and "$\cdot$" for product. As we have noted above, the "$\cap$" is usually considered the symbol for the product of two classes or properties and the "$\cdot$" is usually considered the sign of conjunction between propositions. The conjunctive relation between propositions is analogous to, but by no means identical with, the relations of product

between classes.

Following Quine, "The Class $\hat{z}(z \in x \cdot z \in y)$, which has as members the common members of x and y is called the <u>logical product</u> of x and y and designated by the abbreviated symbolism '$x \cap y$' ".[5] This defination can be stated symbolically as

$$x \cap y = \text{df } \hat{z}(z \in x \cdot z \in y)$$

In describing further the symbol "$\cap$", Quinne says "Like all binary connectives '$\cap$' is to be understood as carrying with it a pair of parentheses . . . . . In practice, however, the parentheses will be dropped when there is no danger of confusion".[6] Quine, of course, is here referring to his own practice in his book, <u>Mathematical Logic</u>; but the missing parentheses do occasion confusion in the series of papers by Dr. Perry and his associates and in the paper by Tyler, Kuipers, and Myers. Again referring to the passage from Machine Literature Searching VI, we find that the authors write "$y \cap (x \cap z)$" with one set of parentheses whereas the rigorous form would be "$[y \cap (x \cap z)]$". But when they write the logical product "$Y \cdot X \cdot Z$", they omit all parentheses. Actually, the use of parentheses does not affect the meaning of a series of logic products, e. g., $Y \cdot X \cdot Z$ is identical with $Y \cdot (X \cdot Z)$ and $(Y \cdot X) \cdot Z$, etc. The theorem which states the associativety of the relation logical product is

$$(z)\ (y)\ (x) \quad (x \cap y) \cap z = x \cap (y \cap z) \quad \text{(Quine, Theorem 286, page 181).}$$

This failure to carry through the associativety (and the idempotence) of the product relation leads, in Machine Searching VI, to some unfortunate used of parentheses, e.g., $(A \cdot B \cdot C)\ (C \cdot B \cdot F) - H$.[7] Since

the authors have earlier in the paper introduced the symbols " $\langle\ \rangle$ " as indicating the requirement that the elements constituting the product be in a certain order, the parenthesis can only be used here to indicate grouping or what both January papers refer to as "association". But even in elementary arithmetic and ordinary algebra we learn that grouping has no effect on the product of a series of elements. The expression [2 x 3 x 4 (5 x 6)] [7 x 8] is exactly equal to the expression, 2 x 3 x 4 x 5 x 6 x 7 x 8. Similarly, in the algebra of classes, the expression $(A \cdot B \cdot C)\ (C \cdot B \cdot F) - H$ is equal to the expression $(A \cdot B \cdot C \cdot F) - H$. [In the algebra of classes, the idempotence of " • " means that $C \cdot C = C$ and not C^2].

The symbols $\langle\ \rangle$ as enclosing a logical product also occasions some dismay. We have previously been told, and quite correctly, that the relation of logical product is commutative, that $X \cdot Y \cdot Z$ is identical with $Y \cdot X \cdot Z$ and $Z \cdot Y \cdot X$. What then does the " • " symbolize in the expression $\langle A \cdot B \rangle$? If it still means logical product, then $\langle A \cdot B \rangle$ is a contradiction, because the expression then says that $\langle A \cdot B \rangle$ is identical with and not identical with $\langle B \cdot A \rangle$.

It is when we come to Machine Literature Searching VII and the paper by Tyler, Myers, and Kuipers that this misuse of symbols has its most serious consequences. Apparently, the parentheses and brackets are in some way to distinguish between the supposed <u>mere</u> juxtaposition or succession of characteristics in some types of coordinate indexing on one hand, and the complex searching possibilities of the type of machine literature searching envisioned by Perry and his associates, and described

by Tyler, Myers, and Kuipers, as characterizing the Kodak Minicard System on the other. Thus, Machine Literature Searching VII gives the following figure "• • • •" as indicating mere succession and the figure [• • • (• •)][• •] as indicating "relationships between associated symbols".[8] Machine Literature Searching VII refers to the Minicard paper on this point where it is stated more fully:

> "Up to the present, many searching methods have used index data in a system as unassociated symbols. Some types of 'co-ordinate indexing' are examples of this kind of approach which corresponds to Case I shown in Figure 10. Index information, when transformed from ordinary language to the symbols in a system of this type, can convey only a certain percentage of the information content. In Case II, also shown in Figure 10, some elementary syntax has been added. Boundaries may specify that certain symbols are associated. With these boundaries, it is often possible to convey more of the information content of the index data than is possible in Case I. The Minicard System's capability corresponds to what has been indicated in Case II."

The "Figure 10" referred to above represents Case I as

• • • Unassociated Symbols	and Case II as	[• • • (• •)][• •].

The fact is that succession and juxtaposition are spatial and not logical relations and are meaningless when applied to any indexing operation. When we write out "• • •" we are writing an elliptical expression either for the sum or the product of the elements, most often, the product. In ordinary algebra A B is read or understood as A x B.

And as a product "• • •" can be written (• •) •, or, [• (• •)] without in any way affecting its meaning. Similarly, the expression [• • • (• •)][• •] is identical in any logical or meaningful sense with the expression "• • • • • • •".

It would be regrettable if this poor symbolism and worse logic should prejudice anyone against the Kodak Minicard System or the general possibility of machine literature searching. These remain as solid promises of the future even though they are limited, as we all must be, by the requirements of a rigorous logic.

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1. Perry, J.W., Berry, M.M., and Kent, Allen. Machine Literature Searching VI. Class Definition and Code Construction. "American Documentation", Vol. 5, No. 4, October, 1954. pp. 238-244. Referred to hereafter as Machine Literature Searching VI.

2. Luehrs, F.V. Jr., Kent, Allen, Perry, J.W., and Berry, M.M., Machine Literature Searching VII. Machine Functions and Organization of Semantic Units. "American Documentation", Vol. 6, No. 1, January, 1955, pp 33-39. Referred to hereafter as Machine Literature Searching VII.

   Tyler, A.W., Myers, W.L., and Kuipers, J.W. The Application Of The Kodak Minicard System To Problems Of Documentation. Ibid, pp 18-30.

3. Machine Literature Searching VI, p. 239.

4. Quine, W.V. Mathematical Logic, Cambridge Harvard University Press, 1951, pp. 120-121.

5. Ibid, p. 179.

6. Ibid, p. 180.

7. Machine Literature Searching VI, p. 242.

8. Machine Literature Searching VII, p. 33, 34.

9. Tyler, Myers, and Kuipers. Op. Cit. pp. 29-30.

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