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SOME THEORETICAL ASPECTS OF THE LOT PLOT SAMPLING INSPECTION PLAN

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Introduction.

Industrial sampling plans for the most part are of one of two types.

1. Attributes plans: Here a sample is taken and each item in the sample is adjudged defective or nondefective. (for example, is a rivet long enough, or will a bolt pass through this hole but not pass through this smaller hole?) If sufficiently few items in the sample are defective, the lot is accepted since the evidence indicates a small proportion of defective items in the lot.
2. Variables plans: Here a sample is taken and each item in the sample is measured. (The question is, "How long is this rivet?," rather than "Is it long enough?") From the mean of the observations and some measure of product variability, such as the sample standard deviation, or range, or a known value of the true process standard deviation, a decision is made as to the fraction of defective items in the lot. Making this decision involves the assumption that the distribution of measurement in the lot is normal.

Variables plans ordinarily require many fewer observations than attributes plans because they exploit the information about the shape of the distribution. On the other hand if the distribution is not normal this may vitiate the decisions reached from a variables plan; whereas, the decisions reached from an attributes plan are in no way disturbed.

Both types of sampling plans may be regarded as alternatives to 100% inspection. Where it is imperative to assure a very small fraction defective remaining in accepted lots, 100% inspection may be used. However, much experience indicates that 100% inspection is often less than 100% effective - that defective items, though inspected, may not be identified and removed. Since this sort of condition arises largely from monotony and inspector fatigue, it is often felt that for large lots a good sampling plan is a better recourse than 100% inspection. In addition 100% inspection is ordinarily much more expensive than sampling inspection.

During recent years there has been widespread use of a third sort of substitute for 100% inspection. This is the Lot Plot Plan, developed by Dorian Shainin, which is a sort of modified variables plan. It has a novel feature however; it is intended to have the advantages of a variables plan without being sensitive to non-normality. In striving to achieve this objective, much reliance is placed on the appearance of the sample histogram (the sample consists of 50 items). The Lot Plot Plan is specifically proposed as a method for assuring smaller fraction defective in accepted lots. than can be achieved by 100% inspection - or even multiple 100% inspection [10].

It is the purpose of this paper to consider theoretical aspects of the Lot Plot procedure. The reader is also referred to a paper by C. C. Craig [3].

Brief Description of Plan.

The inspector draws a random sample of 50 items from the lot. He records the measurements in sets of five, at the same time plotting them on a conveniently arranged specially prepared form. When all 50 have been

measured and recorded he has a histogram of from 7 to 16 intervals[1] (which facilitates quick calculation of $\bar{x}$) and the data themselves recorded in consecutive sets of five (which facilitates the calculation of $\bar{r}_5$ - the average of the ranges of the 10 subsamples). At this stage the inspector has three things:

1. The histogram (the Lot Plot) on which are shown the specification limits.
2. $\bar{x}$, the sample mean
3. $\bar{r}$, the average range.

How the statistics are used in reaching a decision depends upon how the histogram looks. The histogram may appear to be one of several types listed by Shainin:

a. Normal

b. Skewed

c. Multimodal

d. One sided (for concentricity parallelism, etc.)

e. Flat topped

f. Peaked

g. Truncated

h. With strays

If the histogram looks normal then the "lot limits" are calculated as

$$ULL = \bar{x}-1.3\bar{r}$$

$$LLL = \bar{x}-1.3\bar{r}$$

and the lot is accepted if both lot limits lie inside specification limits.

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[1] After the first five observations have been drawn the inspector applies a rule of thumb given him in order to decide whether the interval of measurement is too coarse or too fine to result in such a number of intervals for the histogram.

If a lot limit lies beyond a specification limit, then the fraction of the lot lying beyond that limit is estimated (using a convenient chart) from the normal distribution, taking $\bar{x}$ for the mean and $\bar{r}$ for 2.326σ. After the fraction defective is estimated and entered on the lot plot, the entire sheet is sent to a Salvage Review Board who decides disposition.2/

If instead of looking normal the histogram looks skewed, the inspector calculates the two lot limits differently. He considers the mode as the origin and computes $\hat{\sigma}_U$ the root-mean-square average of the observations greater than the mode (including half of the frequency at the modal cell) and sets the upper lot limit at mode + $3\hat{\sigma}_U$; similarly he constructs the lower limit at mode-$3\hat{\sigma}_L$ where $\hat{\sigma}_L$ is the root-mean-square average of the observations below the modal cell, (including half of the frequency at the modal cell). If both lot limits lie within the specification limits the lot is accepted. Otherwise, the proportion defective at each specification is estimated as before, but taking the mode for the mean and the appropriate one of $\hat{\sigma}_U$ or $\hat{\sigma}_L$ for the standard deviation.

If instead of looking normal the histogram looks multimodal, the inspector calculates the lot limits still differently. In this case he considers the two "outboard modes" and if there are sufficient cases beyond them he calculates separately a $\hat{\sigma}_U$ at the upper one and a $\hat{\sigma}_L$ at the lower one and sets:

$$\text{ULL} = \text{largest mode} + 3\hat{\sigma}_U$$

$$\text{LLL} = \text{smallest mode} - 3\hat{\sigma}_L$$

If there are only a few cases lying beyond either or both of the

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2/ It is to be observed that in some plants the sample standard deviation, s, is calculated and used for an estimate of the process standard deviation. In this case the lot limits are $\bar{x} \pm 3s$. [1]

outboard modes, he first draws some additional observations and then computes the lot limits. If one of the lot limits lies beyond a specification limit the percent defective there is estimated by taking the relevant one of $\hat{\sigma}_U$ and $\hat{\sigma}_L$ and the appropriate mode as the standard deviation and mean, respectively, of a normal distribution.

The other cases listed above carry specific instructions but will not be sketched here because:

1. They are cases which arise less frequently in practice than the three above [4].
2. Description of these cases is not essential to an exposition of the evaluative work being reported on here.

The following remarks are in order at this point

1. If sample items lie outside the lot limits, the lot is not regarded as "normal", but either as "long tailed" or as "with strays."
2. If a histogram looks normal and lies well within the specification limits, the inspector may accept without caluculating any statistics at all.
3. The inspector never <u>rejects</u> a lot. He refers it (via the executed Lot Plot form) to the Salvage Review Board (often interdepartmental in composition). This board weighs the evidence in the Lot Plot form and economic considerations such as production line needs, existing inventories, disassembly costs, etc., in arriving at a disposition such as screen, return to vendor, accept, scrap, remove some good items and rework others, etc.
4. The apparent vagueness in such phrases as "looks normal" is a

real and genuine vagueness in the plan as published. In general, objective criteria (except for (1) above) are not given. It is thus entirely possible for two inspectors to arrive at different dispositions of the same lot from the <u>same sample</u>.

<u>Nature of a Full Evaluation of the Plan.</u>

It has already been said that the plan is intended to be applicable regardless of the distribution of the lot. That is, for any kind of lot - or at least for any kind of lot which might "reasonably" arise in industry - the plan is intended to give the (essentially) same tight operating characteristic.

In evaluating the plan it is thus necessary to consider how it works where the lot may have any one of various sorts of distributions. Since only a sample is inspected, a lot which is actually, say, skewed will sometimes give a histogram which appears to be normal, sometimes skewed, sometimes flat-topped, sometimes bimodal, etc. Then after a certain type of analysis is decided upon (from looking at the histogram) the sample may give lot limits leading to acceptance, or it may not; and the probability of acceptance presumably should be different for the same lot depending on what form of analysis is used.

Thus, if $f(\lambda_1 \lambda_2 \ldots \lambda_k)$ is a given distribution type with parameters $\lambda_1, \lambda_2, \ldots \lambda_k$, a full evaluation for this distribution type would require determining:

$P_f(\alpha | \lambda_1, \lambda_2, \ldots \lambda_k)$. The probability that the histogram would "look like one of type α".

$L_f(\lambda_1, \lambda_2, \ldots \lambda_k | \alpha)$ the probability of acceptance (as a function of the parameters), given that the histogram looked like one of type α.

It would then be possible to obtain the overall probability of acceptance for the distribution type f (as a function of the parameters) by the equation.

$$L_f(\lambda_1\lambda_2\ldots\lambda_k = \sum_{\alpha} L_f(\lambda_1,\lambda_2,\ldots,\lambda_k|\alpha)P_f(\alpha|\lambda_1,\lambda_2,\ldots,\lambda_k)$$

when the summation is overall lot plot "types."

There would then remain the question, "Is $L_f(\lambda_1,\lambda_2,\ldots\lambda_k)$ actually only a function of p, the fraction defective?" If so, it could be written as $L_f(p)$.

If this sort of investigation were then done for many different distribution types, a very complete account of the plan's properties under the circumstances for which it was designed would be obtained.

Carrying through such a complete study is at best horrific in principle. In this case it is also actually impossible because of the fact that the plan gives no objective criteria for concluding that a lot plot "looks skew", etc.

Necessarily then this paper will give a more modest evaluation than a complete analysis would provide.

A Counter Example.

The Lot Plot Plan cannot achieve the objective of giving a tight O. C. for every distribution. In fact it is easy to show that for certain kinds of distributions it must behave almost exactly like an attribute plan calling for 50 observations and rejection if one or more defective items are found. Consider a lot with the following composition: it contains N items whose distribution is centered half way between the two specifications, the standard deviation is so small that the distance between the specification limits is many standard deviations (such as 10 or 20), and its shape is as nearly normal as is possible for a discrete

distribution; in addition it contains k "mavericks" - items which are outside the specification limits in any way whatever. Now consider the operating characteristic. Whenever the sample of 50 items includes none of the k mavericks, the Lot Plot will lead to acceptance. Whenever the sample contains one or more of the k items the Lot Plot will say "refer to salvage". The plan will thus behave exactly like the attributes acceptance plan mentioned above.

A stronger statement can be made. It is impossible to construct <u>any</u> plan, using 50 observations, which has both of the following properties:

a. There exist lots of such high quality that the plan is nearly sure to accept them.

b. The plan is materially tighter than a 50-observation attribute plan for every kind of lot distribution.

A formal statement and proof is as follows:

Any acceptance plan which requires n observations and has the property that for every $N \geq n$ and every $\epsilon > 0$ there exist lots (of "high quality") such that the probability of acceptance, A_p exceeds $1-\epsilon$, also has the property that for any given p there exist lots with fraction defective greater than p for which $A_p > A_A - \epsilon$ where A_A is the probability of acceptance using an attribute plan with sample size n and acceptance number zero.

Proof:

Let L be a lot of size $N \geq n$ such that $A_p(L) > 1-\epsilon$.

Consider the lot $L'(K)$ formed by adding K defectives to L where K is chosen to ensure $\frac{K}{N+K} > p'$

Then

$$A_p(L') \geq \text{Prob}\left\{\begin{array}{l}\text{Sample consists only of items}\\ \text{from L and is accepted}\end{array}\right\}$$

$$= \text{Prob}\left\{\text{n observations from L}\right\} A_p(L)$$

$$> \frac{\binom{N}{n}}{\binom{N+k}{n}}(1-\epsilon) > \frac{\binom{N}{n}}{\binom{N+k}{n}} - \epsilon = A_A(L') - \epsilon$$

The nature of the counter example deserves some consideration from the practical point of view. It surely indicates that where a small fraction of strays constitutes a serious problem, the Lot Plot Plan cannot afford good protection. On the other hand it leaves open the question how should it behave in situations where we have normality, or where the nonnormality is not a matter of strays?

<u>The Use of a 50 Observation Histogram to Detect Nonnormality.</u>

The most distinctive feature of the plan is the use of the histogram to warn the user when the normality assumption is invalid and to cause him to employ special techniques. Thus it is natural to enquire as to the efficacy of the plan in this respect.

Craig [3] reports the results of some sampling experiments in which he forcefully raises the question what <u>can</u> one learn about the shape of a distribution from a histogram based on 50 observations? Other sampling experiments have been done [4] again casting severe doubt on the possibility of greatly profiting from a 50-observation histogram. Here a more theoretical view will be taken.

Because the Plan is very vague as to what sorts of lot plots are to be regarded as skew, what sorts as normal, what sorts as bimodal, etc., it is hardly possible to answer the question "How reliably will the Plan protect against nonnormality?" Instead we will ask how well can <u>any</u>

procedure using 50 observations in grouped frequencies identify non-normality?

Since it is the aim of the Plan to give protection against <u>any</u> kind of departure from non-normality it is fair to focus our attention on "omnibus" or "shotgun" tests. We know of one such test, whose power we can compute, and which has the encouraging property of being a likelihood ratio test (for grouped frequencies against the class of all alternatives). This test is the χ^2 test. Accordingly we computed the power of the χ^2 test against various non-normal alternatives. See Figures I, II, III, IV. This was done in the following way. The null hypothesis specified a unit normal distribution; the cells for the χ^2 test were those given by the deciles of the unit normal (this should give a <u>more</u> sensitive 10-cell test than equal length cells according to results of Mann and Wald [6]). Under the null hypothesis the expected cell frequencies are all 5. For various competing distributions having mean zero and unit variance the cell expectations were evaluated. From these the power of the 10-cell χ^2 test could be evaluated using methods given by Patnaik [7]. The results are shown in table 1.

Another approach to the problem was also taken. The same competitors and cell intervals were used but the test was allowed in each case to be tailor made for maximum power against the alternative. This is accomplished by using the Neyman-Pearson criterion for testing a simple hypothesis against a simple alternative. The hypothesis is that the chosen intervals have the probabilities $\{p_i\}$ specified by the normal distribution; the alternative is that the chosen intervals have the probabilities $\{p_i'\}$ specified by the particular alternative distribution. The Neyman-Pearson criterion then gives the optimum test of the hypothesis against the alternative; optimum

in the sense that no more sensitive test can possibly be constructed.[3/]

Let the observed frequencies be

$$n_i \qquad i=1,\ldots 10.$$

The Neyman-Pearson test is to reject for sufficiently small values of

$$\log \prod_{i=1}^{10} \left(\frac{p_i}{p_i'}\right)^{n_i} = \sum_{i=1}^{10} n_i \log\left(\frac{p_i}{p_i'}\right)$$

Or if we define $w_i = \log \frac{p_i}{p_i'}$ we reject for sufficiently large values of

$$\sum_{i=1}^{10} n_i w_i$$

Since the n_i have a symmetric multinomial distribution under the null hypothesis we should expect the distribution of the weighted sum to be fairly well approximated by the normal distribution with

$$\text{mean} = 50\sum w_i p_i$$

$$\text{variance} = 50\sum w_i^2 p_i - 50\left(\sum w_i p_i\right)^2$$

Thus the test with significance level α becomes, Reject if:

$$\sum_{i=1}^{10} n_i w_i > 5\sum w_i + z^{(1-\alpha)}\sqrt{5\sum w_i^2 - \tfrac{1}{2}\left(\sum w\right)^2}$$

when we define $z^{(1-\alpha)}$ by:

$$\epsilon = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z(\epsilon)} e^{-t^2/2}\, dt$$

Under the alternative hypothesis, $\{p_i'\}$, the statistic $\sum_{i=1}^{10} n_i w_i$ will have

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3/ The power of this test for any given significance level would necessarily exceed that of the Lot Plot test of normality if the Lot Plot used the same choice of cell intervals.

$$\text{mean} = 50\sum w_i p_i' \text{ , and}$$

$$\text{variance} = 50\sum p_i' w_i^2 - 50(\sum p_i' w_i)^2$$

again using a normal approximation, the power of the test which is:

$$P\left\{\sum w_i n_i > 5\sum w_i + z^{(1-\alpha)}\sqrt{5\left(\sum w_i^2 - \frac{(\sum w_i)^2}{10}\right)} \,\middle|\, \{P_i\}\right\}$$

may be approximated as:

$$P\left\{z > \frac{-50\sum w_i(p_i'-p_i) + z^{(1-\alpha)}\sqrt{5\left(\sum w_i^2 - \frac{(\sum w_i)^2}{10}\right)}}{\sqrt{50\sum p_i' w_i^2 - 50(\sum p_i' w_i)^2}}\right\}$$

where z has a unit normal distribution.

The power of this test against the same alternative distributions is shown in table 2. Admittedly study of the values in tables 1 and 2 does not tell us directly how the Lot Plot "test of normality" will behave for these distributions. However, both of the tests considered do enjoy certain optimum properties, and there is every reason to suppose that the combination of an inspector and a Lot Plot diagram will yield a test of normality less sensitive than the second of these and probably less sensitive than either. If this be granted then, from a study of either table 1 or 2 we reach two conclusions.

1. The Lot Plot Plan, relying on a 50-observation histogram to identify non-normality, cannot be relied upon to protect from distributions as nearly normal as these.
2. Distributions as nearly normal as these will <u>very often</u> be regarded as normal, and it is worthwhile to see what sorts of decisions will be made when the normal analysis is applied to them.

Effects of using the "Normal Analysis."

A. General remarks.

We now concern ourselves with examining the results of an acceptance procedure which accepts if both

$$\bar{x} + 3\frac{\bar{r}}{d2} \quad \text{and} \quad \bar{x} - 3\frac{\bar{r}}{d2}$$

lie between the specification limits, and which otherwise rejects. Since the Lot Plot is sometimes used with the sample standard deviation rather than $\bar{r}/d2$ for the estimate of σ, the problem will also be investigated for that case. The behaviour of these procedures will be examined for the normal distribution and a few members of four types of non-normal distributions (which first appeared in the last section). These four types all share the property that the normal distribution is a limiting case. As the parameter in either the t or Γ distribution increases, the shape of the distribution increasingly resembles the normal; similarly, as the two parameters of the symmetric Beta distribution increase it goes to the normal. Finally, as the parameters μ_1 and μ_2 tend to zero in the mixed normal with $\sigma_1^2 = \sigma_2^2$, the distribution tends to the normal. In addition, all of these distributions except for the extreme mixed normal are "bell-shaped", and all of them except for the Gamma distributions are symmetric. We have already seen that if indeed the distribution of the lot is one of these types there is a great probability that it <u>will</u> be subjected to the normal analysis.

In a certain sense [4] it can be said that the plan "aims" to accept

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[4] If $\bar{x} \pm 3s$ be used, as the sample size becomes large the probability tends to one that the lot will be accepted or rejected as $\mu \pm 3\sigma$ both lie (strictly) within the specification or one at least lies (strictly) outside.

if the points $\mu + 3\sigma$ and $\mu - 3\sigma$ both lie within the specifications, and otherwise to reject. Now it is clear that "3-sigma" criteria mean very different things for different distributions. Table 3 displays the probability lying beyond $\mu + 3\sigma$ and $\mu - 3\sigma$ for each of several non-normal distributions. From perusal of these values we would guess that as $\bar{x} \pm 3s$ or $\bar{x} \pm 3\frac{\bar{r}}{d2}$ were successful in approximating $\mu \pm 3\sigma$, so also would lots of identical quality tend to be accepted with widely different probabilities, depending upon the shape of the distribution.

b. <u>The case of normally distributed product.</u>

Where the product is normally distributed the joint distribution of $\bar{x}$, s is of course exactly known and numerical integration enables the probability of acceptance to be evaluated for any relation of μ and σ to the specification limits U and L. As is well known [11], the probability depends not only on p, the fraction defective, but on how the fraction defective is divided; if $p/2$ lies beyond each specification, then the probability of acceptance is greater than if all of it lies beyond one specification. (Variables plans exist for which the O.C. effectively depends on p alone [2,9]; the Lot Plot Plan is inferior to them in this regard). Figure V shows the O.C. as a function of p for various relations of μ to the specification limits..

If $\bar{x}$ and $\bar{r}$ be used, then the exact distribution of $\bar{r}$ is not available. However, an approximation due to Patnaik [8] is known to be excellent. Further $\bar{x}$ and $\bar{r}$ are independently distributed in the normal case. Thus the O.C. can be obtained by numerical integration.

In either of the above cases another method of approximately evaluating the O.C. is available. We know that $\bar{x}$ is exactly normally distributed; $\bar{r}$, being the average of ten independent ranges, may be considered approximately

normally distributed, and s the sample standard deviation is asymptotically normally distributed. Thus we may consider taking $\bar{x}+3s$ and $\bar{x}-3s$ to have a bivariate normal distribution. Similarly we may consider taking $x \pm 1.3\bar{r}$ to have a bivariate normal distribution. Table 4 shows the values of p for which the probability of acceptance (from normal lots) equals .95, .90, .10, .05 as calculated from the bivariate normal approximation, and by the exact method for s, by Patnaik's approximation for $\bar{r}$. The agreement is seen to be good.

c. Methods used for case of non-normal distributions.

When product is not normally distributed the exact distributions of $\bar{x}$, s and $\bar{r}$ are all unknown for the distributions considered here. This means that either approximation to these distributions or empirical sampling must be used to evaluate the probability that the lot limits (using either s or $\bar{r}$) will lie inside the specification limits. The course chosen in this investigation was to take the lot limits as having bivariate normal distribution. It has already been seen that this approximation is excellent for the normal, and it can fairly be hoped that it will not lead to grossly misleading results for these distributions which are "nearly" normal.

To apply the bivariate normal approximation it is necessary to obtain its parameters. The mean and variance of $\bar{x}$ are of course μ and σ^2_{50}, when μ and σ^2 are the mean and variance respectively of the distribution postulated for the lot. The mean and variance of r_5, the range of five observations, were obtained by numerical evaluation on the CPC of the integral expressions (given in [5, p.233]) which are respectively:

$$E(r_5) = \int_0^\infty [1-(1-F(u))^5-(F(u))^5]du$$

$$E(r_5)^2 = \iint_{u>v} \left\{[F(u) - F(v)]^5 - [1-F(v)]^5 - [F(u)]^5 + 1\right\} dudv$$

In these expressions F(u) is the cumulative distribution evaluated at u. Values of $E(\frac{r_5}{\sigma})$ and $\sigma(\frac{r_5}{\sigma})$ for distributions are given in table 5. It is interesting to observe how stable is the ratio $E(\frac{r_5}{\sigma})$ from distribution to distribution.

In the case of symmetric distributions the correlation between the range and the sample mean is zero since they are, after translation, even and odd functions, respectively. For the Gamma distributions the covariance between the mean and range of five observations was obtained by numerical evaluation of the following integral expression:

$$E(\bar{x}_5 r_5) = \frac{(20p+24)(2p+1)!}{2^{2p+2}p!p!} - (60p+72)\int_0^\infty F(x)(1-F(x))xf^2dx$$

when the Gamma density and cumulative function are, respectively:

$$f(x) = \frac{1}{p!} e^{-x}x^p$$

and

$$F(x) = \frac{1}{p!}\int_0^x e^{-t}t^p dt.$$

In terms of these parameters of the joint distribution of $\bar{x}_{50}$ and $\bar{r}_5$, the bivariate normal approximation to the joint distribution of the lot limits is to take:

$$u = \frac{\bar{x}_{50} + 1.3\bar{r}_5 - \mu - 1.3E(r_5)}{\sqrt{\frac{\sigma^2}{50} + 1.69\,\frac{\sigma^2_{r_5}}{10} + \frac{2.6}{10}\,\mathrm{cov}(\bar{x}_5 r_5)}}$$ and

$$v = \frac{\bar{x}_{50} - 1.3\bar{r}_5 - \mu + 1.3E(r_5)}{\sqrt{\frac{\sigma^2}{50} + 1.69\ \frac{\sigma^2_{r_5}}{10} - \frac{2.6}{10}\ \mathrm{cov}(\bar{x}_5 r_5)}}$$

as having a bivariate normal distribution with means zero, variances one, and correlation coefficient:

$$\rho_{uv} = \frac{\frac{\sigma^2}{50} - 1.69\ \frac{\sigma^2_{r_5}}{10}}{\sqrt{\left(\frac{\sigma^2}{50} + 1.69\ \frac{\sigma_{r_5}^2}{10}\right)^2 - \left(\frac{2.6}{10}\mathrm{cov}\ \bar{x}_5 r_5\right)^2}}$$

All the expressions under the square root signs simplify somewhat for the symmetric distributions.

A similar approximation was applied to the problem when the lot limits are taken as $\bar{x}+3s$ and $\bar{x}-3s$. Application of standard methods [5, ch.9] yields the following results:

$$E(\bar{x}) = \mu$$

$$E(s) = \sigma\left(1+\frac{1}{8N}\ \frac{\mu_4}{8N\sigma^4}\right)$$

$$\mathrm{cov}(\bar{x},s) = \frac{\mu_3}{2N\sigma}$$

$$\mathrm{var}\ \bar{x} = \sigma^2/N$$

$$\mathrm{var}\ s = \frac{\mu_4-\sigma^4}{4(n-1)\sigma^2}$$

In terms of these parameters (approximate in the cases involving s) the bivariate normal approximation to the joint distribution of the lot limits is to take:

$$u' = \frac{\bar{x}_{50} + 3s - \mu - 3\sigma\left(1 + \frac{1}{400} - \frac{\mu_4}{400\sigma^4}\right)}{\sqrt{\frac{\sigma^2}{50} + \frac{9}{4}\ \frac{\mu_4-\sigma^4}{(49)\sigma^2} + \frac{3\mu_3}{50\sigma}}}$$

and

$$v' = \frac{\bar{x}_{50} - 3s - \mu + 3\sigma\left(1 + \frac{1}{400} - \frac{\mu_4}{400\sigma^4}\right)}{\sqrt{\frac{\sigma^2}{50} + \frac{9}{4}\,\frac{\mu_4 - \sigma^4}{49\sigma^2} - \frac{3\mu_3}{50\sigma}}}$$

as having a bivariate normal distribution with means zero and variances one, and correlation coefficient:

$$\rho_{u'v'} = \frac{\frac{\sigma^2}{50} - \frac{9}{4.49}\,\frac{\mu_4 - \sigma^4}{\sigma^2}}{\sqrt{\left[\frac{\sigma^2}{50} + \frac{9}{4.49}\left(\frac{\mu_4 - \sigma^4}{\sigma^2}\right)^2\right] - \left[\frac{3\mu_3}{50\sigma}\right]^2}}$$

Again there is simplification for the symmetric distributions since the third central moment, μ_3 is then zero.

d. <u>Results for non-normal distributions</u>.

The distribution theory whose development has just been sketched, enables one to evaluate the probability that the lot limits will lie between any given lower and upper specification limits, L and U respectively. Since for any given distribution the fraction defective p is defined as the probability lying outside the interval L to U we can find both the value of p and the probability of acceptance associated with any pair of specification limits.

In the normal case it was pointed out that the probability of acceptance is not a function of p alone, but also of <u>how</u> p, that fraction defective, is divided between the two ends of the distribution. In general we should expect this condition to obtain for other distributions as well. Therefore, in the case of each of the symmetric non-normal distributions taken as illustrative examples in this study two of the infinitely many divisions

of p were considered: equal division of p between the two ends, and all at one end. In the case of the non-symmetric distributions there were <u>three</u> cases investigated: equal division of p, all at the left end, all at the right end.

Table 6 presents the results obtained. The values of p leading to probability of acceptance (if the "normal analysis" is always used) [5/] equal .9 and .1 are shown there for several distributions, for symmetric and one-sided division of p, and for both "range" and "standard deviation" lot limits.

Study of the figures in this table indicates several things:

Use of range or standard deviation leads to nearly identical results.

The plan is extremely "tight" for the normal distribution.

The plan is far tighter for some others of these distributions (for example the B(3,3) distribution lies entirely between U and L (p=0) and still the probability of acceptance is less than .1).

The plan is very much <u>less</u> tight for some distributions (such as the t or the Γ with all the fraction defective in the upper tail).

Skewness results in a violently strong dependence of L(p) upon the division of the fraction defective between the tails.

In considering the probability that the lot limits lie between the specification limits as a function of <u>p</u> we find a great diversity, even restricting ourselves to the symmetric distributions. Instead we may examine the probability that the lot limits lie between the specification

---

5/ That is, the probability that the lot limits lie between U and L.

limits as a function of how wide apart they are (in terms of the lot standard deviation). This is done in table 7. Here we see a rather surprising degree of uniformity. All these O.C.'s could easily be plotted on the same chart; in fact their 50% points all lie betwee .95 $\frac{6\sigma}{U-L}$ and 1.00 $\frac{6\sigma}{U-L}$.

This observation together with a remark made much earlier helps to make sense of the behavior of the plan. We have just seen that the probability of obtaining lot limits between the specification limits depends upon the distance between the specification limits $\frac{U-L}{6\sigma}$ in much the same way for all these "nearly normal" distributions. On the other hand, we earlier saw that $\mu \pm 3\sigma$ limits have greatly different values of p associated with them. From these two facts we could predict what was shown in table 6, that the behavior of the plan <u>in terms of the proportion defective</u> depends strongly upon the type of distribution.

e. <u>Estimation</u>.

When the Lot Plot looks normal, but the lot limits do not both lie between the specification limits the lot is referred to salvage with an estimate of the fraction defective beyond each limit. These extimates are:

$$\hat{p}_U = \frac{1}{\sqrt{2\pi}}\int_{\frac{U-\bar{x}}{\hat{\sigma}}}^{\infty} e^{-t^2/2}\,dt \text{ and } \hat{p}_L \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{L-\bar{x}}{\hat{\sigma}}} e^{-t^2/2}\,dt$$

where $\hat{\sigma}$ is either s or $\bar{r}/d2$.

The question naturally arises, what properties have these estimates? The answer would seem to be that for distributions, such as the t, with much probability in the tails, the estimates above will tend to be under-estimates, and for distributions with little in the tails (such as the Beta)

the estimates will tend to be overestimates. For example we see from table 6 that if the distribution of product is B(3,3) with actually zero fraction defective we may still accept with probability less than .1; the case shown in the table is for the specification limits at the ends of the (finite) range of the distribution. In such a case whenever we reject we estimate some positive value for $\hat{p}_U$ or for $\hat{p}_L$, of for both; whenever the lot is accepted the estimate for both is zero. Clearly the expected value of $\hat{p}_U+\hat{p}_L$ exceeds zero, the actual fraction defective. Similar reasoning confirms the statement made for long tailed distributions such as the t. The simple fact would seem to be that though one can estimate μ and σ with fair success for many distributions, since identical pairs of values for μ and σ denote greatly different probabilities in the tails for different distributions, we cannot hope to estimate these probabilities with uniform success by always pretending the distribution is normal.

Non Normal analysis:

Although, as we have seen, for non-normal distributions of our illustrative types the "normal analysis" will often - or indeed, usually - be used, it is profitable to investigate what will be the effects of another type of analysis when used.

In all, about 11 different sorts of analysis are distinguished in the plan. Only two of these, besides the normal analysis, receive attention in this study. These are the "skew" and multimodal cases. Since the special procedures in both these cases depend upon the mode it is very difficult to make any study of sample behavior; little if anything is known about the sampling distribution of the mode. However, we can again "look at the problem in the parameters." If the inspector knew the lot was skewed(or

multimodal) knew the mode(s), and knew the true values of σ_U and σ_L, then how would lots of various quality fare under the rule(s) which set the lot limits at

$$\text{mode} - 3\sigma_L \text{ and mode} + 3\sigma_U$$

for skewed lots, or

$$\text{smallest mode} - 3\sigma_L \text{ and largest mode} + 3\sigma_U$$

for multimodal lots? Table 8 shows that the rule - <u>if the parameters were known</u> would lead to very different results depending upon what distribution, or which end of the distribution, is being considered.

It can certainly be said that there are grounds for doubting that the special rules for these two cases, at least, will go far toward making the operating characteristics of the plan independent of the distribution of the lot.

<u>Summary and discussion.</u>

The results can be summarized as follows:

1. The lot plot plan fails - as must any 50-observation plan - in its objective of being uniformly tight regardless of lot distribution.
2. The 50 observation histogram will usually judge to be normal lots which in fact have fraction in their tails appreciably larger, or appreciably smaller than has the normal.
3. The probability that lot limits will lie between the specification limits depends upon the distance (in standard deviation units) between the specification limits in a way which is not strongly dependent upon the lot distribution; but (almost as a result) the probability of acceptance depends very differently upon the

fraction defective for different distributions. (That value of the fraction defective for which the probability of acceptance is .9 varies for cases considered between 0 (for B(3,3)) and .006 (for L_5); fractions defective with 10% probability of acceptance range from 0 (for B(3,3)) to .045 (for attributes plan n=50 c=0).

4. The procedures for estimating fraction defective will lead to systematic overestimates for some distributions, systematic underestimates for others.
5. The procedure for dealing with non-normal distribution are questionable.

Since many of these conclusions apply entirely or in some degree to any variables plan it is fair to ask how does the lot plot plan compare with other variables plans?

It is clear that when all lots are normal, existing plans are superior for several reasons. The lot plot procedure will unfortunately from time to time lead to treating a sample as coming from some other kind of distribution - say bimodal; existing variables plans will routinely treat all samples by those methods which are for the normal distribution the optimum ones. Further, plans which give one probability of acceptance for each fraction defective regardless of how it is divided between the two ends of the distribution are preferable on those grounds alone.

If lots are not normal then either lot plot or other existing variables plans may behave in strange ways. When the user requires "iron-clad protection" regardless of the lot distribution an attributes plan is needed.

Finally, there is a great inflexibility in approaching every acceptance sampling problem with the same decision procedure - "Take 50 observations at random, and.... ." Different problems may legitimately call for different

procedures. This option is not a part of the Lot Plot Plan as it is of Mil Standard 105a, or NavOrd 80 or other standard plans which give choice of AQL and sample size.

Despite the various shortcomings of the plan which have received emphasis here, it has undoubtedly been successful in many installations. What features of the plan may have contributed to its success? There is no doubt that the plan has a very definite psychological appeal arising from literally "seeing a picture" of the sample. The provision for always taking the same size sample is administratively (and psychologically) convenient, however inadequate it may be from some points of view.

If lots are usually normal and the user actually needs a very tight plan it is not a bad approximation to existing good unknown standard deviation plans, and the usual benefits of well chosen variables plans will largely accrue to the Lot Plot user under these circumstances. In cases of gross bimodality, for example, the histogram will give definitely useful information about the process which generated the lot.

Finally, the provision that though an inspector may <u>accept</u> the lot it requires the salvage review board to dispose of one in any other way should have important administrative advantages in many settings: economic factors will tend to be systematically weighed at the time of disposition of the lot; if the salvage review board contains representation from the departments which (in addition to inspection) are concerned, then, fewer dispositions of lots should result in difficulties such as production flow problems.

Table 1.

Power of χ^2 test against various non-normal alternatives.

Distribution	Level of significance	
	.05	.25
t_5	.097	.358
t_8	.062	.288
t_{13}	.053	.260
$\beta(3,3)$	.062	.284
$\beta(10,10)$	.051	.253
$\Gamma(3)$	.210	.533
$\Gamma(8)$	.097	.357
$\Gamma(15)$	.072	.307
$MN^*(\frac{1}{2};1,1;\frac{1}{2},-\frac{1}{2})$	.050	.250
$MN(\frac{1}{2};1,1;1,-1)$	.064	.288
$MN(\frac{1}{2};1,1;\frac{3}{2},-\frac{3}{2})$	.219	.546

* Mixed Normal $(p;\ \sigma_1^2,\ \sigma_2^2;\ \mu_1,\ \mu_2)$;

density function is

$$p\frac{1}{\sigma_1\sqrt{2\pi}}\ e^{\frac{-(x-\mu_1)^2}{2\sigma_1^2}} + (1-p)\frac{1}{\sigma_2\sqrt{2\pi}}\ e^{\frac{-(x-\mu_2)^2}{2\sigma_2^2}}$$

Table 2.

Power of Neyman-Pearson Test of

Normality against Various Alternatives.

Distribution	Level of significance	
	.05	.25
t_5	.310	.689
t_8	.153	.483
t_{13}	.098	.376
$\beta(3,3)$	.160	.489
$\beta(10,10)$	.075	.319
$\Gamma(3)$	.605	.899
$\Gamma(8)$	.312	.685
$\Gamma(15)$	.210	.565
$MN(\frac{1}{2};1,1;\frac{1}{2},-\frac{1}{2})$	.058	.273
$MN(\frac{1}{2};1,1;1,-1)$	.167	.505
$MN(\frac{1}{2};1,1;\frac{3}{2},-\frac{3}{2})$	.627	.912

Table 3.

Probability lying beyond $(\mu - 3\sigma, \mu + 3\sigma)$ for various distributions.

Distribution	Probability
Normal	.0027
t_5	.0117
t_8	.0085
t_{13}	.0062
$\beta(3,3)$	0.
$\beta(10,10)$	.0009
$\Gamma(3)$	0. below .0103 above
$\Gamma(8)$	0. below .0071 above
$\Gamma(15)$	.0000 below .0054 above
$MN(\frac{1}{2};1,1;\frac{1}{2},-\frac{1}{2})$	.0022
$MN(\frac{1}{2};1,1;1,-1)$	.0006
$MN(\frac{1}{2};1,1;\frac{3}{2},-\frac{3}{2})$	.0000

Table 4.

Values of p for which probability of acceptance is equal to .95, .90, .10, .05, calculated for range and standard deviation plans by both exact and approximate methods

		One-sided				Two-sided			
	Probability of acceptance	.05	.10	.90	.95	.05	.10	.90	.95
Range	Exact	.0086*	.0060*	.00024*	.00013*	.0116	.0081	.00034	.00019
	Normal Approximation	.0084	.0057	.00022	.00012	.0115	.0078	.00031	.00018
Standard Deviation	Exact	.0073	.0052	.00032	.00020	.0098	.0070	.00046	.00030
	Normal Approximation	.0071	.0051	.00030	.00019	.0101	.0070	.00046	.00032

*Patnaik approximation

Table 5.

Values of $E(\frac{r_5}{\sigma})$ and $\sigma(\frac{r_5}{\sigma})$ for various distributions.

Distribution	$E(\frac{r_5}{\sigma})$	$\sigma(\frac{r_5}{\sigma})$
t_5	2.24775	1.08931
t_8	2.29378	0.98788
t_{13}	2.31067	0.93279
t_∞	2.32593	0.86409
$\beta(1,1)$	2.30940	.61721
$\beta(3,3)$	2.32105	.79568
$\beta(10,10)$	2.33120	.82743
$\beta(\infty,\infty)$	2.32593	.86409
$\Gamma(3)$	2.25928	.96704
$\Gamma(8)$	2.29588	.91209
$\Gamma(15)$	2.30895	.89157
$\Gamma(\infty)$	2.32593	.86409
$MN(\frac{1}{2};1,1;\frac{3}{2},-\frac{3}{2})$	2.31450	.68094
$MN(\frac{1}{2};1,1;1,-1)$	2.33028	.77979
$MN(\frac{1}{2};1,1;\frac{1}{2},-\frac{1}{2})$	2.32751	.85529
Normal	2.32593	.86409

Table 6.

Values of p leading to probability of acceptance, L(p) = .90 and L(p) = .10 for various distributions for symmetric and one-sided division of p, for both range and standard deviation plans, under lot-plot "normal analysis".

	Two-sided (symmetric limits)				One-sided			
	R		S		R		S	
Distribution	L(p) = .90	L(p) = .10	L(p) = .90	L(p) = .10	L(p) = .90	L(p) = .10	L(p) = .90	L(p) = .10
Normal	.00034	.0081	.00046	.0070	.00024	.0060	.00032	.0052
t_5	.0055	.0250	.0043	.0329	.0081	.0151	.0024	.0198
t_8	.0030	.0185	.0030	.0182	.0018	.0115	.0018	.0115
t_{13}	.0016	.0139	.0020	.0135	.0010	.0092	.0012	.0089
$\beta(3,3)$	.00000	.00000	.00000	.00000	0.	.00004	0.	.00000
$\beta(10,10)$	.00002	.0037	.00005	.0034	.00002	.0035	.00004	.0032
					UL* .0045	.0276	.0042	.0264
$\Gamma(3)$	.00000	.00000	.00000	.00000	LL 0.	0.	0.	0.
					UL .0026	.0194	.0027	.0187
$\Gamma(8)$	.00000	.00003	.00000	.00002	LL 0.	.00001	0.	.00001
					UL .0017	.0155	.0020	.0149
$\Gamma(15)$	.00000	.00071	.00000	.00057	LL .00000	.00036	.00000	.00029
$MN(\frac{1}{2};1,1;\frac{1}{2},-\frac{1}{2})$	.00021	.0066	.00033	.0062	.00016	.0050	.00024	.0047
$MN(\frac{1}{2};1,1;1,-1)$	.00002	.0020	.00005	.0018	.00002	.0020	.00004	.0018
$MN(\frac{1}{2};1,1;\frac{3}{2},-\frac{3}{2})$	.00000	.00030	.00000	.00020	.00000	.00034	.00000	.00025

*UL: upper limit
LL: lower limit

Table 7.

Probability of Acceptance by Lot Plot "Normal Analysis" as a Function of the Standard Deviation

Distribution		$\frac{6\sigma}{U-L}$							
		.80	.85	.90	.95	1.00	1.05	1.10	1.15
t_5	range	.94	.86	.73	.60	.46	.35	.25	.17
	standard deviation	.87	.78	.67	.58	.47	.38	.30	.23
t_8	range	.94	.85	.72	.56	.41	.28	.19	.13
	standard deviation	.94	.85	.74	.59	.41	.28	.19	.13
t_{13}	range	.95	.84	.69	.53	.38	.25	.16	.10
	standard deviation	.97	.88	.75	.58	.40	.26	.15	.09
$\beta(3,3)$	range	.97	.88	.73	.54	.35	.21	.12	.06
	standard deviation	.99	.95	.82	.58	.34	.17	.08	.03
$\beta(10,10)$	range	.96	.86	.71	.52	.35	.22	.12	.07
	standard deviation	.99	.93	.79	.59	.38	.21	.10	.05
$MN(\frac{1}{2};1,1;\frac{1}{2},-\frac{1}{2})$	range	.95	.86	.70	.52	.34	.22	.13	.08
	standard deviation	.98	.92	.78	.57	.37	.23	.12	.06
$MN(\frac{1}{2};1,1;1,-1)$	range	.96	.88	.72	.52	.33	.19	.12	.06
	standard deviation	.99	.94	.79	.59	.38	.19	.10	.04
$MN(\frac{1}{2};1,1;\frac{3}{2},-\frac{3}{2})$	range	.99	.92	.76	.54	.30	.17	.09	.04
	standard deviation	1.00	.96	.83	.60	.33	.14	.05	.02

Table 8.

Probability above Upper mode + $3\sigma_U$

and below Lower mode - $3\sigma_L$ for various distributions.

Distribution	Probability above	Probability below
$\Gamma(3)$	.0050	0.
$\Gamma(5)$	.0036	.00005
$\Gamma(8)$	.0030	.00023
$MN^*(\frac{1}{2};1,1;\frac{3}{2},-\frac{3}{2})$	.00067	.00067

*$MN(\frac{1}{2};1,1;\mu,-\mu)$ is unimodal for $\mu \leq 1$.

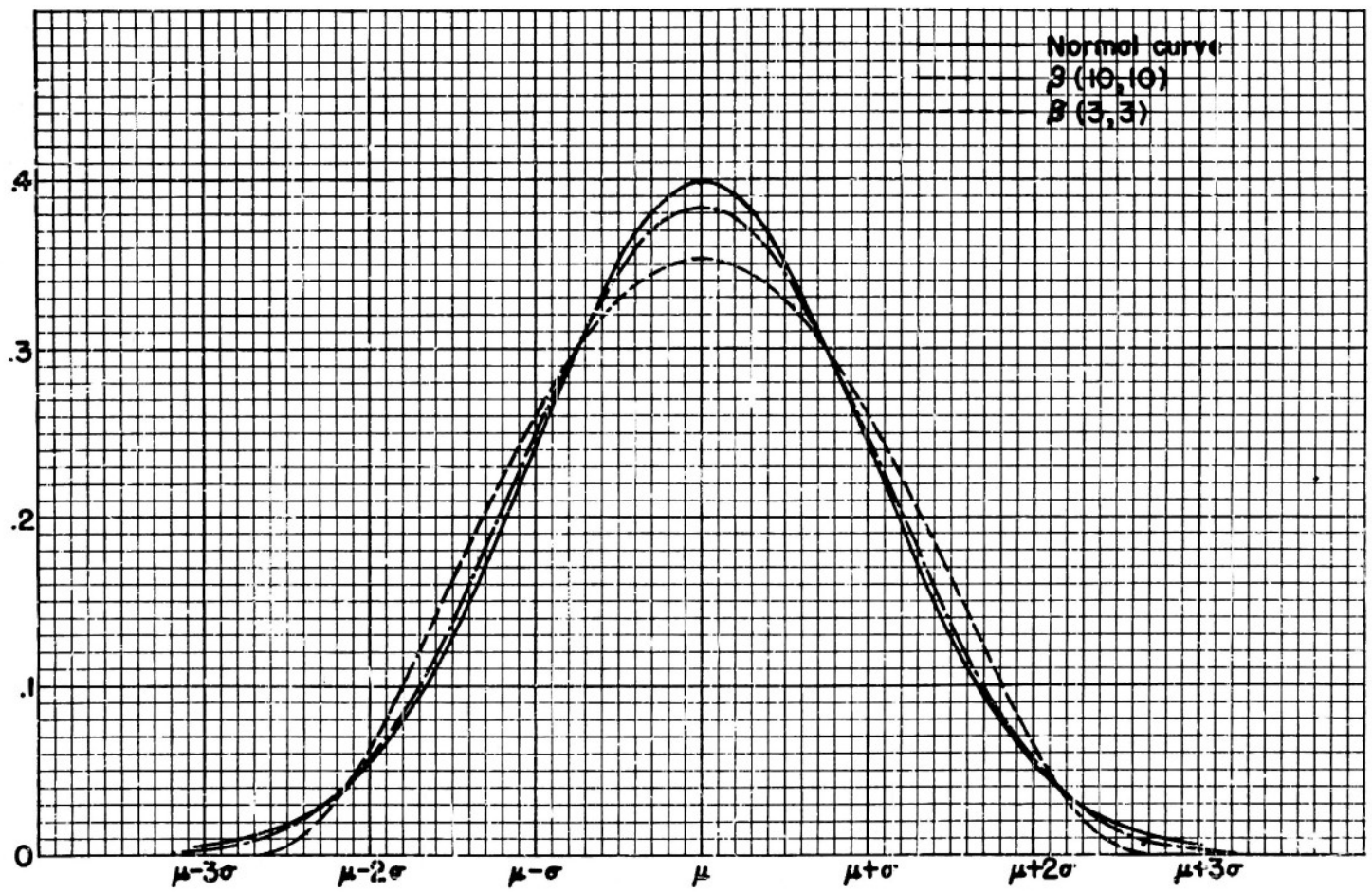

Figure I

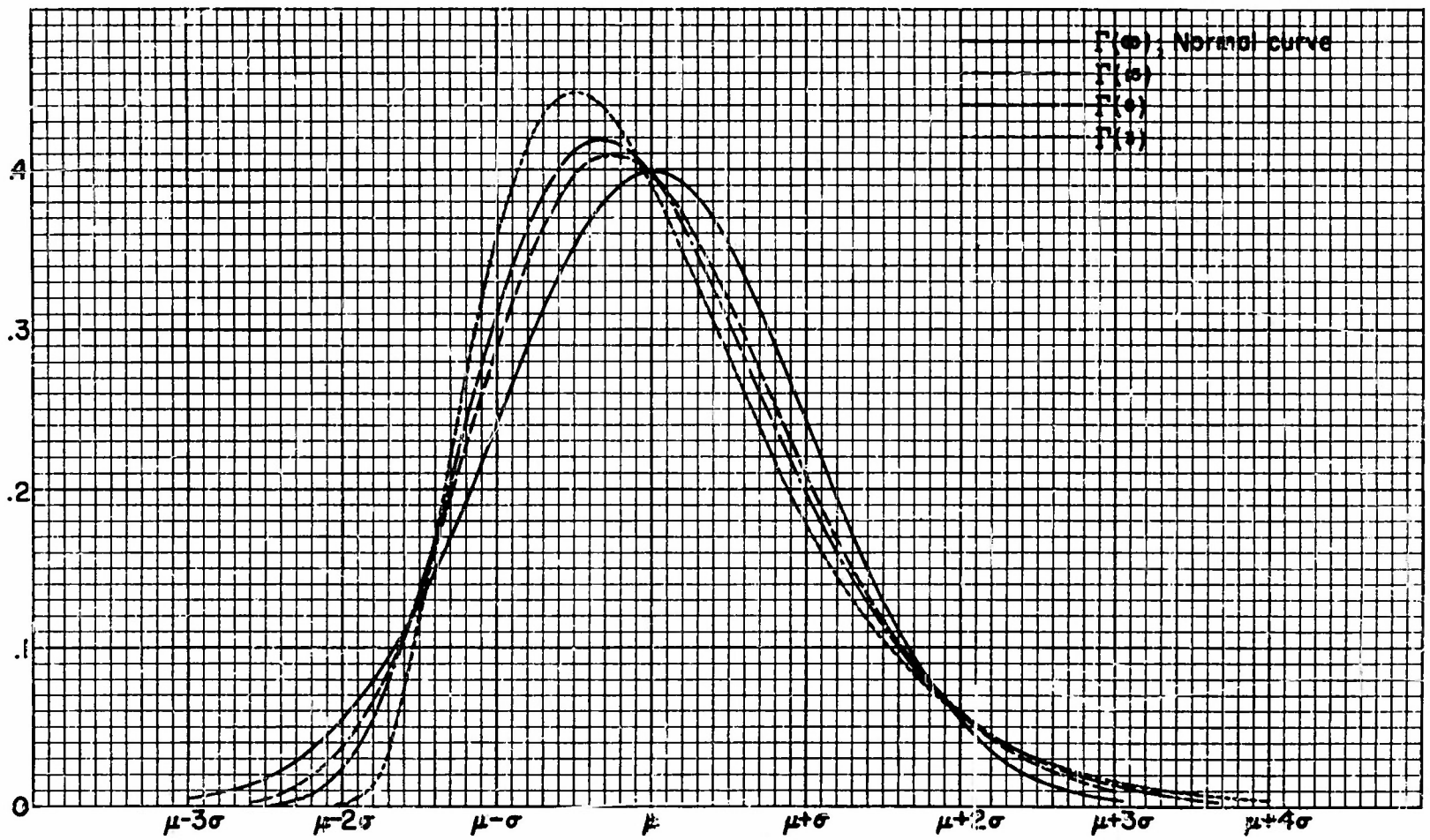

Figure II

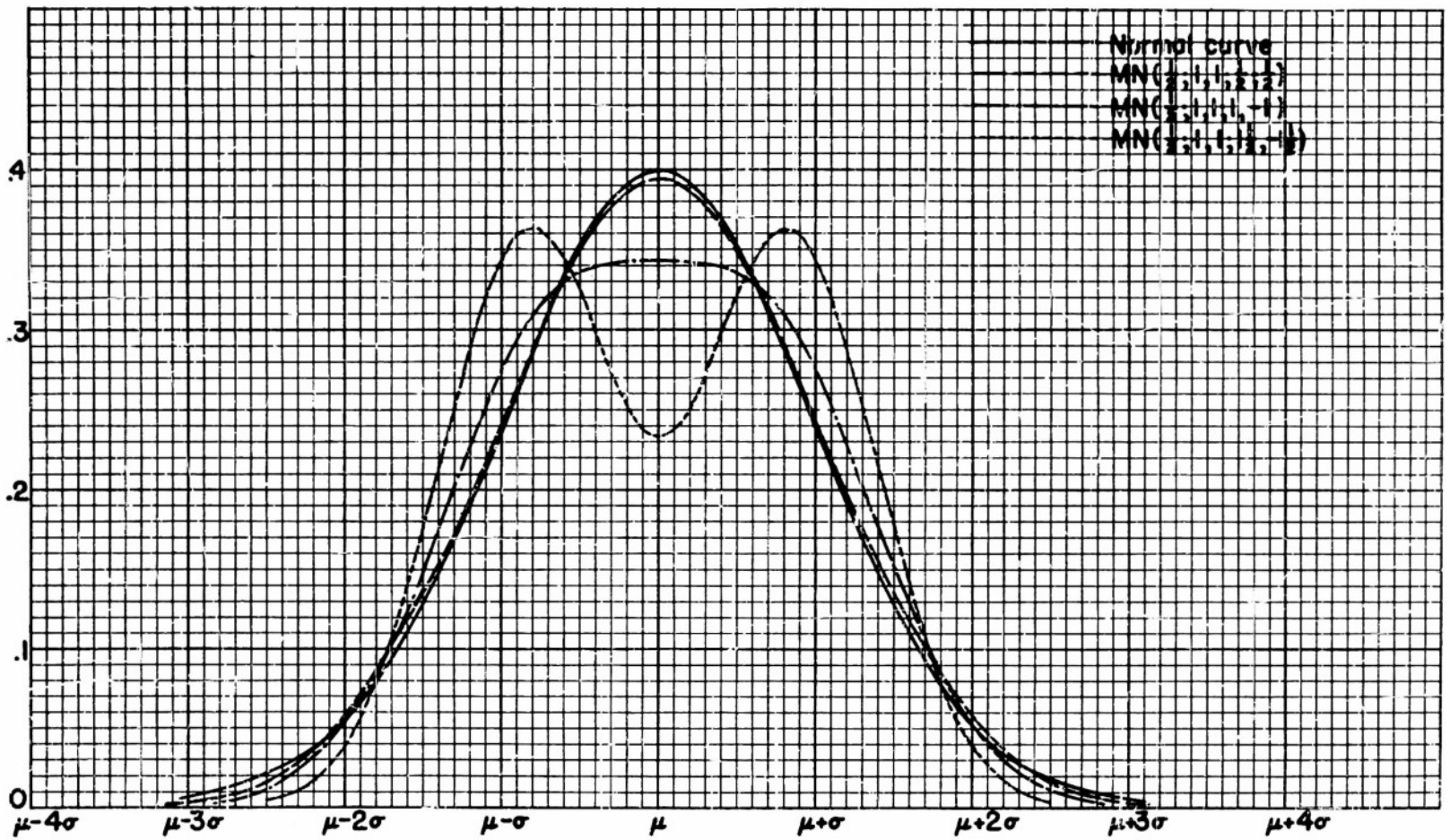
Normal curve
MN(1/2, 1, 1, 1/2, 1/2)
MN(1/2, -1, 1, 1, -1)
MN(1/2, 1, 1, 1/2, -1/2)
.4
.3
.2
.1
0
μ-4σ
μ-3σ
μ-2σ
μ-σ
μ
μ+σ
μ+2σ
μ+3σ
μ+4σ

Figure III

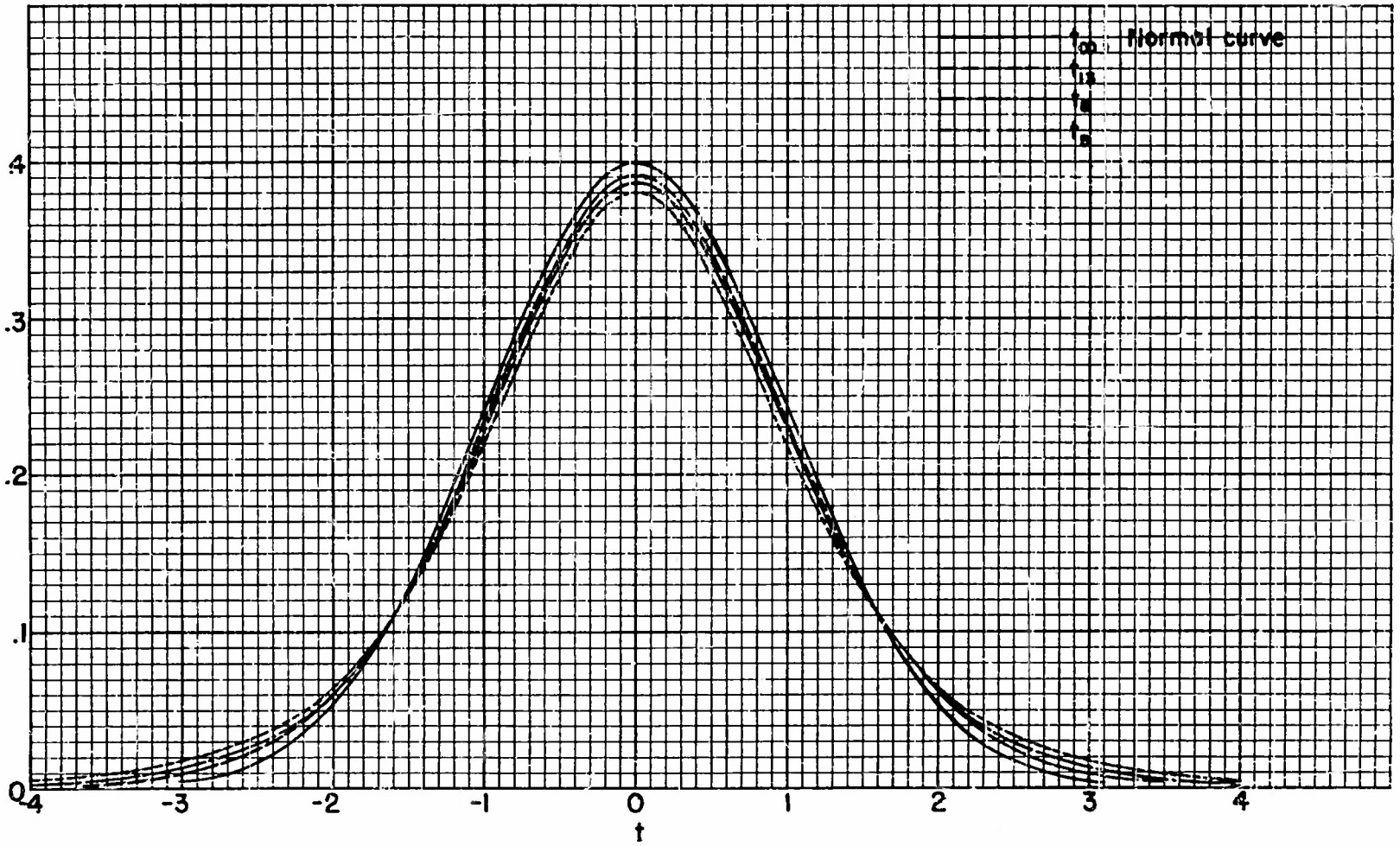

Figure IV

OPERATING CHARACTERISTICS OF LOT PLOT PLAN FOR LOTS FROM NORMAL DISTRIBUTIONS WHICH ARE TREATED AS IF NORMAL, FOR VARIOUS RELATIONS OF μ, σ TO THE TOLERANCES U AND L

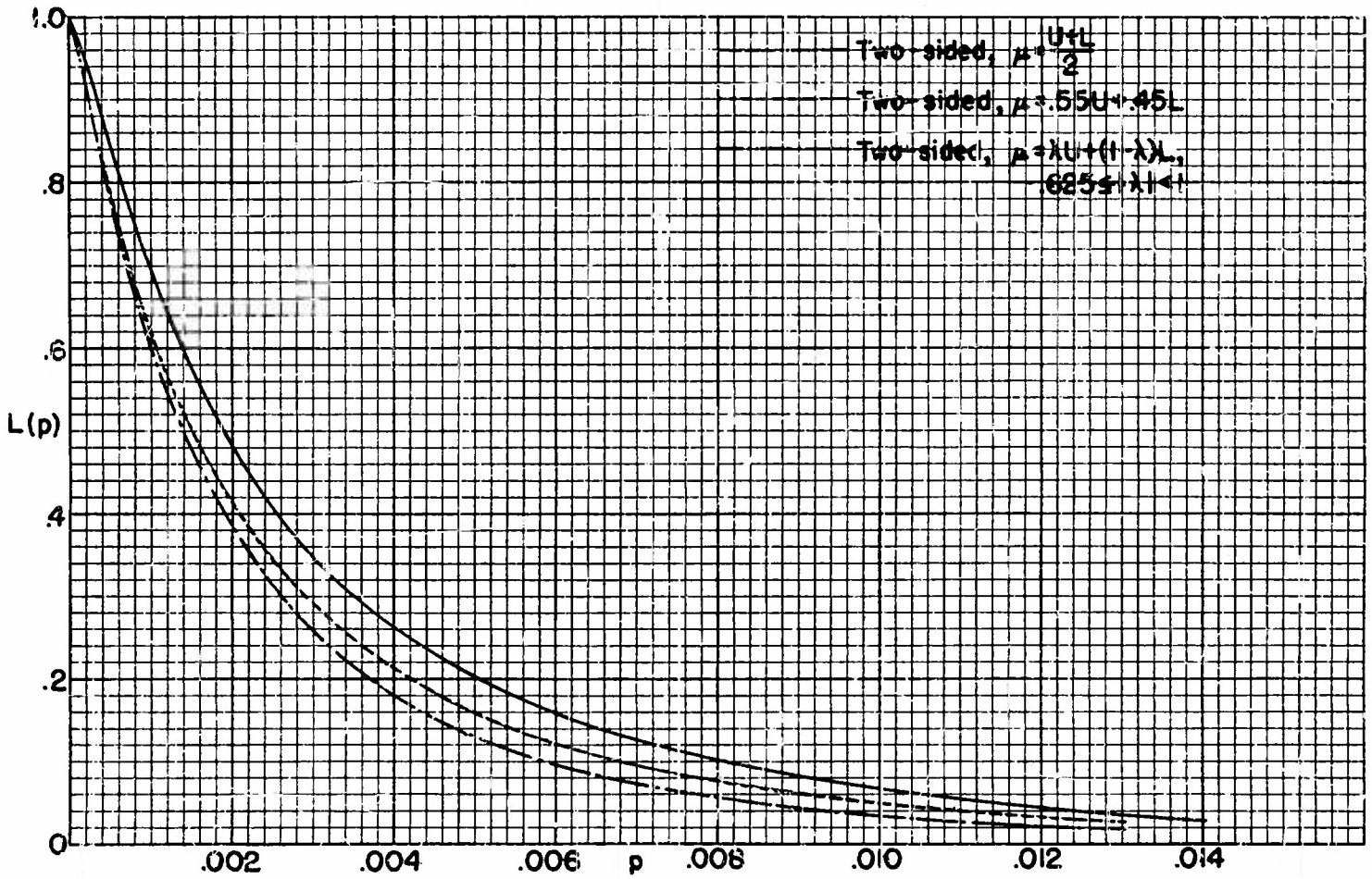

Figure 7

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