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THE DAVID W. TAYLOR MODEL BASIN

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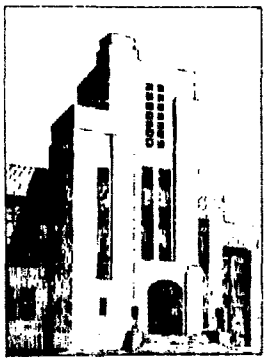
ANALYSIS OF STRESSES AT THE REINFORCED INTERSECTION OF CONICAL AND CYLINDRICAL SHELLS

by

Edward Wenk Jr., Dr. Eng. and C.E. Taylor



March 1953

Report 826
NS 731-038

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NOTATION

A	Area of stiffener
$a, b, c, d, f, g, k, m, n$	Coefficients representing edge rotation and displacement per unit edge or normal load
D	Flexural rigidity $= \frac{Eh^3}{12(1-\nu^2)}$
E	Young's modulus
H	Force normal to axis of cone, at edge
H_x	Force normal to axis of cone
h	Thickness of cone
I	Moment of inertia of the cross section of stiffener
J_0'	Derivative with respect to the argument of Bessel function of first kind and of zero order
M	Moment in a meridional plane, at edge
M_x	Moment in a meridional plane
M_ϕ	Moment in a transverse plane
N_x	Stress resultant in x-direction
N_ϕ	Stress resultant in ϕ-direction
N_0'	Derivative with respect to the argument of Bessel function of second kind and of zero order
p	Pressure acting on the outer surface of the cone
Q_x	Shearing forces
R	Distance from axis of symmetry
u	Displacement in x-direction
w	Displacement in z-direction
$\bar{w}$	Displacement normal to axis of cone, taken positive inward
$\bar{w}_0$	Same, at edge of cone
x	Coordinate taken along generator directed from tip of cone
x_0	x-coordinate of edge of cone
z	Coordinate perpendicular to generator, directed inward
α	Angle between axis of cone and generator
β	$\frac{\xi}{\sqrt{2}} - \frac{\pi}{8}$

γ	$\frac{\xi}{\sqrt{2}} + \frac{\pi}{8}$
ϵ	Strain
η	$\frac{e^{\xi/\sqrt{2}}}{\sqrt{2\pi\xi}}$
θ	Angle of rotation of tangent to meridian due to deformation
λ	$\sqrt[4]{\frac{12(1-\nu^2)}{h^2 \tan^2 \alpha}}$
ν	Poisson's ratio
ξ	$2\lambda\sqrt{x}$
σ	Stress
$\psi(\xi)$	Asymptotic values of Schleicher functions

ABSTRACT

Approximate equations are developed for the equilibrium of complete conical shells, and the results are extended for the practical stress analysis of cone-cylinder intersections reinforced with ring stiffeners. The effects of the approximation are explored to determine geometries for which the analysis is valid.

For several examples of cones of certain shapes, the approximate theory gave results in good agreement with the relatively exact solution of Dubois.

The derived coefficients of edge displacements and rotations provide a convenient method for analyzing the strength of pressure vessels which incorporate conical components.

INTRODUCTION

Conical shells are frequently used in the pressure hulls of submarines; hence design equations are required for the analysis of both the stability and equilibrium of the cone elements. Research directed toward the establishment of such criteria has been conducted at the David Taylor Model Basin as a project assigned by the Bureau of Ships and designated by the Taylor Model Basin as PROJECT CYLICONE.[1] This report presents the results of one theoretical phase of the problem related to the analysis of stresses at a reinforced intersection of a cylindrical and a conical shell.

The differential equations of equilibrium for a conical shell were developed by Dubois,[2] and this solution in terms of his specially tabulated functions has been widely accepted. These results were applied by Watts and Burrows[3] to the stress analysis of conical shells and of cone-cylinder intersections, and their equations have found frequent application in design of naval vessels. However, the numerical operations which were required are somewhat lengthy, especially if the intersection is reinforced.

As a consequence, Watts and Lang[4] recently published a tabulation of stresses developed near cone-cylinder intersections for a large number of cones of different shapes and sizes. Practical application to specific cases requires considerable interpolation, and no consideration is given to reinforcement at the joint.

In efforts to simplify the theoretical analysis, approximate solutions were obtained by Hetényi[5] who considered the cone equivalent to a set of tapered longitudinal beams on elastic foundations. Wetterstrom[6] developed a different approximation by replacing the cone with an "equivalent" cylinder. In both cases, the differential equations resemble the more exact equation of Dubois, but the approximations are not clearly defined and the errors represented by omission of terms in the exact equation have not been evaluated. Consequently, even though these approximate solutions improved the facility of stress analysis, the validity of the results remained unproved.

---

[1] References are listed on page 18.

In this report, another set of approximate solutions is obtained for the discontinuity stresses occurring near the intersections of closed conical shells with other elements of pressure vessels. The derivation makes use of Dubois' original equations for a cone. Although recognition is given to the practical needs of the engineer for generally simple expressions, approximations and departures from rigor where required in this derivation are specifically noted and evaluated. Thus errors in solution or limitations in application are specified in terms of a geometric parameter of the cone. With this analysis, many special cases of cone intersections can be treated satisfactorily. The solutions are applied particularly to the case of the cone-cylinder intersection reinforced with a ring stiffener, and equations are presented for the design of pressure vessels.

The validity of these equations is currently being investigated at the Taylor Model Basin by a comprehensive series of tests of pressurized conical shells using a wide range of geometries. Early results show excellent agreement with the theory. Results will be published in a subsequent report.

DEVELOPMENT AND SOLUTION OF APPROXIMATE DIFFERENTIAL EQUATION

From considerations of equilibrium of the shell element, Dubois established by standard elastic analysis an expression for the normal displacements of a conical shell in terms of a fourth-order differential equation. With only external pressure acting on the shell, this equation is expressed as:

$$x^2 w_{xxxx} + 2x\, w_{xxx} - 2w_{xx} + \frac{12(1-\nu^2)w}{h^2 \tan^2\alpha} = \frac{9(1-\nu^2)}{h^3 E}\, px^2 + \text{constant} \qquad [1]$$

where x is the coordinate taken along the generator from the tip of the cone,

w is the displacement in the z-direction, the subscripts indicating differentiation with respect to x,

ν is Poisson's ratio,

h is the thickness of the cone,

α is the angle between the axis of the cone and the generator,

E is Young's modulus, and

p is pressure acting on the outer surface of the cone; see Figure 1.

The stress resultants are:

$$M_x = D\left(w_{xx} + \frac{\nu}{x}\, w_x\right) \qquad [2]$$

$$M_\phi = D\left(\nu\, w_{xx} + \frac{w_x}{x}\right) \qquad [3]$$

$$N_x = D \tan\alpha \left(w_{xxx} + \frac{w_{xx}}{x} - \frac{w_x}{x^2}\right) - \frac{p\, x \tan\alpha}{2} \qquad [4]$$

$$N_\phi = \frac{d}{dx}\left(x\,N_x\right) \qquad [5]$$

$$Q_x = D\left(w_{xxx} + \frac{w_{xx}}{x} - \frac{w_x}{x^2}\right) \qquad [6]$$

where the flexural rigidity $D = \dfrac{E\,h^3}{12\,(1-\nu^2)}$ [7]

M_x is the moment in a meridional plane,

M_ϕ is the moment in a transverse plane

N_x is the stress resultant in the x-direction,

N_ϕ is the stress resultant in the ϕ-direction, and

Q_x is the shearing force.

The membrane equations are obtained if, after multiplying Equation [1] by h^3, terms containing the second or higher order in h are neglected in all equations. The particular integral of the membrane equations so obtained, which is also a particular integral of Equation [1], is as follows:

$$w_{membrane} = \frac{3}{4\,h\,E}\,p\,x^2\tan^2\alpha + \text{constant} \qquad [8]$$

Since the constant in this equation represents translation parallel to the axis, it may be neglected without affecting the elastic analysis.

The homogeneous form of Dubois' equation is:

$$x^2\,w_{xxxx} + 2\,x\,w_{xxx} - 2\,w_{xx} + \lambda^4\,w = 0 \qquad [9]$$

where

$$\lambda = \sqrt[4]{\frac{12(1-\nu^2)}{h^2\tan^2\alpha}} \qquad [10]$$

Dubois found solutions of this equation in terms of special functions and tabulated these functions and their asymptotic forms as infinite series. Nevertheless, much elaborate numerical computation is required for practical application.

From examination of the literature it was found that solutions in terms of known rather than special or complex functions could be obtained if the second-order term of Equation [9] were omitted; see Reference 5 and pages 406 to 420 of Reference 7. That is, the equation

$$x^2\,w_{xxxx} + 2\,x\,w_{xx} + \lambda^4\,w = 0 \qquad [11]$$

has as a solution

$$w = \xi\left[A_1\,J_0'\left(\pm\sqrt{i}\,\xi\right) + A_2\,J_0'\left(\pm\sqrt{-i}\,\xi\right) + A_3\,N_0'\left(\pm\sqrt{i}\,\xi\right) + A_4\,N_0'\left(\pm\sqrt{-i}\,\xi\right)\right] \qquad [12]$$

where

$$\xi = 2\lambda\sqrt{x} \qquad [13]$$

and J_0' and N_0' indicate derivatives with respect to the argument ξ of Bessel functions of the first and second kind, both of zero order.

These functions may be replaced by the following functions of Schleicher,[8] which have been tabulated and for which simple asymptotic formulas are known,

$$\psi_1(\xi) = \frac{1}{2}\left[J_0\left(\sqrt{i}\,\xi\right) + J_0\left(\sqrt{-i}\,\xi\right)\right] \qquad [14a]$$

$$\psi_2(\xi) = -\frac{i}{2}\left[J_0\left(\sqrt{i}\,\xi\right) - J_0\left(\sqrt{-i}\,\xi\right)\right] \qquad [14b]$$

$$\psi_3(\xi) = \psi_1(\xi) + \frac{i}{2}\left[N_0\left(\sqrt{i}\,\xi\right) - N_0\left(\sqrt{-i}\,\xi\right)\right] \qquad [14c]$$

$$\psi_4(\xi) = \psi_2(\xi) + \frac{1}{2}\left[N_0\left(\sqrt{i}\,\xi\right) + N_0\left(\sqrt{-i}\,\xi\right)\right] \qquad [14d]$$

If the particular integral [8] is included, then the complete solution of Equation [9] becomes

$$w = \sqrt{x}\left[C_1\,\psi_1'(\xi) + C_2\,\psi_2(\xi) + C_3\,\psi_3'(\xi) + C_4\,\psi_4'(\xi)\right] + \frac{3\,p\,x^2}{4\,h\,E}\tan^2\alpha \qquad [15a]$$

Since this report deals only with closed cones, displacements at the tip may be assumed to vanish. The functions $\psi_1(\xi)$ and $\psi_2(\xi)$ are zero at the origin, but $\psi_3'(\xi)$ and $\psi_4'(\xi)$ are not. This requires that two of the constants of integration must be zero, $C_3 = C_4 = 0$ so that:

$$w = \sqrt{x}\left[C_1\,\psi_1'(\xi) + C_2\,\psi_2'(\xi)\right] + \frac{3\,p\,x^2}{4\,h\,E}\tan^2\alpha \qquad [15b]$$

The successive derivatives of w are

$$w_x = \lambda\left[C_1\,\psi_2(\xi) - C_2\,\psi_1(\xi)\right] + \frac{3\,p\,x}{2\,h\,E}\tan^2\alpha \qquad [16]$$

$$w_{xx} = \frac{\lambda^2}{\sqrt{x}}\left[C_1\,\psi_2'(\xi) - C_2\,\psi_1'(\xi)\right] + \frac{3p}{2hE}\tan^2\alpha \qquad [17]$$

$$w_{xxx} = -\frac{\lambda^3}{x}\left\{C_1\left[\psi_1(\xi) + \frac{2}{\xi}\,\psi_2(\xi)\right] + C_2\left[\psi_2(\xi) - \frac{2}{\xi}\,\psi_1(\xi)\right]\right\} \qquad [18]$$

$$\begin{aligned} w_{xxxx} = \frac{\lambda^3}{x^2}\Big\{ & C_1\left[2\,\psi_1(\xi) - \frac{\xi}{2}\,\psi_1'(\xi) + \frac{4}{\xi}\,\psi_2'(\xi)\right] \\ & + C_2\left[2\,\psi_2(\xi) - \frac{\xi}{2}\,\psi_2'(\xi) - \frac{4}{\xi}\,\psi_1'(\xi)\right]\Big\} \end{aligned} \qquad [19]$$

C_1 and C_2 are constants of integration to be evaluated from boundary conditions at the edge of the cone. In computing these consecutive derivatives the following relationships were employed:[7]

$$\psi_1''(\xi) = \psi_2(\xi) - \frac{1}{\xi}\psi_1'(\xi) \qquad [20a]$$

$$\psi_2''(\xi) = -\psi_1(\xi) - \frac{1}{\xi}\psi_2'(\xi) \qquad [20b]$$

The following expressions for the asymptotic form apply with sufficient accuracy when $\xi > 6$ and these expressions are employed in the subsequent analysis:

$$\psi_1(\xi) \approx \eta \cos\beta \qquad [21a]$$

$$\psi_1'(\xi) \approx \eta \cos\gamma \qquad [21b]$$

$$\psi_2(\xi) \approx -\eta \sin\beta \qquad [21c]$$

$$\psi_2'(\xi) \approx -\eta \sin\gamma \qquad [21d]$$

where

$$\eta = \frac{e^{\xi/\sqrt{2}}}{\sqrt{2\pi\xi}} \qquad [22]$$

$$\beta = \frac{\xi}{\sqrt{2}} - \frac{\pi}{8} \qquad [23a]$$

$$\gamma = \frac{\xi}{\sqrt{2}} + \frac{\pi}{8} \qquad [23b]$$

EVALUATION OF THE APPROXIMATION

Since these solutions were found to be so much simpler than those of Dubois, the adoption of the approximate differential equation [11] appeared quite attractive. Consequently, the validity of this approximation has been tested by computing the error introduced by omission of the term $(-2\,w_{xx})$ from the complete equation [9]. As may be noted, terms in the equation are oscillatory and are not in phase. Thus it was not feasible to compute the error from the ratio of the omitted terms to each of the terms retained because these remaining terms periodically become zero.

Therefore the amplitude of the term omitted is compared with the amplitude of each of those retained. For example:

$$\frac{-2w_{xx}}{2xw_{xxx}} = \frac{C_1\, 4/\xi\ \psi_2'(\xi) - C_2\, 4/\xi\ \psi_1'(\xi)}{C_1[2\psi_1(\xi) + 4/\xi\ \psi_2'(\xi)] + C_2[2\psi_2(\xi) - 4/\xi\ \psi_1'(\xi)]} \qquad [24a]$$

For $\xi > 6$, the maximums of $\psi_1(\xi)$, $\psi_2(\xi)$, $\psi_1'(\xi)$, and ψ_2' are equal. If we note that $C_1 =$ constant $\times\, C_2$, then

$$\frac{\left|-2w_{xx}\right|_{max}}{\left|2xw_{xxx}\right|_{max}} = \frac{2}{\xi - 2} \qquad [24b]$$

The relative magnitude of the omitted term is given by this quotient and is limited to 3 percent, if $\xi > 69$.

It may be shown similarly that

$$\left|2xw_{xxx}\right|_{max} << \left|x^2 w_{xxxx}\right|_{max} \qquad [25]$$

so that the omitted second-order term is always much less than any of the remaining terms. From the inequality [25], it would appear that the original differential equation could be further simplified, but since the solution is not correspondingly facilitated, the third-order term has been retained. For use in subsequent analysis, the following two inequalities were established similarly:

$$\frac{\left|\frac{y}{x} w_x\right|_{max}}{\left|w_{xx}\right|_{max}} \approx \frac{2y}{\xi}, \text{ or } \frac{y}{x} w_x << w_{xx} \qquad [26a]$$

and

$$\frac{\left|\frac{w_x}{x^2}\right|_{max}}{\left|w_{xxx}\right|_{max}} \approx \frac{4}{\xi^2 - 2\xi}, \text{ or } \frac{w_x}{x^2} << w_{xxx} \qquad [26b]$$

CONSTANTS OF INTEGRATION FOR VARIOUS BOUNDARY CONDITIONS

With the verification of acceptably small error associated with the approximate differential equation, consideration is given to boundary conditions from which the constants of integration may be evaluated. Three cases have been selected from which other cases may be obtained by a process of superposition. These boundary conditions are:

A. Edge of unloaded cone subject to uniformly distributed forces H perpendicular to axis.

B. Edge of unloaded cone subject to uniformly distributed moment M.

C. Cone subject to normal pressure, with edge simply supported uniformly in an axial direction.

Case C may be treated by superposing the particular solution upon a solution for Case A in which H does not vanish. These edge loads are shown in Figure 1.

These three conditions lead to the following constants of integration:

Boundary Condition	C_1	C_2
Case A		
$p = 0$ $M_x \, I_{x=x_0} = 0$ $H_x \, I_{x=x_0} = H$	$\dfrac{-\sqrt{2}\, x_0 \cos\alpha}{D\lambda^3 \eta_0} \cos\gamma_0 \times H$	$\dfrac{\sqrt{2}\, x_0 \cos\alpha}{D\lambda^3 \eta_0} \sin\gamma_0 \times H$ $\dfrac{C_1}{C_2} = \dfrac{{}_0\psi_1'}{{}_0\psi_2'}$
Case B		
$p = 0$ $M_x \, I_{x=x_0} = M$ $H_x \, I_{x=x_0} = 0$	$\dfrac{-\sqrt{2\, x_0}}{D\lambda^2 \eta_0} \left[\sin\beta_0 + \dfrac{2}{\xi_0} \cos\gamma_0\right] M$	$\dfrac{-\sqrt{2\, x_0}}{D\lambda^2 \eta_0} \left[\cos\beta_0 - \dfrac{2}{\xi_0} \sin\gamma_0\right] M$ $\dfrac{C_1}{C_2} = -\dfrac{{}_0\psi_2 - 2/\xi_0 \, {}_0\psi_1'}{{}_0\psi_1 + 2/\xi_0 \, {}_0\psi_2'}$
Case C		
$p = p$ $M_x \, I_{x=x_0} = 0$ $Q_x \, I_{x=x_0} = \frac{1}{2} p\, x_0 \sin^2\alpha$ $N_x \, I_{x=x_0} = -\frac{1}{2} p\, x_0 \sin\alpha \cos\alpha$	$\dfrac{-\sqrt{2}\, x_0^2 \sin^2\alpha}{2D\lambda^3 \eta_0} \cos\gamma_0 \times P$	$\dfrac{\sqrt{2}\, x_0^2 \sin^2\alpha}{2D\lambda^3 \eta_0} \sin\gamma_0 \times P$ $\dfrac{C_1}{C_2} = \dfrac{{}_0\psi_1'}{{}_0\psi_2}$

The subscript 0 preceding or following a symbol designates the value of the function at $x = x_0$.

ANALYSIS OF STRESSES IN CONICAL SHELLS

In terms of the edge forces H, edge moments M, and pressure p, the displacement w perpendicular to a generator is

$$w = \frac{-\sqrt{2x}}{D\lambda^3}\,\frac{\eta}{\eta_0}\left\{\left[x_0 H\cos\alpha + \frac{x_0^2\sin^2\alpha}{2}P + \sqrt{x_0}\,\lambda\left(\frac{2}{\xi_0} - \frac{\sqrt{2}}{2}\right)M\right]\cos\left(\frac{\xi_0-\xi}{\sqrt{2}}\right)\right.$$
$$\left. + \frac{\sqrt{2}}{2}\sqrt{x_0}\,\lambda M\sin\left(\frac{\xi_0-\xi}{\sqrt{2}}\right)\right\} + \frac{3}{4hE}\,\rho x^2\tan^2\alpha \qquad [27]$$

Successive derivatives may be similarly computed in terms of H, M, and p, and the stress resultants may be obtained from these. From the inequalities of [26], it has been considered justified to adopt the following approximate expressions in lieu of Equations [2] to [6].

$$M_x = D\,w_{xx} \qquad [28]$$

$$M_\phi = D\,\nu\,w_{xx} \qquad [29]$$

$$N_x = D\tan\alpha\,w_{xxx} - \frac{px}{2}\tan\alpha \qquad [30]$$

$$N_\phi = D\tan\alpha\,x\,w_{xxxx} - px\tan\alpha \qquad [31]$$

$$Q_x = D\,w_{xxx} \qquad [32]$$

Expressions for the meridional moment M_x and transverse shear Q_x are then given as follows:

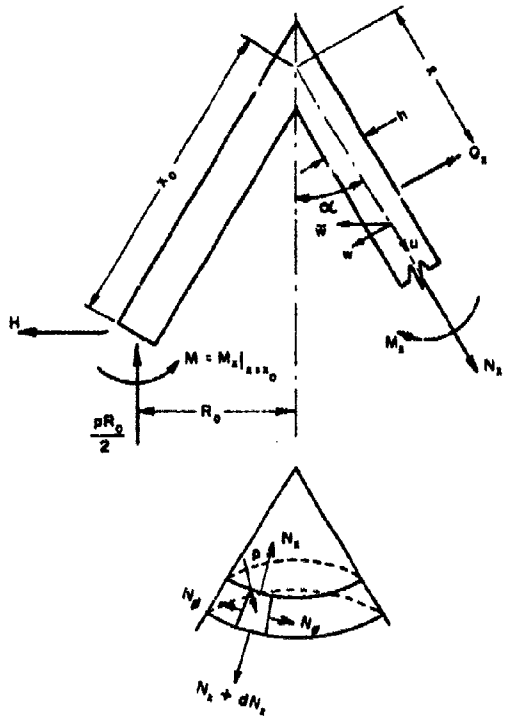

Figure 1 - Nomenclature and Sign Convention Used for Cones

$$M_x = \frac{-\eta}{\eta_0\xi}\left\{\left[2\sqrt{2}\,x_0 H\cos\alpha + \sqrt{2}\,x_0^2 p\sin^2\alpha\right.\right.$$
$$\left.\left. - \sqrt{x_0}\,\lambda\left(2 - \frac{4\sqrt{2}}{\xi_0}\right)M\right]\sin\left(\frac{\xi_0-\xi}{\sqrt{2}}\right) - 2\sqrt{x_0}\,\lambda M\cos\left(\frac{\xi_0-\xi}{\sqrt{2}}\right)\right\} \qquad [33]$$

$$Q_x = \frac{\eta}{\eta_0\xi^2}\left\{\left[2\lambda^2\left(x_0^2 p\sin^2\alpha + 2x_0 H\cos\alpha\right)\right]\left[\cos\left(\frac{\xi_0-\xi}{\sqrt{2}}\right)\right.\right. \qquad [34]$$
$$\left. + \left(\frac{2\sqrt{2}}{\xi} - 1\right)\sin\left(\frac{\xi_0-\xi}{\sqrt{2}}\right)\right] + 4\sqrt{2}\,\sqrt{x_0}\,\lambda^3 M\left[\left(1 - \frac{\sqrt{2}}{\xi_0} - \frac{\sqrt{2}}{\xi} + \frac{4}{\xi_0\xi}\right)\sin\left(\frac{\xi_0-\xi}{\sqrt{2}}\right)\right.$$
$$\left.\left. + \left(\frac{\sqrt{2}}{\xi_0} - \frac{\sqrt{2}}{\xi}\right)\cos\left(\frac{\xi_0-\xi}{\sqrt{2}}\right)\right]\right\}$$

Stresses in the outer fiber of the shell may be simply computed from the following relations

$$\sigma_x = \frac{Q_x \tan \alpha}{h} - \frac{px}{2h} \tan \alpha \pm \frac{6 M_x}{h^2} \qquad [35]$$

The circumferential stress σ_ϕ may similarly be computed in terms of N_ϕ and M_x. However, the computation of N_ϕ involves a large number of terms, and resort is made to still another approximation.

It is known that

$$\epsilon_\phi = -\frac{\bar{w}}{R} \qquad [36]$$

where R is the distance from the axis and $\bar{w}$ is the displacement perpendicular to the axis of the shell. From Hooke's law then:

$$\sigma_\phi = -\frac{E \bar{w}_a}{R} + \nu \sigma_x \qquad [37]$$

where $\bar{w}_a$ is taken as

$$\bar{w}_a = w_b \cos \alpha + \frac{(1-\nu/2) R^2 p}{Eh \cos \alpha} \qquad [38]$$

Here w_b is that part of the normal displacement w exclusive of the membrane component in x^2, and R replaces $x \sin \alpha$. This relationship is derived as follows.

From consideration of components of displacement

$$\bar{w} = w \cos \alpha - u \sin \alpha \qquad [39]$$

The error introduced by neglecting the u displacements is now investigated. From the relation

$$\bar{w} = -R \epsilon_\phi = -\frac{R}{Eh} (N_\phi - \nu N_x) \qquad [40]$$

and formulas previously written for N_ϕ and N_x,

$$\bar{w} = -\frac{\sin \alpha \tan \alpha}{Eh} \left\{ D \left[x^2 w_{xxxx} + x(1-\nu) w_{xxx} \right] - \left(1 - \frac{\nu}{2}\right) x^2 p \right\} \qquad [41]$$

On the other hand from Equation [1], with the w_{xx} term omitted

$$w \cos \alpha = -\frac{\sin \alpha \tan \alpha}{E h} \left\{ D \left[x^2 w_{xxxx} + 2 x w_{xxx} \right] - \frac{3 x^2 p}{4} \right\} \qquad [42]$$

It can be shown that the terms within the brackets differ in the two expressions by a small amount if ξ is sufficiently large. However, the membrane terms appear in the ratio of

$\left(1-\frac{\nu}{2}\right)\Big/\frac{3}{4}$, independent of ξ, so that complete substitution of $w \cos \alpha$ for $\overline{w}$ would introduce unacceptable error. Thus, $\overline{w}_a$, the approximate expression for $\overline{w}$, may be written as given in Equation [38] where w_b is that part of the normal displacement w exclusive of the membrane component in x^2, and R replaces $x \sin \alpha$.

ANALYSIS OF STRESSES AT CONE INTERSECTIONS

COEFFICIENTS FOR EDGE DISPLACEMENTS AND ROTATIONS

With the available expressions, stresses in any cone can be computed for any given H, M, and p.

If the cone is attached to any contiguous elastic structure, then H and M represent the discontinuity shears and moments at the intersection and must be computed in terms of the geometry and elastic constants of the two intersecting structures.

In general, this is accomplished by consideration of the compatibility of rotations and displacements at the joint of the two elements. For analysis, the pertinent quantities required at the open end of the cone are $\overline{w}_0$ and θ_0. For use in subsequent computations, these rotations and displacements were found to be

$$a = \frac{\theta_0^M}{M} = \frac{U^2 \tan \alpha}{E h^2}\left[U\sqrt{\frac{2R}{h \tan \alpha \sin \alpha}} - 1\right] \approx \frac{U^3}{E}\sqrt{\frac{2R}{h^5 \cos \alpha}}$$

$$b = \frac{\theta_0^H}{H} = -\frac{U^2}{E}\frac{R}{h^2}$$

$$c = \frac{\theta_0^p}{p} = -\frac{U^2}{2E}\frac{R^2 \tan \alpha}{h^2} + \frac{3}{2E}\frac{R}{h}\frac{\tan \alpha}{\cos \alpha} \qquad [43]$$

$$d = \frac{\overline{w}_0^M}{M} = \frac{U}{E}\sqrt{\frac{R}{h^3}\frac{\sin^2 \alpha}{\cos \alpha}}\left[U\sqrt{\frac{R}{h \tan \alpha \sin \alpha}} - \sqrt{2}\right] \approx \frac{U^2 R}{E h^2}$$

$$f = \frac{\overline{w}_0^p}{p} = -\frac{U}{E}\sqrt{\frac{R^5}{2h^3}\frac{\sin^2 \alpha}{\cos \alpha}} + \frac{(1-\nu/2)}{E}\frac{R^2}{h \cos \alpha}$$

$$g = \frac{\overline{w}_0^H}{H} = \frac{-\sqrt{2}\,U}{E}\sqrt{\frac{R^3}{h^3}\cos \alpha}$$

where

$$U = \sqrt[4]{12\,(1-\nu^2)} \qquad [44]$$

The rotations and displacements at the edge for combined loading are obtained by superposition as

$$\theta_0 = aM + bH + cp \qquad [45]$$

$$\bar{w}_0 = dM + gH + fp \qquad [46]$$

Equations [43] thus represent important coefficients which are very convenient in the analysis of any composite structure involving conical heads. They are especially useful in problems concerning pressure vessels.* Similar displacement and rotation coefficients for hemispherical shells, cylindrical shells, rings, and flat plates have been derived by others (see Reference 7, or Roark's "Formulas for Stress and Strain"). The expressions obtained for the cone are found in the limiting cases, $\alpha \to 0$, to agree with the well-known results of Timoshenko (pages 393-406 of Reference 7) for a cylinder, provided the origin of the x-coordinate is transformed to the open end.

ANALYSIS WITH NORMAL PRESSURE LOADING

Two Intersecting Cones

These results are now applied to the general case of two closed cones with the same base circle which are joined and subjected to external pressure; see Figure 2.

By requirements of compatibility and static equilibrium at the joint:

$$\bar{w}_1 = \bar{w}_2\,,\ \theta_1 = -\theta_2\,,\ H_1 = -H_2\,,\ M_1 = M_2 \qquad [47]$$

where subscripts 1 and 2 denote the two shells. Using Equations [45] and [46] and defining

$$\bar{a} = a_1 + a_2\,, \qquad \bar{b} = b_1 - b_2\,, \qquad \bar{c} = c_1 + c_2\,,$$

$$\bar{d} = d_1 - d_2\,, \qquad \bar{f} = f_1 - f_2\,, \qquad \bar{g} = g_1 + g_2\,. \qquad [48]$$

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*Although corresponding coefficients for the cone have been obtained by Watts and Lang[4] using Dubois' analysis, their expressions which involve *ber* and *bei* functions appear to be much more difficult and time-consuming to evaluate.

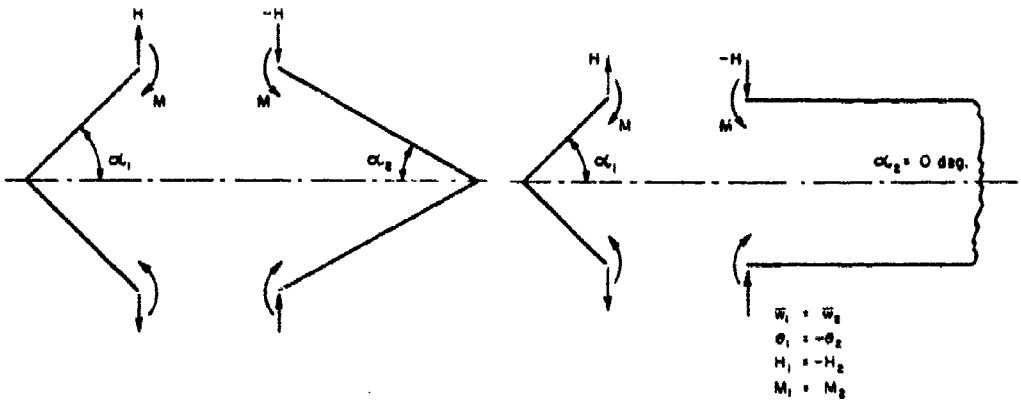

Figure 2 - Free-Body Sketch for Intersection of Two Cones or of a Cone and Cylinder

the discontinuity forces H and moments M become

$$M = \frac{\bar{f}\,\bar{b} - \bar{c}\,\bar{g}}{\bar{a}\,\bar{g} - \bar{b}\,\bar{d}}\,p \qquad [49]$$

$$H = \frac{\bar{c}\,\bar{d} - \bar{a}\,\bar{f}}{\bar{a}\,\bar{g} - \bar{b}\,\bar{d}}\,p \qquad [50]$$

These values substituted in expressions [27] and [33] to [38], completely describe stresses, strains, and displacements in both cones.

Cone-Cylinder Intersection

The case of a cone-cylinder intersection is developed by considering a cylinder as a limiting geometry of one of the cones, i.e., $\alpha_2 \to 0$ degrees. Then

$$c_2 = 0$$

Furthermore, if the thickness of the shells is the same, $h_1 = h_2$ then $\bar{b} = \bar{d} = 0$ and

$$M = \frac{(-c_1)\,p}{a_1 + a_2} = \frac{\sqrt{Rh}\;p \tan\alpha}{2\sqrt{2}\;\sqrt[4]{12(1-\nu)^2}\,[1+\sqrt{\cos\alpha}\,]}\left[R\sqrt{\cos\alpha} - \frac{3h}{\sqrt{12(1-\nu^2)\cos\alpha}}\right] \qquad [51]$$

$$H = \frac{(-f_1 + f_2)p}{g_1 + g_2} = -\frac{R p \sin\alpha}{2(\cos\alpha + \sqrt{\cos\alpha})} + \frac{(1-\nu/2)\sqrt{RH}}{\sqrt{2}\,\sqrt[4]{12(1-\nu^2)}}\left(\frac{1-\sqrt{\cos\alpha}}{\cos\alpha}\right)p \qquad [52]$$

Two Intersecting Cones with Reinforcement at the Joint

Consideration is now given to a stress analysis of the intersection of two cones when the joint is reinforced by a ring stiffener; see Figure 3.

If the ring stiffener is treated in the limiting case as an inextensible membrane, then

$$\begin{aligned} \overline{w}_1 &= \overline{w}_2 = 0 \\ \theta_1 &= -\theta_2 \\ M_1 &= M_2 \\ H_1 &\neq H_2 \end{aligned} \qquad [53]$$

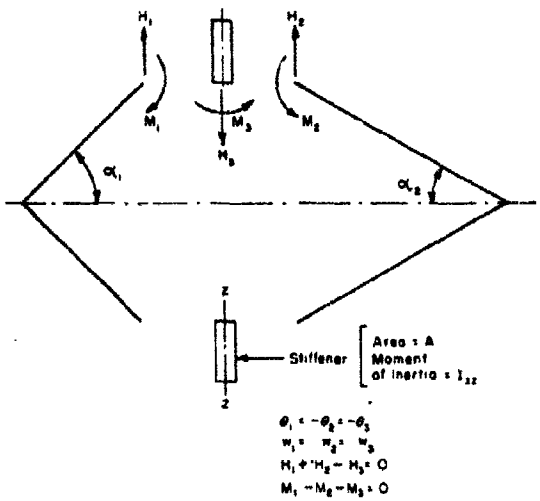

Figure 3 - Free-Body Sketch for Reinforced Intersection of Two Cones

and values may be obtained for M, H_1, and H_2 as before.

It is also possible that such a reinforcement exercises considerable restraint to rotation at the joint. In the limiting case, the cones can be considered as having complete fixity at the edge, that is, for either cone

$$\begin{aligned} \theta &= 0 \\ \overline{w} &= 0 \end{aligned} \qquad [54]$$

then

$$H = \frac{cd - af}{ag - bd}\,p = -\frac{R}{2}\tan\alpha\, p - \frac{3h\tan\alpha}{2\sqrt{12(1-\nu^2)}\cos\alpha}\,p + \left[\frac{(1-\nu/2)}{\sqrt[4]{12(1-\nu^2)}}\sqrt{\frac{2hR}{\cos^3\alpha}}\right]p \qquad [55]$$

$$M = \frac{fb - cg}{ag - bd}\,p = \frac{(1-\nu/2)Rh}{\sqrt{12(1-\nu^2)}\cos\alpha}\,p - \left[\frac{3\sqrt{2}}{2\left[12(1-\nu^2)\right]^{\frac{3}{4}}}\sqrt{\frac{Rh^3\tan^2\alpha}{\cos\alpha}}\right]p \qquad [56]$$

If $\alpha = 0$, the cone becomes a cylinder with fixed edge and

$$M = \frac{(1 - \nu/2)Rh}{\sqrt{12(1-\nu^2)}}\, p \qquad [57]$$

A similar procedure may be used for studying an intersection reinforced with a finite stiffener.* The directions assumed positive for the forces and moments are shown in Figure 3. The conditions for continuity and compatibility at the intersection become

$$\begin{aligned} \overline{w}_1 &= \overline{w}_3 \\ \overline{w}_2 &= \overline{w}_3 \\ \theta_1 &= -\theta_3 \\ \theta_2 &= \theta_3 \end{aligned} \qquad [58]$$

For static equilibrium at the intersection, the following expressions are valid

$$H_1 + H_2 = H_3$$

$$M_1 - M_2 = M_3$$

If the dimensions of the cross section of the ring are small when compared with its radius, the angle of rotation and the moment in the ring are related as shown by Timoshenko[9]

$$\theta_3 = \frac{M_3 R^2}{EI} \qquad [59]$$

where I is the moment of inertia of the cross-sectional area of the ring with respect to a centroidal axis as shown in Figure 3.

It can easily be shown that

$$H_3 = \frac{\overline{w}_3 EA}{R^2} \qquad [60]$$

where A is the area of the stiffener. By taking

$$\begin{aligned} k_1 &= \frac{R^2}{EA} \\ k_2 &= \frac{R^2}{EI} \end{aligned} \qquad [61]$$

*Note that in this treatment, the stiffener is assumed to have line contact at the intersection.

It is found from Equations [58] to [60] that

$$\overline{w}_3 = \frac{R^2}{EA}(H_1 + H_2) = k_1(H_1 + H_2) \qquad [62]$$

and

$$\theta_3 = \frac{R^2}{EI}(M_1 - M_2) = k_2(M_1 - M_2) \qquad [63]$$

From Equations [43], [45], [46], [62], and [63], Equations 58] may be written:

$$d_1 M_1 + m_1 H_1 - k_1 H_2 = -f_1 p \qquad [64]$$

$$d_2 M_2 - k_1 H_1 + m_2 H_2 = -f_2 p \qquad [65]$$

$$n_1 M_1 - k_2 M_2 + b_1 H_1 = -c_1 p \qquad [66]$$

$$-k_2 M_1 + n_2 M_2 + b_2 H_2 = -c_2 p = 0 \qquad [67]$$

where

$$m_1 = g_1 - k_1 \qquad n_1 = a_1 + k_2$$
$$m_2 = g_2 - k_1 \qquad n_2 = a_2 + k_2 \qquad [68]$$

The determinant of the coefficients for these equations may be written:

$$\begin{vmatrix} d_1 & 0 & m_1 & -k_1 \\ 0 & d_2 & -k_1 & m_2 \\ n_1 & -k_2 & b_1 & 0 \\ -k_2 & n_2 & 0 & b_2 \end{vmatrix} = \Delta \qquad [69a]$$

Replacing the first column by the constant terms

$$\begin{vmatrix} f_1 & 0 & m_1 & -k_1 \\ f_2 & d_2 & -k_1 & m_2 \\ c_1 & -k_2 & b_1 & 0 \\ 0 & n_2 & 0 & b_2 \end{vmatrix} = \Delta_{M_1} \qquad [69b]$$

From these

$$M_1 = \frac{\Delta_{M_1}}{\Delta} p \qquad [70]$$

M_2, H_1, and H_2 may be computed similarly, and the stresses may be calculated by using Equations [27] and [33] to [38].

To illustrate the manner by which the stiffener modifies stresses developed at a cylinder-conical intersection, a numerical example is provided.

Let $h_1 = h_2 = h = 0.097$ in., $R_0 = 7.6$ in., $\alpha = 30$ degrees. Several different sized stiffeners are chosen as follows:

Case	A, in.^{2}	I, in.^{4}
a	0.0396	0.323×10^{-4}
b	0.0396	5.38×10^{-4}
c	0.0792	0.323×10^{-4}
d	∞	0.323×10^{-4}
e	0.0396	∞
f	∞	∞

The resulting circumferential and longitudinal strains are plotted in Figure 4. Of considerable interest is the much greater influence which variation in area of stiffener exercises on the stresses as compared with the influence of the variation in moment of inertia.

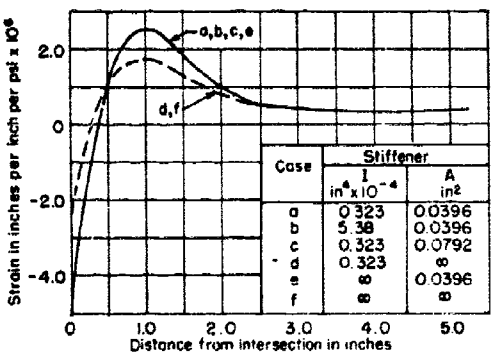

Figure 4a - Longitudinal Strains

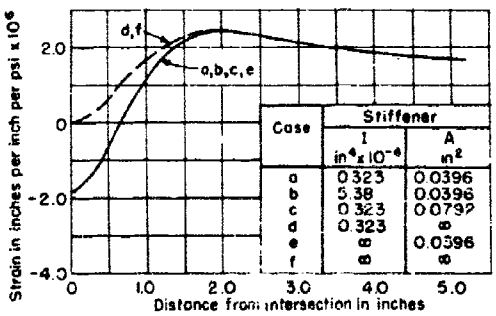

Figure 4b - Circumferential Strains

Figure 4 - Effect of Properties of Stiffener on Strains Near Reinforced Intersection of Cylinder and Cone Subjected to Internal Pressure

COMPARISON WITH DUBOIS' SOLUTION FOR SEVERAL NUMERICAL EXAMPLES

Because of the approximations required in this analysis, a comparison has been made with the results of Dubois' more exact equations for several numerical cases. These are listed in Table 1.

TABLE 1

Case I	$h = 3/32$ in.		$2R_0 = 13.375$ in.			
	$\alpha = 30$ deg.		$\xi_0 = 56.2$			
Loading	H		M		p	
	$\theta_0 E$		$\theta_0 E$		$\theta_0 E$	
Dubois	2429		8499		4730	
Wenk-Taylor	2514		8774		4783	
Case II	$h = 5$ cm		$2R_0 = 150$ cm			
	$\alpha = 60$ deg.		$\xi_0 = 11.5$			
Loading	H		M		p	
	$\bar{v}_0 E$	$\theta_0 E$	$\bar{w}_0 E$	$\theta_0 E$	$\bar{w}_0 E$	$\theta_0 E$
Dubois	94.6	9.48	9.48	1.84	4550	584
Wenk-Taylor	105.6	9.91	9.91	1.86	4935	566

The agreement is rather good, even for $\xi = 11.5$.

DISCUSSION AND CONCLUSIONS

As noted in the development, departures from rigor occurred:

A. With use of an approximate differential equation,

B. With use of approximate expressions for stress resultants,

C. With assumption that u displacements can be neglected when computing $\bar{w}$ for bending effects.

D. With simplification of coefficients representing edge rotations and displacements.

E. With assumption that stiffener has a line instead of finite contact at the intersection.

Each of the approximations appears to introduce small errors which could be additive rather than compensating. However, it can be noted that the expression for Q_x in this analysis (from Equations [18] and [32]) is identical with the Q_x from the exact solution by Dubois[11] because of compensating errors of the approximations in the differential equation and in the

shearing force. Maximum errors in stress analysis due to use of the approximate theory are estimated as a function of ξ,

ξ	Estimated Maximum Error percent
10	25
20	15
30	9
40	7
60	5
100	2

Obviously ξ must exceed 6 to permit use of the asymptotic form of the solution of the differential equations.

From these considerations, it is concluded that these results can be employed with satisfactory accuracy for engineering calculations on all but very flat or very thick cones. The rotation and displacement coefficients given in Equations [43] may be conveniently used in the analysis of pressure vessels in a manner similar to that now popular for other components.

With the one numerical example of a reinforced cone-cylinder intersection, the area of the stiffener was shown to have a much greater influence in reducing strains than did the moment of inertia. Although this is not shown to be generally true, these limited results suggest that the shape of stiffener may not be significant.

ACKNOWLEDGMENTS

Much helpful advice in the investigation of the rigor of existing analyses and in the development of results was provided by Dr. E.H. Kennard, Chief Scientist of the Structural Mechanics Laboratory.

The computations were ably performed by Messrs. J.L. Bigio, R.L. Waterman, P.P. Gillis, and Miss T.M. Morrow of the David Taylor Model Basin Staff.

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