TITLE: Life Extension Methodologies and Risk-Based Inspection in the Management of Fracture Critical Aeroengine Components

DISTRIBUTION: Approved for public release, distribution unlimited
Availability: Hard copy only.

This paper is part of the following report:
TITLE: Ageing Mechanisms and Control. Specialists’ Meeting on Life Management Techniques for Ageing Air Vehicles [Les mecanismes vieillissants et le controle] [Reunions des specialistes des techniques de gestion du cycle de vie pour vehicules aeriens vieillissants]

To order the complete compilation report, use: ADA415672

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:
ADP014058 thru ADP014091
Life Extension Methodologies and Risk-Based Inspection in the Management of Fracture Critical Aeroengine Components

A.D. Boyd-Lee, D. Painter and G.F. Harrison
Structures and Materials Centre, QinetiQ, Farnborough, Hants GU14 0LX, U.K.
Phone: +44-1252-392000
Fax: +44-1252-397298
Email: adblee@qinetiq.com, dipainter@qinetiq.com and gfharrison@qinetiq.com

1 ABSTRACT
This paper briefly describes the main methodologies used in the assessment of fracture critical parts. The development of a procedure for the quantitative assessment of non-finite results is discussed and typical life extension levels are illustrated. Risk assessment is considered within the context of the safe life methodology. It is used to quantify the potential risks associated with the short-term continued operation of life-expired parts and to allow their managed withdrawal. The paper then considers risk assessment applied to the damage tolerance and retirement for cause life assessment. The significance benefits of risk based inspections intervals over standard fixed inspections are illustrated. Finally with regard to retirement for cause, it is shown that a risk based approach automatically sets an economical limit to retirement for cause but avoids the progressive risk levels associated with the current embodiment.

2 BACKGROUND
Aeroengine components are classified into fracture ‘critical’ or ‘non-critical’ depending on the consequences that a malfunction might have on the integrity of the aircraft. Turbine and compressor discs and shafts are identified as the major fracture critical components. They experience extreme thermo-mechanical transient loadings, which, in the absence of an adequate life prediction policy, eventually lead to low cycle fatigue (LCF), creep or corrosion failures. Since it is not practical to design engine casings capable of containing such events, it is essential to ensure that their occurrence in service is an extremely remote possibility. Hence a fundamental requirement of both military and civil for engine certification is therefore to ensure the continued airworthiness and integrity of fracture components such as discs and shafts throughout their service operation. Such events are non-random and hence as service operation progresses, both the risk of failure and the rates at which risks accumulate rise rapidly.

In earlier times, prior to aircraft reaching their full declared lives, the introduction of next generation aircraft led to fleet withdrawal prior to many major components becoming life expired. This is certainly no longer the case and lifing methodologies must ensure that thoughout the declared service lives of the components the likelihood of failure is extremely remote. Within the U.K., the inclusion of a requirement to fatigue test ex-service discs has significantly reduced the risk of the unforeseen. The current paper will review briefly the major methodologies applied to meet these requirements. Additionally, it will illustrate the concepts and application of several methods which allow safe life extension. Within the general context of Damage Tolerance, the benefits of risk based inspection intervals over conventional procedures will be demonstrated.

3 LIFE-TO-FIRST-CRACK
Under current UK Military Defence Standards (Def. Stan. 00-971) [1] and European Civil Joint Airworthiness Requirements (JAR-E) [2], the service lives for fracture-critical components are derived from spin-rig test results of actual engine discs under cyclic loading at stress and temperature conditions similar to those experienced in engine operation. These regulations are based on a safe-life policy wherein component fatigue ‘failure’ is defined as the occurrence of an ‘engineering crack’ of 0.38mm radius.

If the total fleet dysfunction distribution were known i.e. all components were tested to failure, the mean dysfunction life would be known exactly. In practice a sample is selected from this distribution and tested until significant crack growth or burst. The mean life for the sample provides an estimate of the mean life of the rest of the fleet.

A consequence of the high cost of disc spin testing is that it is only practical to test a very small sample of the discs. Hence such tests are extremely expensive and few results are available, dealing with the probabilistic nature of fatigue is an essential part of any life assessment. To address the statistical difficulties associated with small sample sizes, the lifing regulations define the procedures for the safe interpretation of these results and for the calculation of the declared service lives. Certain assumptions have therefore to be made concerning the statistical form of the dysfunction population distribution. From past UK experience the failure distribution is assumed to be lognormal and significantly the assumption is also made that the scatter associated with the distribution is also known. It is therefore assumed that the ratio in life (scatter) between the +3σ and -3σ quantiles (749/750 and 1/750) is known. This assumption has major implications regarding the analysis performed on the test sample. Hence the mean life from a small sample used as an estimate of the population mean life will only be approximate, provided that the population scatter factor is known, for any test sample taken from the population the sampling error must be established absolutely. For example, if the whole of the population was randomly split-up into samples of any specified size (e.g. 5), tested, and the mean lives from all the samples plotted, the distribution that would be obtained can be predicted statistically without any testing. Hence by associating the known distributions to the actual test sample, the magnitude of the sampling error is also known. In other words from the mean life of the test sample the population mean life can be established to any required level of confidence e.g. 95%. Also if the mean life of the population is known, the mean life to any other quantile is also known.

This is the basis of the UK statistical models applied to both (Life to First Crack) ltfc and to the dysfunction distribution.

Having established the burst distribution, risk assessment is about identifying where, at any specific time, individual discs in service lie relative to the failure distribution. In practice, it is easier to attach the error function to these points rather than the full burst distribution but in statistical terms the effects are identical. The statistical model is the same as that applied in ltfc and 2/3 dysfunction approaches. That is, a predicted safe cyclic life (PSCL) is calculated by applying statistically-derived safety factors to the geometric mean (GM) of the respective test results, such that at this life, not more than 1 in 750 service discs would be expected to contain an ‘engineering crack’, to 95% confidence. It is from this base that the life-extension procedures discussed in the paper are quantitatively assessed.

Experience has shown that typically disc fatigue lives are distributed according to a lognormal density function. Also the assumption is made that the ratio of the fatigue lives at the +3σ and -3σ points on the life-to-first-crack (ltfc) distributions is 6. Given an assumed scatter factor of 6, the life corresponding to the lower 1/750 quartile is automatically located at a factor of 3 standard deviations (i.e. $\sqrt{6} (=2.449)$) below the geometric mean (GM) ltfc obtained from the sample. A 95% confidence level allows for the effect of component test sampling error on the safety level inherent in the calculation. This lower confidence bound corresponds to a safety factor in life of:

$$y = \frac{1.645}{\sqrt{6}}$$

Finally, a predicted safe cyclic life (PSCL) or $Ar$ is obtained by dividing the log mean test cycles (converted to equivalent reference cycles) by the factor $y$. 

$$Ar = \frac{6^{\frac{1.645}{6\sqrt{6}}}}{\sqrt{6}}$$
where ‘Ni,’ are the individual lifec test results. It can be seen that equation 2 is an increasing function with respect to decreasing sample size. Application of factors involved in determining ‘Ar’ is illustrated in Figure 1.

\[ Ar = \frac{\sqrt[n]{\prod_{i=1}^{n} N_i}}{y} \]  

(3)

Figure 1. Derivation of the material design curve and predicted safe cyclic life.

Through using an ‘engineering crack’ as the basis for the calculation, the additional life taken to grow to the critical size associated with dysfunction (failure) acts as a further margin of safety. Ultimately, it is this, in combination with the factors in the denominator of equation 2 which define the actual safety level inherent in the application of this method.

4 LIFE REVISION OF AGEING ENGINES

A consequence of the high cost of disc spinning tests is that once a specified (required) design life has been demonstrated, there is great pressure to discontinue the respective test programme, even though the discs have not reached ‘failure’ (defined as the occurrence of an engineering crack). Results of this kind are called ‘non-finite’. Non-finite results can also arise for several other reasons. Amongst these, a common cause is a change in the identified failure location within a component as the result of either service experience or reanalysis of the basic design. The current lifing regulations were originally derived to accept only finite results and this has caused non-finite results to be either rejected unnecessarily or accepted overconservatively by assuming ‘failure on the next load cycle’. The corollary is, that where the newly established failure location is now identified as having experienced little or no overstress, the service life that can now be declared would need to be reduced by a factor of up to 4, depending on the number of test results available.

4.1 Improved statistical analysis of non-finite results

In the current paper an analysis methodology for samples comprising of only non-finite results, is illustrated. However, in addition, the general methodology can also handle samples comprising of mixed finite and non-finite data. To illustrate the principles, in the following analysis it is assumed that the results belong to a lognormal lifec distribution having a known scatter factor. As is the case for samples of finite results, the geometric mean (GM) of the population lifec distribution is estimated to 95% confidence. Once this value is obtained, the 1/750 (-3\sigma) quantile is selected to provide a safety factor to give a value
for the required safe-service life, \( Ar \) (cycles). To identify a conservative confidence interval, the probability of obtaining a non-finite result at \( N_i \) cycles is established based on a given value of the GM (the minus superscript indicates that the value is non-finite).

A 95% lower confidence bound for the true GM can be obtained by identifying that mean value for which the probability of a component not surviving \( N \) cycles is 5%. That is, there is a 95% chance that the defined likelihood lies within the interval 0.05 to 1. Therefore, given the value of \( N \), a 95% lower confidence bound for GM can be obtained when the following equation is satisfied.

\[
p(\text{non-finite at } N \text{ cycles}) = 0.05
\]

(4)

As the sample size is equal to one, this confidence bound leads to the same formula as would apply if the result were finite. For a sample of size \( n \) the confidence test generalises to

\[
\prod_{j=1}^{n} p(\text{non-finite at } N_j \text{ cycles}) = 0.05
\]

(5)

That is, the probability of obtaining all non-finite results is equal to the product of the probabilities of obtaining the individual non-finite results in the sample. As for a sample of size one, a confidence interval on the GM is chosen such that the probability of obtaining the sample is equal to a value lying between 0.05 and 1. Thus, a conservative 95% lower confidence bound for the GM is given as that value for which this product is equal to 0.05 (see equation 4). This equation embodies a 'next-cycle-failure' assumption for up to one of the non-finite results. It is still slightly overconservative since the probability of even one of the non-finite results reaching dysfunction on its next cycle is extremely low. However, in general, removal of this overconservatism is complex and (without further information) does not result in a significant life increase.

It remains to substitute an expression for the failure distribution into equation 4. Firstly a change of variable is applied such that the parameter \( N_j \) is the resulting non-finite value associated with test result \( J \), expressed in its logarithmic form, divided by the geometric mean life, \( N_\mu \), and rescaled to units of standard deviation. (Division by the population geometric mean life, \( N_\mu \), has the effect of placing the origin at the population GM life which has an indeterminate fixed value.) The following solution is obtained:

\[
\prod_{j=1}^{n} 1 - \text{normal}\left( \frac{\log\left( \frac{N_j}{N_\mu^{95\%}} \right)}{\frac{1}{6} \log\left( \frac{N_{+3\sigma}}{N_{-3\sigma}} \right)} \right) = 0.05
\]

(6)

where the \( N_\mu^{95\%} \) value is a 95% confidence estimate of the GM of the population fatigue life distribution. This is the only unknown in equation 6, and hence can be solved by substitution of the known values and iteration until the equation is satisfied.

Once \( N_\mu^{95\%} \) has been established, the standard safety factor can be applied (as illustrated in equation 2) to ensure that not more than 1/750 components reach the defined dysfunction point. That is,

\[
Ar = \frac{N_{-3\sigma}^{95\%}}{N_\mu^{95\%}}
\]

(7)
By way of illustration, the application of the above expression to 5 non-finite results of 10,000 cycles gives $N_{95}^t = 10,374$ cycles and substitution of this value in equation 7 gives an Ar value of 4,235 cycles.

Thus, in this example and without any compromise to safety, the improved statistical analysis supports a 29% increase in the service life relative to that declared via current regulations. In service, the life extensions given by the method have ranged between 5% and 50%.

At one extreme a sample may contain all finite results; at the other all non-finite results. Consistency between the solution for mixed results and these two extremes can be demonstrated. Firstly for mixed results the solution copes with cases where all the results in the sample are finite and hence the confidence test used for the analysis of all finite results gives exactly the same results as that developed for samples of totally finite results. There is a subtle difference in that this confidence test uses a change of variable to a logarithmic normal density functional form as the reference datum, whereas the earlier test uses the mean of the sample as the reference datum. However, as this difference does not change the statistical model, it makes no difference to the solution. For illustration, at the other extreme (mostly non-finite) when all the results in the sample have the same life and all but one is non-finite, then the mixed results confidence test yields the same equation as is used in the all non-finite case. It can therefore be verified that there is complete consistency between the solutions for samples of mixed results and for samples of all finite results. Also, there appears to be a consistent level of conservatism between the solution for samples of all non-finite results and the solution for samples of mixed results. The confidence test used in the well-established method for handling samples of all finite results is fully optimised. However, as discussed above, the confidence test for samples of mixed results is essentially a more generalised version of that used to handle samples of all finite results. Thus, this is also fully optimised.

Finally, for a sample of non-finite results, a different confidence test has to be used. The one currently demonstrated is marginally overconservative for two reasons. Firstly, the additional complexity required to exploit fully the non-finite results is probably not justified in terms of the small increase in life it would give. Secondly, it enables the approach to be related to the familiar 'minimum life formula' and the Weibayes formula. This is important for verification of the theory.

The solution for mixed results is well behaved between the extremes of all finite and all non-finite results. One way of demonstrating this is to compare the variation of the safe service life as a function of the proportion of the results in the sample which are non-finite. For example for the sample of 5 results all of 10,000 cycles that were considered above, as the sample moves from all finite to only one finite result, the available safe life increases as illustrated:

<table>
<thead>
<tr>
<th>No. of Finite Results</th>
<th>No. of Non-Finite Results</th>
<th>PSCL (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>4,235</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3,857</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3,611</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>3,427</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>3,278</td>
</tr>
</tbody>
</table>

Table 1 Influence of non-finite results on predicted safe cyclic life. (All tests discontinued at 10,000 cycles).

Although somewhat obvious, the lower the value of a non-finite result; the less its significance. However, this property allows a further verification of the mixed results methodology (see Table 1). For example, suppose there is a sample of 2 results one has a finite value of 10,000 cycles and the other a non-finite value of 10,000 cycles. Then a safe service life of 3,254 cycles is given by the methodology (see row 1 of the Table 2). Suppose instead the non-finite value of the second result is 8,000 cycles, then a safe life of 3,000 is given (see row 2 of Table 2). This calculation is repeated with successively smaller values for the second result. As expected, eventually the safe life converges on a value of 2,498 cycles (the same value as if the second result did not exist at all).
<table>
<thead>
<tr>
<th>First Test Result (cycles)</th>
<th>Second Test Result (cycles)</th>
<th>Third Test Result (cycles)</th>
<th>Predicted Safe Cyclic Life (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>10,000 n.f.</td>
<td>3,693</td>
</tr>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>8,000 n.f.</td>
<td>3,505</td>
</tr>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>6,000 n.f.</td>
<td>3,346</td>
</tr>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>4,000 n.f.</td>
<td>3,264</td>
</tr>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>2,000 n.f.</td>
<td>3,254</td>
</tr>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>1,000 n.f.</td>
<td>3,254</td>
</tr>
</tbody>
</table>

Table 2. Influence of low non-finite results on predicted safe cyclic lives (samples of 2)

A further example of the capability of the methodology to handle low non-finite results as expected is given in Table 3, this time using a sample size of 3.

<table>
<thead>
<tr>
<th>First Test Result (cycles)</th>
<th>Second Test Result (cycles)</th>
<th>Third Test Result (cycles)</th>
<th>Predicted Safe Cyclic Life (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>10,000 n.f.</td>
<td>3,693</td>
</tr>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>8,000 n.f.</td>
<td>3,505</td>
</tr>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>6,000 n.f.</td>
<td>3,346</td>
</tr>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>4,000 n.f.</td>
<td>3,264</td>
</tr>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>2,000 n.f.</td>
<td>3,254</td>
</tr>
<tr>
<td>10,000 f.</td>
<td>10,000 n.f.</td>
<td>1,000 n.f.</td>
<td>3,254</td>
</tr>
</tbody>
</table>

Table 3. The influence of lower non-finite results on predicted safe cyclic lives (sample size 3)

Further examples, comparing mixed samples of size 3.

As for any lifing calculation, safe application of the non-finite methodology depends on the validity of the assumptions made. Primarily this means that the assumed scatter factor (i.e. the factor in life between the plus and minus 3σ points on the population failure distribution) must be conservative. Wherever possible a Chi-squared test should be undertaken verify that the assumed population scatter is consistent with that associated with the sample. In dealing with non-finite results, via an inequality condition a lower bound is established for the finite value associated with the non-finite result.

4.2 2/3 Dysfunction failure criterion

Experience over many years has shown that the ltfc approach is very conservative and disc failures in service are extremely remote with rates of only one or two for every 100 million flying hours. The approach has an in-built factor of safety associated with the cycles required to grow a crack from 0.78mm to the dysfunction crack size. Since this value is both material and geometry dependent, the method does not provide an overall quantifiable level of safety. To address this, an extension of this approach, the crack initiation life has been replaced by a set fraction of the total cycles to dysfunction. A figure of 'two-thirds' has been established in the UK and increasingly throughout Europe.

RB199 engine for which it has been shown that crack propagation β-factors can be 2-3 times greater than crack initiation β-factors [3]. In this paper, we consider a typical case in which $\beta_p/\beta_i=2.5$.

4.3 Crack tolerant designs

In crack tolerant designs, components have 2/3 dysfunction lives that can be significantly greater than those established via ltfc. This difference represents an additional safe crack growth phase which can be
exploited. Assessments of life-limiting areas such as drive arm vent holes and disc rim features are typical of design features that can exhibit significant crack tolerance.

For such crack tolerant components, service lives can be safely extended beyond 100% \( \text{Ar} \), without exceeding the extremely remote risk levels associated with conventional failure locations. However, to ensure a consistent level of safety, the associated 'never exceed' 2/3 dysfunction margin of safety must be imposed on the useable crack growth life. Thus, relative to component lives based on ltfc, life extension is available when the crack growth life is greater than 50% of the ltfc value. To ‘cash in’ the benefits of such designs, it is necessary to be able to predict accurately this crack growth phase and hence it is necessary to apply fracture mechanics procedures based on establishing operating stress intensity levels.

Although service lives are expressed in terms of the major reference cycles experienced, the damage induced by the large number of minor cycles must also be accounted for. Minor cycles occur as a result of adjustments to the thrust requirements during various stages of the mission flown by the aircraft. These are accounted for as the additional number of reference cycles per hour of flight that would inflict equivalent damage to one hour of the minor cycle loading. Then the exchange rate \( \beta \) is defined as the total number of reference cycles (major and minor) consumed during one hour of flight.

Both theoretical analyses and experimental test programmes have shown that, under an identical mission loading sequence, minor cycles are relatively more damaging during crack propagation than during the ltfc initiation stage. This can be explained in terms of the difference in the stress exponents associated with the life-to-first-crack curve and with the stress intensity exponents associated with the crack growth curve. Below the fatigue endurance limit, minor sub-cycles have virtually no influence on crack initiation, however they can contribute during the propagation phase. For the crack initiation phase (ltfc), a weighted average consumption rate of reference cycles per engine flying hour, \( \beta_i \) must be established. Similarly, during crack propagation \( \beta_p \) is the weighted average consumption rate of reference cycles per engine flying hour, \( \beta_p \) must also be established. A consequence of crack initiation and crack propagation models is that \( \beta_p \) is generally greater than \( \beta_i \). And typically, \( \beta_p/\beta_i \) has a value equal to 2.5. These observations have been confirmed recently for the Tornado aircraft.

Although life consumption and remnant life calculations are evaluated in terms of reference cycle damage, the end user is interested in the service hours available for specific components. In general to get this information, it is simply a matter of dividing the release lives (in cycles) by the relevant \( \beta \)-factor. The value obtained identifies the authorised service life in engine flying hours. For the situation in which the 2/3 dysfunction life exceeds the ltfc, this difference in reference cycles needs to be divided by the propagation \( \beta \)-factor to establish the authorised additional life in engine flying hours. Extensions of about 40% have been demonstrated for RB199 IPC and IPT rotor components. Details of the procedures involved in determining safe crack growth lives are given elsewhere (AGARD) [4].

### 4.3.1 Fracture Mechanics Based Procedures

A consequence of the development of very high strength disc alloys is that the critical crack size for the onset of rapid fatigue crack growth can be smaller than the 0.78mm associated ltfc. Additionally, aeroengine discs can have several life limiting features dependent on the specific design and operational requirements. The net effect is that it is impossible to construct a common databank for all these features on an ltfc basis. However a rationalised design approach can be developed via a fracture mechanics approach. In such cases, it is assumed that failure occurs either from initiation and growth of cracks induced by the imposed service loading sequences or as the result of crack propagation from inherent defects. Currently there are three fracture mechanics based life assessment methodologies approved for the certification of aeroengine fracture critical parts. These are Databank Lifing [5], Damage Tolerance Lifing and Retirement-for-cause [6]. The first approach is approved for use in UK Civil and Military engines and the others apply to US Military engines.

### 4.3.2 Databank Life Assessment

This procedure assumes that all discs contain small “pseudo” defects that grow under fatigue loading in a predictable manner from the first cycle. For the various features for which failure lives and final crack lengths are known, the ‘Paris’ crack growth equation is used in a back calculation mode to determine a “pseudo” crack size present in the component/test piece at cycle one. Since the fracture mechanics analysis accounts for variations in component geometry, stress field and crack shape, the approach offers
a means of combining the results of different disc designs and large specimens into a common data bank. Statistical analysis procedures define the maximum effective “pseudo” crack size likely to be present in the total population. This initial flaw is then used as the starting point in a standard Paris summation to calculate the maximum allowable service life. The effective initial flaw sizes have no physical meaning, they simply provide a suitable parameter to enable the lives of different component geometries to be combined into a common data set. The inverses of the pseudo crack sizes are plotted in terms of a three-parameter Weibull model. Accurate estimates of component life can be established at the design stage thereby allowing greater design optimisation and more effective use of materials. Both CAA and FAA Airworthiness Authorities have approved fracture mechanics databank methods in declaring lives of civil engine components.

4.4 Damage Tolerance Life Assessment and Retirement for Cause

In this philosophy, damage is again assumed to pre-exist in newly manufactured components, but the starter crack size for residual life calculations is based on proven NDE capability. The residual life of the component is determined by applying a fracture mechanics approach to calculate the number of cycles required to grow from the starter crack size to a critical length. By assuming that damage exists in components as manufactured, the damage tolerance method avoids problems associated with the statistical distribution of crack initiation. Achievements of an acceptable level of safety now depends both on the reliability of the NDE system and on how often cracks or defects of a given size occur in practice. The most widely applied damage tolerance methodology is that developed by the US Airforce under its Engine Structural Integrity Programme (ENSIP). Here, the declared service life is based on an NDE crack detection size set to achieve a detection level of 90% with 95% confidence. Mean crack growth data are used to determine the available growth life to dysfunction and the declared safe service life is normally set at half this value.

4.4.1 Retirement for Cause

This is an extension of the damage tolerance approach which has been applied to high performance military components in the U.S. In contrast to the ltfc method, retirement is not implemented until actual cracks have been identified in individual discs. The safety of the approach is strongly dependent on the effectiveness of the NDE technology.

5 EXTENDED COMPONENT SERVICE LIVES VIA RISK ASSESSMENT AND RISK MANAGEMENT

5.1 Risk analysis

In the safe life methodology, the predicted safe cyclic life of a fracture-critical component is set to ensure that to 95% confidence not more than 1/750 service components suffer an engineering crack (>0.38mm radius). However, in applying this definition to the lifing of service components, several limitations have been encountered. Hence, although, ultimately safety should be concerned with ensuring that the possibility of fracture critical parts failure (dysfunction) must be extremely low, the risks associated with this event are not addressed directly. Also, in general, the declared life-to-first-crack is not a fixed fraction of the dysfunction life and consequently an inconsistent level of safety results. Additionally, it is found that minor cycles are usually more damaging in crack propagation than they are during life-to-first-crack. All these issues have important consequences for the implementation of a safe life, ltfc lifing policy. They have even greater significance for damage tolerance approaches.

Another aspect that has to be addressed is that in current methodologies little consideration is given to rate of increase in risk with increased component service life and hence these traditional approaches do not always provide the required statistical information for the managed withdrawal of service components. Current methods do not allow exploitation of the full life capability of components with long crack propagation lives.

Although the pscl methodology can loosely be considered to be a form of risk model, it is not sufficiently flexible to handle the wide variety of issues that arise in service. Indeed, when a significant downward life reduction occurs, strict implementation of the safe life criterion could result resulted in the grounding of aircraft fleets. Hence, within the context of airworthiness, it is now proposed that safety should be
related to the predicted dysfunction distribution and safety factors associated with ‘95% confidence, 1-in-750’ should be replaced by the determination of maximum risk levels (e.g. for components a reasonable level might be set at $10^{-5}$ per engine flying hour).

To address these diverse issues it is strongly advocated that there should be a move towards the development and adoption of a more robust risk based life assessment methodology. Such an approach should include the identification of the component life-to-first-crack and propagation life distributions and account for all factors that affect them (inspection, fabrication knowledge etc.). These distributions should then be combined to obtain the estimated dysfunction distribution, whilst taking full account of sampling error. This should allow the instantaneous risk/hour to be derived from the estimated dysfunction distribution and hence allow the safety criteria to be defined in terms of risk.

Useful management parameters available from such an approach include the identification of an individual aircraft peak risk/hour by which time components must be inspected or else retired and also identification of a maximum fleet cumulative risk per annum. It should then be possible to correlate the identified risk to other available management tools such as the RAF Hazard Risk Index.

5.2 Identification of the component distributions

For aeroengine discs, the lives-to-first-crack form an approximate lognormal distribution. The associated probability density function can be expressed as

\[
PD' = 0.5 \cdot \text{Erf} \left[ \frac{\ln \left( \frac{N^i}{\bar{N}} \right)}{\sqrt{2} \cdot \ln (s^i)} \right] + 0.5
\]

where ‘Erf’ is the mathematical error function, ‘$N^i$’ is the life-to-first-crack (cycles) of a given component, ‘$\bar{N}$’ is the log-mean of the ltfc distribution and ‘$s^i$’ is the scatter factor of the ltfc distribution. By replacing the occurrences of the superscript ‘i’ with the superscript ‘p’, a propagation life distribution is described. Similarly by replacing the superscript ‘i’ by the superscript ‘d’, a dysfunction distribution is defined.

5.3 Estimation of the dysfunction distribution

Often the limited available data are such that it is preferable to derive the dysfunction distribution from the life-to-first-crack and propagation life distributions for the component. The dysfunction life ‘$N^d$’ is equal to the sum of its life-to-first-crack ‘$N^i$’ and its propagation life ‘$N^p$’.

\[
N^d = N^i + N^p
\]

From a purely statistical viewpoint the two extremes are that the two component distributions are totally dependent or they are independent (total dependence means that a component with a long ltfc will have a proportionately long propagation life). Factors that contribute to the net effect include chemical composition, manufacturing and machining, heat treatment and surface residual stresses. The life-to-first-crack distributions and the propagation lives obtained in the nickel-base superalloys used in aeroengine discs appear to exhibit an interdependence. That is they are partly dependent and partly independent. Current evidence indicates that assuming 50-50 dependence-independence is conservative. Although the degree of interdependence does not affect the geometric mean of the dysfunction distribution and it has a relatively minor effect on the distribution form, it has a significant effect on the dysfunction scatter factor as illustrated in Table 4.
<table>
<thead>
<tr>
<th>Nature of component</th>
<th>Individual SF</th>
<th>Combined Scatter Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Itfc</td>
<td>Prop.</td>
</tr>
<tr>
<td>Typical (NP=0.5N')</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Surface sensitive (NP=0.5N')</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>Crack Tolerant (NP=2N^2)</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Crack Tolerant (NP=2N^2) but surface sensitive</td>
<td>10</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 4. Influence of initiation to propagation life ratio, and initiation scatter on the combined scatter factors

5.4 Estimation of the dysfunction distribution subject to inspection (Effect of inspection on remnant dysfunction distribution)

If a component is subject to inspection, then a dysfunction distribution can be calculated for each inspection interval. Until the first inspection occurs the dysfunction distribution is derived as described above. For the second inspection interval, suppose that the probability of detection is ‘POD’. Then POD*100% of the life-to-first-crack distribution is truncated and 100(1-POD)% is not truncated. The effective life-to-first-crack distribution is thus given by the following expression.

\[
P(D) = POD \cdot P(D)[\text{truncated}] + (1-POD) \cdot P(D)[\text{not truncated}]
\]

The modified life-to-first-crack distribution can then be combined with the propagation life distribution to obtain the dysfunction distribution using the approach described in the previous section. For simplicity, in the results to be presented later, 100% independence has been assumed. Numerical methods can be used to evaluate the required convolution integral.

Assuming 100% detection of all cracks exceeding 0.38mm surface length, inspection at ‘n’ cycles can be represented by a sharp truncation of the life-to-first-crack distribution at that point. Figure 2 shows the effect of inspection on the Itfc density distribution. (For the case shown, the geometric mean initiation life is 10,000 cycles and scatter factor (defined as the ratio between the plus and minus 3σ quantiles) has a value of ‘6’.

![Figure 2. Truncated life-to-first-crack distribution (representative of perfect inspection.)](image-url)
If crack detection is perfect prior inspections are irrelevant because the current inspection detects any crack missed by a previous inspection. If no cracks are detected, then the risk curve associated with the previous inspection applies. Thus the risk curve for a subsequent inspection ‘i’ can be expressed as

$$R_i[efh] = POD \times R_p[efh] + (1 - POD) \times R_{i-1}[efh]$$  \hspace{1cm} (11)

An imperfect inspection can be represented as a mix of the truncated and non-truncated life-to-first-crack distributions.

### 5.5 Determination of instantaneous risk per flight hour

This section presents a model for determining individual aircraft dysfunction risk per aircraft flying hour and then indicates how the cumulative risk can be expressed via an integral form of this equation. Since in UK, most of the service components are lifed according to the PSCL approach described earlier, the following risk model is expressed in terms of the safe service life ‘Ar’ since originally, the risk equation was developed to allow management of components operating beyond their declared life, Ar.

The initial stage in the assessment therefore quantifies the safety factors associated with Ar. A procedure for determining the safety factor ‘y’, associated with a component that has reached its safe service life, Ar, was discussed. This factor is with respect to the crack initiation distribution and when related to burst a further factor has to be applied. In the absence of onboard usage recorders, service lives are expressed in terms of hours and mission exchange rates are used to convert these hours into reference cycles. For the typical case illustrated previously where the mean of the burst distribution is a factor of 1.5 times the mean life, exchange rates for initiation and for propagation are based on different criteria and typically propagation rates are about a factor of 2.5 times initiation rates. This correlation allows propagation cycles to be converted to equivalent initiation cycles and hence allows the application of a single exchange rate to risk assessments. The propagation cycles can be converted into equivalent available initiation cycles by dividing them by the factor of ‘2.5’ thus, to relate the safety factor ‘y’ to the burst distribution it has to be modified by the ratio ‘1.2’ (=1+0.5/2.5). For any other service life in hours, the additional safety factor can be related to the safety factor at Ar through the multiplication factor Ar/(Hβ). Hence

$$S = y \cdot \frac{Ar}{H \beta} = 1.2 \times 2.449 \times 6^{(1.645/\sqrt{6})} \times \left(\frac{Ar}{H \times \beta_i}\right)$$  \hspace{1cm} (12)

where H is the component life in hours.

The risk per engine flying hour is defined as the rate of increase of probability of failure with respect to increased engine flying hours.

$$\frac{\text{risk}}{\text{efh}} = \frac{\partial \{p(\text{fail})\}}{\partial H}$$  \hspace{1cm} (13)

To simplify the derivation, the assumed lognormal dysfunction distribution is transformed to a Gaussian distribution and to map the fatigue lives of the test results to the transformed distribution, the logarithm of their values is taken. Since it is easier to consider the risk in terms of the rate of increase of the transformed variable $S'$ (i.e. log S), the following partial derivative is used:

$$\frac{\text{risk}}{\text{efh}} = \frac{\partial \{p(\text{fail})\}}{\partial S'} \cdot \frac{\partial S'}{\partial H} = -\frac{1}{H} \cdot \frac{\partial \{p(\text{fail})\}}{\partial S'}$$  \hspace{1cm} (14)

For the artificial case of an infinite sample of component test results, there would be no sampling error. In this case, the location of the component service life relative to the GM of the burst distribution would be precisely determined as shown in Figure 3. Therefore, the risk per engine flying hour (risk/efh) is simply equal to the shaded area in the figure.
In current UK lifting procedures, for fracture critical components both the form and the scatter of the population burst distribution are assumed to be identified [4]. A consequence of these assumptions is that, given a known failure distribution of an infinite population, each result can be considered as the mean of a sample of one. If groups of samples of 2, 3, 4...n are randomly selected from the total population, the means of the individual samples of size ‘n’ will also have a known distribution. Hence, a single component test sample of size ‘n’ will have a known error band associated with its use in the estimation of the mean of the total population. This is particularly important in risk analysis since the location of the population mean solely from the mean of the sample and without consideration of sampling error can lead to an under-prediction of the ‘best estimate’ of the risks associated with individual service components. The sampling error x’, is equal to the difference between the sample GM and the population GM. In other words, since the population mean life has been determined from the test sample mean life, the location of a component service life relative to the burst distribution is not precisely identified. Equally, the analysis can be considered from the viewpoint of an error in the service component life location relative to a fixed dysfunction distribution. This approach yields identical risk values as obtained with the assumption of dysfunction distribution error, but is simpler to illustrate, Figure 4.

It follows that for a specified service life and its associated sampling error, x’, the risk/efh is equal to the integral of the probability of burst distribution over the interval (efh) to (efh+1). The total risk/efh is now the summation, or the integral of these risks multiplied by the probability that the sampling error is x’. Hence, to account for the sampling error x’, the risk/efh is equal to the integral of the risk/efh given the location x’ of the sample mean on the sampling distribution, times the probability of the sample mean being located at x’ on the sampling distribution. Since the probability density function for the sampling error x’ can be determined, then the following standard statistical equation can be used to solve the risk/efh.

\[
P(y) = \int_{-\infty}^{\infty} P(y \mid x’).PD(x’).dx’
\]

where ‘y’ is the risk/efh. The solving of the above equations to give an expression in terms of risk levels associated with individual life expired discs is explained elsewhere [4]. Equation 16 is a basic form of this solution which has been used in some recent risk assessments.

\[
\frac{\partial(risk)}{\partial(H)} = 1.5 \left( \frac{n}{n+1.239} \right) \times \text{Exp} \left[ -6.94 \left( \frac{n}{n+1.239} \right) \left( \ln \left( \frac{H}{Ar \times \beta_i} \right) + 0.49094 \right) \right]
\]

\[
(n+1.239)(16)
\]
where ‘H’ is the life of the component service life in efh and ‘n’ is the component test sample size.

**Figure 4. Incorporation of disc sampling error into risk assessment.**

### 5.6 Risk based managed withdrawal of over-lifed parts

From an operational perspective, two of the most useful measures of the risk of fatigue failure of fracture-critical components are a) individual aircraft instantaneous risk per flight hour, and b) fleet cumulative risk per annum (or other suitable management period).

#### 5.6.1 Individual aircraft instantaneous risk of fatigue per flight hour

As components are exposed to continued service usage, the instantaneous risk of failure increases at a highly non-linear rate. A consequence of this is that, in the event of an unforeseen component life reduction, service engines may suddenly have life-expired parts and the operator needs to know the risks that leading parts are incurring. Hence, for example, in a recent service case, the original declared safe service life of 10,000 cycles was calculated from a test sample of $3.7 \times 10^6$. With a cyclic exchange rate of 2.0, at the full declared life, $H$, substitution into equation 16 gives an estimated risk/efh of $2.8 \times 10^{-8}$. A subsequent requirement reduced the declared safe service life of the component by 20%, to 8,000 cycles. However, at the original life of 10,000 cycles, the risk carried by such a component has now risen to $2.1 \times 10^{-7}$/efh. And hence for an individual disc close to the old service life, if this level of risk is deemed acceptable, then such a service life could be maintained for a limited period whilst replacement parts are procured.

An illustration of the form in of results obtained from a risk analysis of a typical aeroengine component, are illustrated in Table 5 (in this case the safe service life, $H$, equals 5,200 cycles). The table provides the calculated risk per efh at the service life for the component and provides estimates of the ‘available’ additional service hours until the operating risks reach the identified levels.
Table 5. Estimated lives corresponding to specified risk level/efh/engine using a sample size of 5, and a mission exchange rate equal to 3. (Propagation rates have been assumed to be 2.5 times initiation).

<table>
<thead>
<tr>
<th>Total cycles</th>
<th>Cycles over PSCL</th>
<th>Additional Efh</th>
<th>Risk/efh/engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,201</td>
<td>0</td>
<td>0</td>
<td>7 \times 10^{-8}</td>
</tr>
<tr>
<td>5,344</td>
<td>143</td>
<td>44</td>
<td>1 \times 10^{-7}</td>
</tr>
<tr>
<td>5,828</td>
<td>627</td>
<td>194</td>
<td>3 \times 10^{-7}</td>
</tr>
<tr>
<td>6,464</td>
<td>1,263</td>
<td>391</td>
<td>1 \times 10^{-6}</td>
</tr>
</tbody>
</table>

5.6.2 Effect of Fixed Inspection Intervals

In both Damage Tolerance and Retirement for Cause life assessment methods, inspection is perceived to be the essential parameter that underpins safety. A detection limit is set such that to 95% confidence there is a 90% probability that cracks of the maximum specified size will be detected if present. Using this crack size as the input parameter for finite element crack growth analysis, the allowable inspection interval is then set at 50% of the calculated mean crack growth life. It is asserted that the procedure has a built-in safety in that should a crack be missed a further inspection will have occurred prior to dysfunction.

The above illustrations although strictly accurate do not reflect the true safety levels associated with damage tolerance. Indeed safety comes from the fact that extremely few parts are likely to be cracked at the specified inspection interval (although for high strength materials severe quenching, etc. can crack inherent defects at the microscopic level). Given either case, damage tolerance does not optimise the inspection interval. For conventional superalloys, the ltc approach has demonstrated that typically the dysfunction life is 50% above ltc. Assuming a crack initiation size of 0.38 mm as a realistic level at which cracks will not be missed, wide European experience indicates that exchange rates for growth beyond the 0.38mm size are not less than a factor of 2.5 greater the rates up to this crack size.

In a damage tolerance context, the inspection interval would normally be based half the crack growth life, that is 25% of the initiation life. However, since a crack growth mission exchange rate has to be assumed, this 25% in cycles translates to only 10% in equivalent service hours that can be flown. It follows that under damage tolerance a disc could be inspected up to 10 times prior to it reaching the European predicted safe cyclic life.

Specifically with regard to retirement for cause and set inspection intervals, Figure 5 illustrates the increased risk associated with running all components until cracks are identified. In this example, the inspection interval has been set at half the propagation life. Inspections 1-3 are unnecessary and inspections 5-7 are too infrequent to ensure acceptable risk management.

![Figure 5. This is the effect of set inspection intervals on component service risk](image-url)
5.6.3 Risk governed inspection intervals

An extension of the risk model allows new components to be left in service until the dysfunction risk/reaches a specified limit. At this point component inspection is used to reduce the risk of failure. The model is again used to predict the dysfunction risk during subsequent service and hence the life extension prior to the set risk level being reached and the service intervals for all subsequent inspections. Figure 6 shows that the safe inspection intervals get progressively shorter until the life extension falls below a minimum economical viable service life. Although significant life extension beyond the pscl may be obtained using this approach while still ensuring that the risk of LCF failure does not exceed the specified level, the major benefit comes from identifying a safe service life beyond which retirement for cause would be unsafe. Hence the approach automatically sets an economic limit for retirement for cause but avoids the progressive risk levels associated with its current embodiment.

![Figure 6](image.png)

Figure 6. Prediction of the risk associated with application of Risk Based Damage Tolerance. Inspection intervals set to keep to remain within a set risk level of $10^{-7}$ per efh. (Note the long interval before inspection 1 and that the PSCL in this example is 2,880 efh).

6 FINAL COMMENTS

The paper has briefly summarised the common life assessment procedures for fracture critical parts. A procedure which allows the maximum benefit to be extracted from non-finite fatigue results whilst still maintaining safety. The application of risk based life assessment methods have been shown to provide more consistent levels of safety than standard procedures. The approach also enables the risks associated with the continued running of life expired parts and enables informed management decisions to be taken. The model has significant potential for application within damage tolerance lifing procedures and should allow inspection intervals to be set on the basis of defined risk levels. In this context the approach automatically sets an economic limit for retirement for cause but avoids the progressive risk levels associated with its current embodiment.

7 REFERENCES

2. Joint (European) Airworthiness Requirements - Engines (JAR-E), Civil Aviation Authority, 1986.