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# ROTOR STATOR ACOUSTIC INTERACTION, DEATH AND BIRTH OF RESONANCE FREQUENCY

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Guiding and resonance phenomenon near a periodic double cascade of plates has been investigated. Much attention has been paid to the difference from the oscillations near single periodic cascade of plates. It has been demonstrated that the mutual influence of the two periodic cascades decrease rapidly with the increasing distance between them, but at short distances the distortions in the oscillations localised near one periodic cascade of plates caused by the interference of the other become substantial.

## PROBLEM FORMULATION

In the present paper an elementary double cascade of plates resulting from shifts multiple to 1 of the fundamental area of the group of translations along Y axis is studied. (Fig 1)

Steady state oscillations near the structure are described with the function $u(x,y)$ which is the potential of acoustic speed perturbation or the pressure field. In the oscillation area this function satisfies the Laplace equation:

$$(\Delta + \lambda^2)u = 0, \tag{1}$$

Here $\lambda$ is a dimensionless oscillation frequency and it is assumed that $(\lambda \geq 0)$. On the cascade elements the Neuman conditions should be fulfilled:

$$\left.\frac{\partial u}{\partial n}\right|_{(G_1 \cup G_2)} = 0 \tag{2}$$

In any boundary area $\Omega_a$ which is a subarea of $\Omega$ the local energy finiteness condition should be fulfilled:

$$\int_{\Omega_a} \left(|\nabla u|^2 + |u|^2\right) < \infty \tag{3}$$

As the structure according to the problem formulation has a translational symmetry, the function $u(x,y)$ should satisfy the condition:

$$u(x, y+1) = e^{i\xi} u(x,y) \tag{4}$$

Where function $v(x,y)$ satisfies the condition $v(x,y) = v(x,y+1)$. Here we suppose that $0 < \xi \leq \pi$. The problem (1) – (4) will be further referred to as problem $B(\xi)$ .

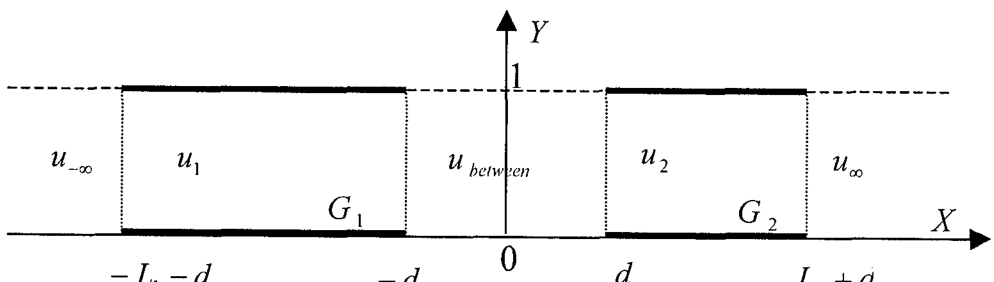


***Fig 1.***

Definition. *Waveguiding function* of the problem $B(\xi)$ is a generalized eigenfunction localized in the neighborhood of the cascade of plates, i.e. $u(x,y) \underset{|x|\to\infty}{\to} 0$

**THEOREM OF EXICTENCE**

Definition. The solutions of problem $B(\xi)$ u(x,y), which satisfy the condition **u(-x,y)=u(x,y)** or **u(-x,y)=-u(x,y)**, are called **symmetrical** $(\alpha)$ or **anti-symmetrical** $(\beta)$ modes.

***Theorem 1.*** If $L_1 = L_2$ then there are **symmetrical** modes for any geometrical parameters of the system and for all $\xi$.

***Theorem 2.*** If $L_1 = L_2 > 1$ than there are **anti-symmetrical** modes. If $L_1$ =L2<1 then there always exists such d that **anti-symmetrical** modes exist.

**DISPERSION RELATION**

The dimensionless wave frequencies $\lambda$ can be considered as functions of wave number $\xi$ These functions are so called dispersion relations.

$$\left(1-e^{2i(\theta_1+\lambda L_1)}\right)\left(1-e^{2i(\theta_1+\lambda L_2)}\right)-e^{2\theta_2}\left(1-e^{2i(\theta_1+\lambda L_1-2iarctan\left(\frac{\lambda}{\theta_0}\right))}\right)\left(1-e^{2i(\theta_1+\lambda L_2-2iarctan\left(\frac{\lambda}{\theta_0}\right))}\right)=0 \quad (5)$$

$$\theta_1 = -\frac{2\lambda\ln(2)}{\pi} + 2\arctan\left(\frac{\lambda}{\delta_0}\right) + \sum_{n=1}^{N}\arcsin\left(\frac{\lambda}{2\pi k-\xi}\right) + \arcsin\left(\frac{\lambda}{2\pi k+\xi}\right) - \arcsin\left(\frac{\lambda}{\pi k}\right)$$

$$\theta_2 = -\frac{2\delta_0\ln(2)}{\pi} - 2\delta_0 d + 2i\arctan\left(\frac{\delta_0}{\lambda}\right) + \sum_{n=1}^{N}\arctan\left(\frac{\delta_0}{\delta_n}\right) + \arctan\left(\frac{\delta_0}{\delta_{-n}}\right) - \arctan\left(\frac{\delta_0}{\pi k}\right)$$

These relations were numerically investigated.

**NUMERICAL INVESTIGATIONS**

Fig. 2-5 show the results of numerical investigation of the dispersion relation (5). Fig.2 shows the dependence of eigenwave frequencies $\lambda$ on the wave number $\xi$ provided that $L_1 = L_2 = 2$ and the distance between the cascades $d = 1$. It coincides entirely with corresponding dependence for a singular cascade with the same geometrical parameters [2].

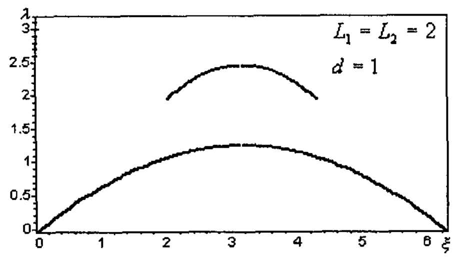


*Fig* 2.

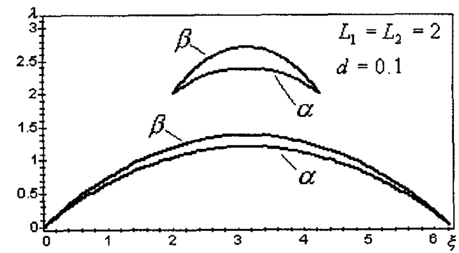


*Fig* 3.

Fig 3 shows the dependence of eigenwave frequencies $\lambda$ on the wave number $\xi$ for $L_1 = L_2 = 2$ and the distance between the cascades *d=0.1*. The frequencies can be seen to part into symmetrical$(\alpha)$ and anti-symmetrical $(\beta)$ ones for each oscillation mode in the contrast to a singular cascade. This process can be called as "birth" of resonance phenomena.

The frequency parting process is shown on Fig. 4. One can see the rapid decrease of the mutual influence.. Fig 5 shows the effect of the symmetry break ( $L_1 = 0.6, L_2 = 0.5$ ) on the waveguiding eigenvalues for $\xi = \pi$

**ACNOLEGMENT**

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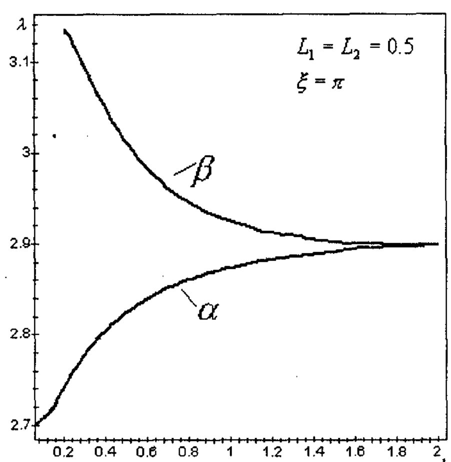


*Fig* 4.

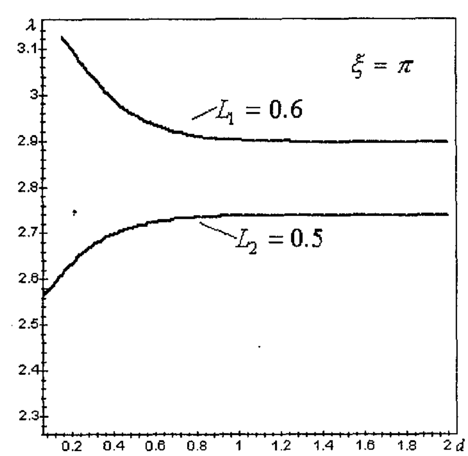


*Fig* 5.

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