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EXISTENCE THEOREMS FOR EIGENOSCILLATIONS IN 3D RECTANGULAR WAVEGUIDES

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ABSTRACT

The paper deals with the problem of eigenoscillations near the obstacle with the arbitrary sufficiently smooth shape of boundary immersed in three-dimensional waveguide of rectangular cross-section. Assumed that the guide and the obstacle are rigid. For a wide range of the obstacle geometry the existence of eigenwaves has been proved and their frequencies are embedded in the continuous spectrum.

INTRODUCTION

The investigation of eigenoscillations in unbounded waveguide regions is very important in many physics fields. First of all, the interest to the given problem is stipulated by the aeroacoustic resonance phenomenon the investigation of which is actual, e. g. when turbomachines designing (gas, vapour and hydraulic turbines, pumps, compressors), pipelines etc. (some experimental works review is in the paper[2]). A lot of papers (we'll mention only some of them [1,4-6,8]) are devoted to the investigation of eigenwaves in guide regions in two-dimensional case. The eigenwaves existence in three-dimensional guide regions has been investigated less completely. Let's note the paper [3] where the existence of the eigenoscillations being localized near sphere of the sufficiently small radius being situated in the center of the waveguide with the constant circular cross-section has been proved and also let's note the papers [7,9], where the cases of the thin-shelled obstacles in waveguides have been considered.

In the given paper by using the variational principle the sufficient conditions of the eigenwaves existence in three-dimensional waveguide with rectangular cross-section, where the obstacle with sufficiently arbitrary geometry possessing some symmetry conditions, have been obtained.
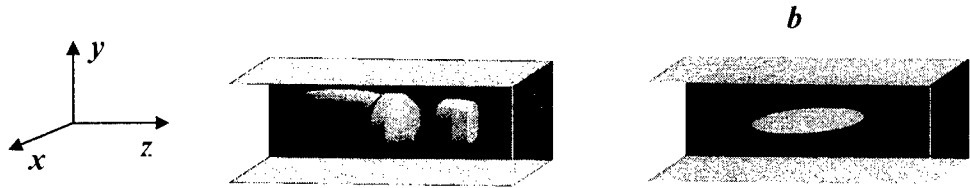
STATEMENT OF THE PROBLEM AND THE MAIN RESULTS

The domain Ω_0 is being considered:
$$\Omega_0=\{(x,y,z)\in \mathbb{R}^3: x \in (-d_1,d_1), y \in (-d_2,d_2), z \in \mathbb{R} \}, d_i{>}0 \ (i=1, 2).$$
The bounded obstacle B is placed in it. This obstacle may be disconnected. According to the obstacle type the following cases will be considered:

A. B is a compact set being bounded by the piecewise smooth surfaces and it is such one that $\mu_3(B){>}0$. (Here and further μ_3 means k-dimensional measure). It is supposed that B is symmetrical with respect to the plane $y=0$.

B. B is an infinitely thin plate with sufficiently smooth boundary, such one that $0{<}\mu_2(B){<}\infty$. It is supposed that B is situated in the plane $y=0$.

C. B is a compact set being bounded by the piecewise smooth surfaces and it is such one that $\mu_3(B){>}0$. It is supposed, that B is symmetrical with respect to the planes $x=0$ and $y=0$.

D. *B* is an infinitely thin plate with sufficiently smooth boundary which is situated in the plane $y=0$ and $0<\mu_2(B)<\infty$. It is supposed, that OZ axis is a line of symmetry for the obstacle *B*.

Fig. 1.

Waveguide regions geometry is represented in figure 1.
We seek the non-trivial solution $u(x,y,z)$ of the boundary value problem for the equation:

$$\Delta u + \lambda u = 0 \quad \left(\lambda \geq 0\right) \quad \text{in} \quad \Omega = \Omega_0\backslash B, \tag{1}$$

satisfying to the Neumann boundary condition on $\partial\Omega$:

$$\frac{\partial u}{\partial \vec{n}} = 0 \tag{2}$$

and to the condition of finite energy:

$$E(u) \equiv \iiint_\Omega \left(|u|^2 + |\nabla u|^2\right)d\Omega < \infty. \tag{3}$$

Here, $\vec{n}$ is a vector of the external normal to $\partial\Omega$.
Further, we'll call the problem (1)–(3) as the problem *N*. Besides, we are going to consider the problem N^{up} – the solution finding $u^{up}(x,y,z)$ of the boundary value problem (1)–(3) which is odd in y for **A** and **B** cases, and the problem N^a – the solution finding $u^a(x,y,z)$ of the boundary value problem (1)–(3) which is odd in x and y for **C** and **D** cases.
The parameter value λ for which the non-trivial solution of the problem *N* (of the problems N^{up} and N^a correspondingly) exists is called the eigenvalue of the problem *N* (of the problems N^{up} and N^a correspondingly). In addition, the self non-trivial solution is called the eigenfunction of the corresponding problem.
It is known, that the Neumann Laplacian possesses a continuous spectrum $[0,+\infty)$ for the domain Ω and the eigenvalues of the problem *N* (if they exist) turn out to be embedded in the continuous spectrum. It is obvious that the eigenvalues of the problems N^{up} and N^a are the eigenvalues of the problem *N*.
Let us introduce the notations:

$$\Omega_0^a = \left\{ (x,y,z) \in \Omega_0 : \quad x > 0, \quad y > 0 \right\}, \quad B^a = B \cap \overline{\Omega_0^a},$$

$$\Omega_0^{up} = \left\{ (x,y,z) \in \Omega_0 : \quad y > 0 \right\}, \quad B^{up} = B \cap \overline{\Omega_0^{up}},$$

$$\Omega^a = \Omega \cap \Omega_0^a, \quad \Omega^{up} = \Omega \cap \Omega_0^{up}.$$

The following lemmas are valid.
Lemma 1. The continuous spectrum of the problem N^{up} (N^a) is the semi-axis $[\Lambda_{up}^2,+\infty)$ ($[\Lambda_a^2,+\infty)$ correspondingly), where $\Lambda_{up}^2 = \pi^2/4d_2^2$, $\Lambda_a^2 = \pi^2/4d_1^2 + \pi^2/4d_2^2$.

Lemma 2. Let

$$\lambda_0^{up,\,a} = \inf_{\substack{\psi \in H_0^1(\Omega^{up.a}) \\ \psi \neq 0}} \frac{\iiint_{\Omega^{up.a}} |\nabla \psi|^2 \, d\Omega}{\iiint_{\Omega^{up.a}} |\psi|^2 \, d\Omega}.$$

Then $\lambda_0^{up} \leq \Lambda_{up}^2$ ($\lambda_0^a \leq \Lambda_a^2$ correspondingly). More over, if $\lambda_0^{up} < \Lambda_{up}^2$ ($\lambda_0^a < \Lambda_a^2$ correspondingly), then λ_0^{up} (λ_0^a) is the lowest eigenvalue of the problem N^{up} (N^a), and if $\lambda_0^{up} = \Lambda_{up}^2$ ($\lambda_0^a = \Lambda_a^2$), then the eigenvalues of the problem N^{up} (N^a) do not exist in the interval $(-\infty, \Lambda_{up}^2)$ ($(-\infty, \Lambda_a^2)$ correspondingly).

Using these lemmas, the existence of eigen waves has been proved.

Theorem. Eigenfunctions of the problem N^{up} exist:

in case **A**, if the following inequality holds:

$$\iiint_{B^{up}} \cos\left(\frac{\pi y}{d_2}\right) d\Omega > 0 ; \tag{4}$$

in case **B** – exist always.

And the lowest eigenvalue λ_0^{up} belongs to the interval $(0, \Lambda_{up}^2)$.

Eigenfunctions of the problem N^a exist:

in case **C**, if the following inequality holds:

$$\iiint_{B^a} \left[\frac{1}{d_1^2} \cos\left(\frac{\pi x}{d_1}\right) + \frac{1}{d_2^2} \cos\left(\frac{\pi y}{d_2}\right) - \left(\frac{1}{d_1^2} + \frac{1}{d_2^2}\right) \cos\left(\frac{\pi x}{d_1}\right) \cos\left(\frac{\pi y}{d_2}\right) \right] d\Omega > 0 ; \tag{5}$$

in case **D** – exist always.

And the lowest eigenvalue λ_0^a belongs to the interval $(0, \Lambda_a^2)$.

Corollary. Eigenfunctions of the problem N^{up} exist in case **A**, if the obstacle B is included in the set $\{(x, y, z) \in \Omega_0 : |y| < d_2/2 \}$. And the lowest eigenvalue λ_0^{up} belongs to the interval $(0, \Lambda_{up}^2)$.

Eigenfunctions of the problem N^a exist in case **C**, if the obstacle B is included in the set $\{(x, y, z) \in \Omega_0 : |y| < d_2/2 , |x| < d_1/2 \}$. And the lowest eigenvalue λ_0^a belongs to the interval $(0, \Lambda_a^2)$.

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