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ON UNIQUE ESTIMATION OF AZIMUTH-ELEVATION-CARRIERS BY VOLUME ARRAYS

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ABSTRACT

The necessary and sufficient conditions in terms of antenna array geometry for unique estimation of azimuth-elevation-carriers (AEC) of narrow-band signals are introduced. These conditions make it possible to solve the problem of joint AEC estimation based on spatial samples taken by the volume array without using the large number of temporal samples.

INTRODUCTION

The problem of joint azimuth-elevation-carriers (AEC) estimation of narrow-band signals arises in communication, radar, radio astronomy, sonar and many other applications. An important issue related to this problem is the unique estimation of signal AECs via array of given geometry. This issue called also as the identifiability problem (see [1]-[3] and references herein). The standard formulation of this problem involves determination of the number of harmonics that can be resolved for a given total sample size. The solutions to be proposed are obtained under conditions of equispaced samples, existence of all samples along all the array dimensions and difference of all harmonics to be resolved. These conditions significantly restrict the practical implementation of the theory. In this paper we present the necessary and sufficient conditions for unique AEC estimation by volume array those are given in terms of the number of the antenna array sensors and array geometry.

DATA MODEL AND PROBLEM FORMULATION

We assume that there are M point sources that are emitting the unknown narrow-band complex deterministic signals $s_m(t)$, $m=1...M$ in the direction of the measurement system. For every signal the parameters of interest are the azimuth β_m, the elevation ε_m and the carrier f_m. To describe the signal parameters let us introduce a vector $\boldsymbol{\mu}_m = f_m \boldsymbol{e}_m / f_{max} \in R^3$, $\boldsymbol{\mu}_m \in U_\mu = \{\boldsymbol{\mu} : \beta \in [0, \pi], \varepsilon \in [0, \pi], f \in [f_{min}, f_{max}]\}$, where $\boldsymbol{e}_m=[cos\beta_m cos\varepsilon_m,\ sin\beta_m cos\varepsilon_m,\ sin\varepsilon_m]^T$ is the unit vector in the Cartesian coordinate system and $(\cdot)^T$ denotes transpose. It is assumed that the measurement system consists of N omnidirectional point sensors. Then the model of antenna array output in noiseless case can be represented as follows:

$$z(t) = \boldsymbol{A}\boldsymbol{s}(t) \in C^{N\times 1},\ t = 1...L, \tag{1}$$

where $A=[a(\mu_1)\ldots a(\mu_M)]\in C^{N\times M}$ is a matrix of steering vectors, $a(\mu_m)=[1, \exp\{j2\pi\, d_2^T\mu_m\}, \ldots, \exp\{j2\pi\, d_N^T\mu_m\}]^T$ is a steering vector, $d_n = d_n i_n c/f_{max}$ is a vector of the nth sensor position ($d_1=0$), d_n is a distance to the nth sensor, i_n is a unit vector toward the nth sensor position, $D=[d_1\ldots d_N]\in R^{3\times N}$ is a matrix that specify the antenna array geometry, $s(t)\in C^{M\times 1}$ is a vector of the signal waveforms, L denotes the number of data snapshots available. We assume, that the sampled covariance matrix of signal waveforms $\hat{P} = (L)^{-1}\Sigma_{t=1}^{L} s(t)s^H(t) \in C^{M\times M}$ has full rank, that is $rank(\hat{P}) = M$, and the number of signals M is known.

The problem to be solved is formulated as follows: to determine the basic requirements for the number N and positions D of array sensors, that guarantee the unique estimation of any distinct set $\mu_1 \neq \ldots \neq \mu_M \in U_\mu$ of signal parameters.

IDENTIFIABILITY OF AEC ESTIMATES

The necessary and sufficient conditions for identifiability of U_μ are [1] *NSC1:* the set of steering vectors $a(\mu)$ is known; *NSC2:* for any $\mu_1 \neq \ldots \neq \mu_M \in U_\mu$ the matrix A has full rank $rank[a(\mu_1)\ldots a(\mu_M)] = min(N, M) = M < N$; *NSC3:* the number of sensors N, the rank R of matrix P and the number Q of parameters per signal satisfy to the following inequality $N \geq R + Q = M + 3$. The *NSC1* and *NSC3* hold if the antenna array is calibrated and its geometry D is known, and the maximum number of impinging signals $M \leq N - 3$. Therefore, the problem to be solved can be reduced to the determination of the requirements for the sensor positions D that guarantee the implementation of the *NSC2*.

Theorem: For any $\mu_1 \neq \ldots \neq \mu_M \in U_\mu$ the matrix A has full rank if: i) the array has a subarray with $K \leq N$ sensors, such that the number N_p of the parallel planes that can be passed through the points $d_1, \ldots, d_K$ satisfy the conditions $N_p = K - 2 > M$, and ii) the maximum distance d_{max} between any nearest sensors of subarray is less then half of minimum wavelength $d_{max} \leq \lambda_{min}/2 = c/(2f_{max})$.

Proof: The proof is based on the fact that the rank of matrix A is defined as follows $rank(A) = min(N - K_r, M - K_c)$, where K_r is the number of rows and K_c is the number of columns that are coincident to any other row or column respectively. Condition ii) means, that the maximum sampling frequency in spatial domain is higher then the Nyquist rate, therefore the vectors μ giving $K_c > 0$ columns that are coincident to the column $a(\mu_1)$ are the roots of the following set of equations

$$i_n^T \mu = c_n, \quad n = 2\ldots K, \tag{2}$$

where $c_n = i_n^T \mu_1$. The vectors $\mu^{(n)}$ that satisfy to the nth equation of (2) represent the set of points lying on a plane $\Xi(i_n, c_n) \in R^2$ that is orthogonal to the vector i_n and located on the distances $|c_n| \leq 1$ from the origin. Therefore the set U'_μ of roots of (2) are the intersection between U_μ and the planes $\Xi(\boldsymbol{i}_n, c_n)$: $U'_\mu(\boldsymbol{i}_2 ... \boldsymbol{i}_K) = U_\mu \bigcap_{n=2}^{K} \Xi(\boldsymbol{i}_n, c_n)$.

Observe, that the set U_μ is a space between two hemispheres of radiuses f_{min}/f_{max} and 1. Therefore, if the vectors $\boldsymbol{d}_1 ... \boldsymbol{d}_K$ do not lie on the same plane, i.e. $rank([\boldsymbol{d}_1 ... \boldsymbol{d}_K]) = 3$, then the set U'_μ is a point and only point $U'_\mu = \{\boldsymbol{\mu}_1\}$. It means, that if the subarray is a volume, then for any $\boldsymbol{\mu}_1 \neq ... \neq \boldsymbol{\mu}_M \in U_\mu$ the inequality $\boldsymbol{a}(\boldsymbol{\mu}_1) \neq ... \neq \boldsymbol{a}(\boldsymbol{\mu}_M)$ holds true and $K_c = 0$. Observe also, that if $K_1 \leq K$ sensors lie on a plane, then the set $U'_\mu(\boldsymbol{i}_2 ... \boldsymbol{i}_{K_1})$ is a section and consists of infinite number of elements. Hence, for any distinct $\boldsymbol{\mu}_1 \neq ... \neq \boldsymbol{\mu}_M \in U'_\mu(\boldsymbol{i}_2 ... \boldsymbol{i}_{K_1})$, K_1 equations of (2) hold and K_1 rows of the matrix $[\boldsymbol{a}(\boldsymbol{\mu}_1) ... \boldsymbol{a}(\boldsymbol{\mu}_M)]$ are equal, i.e. $K_r = K_1$. The $K_r - 1$ rows can be rejected without loss of information. Therefore, to ensure the full rank of the matrix $\boldsymbol{A}$ the number of sensors lying in nonparallel planes has to satisfy the condition $N_p > M$. Since the plane can be passed through any three sensors, the minimum size of the subarray is $K = N_p + 2$.

CONCLUSION

An important corollary of the theorem is that the joint AEC estimation of narrow-band signals can be performed by means of spatial samples taken by volume antenna array. The temporal averaging is needed to ensure a full rank of the covariance matrix of signal waveforms as well as to increase the signal-to-noise ratio. In practice analogue tools can successfully carry out this procedure that is very important for the MW measurements systems. Another conclusion is that the arrays with irregular geometry have advantages in comparison with the periodic arrays from the identifiability point of view.

The possible applications of the considered theory could include the wide-range spectrum monitoring direction finders, Doppler radar systems, wireless communication systems and other.

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