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ADP013961

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## SIMULATION OF A DISCRETE LUNEBURG LENS FED BY A CONFORMAL PRINTED ANTENNA

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### INTRODUCTION

Communications and information technology stimulate a development of various antennas. Because of their simplicity, slot fed circular microstrip antennas (MA) [1] seem to be attractive. Even more attractive are MAs conformally printed on curved surfaces, such as spherical-circular MA (SCMA) because of their higher degree of freedom. However, conventional numerical methods, such as Moment-Method (MM) or FDTD need very high computer resources and do not guarantee a convergence because of ill-conditioned matrices, numerical instabilities, and vulnerability to high-Q resonances. Besides, many applications need special properties: agile scanning beam, multibeam capability, scanning in a large field of view, etc. Here, a very attractive candidate is a discrete Luneburg lens (LL) [2,3,4], which is a layered dielectric sphere. Spherical geometry of both SCMA and LL enables one to simulate them with the same method. Here, we shall use the Method of Analytical Regularization (MAR) [5-7], sometimes called semi-inversion method. Generally, it converts a first-kind singular integral or series equation to a well-conditioned second-kind Fredholm matrix equation, and therefore serves as a perfect pre-conditioner of an ill-posed problem. Then both numerical convergence and efficiency is achieved and matrix-truncation error is controlled.

### ELECTROMAGNETIC MODELING

Suppose that the layer # $i$  is characterized by its material properties  $\varepsilon_{ri}$  and  $\mu_{ri}$ , outside radius  $r_i$ , and the size of corresponding PEC spherical disk,  $\theta_i$ , like in Fig. 1. The used excitation is a tangential magnetic dipole (TMD). Spherical geometry offers to expand electromagnetic field in terms of vector spherical modes involving  $P_n^l$ , associated Legendre functions, with coefficients  $a_n^{s,i}$  and  $b_n^{s,i}$ .

The first step concerns the application of so-called dual boundary conditions. This yields a set of  $4N_{shell}$  coupled dual series equations (DSE): for  $0 \leq \theta < \theta_i, i=1 \dots N_{shell}, \sigma = e, o$ ,

$$\sum_{n \geq 1} \left( C_{an}^Z \cdot X_{an}^\sigma - X_{an}^{\sigma Z, feed} \right) \cdot P_n^1(\cos \theta) = \sigma(-1) \cdot C_1^\sigma \tan \theta / 2$$

$$\sum_{n \geq 1} \left( C_{bn}^K \cdot X_{bn}^\sigma - X_{bn}^{\sigma K, feed} \right) \cdot P_n^1(\cos \theta) = -C_1^\sigma \tan \theta / 2,$$

and also, for  $\theta_i < \theta \leq \pi, i=1 \dots N_{shell}, \sigma = e, o$ ,

$$\sum_{n \geq 1} \left( C_{an}^K \cdot X_{an}^\sigma - X_{an}^{\sigma K, feed} \right) \cdot P_n^1(\cos \theta) = \bar{\sigma}(-1) \cdot C_2^\sigma \cot \theta / 2$$

$$\sum_{n \geq 1} \left( C_{bn}^Z \cdot X_{bn}^\sigma - X_{bn}^{\sigma Z, feed} \right) \cdot P_n^1(\cos \theta) = C_2^\sigma \cot \theta / 2,$$

where  $X_n^{feed}$  are vectors corresponding to the slot feed field modal description,  $C_n^Z$  and  $C_n^K$  are matrices related to the structure without metallic elements. The unknowns are the vectors  $X_n$ ,  $n \geq 1$ ,  $X_{an}^\sigma = [a_n^{4,2} \dots a_n^{4,N_{shell}-1}]^T$ ,  $X_{bn}^\sigma = [b_n^{4,2} \dots b_n^{4,N_{shell}-1}]^T$ .  $C_1^\sigma$  and  $C_2^\sigma$  correspond to auxiliary constants to be determined.  $\sigma(-1) = +1$  if  $\sigma = o$  and  $\sigma(-1) = -1$  otherwise. Besides, the power boundedness condition determines the allowable class of the unknowns as

$$\sum_{n \geq 1} \left\| C_{an}^Z \cdot X_{an}^\sigma / n \right\|^2 < +\infty \quad \sum_{n \geq 1} \left\| n \cdot C_{bn}^K \cdot X_{bn}^\sigma \right\|^2 < +\infty.$$

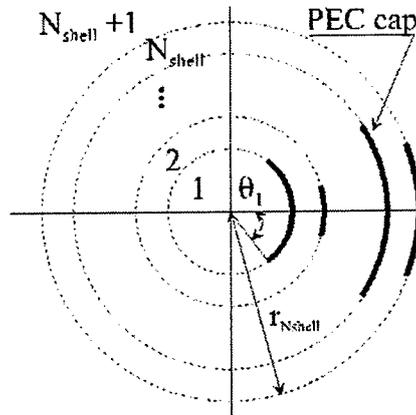


Fig. 1: Definition of the structure.

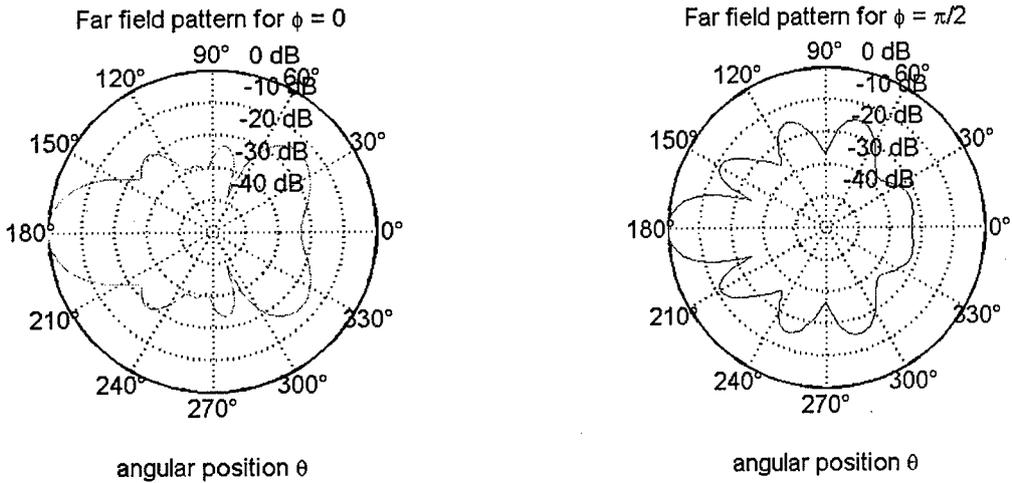


Fig. 2. Far field pattern for a SCMA fed LL with  $k_0 r_{outside} = 10.0$ .

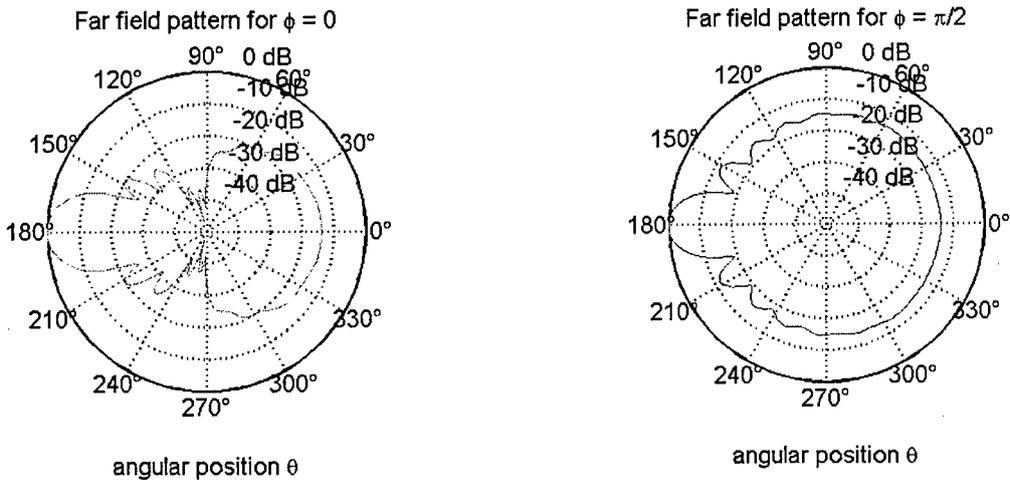


Fig. 3. Far field pattern for a TMD fed 40-layer LL with  $k_0 r_{lens} = 10.0$ .

## MAR TREATMENT

Mentioned above DSEs are collectively the 1<sup>st</sup> kind equation which can be written as  $CX = Y$  in terms of operators. Unfortunately,  $C$  is not directly invertible and its numerical inversion is not convergent. According to [6,7],  $C$  can be cut in such a way that  $C = (C_1 + C_2)C_0$ , with  $C_1^{-1}$  a known operator. Then, the 1<sup>st</sup> kind problem becomes the 2<sup>nd</sup> kind problem  $Z + AZ = Z_0$ , where  $Z_0 = C_1^{-1}Y$ ,  $Z = C_0X$  and also  $A = C_1^{-1}C_2$ .  $A$  is a Fredholm operator provided that  $\|A\|_{L^2} < +\infty$ . In our problem, MAR technique is easily applied thanks to some simple variable changes:  $\Omega_a(I - G_{an})Z_{an}^\sigma = n(n+1)C_{an}^Z X_{an}^\sigma$ , and  $(2n+1)\Omega_b(I - G_{bn})Z_{bn}^\sigma = n(n+1)C_{bn}^K X_{bn}^\sigma$ , for  $0 \leq \theta < \theta_i, i=1 \dots N_{shell}$ , with  $\Omega_a$  and  $\Omega_b$  two constant matrices, and,  $(2n+1)Z_{an}^\sigma = n(n+1)C_{an}^Z X_{an}^\sigma$ , and  $Z_{bn}^\sigma = n(n+1)C_{bn}^Z X_{bn}^\sigma$ , for  $\theta_i < \theta \leq \pi, i=1 \dots N_{shell}$ . By the use of the Mehler-Dirichlet expressions, the set of  $4N_{shell}$  coupled linear equations reduces to a set of 2<sup>nd</sup> kind equations:

$$Z_m^\sigma + \sum_{n \geq 1} A_{m,n} \cdot Z_n^\sigma = Z_m^{\sigma, feed}, \quad \forall m \geq 1.$$

By collecting the unknowns into a double infinite vector  $Z^\sigma = [Z_{a0}^{\sigma T}, Z_{b0}^{\sigma T}, Z_{a1}^{\sigma T}, Z_{b1}^{\sigma T}, Z_{a2}^{\sigma T}, Z_{b2}^{\sigma T}, \dots]^T$ , and also  $Z_0^\sigma = [Z_{a0}^{feed T}, Z_{b0}^{feed T}, Z_{a1}^{feed T}, Z_{b1}^{feed T}, Z_{a2}^{feed T}, Z_{b2}^{feed T}, \dots]^T$ , and by defining an infinite matrix  $A$  having  $(2N_{shell}+2) \times (2N_{shell}+2)$  infinite blocks composed of the  $A_{m,n}$  matrices, the whole set of the 2<sup>nd</sup> kind can be written as  $Z^\sigma + AZ^\sigma = Z_0^\sigma$ . Each  $A_{m,n}$  is a product of two terms,  $A_{m,n} = A_n^s \cdot A_{m,n}^d$  where  $A_n^s$  depend on the shells characteristics ( $r_i, \varepsilon_{r_i}, \mu_{r_i}, i=1 \dots N_{shell}$ ) and  $A_{m,n}^d$  on the disk ones ( $\theta_i, i=1 \dots N_{shell}$ ). Moreover,  $A_n^s$  behave as  $O(1/n)$ , and  $A_{m,n}^d$  as  $O(1/(n-m))$  if  $m \neq n$ , and like  $O(1)$  otherwise. As a consequence,  $A$  is compact as  $\|Z_0\| < +\infty$  that ensures the existence of unique solution, which can be approached as closely as wanted thanks to point-wise convergence.

## RESULTS

In order to ensure a 3-digits accuracy in solving the matrix all the computations were done with  $N = 120$ . Figs. 2 and 3 present far field patterns of LL fed by SCMA+TMD and a simple TMD feeds, respectively. Here, the patch is the inner (smaller:  $\theta_{inside} = 1.3^\circ$ ) conductor, and the ground surface is formed by the outer (larger) metallic cap ( $\theta_{outside} = 3.0^\circ$ ). The focusing effect is clearly seen, as well as the shadowing by SCMA. MAR technique is a very powerful and economic method to study complete SCMA-LL radiation problem. It enables one to highlight with controlled accuracy the effects of finite ground size, curvature, several types of resonances connected with SCMA, ground and coating, and LL focusing, etc.

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