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# THE PLANE H-POLARIZED WAVE DIFFRACTION BY A METAL GRATING WITH A MAGNETOACTIVE PLASMA

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A periodic grating of infinitely thin, perfectly conducting metal strips is considered in the $yOz$ plane. The $d$-spaced strips are extending along the $oZ$ axis, the grating period is $l$. The subspace $x<0$ is occupied by a magnetoactive plasma with the magnetic field having the $oZ$ direction. The plasma is characterized by the tensor

$$\hat{\varepsilon} = \begin{Vmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{Vmatrix},$$

where $\varepsilon_1 = 1 - \dfrac{\chi_p^2}{\chi^2 - \chi_c^2}$, $\varepsilon_2 = -\dfrac{\chi_p^2 \chi_c}{\chi\left(\chi^2 - \chi_c^2\right)}$, $\varepsilon_3 = 1 - \dfrac{\chi_p^2}{\chi^2}$, and $\chi = \dfrac{\omega\, l}{2\pi c}$, $\chi_p = \dfrac{\omega_p l}{2\pi\, c}$, $\chi_c = \dfrac{\omega_c l}{2\pi\, c}$ ; $\omega = kc$ is the incident field frequency, $\omega_p$ and $\omega_c$ are, respectively, the plasma and electron cyclotron frequencies, $c$ is the velocity of light in vacuum.

From above $(x>0)$, a plane $H$-polarized electromagnetic wave $e^{-ikx}$ is normally incident on the grating plate. The time dependence is chosen to be $e^{-i\omega t}$. The field of the wave diffraction by the grating-plasma structure is necessary to find.

The diffraction field (function $V_1(x,y)$ and $V_2(x,y)$) sought in terms of the boundary value problem:

$$\varepsilon_1 \Delta V_2(x,y) + k^2\left(\varepsilon_1^2 - \varepsilon_2^2\right)V_2(x,y) = 0, \quad x<0;$$
$$\Delta V_1(x,y) + k^2 V_1(x,y) = 0, \quad x>0; \qquad (1)$$

$$V_j(x, y \pm l) = V_j(x,y),\ j = 1,2; \qquad (2)$$

$$\left(\frac{\partial V_2(0,y)}{\partial x} - i\tau \frac{\partial V_2(0,y)}{\partial y}\right) = 0, \text{ on metal,} \qquad (3.a)$$

$$\frac{\partial V_1(0,y)}{\partial x} = ik, \text{ on metal,} \qquad (3.b)$$

$$\left(\lambda \frac{\partial V_1(0,y)}{\partial x} - \frac{\partial V_2(0,y)}{\partial x} + i\tau \frac{\partial V_2(0,y)}{\partial y} + i\lambda k\right) = 0, \text{ over grating period,} \qquad (4)$$

$$\left(V_2(0,y) - V_1(0,y)\right) = 1, \text{ in slot.} \qquad (5)$$

In addition, functions $V_1(x,y)$ and $V_2(x,y)$ must fit the Meixner and radiation conditions on any compact set in the $xOy$ plane. Here $\lambda = \frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1}$, $\tau = \frac{\varepsilon_2}{\varepsilon_1}$, and functions $V_1(x,y)$ and $V_2(x,y)$ are related to the scattered field component $H_z(x,y)$ as follows

$$H_z(x,y) = \begin{cases} V_1(x,y); x > 0, \\ \left(\varepsilon_1^2 - \varepsilon_2^2\right)V_2(x,y); x < 0. \end{cases}$$

Satisfying boundary conditions (3)-(5) gives the system of dual series equations

$$\begin{cases} \sum\limits_{(n,n\neq 0)} \frac{1+\tau_n}{1+\tau_n+\lambda}|n|x_n e^{\frac{2\pi}{l}iny} = \frac{\chi i}{1+\sqrt{\lambda}}(x_0+2) + \sum\limits_{(n,n\neq 0)} \frac{1+\tau_n}{1+\tau_n+\lambda}|n|x_n\delta_n e^{\frac{2\pi}{l}iny}, on \quad metal, \\ \sum\limits_{(n)} x_n e^{\frac{2\pi}{l}iny} = 0, in \quad slot, \end{cases} \tag{6}$$

with $x_0 = b_0\left(1 + \frac{1}{\sqrt{\lambda}}\right) - 2$, where $b_0 = \sqrt{\lambda}\left(1 - a_0\right), \tau_n = sign(n)$.

For all $n \neq 0$, $x_n = \left(1 + \frac{\xi_n' + i\tau_n}{\lambda\xi_n}\right)b_n$, $\xi_n = \sqrt{\frac{\chi^2}{n^2} - 1}$, $\xi_n' = \sqrt{\lambda\frac{\chi^2}{n^2} - 1}$, $a_n = -\frac{\xi_n' + i\tau_n}{\lambda\xi_n}b_n$.

The values to find are amplitudes $a_n$ and $b_n$ of the diffraction spectra: $a_0$ and $b_0$ are, respectively, the reflection and transmission coefficients.

The authors' analytical regularization procedure suggested in [1] makes it possible to convert (6) into the infinite system of linear algebraic equations of the type

$$x_n = \sum_{m=-\infty}^{+\infty} B_{nm}x_m + w_n. \tag{7}$$

The matrix elements look like

$$B_{nm} = \begin{cases} \frac{\chi i(1+\varepsilon_1-\varepsilon_2)}{1+\sqrt{\lambda}}A_{00}; n=0, m=0, \\ (-1)^n A_{n0}\frac{\chi i(1+\varepsilon_1-\varepsilon_2)}{1+\sqrt{\lambda}}; n\neq 0, m=0, \\ (-1)^m|m|\delta_m\Lambda_m A_{0m} \quad ; n=0, m\neq 0, \\ (-1)^{n+m}|m|\delta_m\Lambda_m A_{nm}; n\neq 0, m\neq 0, \end{cases} \qquad w_n = \begin{cases} \frac{2\chi i(1+\varepsilon_1-\varepsilon_2)}{1+\sqrt{\lambda}}A_{00}: n=0 \\ \frac{2\chi i(1+\varepsilon_1-\varepsilon_2)}{1+\sqrt{\lambda}}(-1)^n A_{n0}; n\neq 0. \end{cases}$$

The $A_{nm}$ expressions can be found in [1] and $\Lambda_n = \begin{cases} 1: n>0, \\ \frac{1+\varepsilon_1-\varepsilon_2}{1+\varepsilon_1+\varepsilon_2}: n<0. \end{cases}$ For large $m$, the smallness parameter $\delta_m \approx \left(\frac{\chi}{2m}\right)^2 \times \begin{cases} 1+\varepsilon_1-\varepsilon_2 : m>0 \\ 1+\varepsilon_1+\varepsilon_2 : m<0. \end{cases}$

So, it has been shown that matrix $\|B_{mn}\|_{m,n=-\infty}^{+\infty}$ generates the Hilbert-Schmidt operator in $l_2$, and $w_n \in l_2$. Hence the solution of (7) can be obtained by truncation with any preassigned accuracy.

No wave transmission is found if $\chi = \sqrt{\chi_c^2 + \chi_p^2}$ and $\chi = \frac{1}{2}(\sqrt{\chi_c^2 + 4\chi_p^2} \pm \chi_c)$ because at these frequencies the plasma acts as a perfect reflector.

In the long-wave region, the reflection and transmission coefficients take the form

$$a_0 \approx \frac{\chi i A_{00}(1+\varepsilon_1-\varepsilon_2)+1-\sqrt{\lambda}}{\chi i A_{00}(1+\varepsilon_1-\varepsilon_2)-1-\sqrt{\lambda}}, \quad b_0 \approx \frac{2\sqrt{\lambda}}{1+\sqrt{\lambda}-\chi i A_{00}(1+\varepsilon_1-\varepsilon_2)}. \tag{8}$$

For $\dfrac{1+\varepsilon_1-\varepsilon_2}{1+\varepsilon_1+\varepsilon_2} > 0$, $A_{00} = \dfrac{e^{-2\beta\theta}}{\pi} R_\sigma(\beta,\theta) \dfrac{2\theta\varepsilon_2}{1+\varepsilon_1-\varepsilon_2} - \dfrac{e^{-\pi\beta}}{2ch(\pi\beta)}\left(e^{-2\beta\theta} R_\sigma(\beta,\theta) + e^{2\beta\theta} R_\sigma(-\beta,\theta)\right)$,

where $\beta = \dfrac{1}{2\pi} ln \dfrac{1+\varepsilon_1-\varepsilon_2}{1+\varepsilon_1+\varepsilon_2}$, $\theta = \pi\left(1-\dfrac{d}{l}\right)$; $R_\sigma(\beta,\theta) = \sum\limits_{n=-\infty, n\neq 0}^{+\infty} \dfrac{(-1)^n}{n} P_{n-1}(-\beta,\theta)$ with $P_n(\beta,\theta)$ being the Pollachek polynomials. Notice that if $\chi_p$ and $\chi_c$ are both zero together, expressions (8) turn into the standard Lamb formulae for a grating at no plasma medium.

For $\dfrac{1+\varepsilon_1-\varepsilon_2}{1+\varepsilon_1+\varepsilon_2} < 0$,

$$A_{00} = \frac{e^{-\pi\tilde{\beta}}}{2sh(\pi\tilde{\beta})}(1-e^{2\tilde{\beta}\theta}) - \frac{2\varepsilon_2}{\varepsilon_2-\varepsilon_1-1}\frac{\theta}{\pi} + \frac{e^{\tilde{\beta}(\theta-\pi)}}{\pi}\int_0^\theta \varphi \, sin\left(\tilde{\beta}\, ln \frac{sin\frac{\theta+\varphi}{2}}{sin\frac{\theta-\varphi}{2}}\right) d\varphi,$$

where $\tilde{\beta} = \dfrac{1}{2\pi} ln \dfrac{\varepsilon_2-\varepsilon_1-1}{\varepsilon_2+\varepsilon_1+1}$. The integral in $A_{00}$ presents no calculation problems since it can be represented as a well convergent series expansion in the polynomials $B_n$ given in [1]. For $\tilde{\beta} = 0$, $A_{00} = -\dfrac{\theta}{\pi}$.

## REFERENCES

[1] A.V. Brovenko, P.N. Melezhik, and A.Ye. Poyedinchuk, The regularization method to a class of dual series equations, Ukrainian Math. Zh., 2001,v.53, no.10, pp.1320-1327 (in Ukrainian).