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THE SECOND OR THE THIRD HARMONIC GENERATION ON A NONLINEAR FILM IN A BRAGG RESONATOR

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ABSTRACT

A resonator formed by nonperiodic Bragg structure and nonlinear film is considered. Effects of the second or the third harmonic generation are theoretically investigated. System of nonlinear equations for amplitudes of time harmonics is obtained. Phenomenon of resonant increasing of amplitude of the higher harmonic caused by combination of nonlinear and Bragg effects is discovered. Reason of instability of numeric algorithm for "optimal" case is revealed. Ways for overcoming this instability are found. Bistable behavior of scattered waves is observed for cubic nonlinearity.

INTRODUCTION

If a harmonic plane wave falls on a nonlinear film scattered field contains higher harmonics (the second harmonic for quadratic non-linearity or the third one for cubic non-linearity). As a rule values of non-linearity and amplitude of fundamental harmonic is not very large so that amplitudes of generating higher harmonics are usually small [1]. Higher-harmonic amplitude is increased under phase-matching condition that can be achieved more easily in nonlinear spatially periodic waveguiding structure [2]. Amplitude of wave falling onto nonlinear film can be increased also by situating nonlinear film into resonator. Non-periodic Bragg structures used as resonator reflector [3] have flexible frequency characteristics so that they can be useful for devices operating on some frequencies simultaneously.

METHOD OF ANALYSIS

In the paper physical effects in a resonator formed by non-periodic Bragg reflector and non-linear film situated on metal plane are theoretically considered. The boundary-value problem for H_x contains spatial inhomogeneity in wave equation and non-linearity in quasiimpedance boundary condition

$$\frac{\partial^2 H}{\partial y^2} - \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial y} \frac{\partial H}{\partial y} - \frac{\varepsilon(y)}{c^2} \frac{\partial^2 H}{\partial t^2} = 0, \quad (1)$$

$$\left[\frac{\partial H}{\partial y} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} (w_0 H + w_N H^q) \right]_{y=0} = 0. \quad (2)$$

where w_0 is unperturbed values of impedance parameter, w_N is parameter describing non-linearity of film, $q = 2$ (quadratic non-linearity) or $q = 3$ (cubic non-linearity). For quasiperiodic Bragg structure permittivity as function of coordinate y has form:

$$\varepsilon(y) = \varepsilon_a + \beta \sum_j \varepsilon_j(\beta y) \exp[i\psi_j(y)], \quad (0 < y < y_b) \quad (3)$$

$\psi_j(y) = \int \chi_j(\beta y) dy$, y_b is coordinate of Bragg-structure beginning, β is small parameter [3]. If the non-linearity is very small, undepleted pump approximation could be used [4].

This approach is easy for implementation, but it does not allow one to take into account changing energy of fundamental harmonic. We suppose that plane time-periodic wave ($T = 2\pi/\omega$ is time period) falls on the structure. Along with real function $H(y,t)$ we consider complex function $H_c(y,t)$. The solution of the boundary-value problem (1), (2) for $H(y,t)$ and $H_c(y,t)$ will be searched as a sum of monochromatic waves

$$H_c(y,t) = \sum_{n=1}^N a_{cn}(y) \exp(in\omega t), \quad a_{cn}(y) = a_{rn}(y) + ia_{in}(y), \quad (4)$$

$$H(y,t) = \text{Re}[H_c(y,t)] = \sum_{n=1}^N [a_{rn}(y) \cos(n\omega t) - a_{in}(y) \sin(n\omega t)], \quad (5)$$

A linear spatial problem must be solved for each frequency harmonic to obtain transmission matrices of Bragg structure.

$$\frac{d^2 a_{cn}}{dy^2} - \frac{1}{\varepsilon} \frac{d\varepsilon}{dy} \frac{da_{cn}}{dy} + \varepsilon(y) n^2 k_0^2 a_{cn} = 0, \quad k_0 = \omega/c, \quad (6)$$

If $y > y_b$ functions $a_{cn}(y)$ express propagation of direct and opposite plane waves

$$a_{cn}(y) = a_{bdn} \exp[i(y - y_b)k_b] + a_{bon} \exp[-i(y - y_b)k_b],$$

where $k_b = k_0 \sqrt{\varepsilon(y_b)}$, a_{bdn} and a_{bon} are complex amplitudes of direct and opposite plane waves in beginning of Bragg-structure ($y = y_b$).

Result of solving equations (6) is transmission matrices T_n , that couple amplitudes of direct and opposite waves in beginning and end of Bragg structure

$$a_{edn} = T_{n11} a_{bdn} + T_{n12} a_{bon}, \quad a_{eon} = T_{n21} a_{bdn} + T_{n22} a_{bon}, \quad (7)$$

where a_{edn} and a_{eon} are amplitudes of direct and opposite waves on surface of nonlinear

film ($y = 0$). They are connected with values $a_{cn0} \equiv a_{cn}(0)$ and $a'_{cn0} = \left. \frac{da_{cn}}{dy} \right|_{y=0}$ by

following relations:

$$a_{cn0} = a_{edn} + a_{eon}, \quad a'_{cn0} = ik_c(a_{edn} - a_{eon}), \quad k_c = k_0 \sqrt{\varepsilon(0)}. \quad (8)$$

Granting nonlinearity of boundary condition it is more convenient to operate with real function $H(y,t)$ and real spatial functions $a_{rn}(y)$ and $a_{in}(y)$ in (2). Taking into account expression (5), boundary condition (2) can be written in the form

$$F(t) \equiv H_d - k^2 w_0 H_u - k^2 w_n q H_0^{q-2} [(q-1)H_t^2 + H_0 H_u] = 0, \quad (9)$$

where H_d, H_0, H_t, H_u are auxiliary functions of time coordinate t

$$H_d = \sum_{n=1}^N [a'_{r0n} \cos(n\omega t) - a'_{i0n} \sin(n\omega t)], \quad H_0 = \sum_{n=1}^N [a_{r0n} \cos(n\omega t) - a_{i0n} \sin(n\omega t)],$$

$$H_t = -\sum_{n=1}^N n [a_{r0n} \sin(n\omega t) + a_{i0n} \cos(n\omega t)], \quad H_u = -\sum_{n=1}^N n^2 [a_{r0n} \cos(n\omega t) - a_{i0n} \sin(n\omega t)],$$

Real parameters $a'_{rn0}, a_{rn0}, a'_{in0}, a_{in0}$ are expressed in terms of complex parameters a'_{cn0} and a_{cn0}

$$a_{rn0} = \text{Re}(a_{cn0}), \quad a_{in0} = \text{Im}(a_{cn0}), \quad a'_{rn0} = \text{Re}(a'_{cn0}), \quad a'_{in0} = \text{Im}(a'_{cn0}),$$

Introducing frequency-harmonic expansion into boundary condition (9) in discrete time points on time period we get system of non-linear equations with respect to harmonic

amplitudes on output from Bragg structure. Here we consider that amplitude of falling fundamental wave is specified on outer interface of Bragg reflector. Obtained non-linear system has been solved by numerical method based on Newton method with finite-difference approximation of derivatives.

THE PHYSICAL EFFECTS

The most interesting phenomena occur at resonance and middle values (close to optimal ones) of non-linearity. In this case amplitude of fundamental harmonic near the film multiply exceeds the amplitude of falling wave, the amplitude of the reflected fundamental wave on outer interface of the resonator is close to zero, energy of falling wave almost completely transforms to energy of higher harmonics. Unfortunately near optimal values of parameters quasi-Newton method collapses. To overcome difficulty of algorithm convergence we describe the fundamental harmonic not by the wave amplitudes on outer boundary of the Bragg structure but by the amplitudes of the direct and reflected waves on the surface of the non-linear film.

Obtained solutions essentially depend on from type of non-linearity. For cubic non-linearity the amplitude of the wave falling onto the resonator as function of the amplitude of fundamental harmonic on the film is not monotonic thus dependences of amplitudes of all scattered waves have hysteretic character.

CONCLUSIONS

Physical effects of scattering of a monochromatic plane wave on non-linear films situated in Bragg resonator are theoretically considered. Mathematical model of processes of higher-harmonic generation are derived. Influence of phase and amplitude parameters of Bragg reflection on amplitudes of scattered waves is investigated. Conditions of appearance of bistable regime are found. Obtained results can be used for designing generator, transformer and digital devices.

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