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WAVE SCATTERING BY CYLINDRICAL OBSTACLE IN GENERALISED WAVEGUIDE

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ABSTRACT

A novel full-wave solution of the problem of electromagnetic wave scattering by a circular post in a straight or continuously curved rectangular waveguide is presented. A rigorous mathematical model is based on the domain-product technique. It allows to construct an efficient series representation for the field in the interaction region. As a result, the initial boundary value formulation is reduced to a matrix equation with the fredholm operator. The last has the form of a sum of several products of the hilbert-schmidt operators provided the post does not touch the boundaries of the interaction region. The kernel matrix operator predetermines the fast convergence of the numerical approximations for a wide range of curvature variation, any possible radius of the cylinder and arbitrary location of the obstacle in the interior of the guide.

INTRODUCTION

A variety of techniques has been applied to the problem of electromagnetic wave scattering by cylindrical obstacles in the straight rectangular waveguides, which has been examined for many years (see, for instance, [1-4, 6]). The well known works deal exclusively with the special situation of relatively thin and centered posts [1, 2]. Recently these tight constraints have been overcome [3, 4], but, as a rule, the data obtained are validated by computational experiment in the form of "practical convergence" and comparative checks. Rigorous analysis of a post in continuously curved rectangular waveguide did not carry out at all.

We present an alternative rigorous analytical approach for solving the outlined class of problems that is quite straightforward, effective and well substantiated. The method of analysis is the domain-product technique (DPT) [5]. We consider the region of field determination as a common part of several auxiliary domains with separable geometry. After projective procedure the initial formulation is reduced to a matrix operator equation with respect to expansion coefficients that are associated with the auxiliary region related to the post. Functional properties of the matrix operator obtained are a subject of our investigation. The approach is described for a circular perfectly conducting post in a common 2-D waveguide structure, which is a generalisation of both straight and continuously curved rectangular waveguide.

DPT MODEL OF INDUCTIVE POST

The configuration of interest and the co-ordinate systems used are shown in Fig.1a. The circular post of a radius r placed across the guide parallel to the narrow wall and arbitrary centered. The mode incident upon the post is LM_{10} . To take the advantage of the physical symmetry plane the problem has been partitioned into two sub-problems corresponding to the symmetric and anti-symmetric excitations.

The interior of the guide is divided into the interaction region and two regular semi-infinite waveguides. Their fields are bound by the matching conditions on the common boundary lines. According with DPT, a field inside the interaction region can be represented in the form

$$u = u^{(r)} + \sum_{j=1}^4 u^{(j)} \quad (1)$$

as a superposition of partial solutions of the Helmholtz equation for five specially constructed intersecting domains [6]. In the expression (1)

$$u^{(r)}(\rho', \theta') = \sum_{n=-\infty}^{\infty} x_n \exp(in\theta') \frac{H_n^{(2)}(k\rho')}{H_n^{(2)}(kr)}, \quad \rho' \geq r, -\pi < \theta' \leq \pi$$

is a solution in the exterior to the post, which meets the condition at infinity. The expansion coefficients $\{x_n\}$ are sought in the Hilbert space

$$h_1 = \left\{ \mathbf{x} : \sum_{n=-\infty}^{\infty} (|n|+1) |x_n|^2 < \infty \right\}$$

Using continuity of the tangential electric and magnetic fields inside of guide and homogeneous boundary conditions on the conducting parts of the boundaries of the interaction region we obtain the matrix equation

$$\mathbf{x} + \mathbf{A}\mathbf{x} = \mathbf{b}, \quad \mathbf{b} \in h_1 \quad (2)$$

after familiar algebraisation.

PROPERTIES OF MATRIX OPERATOR

The matrix operator $\mathbf{A} : h_1 \rightarrow h_1$ from (2) has the form

$$\mathbf{A} = (\mathbf{T}_1 \mp \frac{1}{2} \mathbf{T}_2 \mathbf{F}_1) \mathbf{D}_1 + (\mathbf{T}_3 \mp \frac{1}{2} \mathbf{T}_2 \mathbf{F}_3) \mathbf{D}_3 \mp \frac{1}{2} \mathbf{T}_2 (\mathbf{F}_2 - \mathbf{G}) \quad (3)$$

It compactness follows from asymptotic estimates of some integrals, which are Fourier's coefficients of the functions being differentiable infinitely many times. For the elements of matrix operators \mathbf{T}_j from (3) we have relations

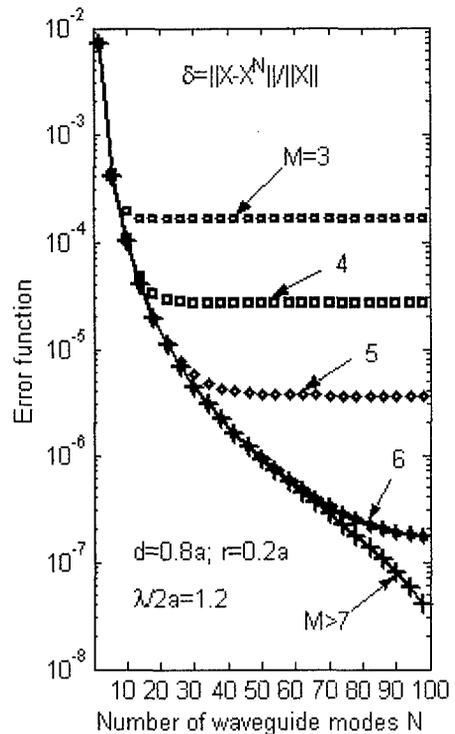
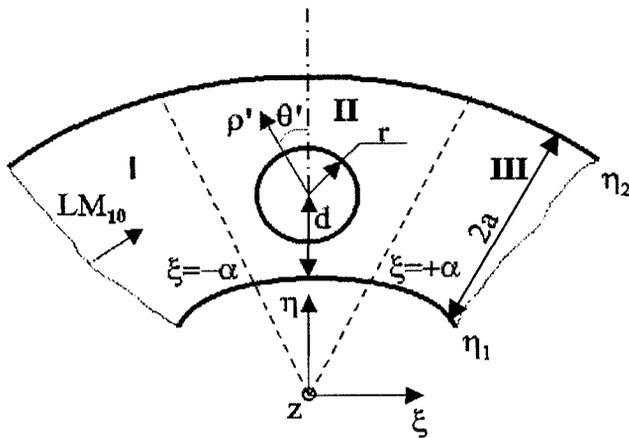
$$\sum_{m,n} m |t_{mm}^{(j)}|^2 = O\left[(1 - \zeta_j)^{-\tau}\right], \quad \tau > 0, \quad j = \overline{1,3}$$

The conditions $\zeta_j < 1, j = \overline{1,3}$, mean that the post does not touch the boundaries of the interaction region. Under the same condition, we get

$$\sum_{m,n} |f_{mm}^{(1,3)}|^2 < \infty, \quad \sum_{m,n} \frac{1}{n} |\beta_j^{(m,m)}|^2 < \infty, \quad j = \overline{1,3}$$

where $\beta^{(m,n)} = (f_{mm}^{(2)}, g_{mm}, d_{mm}^{(1,3)})$. It proves that $\mathbf{A} : h_1 \rightarrow h_1$ is the kernel operator.

Let $\mathbf{x}^{N,M}$ be the solution of a "truncated" counterpart of the matrix equation, then in the sense of the h_1 -norm the relative errors of approximation tend to zero with $M, N \rightarrow \infty$, Fig. 1b.



(a) (b)
 Fig.1. Geometry of the problem (a) and convergence of approximate solution in norm (b)

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