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MINIMIZATION OF THE FIELD DIFFRACTED FROM A CONVEX IMPEDANCE BODY TO THE SHADOW REGION

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INTRODUCTION

Modern means of transport, such as aircrafts, can contain several dozens transmitting and receiving antennas aboard. Each of these antennas is the potential source of the interference for the others. That is why the developers have to take appropriate steps to decrease the undesirable reciprocal effects. This can be made, for example, by means of the optimal positional relationship, or directional patterns correction, or by means of the various coverings.

This work presents a method of the body impedance (or covering) distribution determination, under which the field of the first antenna is the lowest on the second one. It is based on the well-known asymptotic methods of field determination - Geometrical Theory of Diffraction (GTD), Uniform Asymptotic Theory (UAT) [1] and Ritz method for functional minimization [2].

PROBLEM SETTING

Let us consider the following problem: The point of observation M is situated in the shadow region relative to point source $M_0$ (fig. 1). The body is bounded by the smooth curve $l$.

![Diagram](image)

The curvature radius of $l$ is $\rho(s)$, where $s$ is the natural parameter of $l$. The impedance of the body $g(s)$ can be any smooth complex function of $s$ with the only restriction $|g(s)| = O(1)$. This restriction will be explained later. Enter the coordinates $(s, n)$ where $n$ is the length of the perpendicular dropped to the body from a point, and $s$ is the natural parameter of the meet point of the body and the perpendicular. The field at the point $M$ can be calculated using GTD formulas for the creeping waves if both the point source and the observation point are far enough from the body. If the points are close to the...
body, the asymptotic formulas of V.M. Babich and V.S. Buldyrev [1] should be used:

\[ U(M) = \sum_{n=-\infty}^{\infty} \sum_{n=0}^{\infty} \Gamma_p(r_0, \varphi_0; r, \varphi + 2\pi n; k); \]

\[ \Gamma_p(r_0, \varphi_0; r, \varphi; k) = \frac{1}{2} \left( \frac{2}{k} \right)^{1/3} \left[ \rho(s) \rho(s_0) \right]^{1/6} \left[ w'_i(\xi_p) \right]^{2} e^{i(k-s_0)} \cdots \]

\[ \exp \left( i \frac{\alpha_{10}(s) - \alpha_{10}(s_0)}{k^{1/3}} \right) \left[ \int_{s_0}^{s} d\xi \frac{\rho'}{\rho} + \int_{s_0}^{s} d\xi \frac{\rho'}{\rho} g(s) + i \frac{1}{6k^{1/3}} \left[ \frac{\rho'(s)}{\rho(s)} \right] - \frac{\rho'(s_0)}{\rho s_0} \right] \right) \cdots \]

\[ \exp \left( i \frac{\alpha_{10}(s) - \alpha_{10}(s_0)}{k^{1/3}} \right) \left[ \int_{s_0}^{s} d\xi \frac{\rho'}{\rho} + \int_{s_0}^{s} d\xi \frac{\rho'}{\rho} g(s) + i \frac{1}{6k^{1/3}} \left[ \frac{\rho'(s)}{\rho(s)} \right] - \frac{\rho'(s_0)}{\rho s_0} \right] \right) \cdots \]

where \( w_i(x), w'_i(x) \) are the Airy function of first order and its derivative 
\( (w_i(z) = 2e^{\pi z} / 2 \text{Ai}(\text{ze}^{2\pi z})) \); \( \xi_p \) is the p-th root of the equation \( w_i(\xi_p) = 0 \); \( \nu = nk^{2/3} \),
and \( \alpha_{10}, T(\xi, M) \) are defined by the following formulas:

\[ \alpha_{10}(s) = 2\xi_p^2 \left( \frac{1}{60} + \frac{4}{135} \rho(s) - \frac{2}{45} \rho(s) \rho''(s) \right) \]

\[ T(\xi, M) = \xi - \nu \left( \frac{2}{\rho(s)} \right)^{1/3} \frac{i}{\left( k^{2/3} \right)^{-1/3} \rho^{1/3}(s)g(s)} + O(k^{-2/3}) \]

It is evident that the formula (1) can't be used if the impedance becomes too small by absolute value. That is why the present method doesn't allow the passage to the limit \( g(s) \to 0 \).

We are interested in the function \( g(s) \), which minimizes one of the following functionals:

\[ F_1(g(s)) = \left| U(g(s); M, M_0, k) \right| \]

\[ F_2(g(s)) = \max \left| U(g(s); M + \delta M, M_0, k) \right| \]

\[ F_3(g(s)) = \max \left| U(g(s); M, M_0 + \delta M_0, k) \right| \]

\[ F_4(g(s)) = \max \left| U(g(s); M, M_0, k + \delta k) \right| \]

The functionals \( F_{2,3,4} \) can be used if the source, the observation point or the frequency varies within the defined limits. They also allow estimating the stability of the results to the deviations of the initial conditions.

The problem of functional minimization can be reduced to problem of function minimization if we assume that \( g(s) = \sum_p a_p f_p(s) + R_\xi(s) \) where \( f_p(s) \) are the members of a set of orthogonal functions that is complete on the defined space, and \( R_\xi(s) \) is the remainder [2]. There is a great variety of function minimization methods. In the present work the method of Nelder and Mead [3] has been used, though the other methods, for
example [4], are also applicable.

**NUMERICAL RESULTS**

In order to test the present method, it was applied to the problems of scattering on circular, elliptic and parabolic cylinders.

![Fig. 2](image1.png)

**Fig. 2**

![Fig. 3](image2.png)

**Fig. 3**

![Fig. 4](image3.png)

**Fig. 4**

![Fig. 5](image4.png)

**Fig. 5**

The fig. 2,3 shows the field amplitude dependencies in the point M on the real and imaginary parts of the impedance \((s_0 = 0; s = \pi; n = n_0 = 0.1; \rho = 1; k_0 \rho = 10\) ). On the fig. 2 \(\text{Re}(g(s)) = \text{const} = \pm 1\), and on the fig. 3 \(\text{Im}(g(s)) = \text{const} = \pm 2\). The dependencies have extremums at small real (imaginary) parts of the impedance, and have the common limit if \(\text{Re}(g) \to \pm \infty\) \((\text{Im}(g) \to \pm \infty\) ). This allows to solve both the problem for minimization and maximization of the field. It is evidently from fig. 2,3 that the minimum of the field for the circle can be reached only if the real part of the impedance becomes negative at least in a small sector, and the maximum – only if the real part becomes positive at least in a small sector. The fig. 4 shows the impedances that minimize the functional \(F(g(s))\) (the geometry is the same as in the previous example). Both distributions are symmetrical relative to the point \(\pi/2\) accordingly to the reciprocity principle. The optimal imaginary part of the impedance for the parabolic cylinder is shown on fig.5 \((kF=10\) where \(F\) is the focal distance;
$s_0 = -F; s = F; n0 = 0.05F; n = 0.1F; \text{Re}(g(s)) = -1$). The minimum of the field is still observed only for the negative real parts, but the distribution is no longer symmetrical and parabolical.

**CONCLUSIONS**

The proposed method of field minimization in the shadow region has shown that it can be used for various optimization procedures provided that $ka \sim 10$ or larger, where $a$ is the typical dimension of the scatterer. It can also be generalized to the 3-dimensional case and improved by accounting surface waves in the case $\text{Im} g(s) < 0; \text{Re} g(s) > 0$. As the formula (1) actually presents the Green's function, the method can be easily modified to deal with distant sources.

**REFERENCES**