TITLE: A Combination of Up- and Down-Going Plane Waves Used to Describe the Field Inside Grooves of a Deep Grating

A COMBINATION OF UP- AND DOWN-GOING PLANE WAVES USED TO DESCRIBE THE FIELD INSIDE GROOVES OF A DEEP GRATING

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INTRODUCTION

The purpose of the present research is to extend the range of application of Yasuura's method [1,2] in solving the problem of diffraction by a grating. Although alternative terminology for the method (e.g., a least-squares boundary residual method or a modified Rayleigh method) exists, we employ the name throughout this paper.

It is an accepted knowledge [3, 4] that Yasuura's method, in particular, the conventional Yasuura method with Floquet modes as basis functions does not have a wide range of application. Although the convergence of the sequence of solutions obtained by the method is proven, the rate of convergence is often so slow for deep gratings that we cannot find solutions with accuracy. Let D and $2H$ be the period and the depth of a sinusoidal grating made of a perfectly conducting metal. The period is assumed to be comparable to the wavelength, i.e., we are working in the resonance region. For an E-wave (s polarization) problem where $2H/D = 0.5$, taking 71 Floquet modal functions, we can obtain a solution with 1 percent error in both energy conservation and boundary condition. Employment of additional Floquet modal functions easily causes numerical trouble in making least-squares approximation on the boundary. Hence, a practical limit in $2H/D$ in the E-wave case is 0.5 so long as we use conventional double-precision arithmetic. Similarly, the limit in the H-wave (p polarization) case seems to be a little less than 0.4.

To accelerate the convergence of solutions, Yasuura's method is equipped with a smoothing procedure [5, 6]. It has been shown that: in the above problem, we can obtain a solution with 1 percent error using 17-41 modal functions (the number depends on the order of the smoothing procedure and on the polarization). Hence, Yasuura's method with the smoothing procedure is effective in making a systematic research that needs to handle problems with complicated boundaries, e.g., Fourier gratings.

Although Yasuura's method with the smoothing procedure solves most of the problems for commonly used gratings, the limit in $2H/D$ has as yet been scarcely dealt with. There still is a limit at $2H/D = 0.7$ or 0.8 even if we employ the smoothing procedure. This limitation can be removed practically by the following way. Here, *practically* means that we can solve the problem with a profile deep enough for our research work in the direction of our interest.

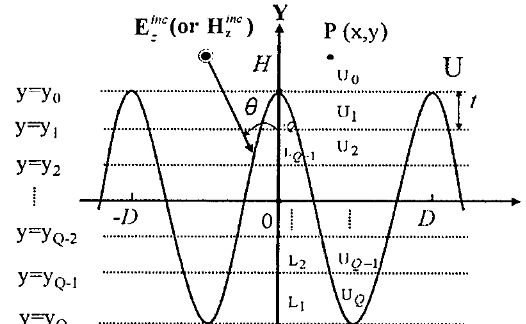

Fig. 1. Geometry of the problem.

STATEMENT OF THE PROBLEM

Let the cross section of the grating be periodic in X as shown in Fig. 1. The depth is in Y and $y = f(x)$ represents the profile. $f(x)$ is a sinusoidal function with a period D and a depth $2H$. The profile is the boundary between two regions $U(Y > f(X))$ and $L(Y < f(x))$. U is a vacuum and L is filled with a dielectric with a relative refractive index n. We consider the problem to seek diffracted waves in U and L assuming a plane E-wave incidence that comes from the positive Y direction.

METHOD OF ANALYSIS

The basic idea of the present method includes two strategies. First, in constructing an approximate solution inside the grooves in U (or in L), we employ down-going (or up-going) Floquet modes in addition to the up-going (or down-going) solutions. This would expand the function space spanned by the modal functions and make the boundary matching easy. Second, in consideration of the ill nature of higher-order evanescent modes (They strongly oscillate in X and rapidly increase or decrease in Y), we divide the regions inside the grooves into a couple of sub-regions and define approximate solutions in each sub-region. This may be understood as a kind of normalization of the modal functions.

To do this, we first separate a groove region U_G $(f(x) < Y < H)$ from a free space region U_0 $(Y > H)$. U_G and U_0 are sub-regions of U having a common border at $Y = H$. Another groove region L_G $(-H < Y < f(X))$ and a homogeneous half plane L_0 $(Y < -H)$ are defined similarly. Approximate solutions in U_0 and L_0 take the form of commonly employed modal expansion satisfying the radiation condition. That is, an approximate solution in U_0 (or in L_0) is a sum of up-going (or down-going) plane waves.

Next, we slice the groove regions to have Q layers. U_G is divided into $\{U_1, U_2,..., U_Q\}$ and a horizontal line $Y = (1 \geq 2q/Q)H$ $(q = 0, 1,..., Q \geq 1)$ is the boundary between U_q and U_{q+1}. Similarly, L_G is divided into $\{L_1, L_2,..., L_Q\}$ by horizontal lines $Y = (2r/Q \leq 1)H$ $(r = 0, 1,..., Q \geq 1)$. Consequently, we have $2Q$ sub-regions in one period $(0 < X < D)$. As a matter of fact, we have $3Q$ sub-regions because either U_G or L_G should be partitioned into two. We, however, regard the groove region consists of $2Q$ sub-regions because the latter partition is not essential. Each sub-region has its own local coordinates and modal functions are defined in each sub-region. It should be noted that: the set of modal functions in U_q includes not only up-going separated solutions but also down-going solutions. Similarly, the set in L_r includes both down- and up-going waves. An approximation in a sub-region (U_q or L_r: q, $r = 1, 2,..., Q$) is a finite sum of up- and down-going modal functions with unknown modal coefficients.

Now we have $2(2N + 1)(2Q + 1)$ unknown coefficients in total: $2(2N + 1)$ for U_0 and L_0; $2Q(2N + 1)$ for U_q; and $2Q(2N + 1)$ for L_r. Here, N is the number of truncation and summation should be taken from $-N$ through N. The coefficients are determined so that the approximate solutions satisfy the boundary conditions. We employ the least-squares method noting that a sub-region is enclosed with two horizontal lines and a part of grating profile.

NUMERICAL RESULTS AND DISCUSSIONS

Results of numerical computation and a couple of comments are itemized as follows:

(1) If $2H/D < 0.5$, the result obtained by the present method agrees well with the results by the conventional Yasuura method.

(2) Comparison with an existing data [7] shows good agreement at $2H/D = 1.0$ for an E-wave incidence (Figs. 2 and 3 ($Q = 4$, $N = 11$, 0.04 % energy error)) and for an H-wave incidence (Figs. 4 and 5 ($Q = 13$, $N = 16$, 0.9 %)).

(3) A personal computer (Pentium 1.7 GHz, RAM 512 MB) can handle an E-wave problem with a depth $2H/D = 1.7$ ($Q = 11$, $N = 7$, 1 %; or 16, 5, 0.4 %). Because this limitation in $2H/D$ comes from memory requirement, we can employ the technique of sequential accumulation [8] to extend the range of application.

(4) If we construct approximations in U_q (or in L_r) employing up-going (or down-going) waves alone, we cannot obtain convergent solutions for $2H/D > 1.0$. This means that the normalization of the modal functions alone is not so effective as the combined strategies.

(5) We have succeeded in establishing a method of modal-expansion that solves the problem of deep gratings. We are planning to employ the method in solving the problem of a stratified grating in which the boundaries between layers have a common period but do not have a common profile.

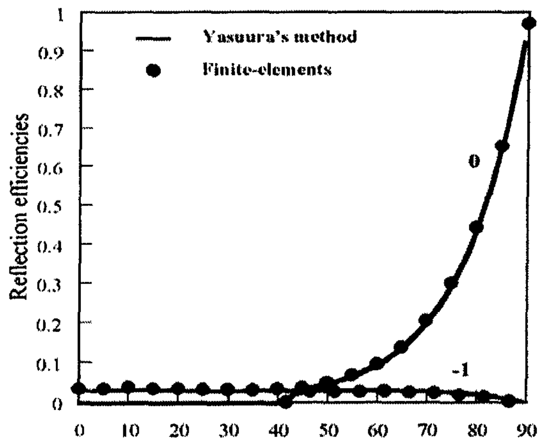

Fig. 2. Reflection efficiency (E-wave)

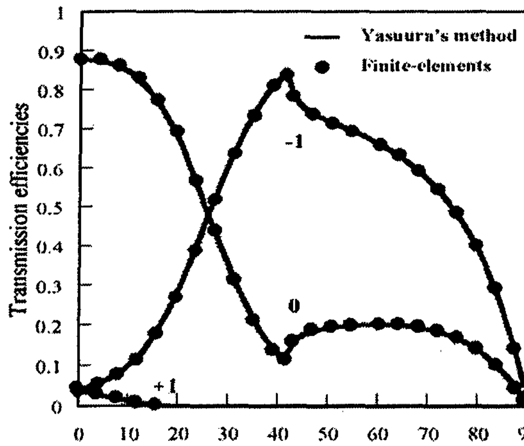

Fig. 3. Transmission efficiency (E-wave)

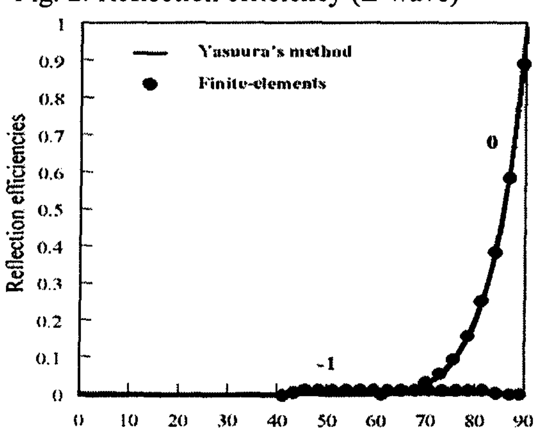

Fig. 4. Reflection efficiency (H-wave)

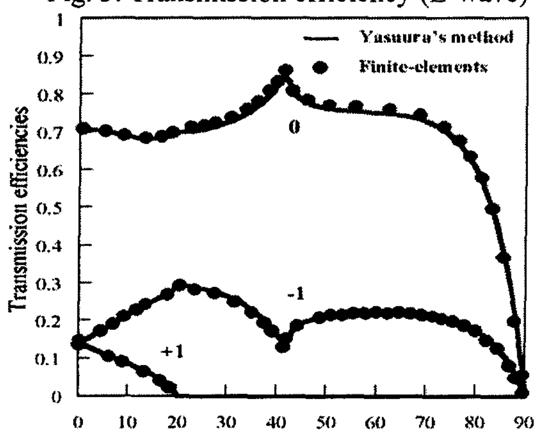

Fig. 5. Transmission efficiency (H-wave)

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