One- and two-colour superradiant lasing in magnetized quantum-well heterostructures

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Abstract. We study generation of femtosecond superradiance pulses due to collective recombination of electron-hole \((eh)\) pairs in quantum wells placed in a strong magnetic field oriented perpendicular to the growth plane. Such a superradiant laser can work under the conditions of continuous pumping and at room temperature due to complete quantization of particle motion, maximum possible spectral density of states, high volume density of cyclotron quantum dots, and partial suppression of the intraband scattering. It is shown that two-colour superradiant generation is possible at frequencies resonant to frequencies of the neighbouring interband transitions defined by \(e, h\) Landau levels.

1. Introduction. Cyclotron quantum dots

As is known [1, 2], superradiant coherent recombination of \(eh\) pairs in semiconductors can occur faster than the processes of nonradiative recombination and intraband scattering of carriers. This provides possibility to obtain femtosecond pulses in lasers based on both standard optoelectronic materials, such as GaAs, and materials with low efficiency of radiative recombination, e.g., wide-gap and even indirect-gap semiconductors. Ideal candidate for an active medium of a superradiant semiconductor laser is a multilayer heterostructure of cyclotron quantum dots created when a quantizing magnetic field \(B \sim 10-50\ T\) is imposed on the multiple quantum-well (QW) structure perpendicular to the growth plane [3, 4].

We will have in mind AlGaAs/GaAs QW structure under the conditions when an electron gyrofrequency satisfies the inequality \(o_{Be} > \{1/T_2, 1/T_2^*\}\), where \(1/T_2, 1/T_2^* < 10\ \text{meV}\) are homogeneous and inhomogeneous broadening of the Landau levels. Note that if the cyclotron energy \(\hbar o_{Be}\) exceeds the energy of the optical phonon, one can expect suppression of the intraband relaxation, i.e. \(1/T_2\).

Maxwell–Bloch equations describing interaction of \(eh\) pairs with radiation have the following form (in the semiclassical and rotating-wave approximations for slowly varying amplitudes of the electric field \(E(t)\) and polarization \(P(t)\)) [1, 2]

\[
\frac{dE}{dt} + E/\tau_E = 2\pi i \omega \Gamma P/\mu^2, \quad (1)
\]
\[
\frac{dP}{dt} + [1/T_2 + i (\omega_0 - \Omega)] P = -i d^2 \Delta N_I E/(2\hbar), \quad (2)
\]
\[
\frac{d\Delta N_I}{dt} + (\Delta N_I - \Delta N_I^0)/T_1 = \text{Im}(E^* P)/(2\hbar). \quad (3)
\]

Here \(\Omega, \Gamma,\) and \(\mu\) are the frequency, optical confinement factor, and refraction index for a given cavity mode. For a moment we neglect inhomogeneous broadening and consider only the transitions at frequency \(\omega_0\) between \(e, h\) Landau levels with the same number \(I\) and given transverse quasimomentum \(k_n\). The time \(T_E\) of the field decay in a cavity of length \(L\) due to radiative losses only is equal to \(4\mu L/c \ln(R_1 R_2)\), where \(c\) is the velocity of light in vacuum, \(R_{1,2}\) are the reflection coefficients on the cavity facets. In the above equations
we introduced the volume density of inversion $\Delta N_l(t)$ for a given interband transition with dipole moment $d$. The quantity $\Delta N_l^0$ is its equilibrium value supported by pumping (in the absence of generation). The relaxation time of inversion $T_1 \geq T_2$. Inversion is achieved when the density $N$ of all pairs in QWs exceeds the value $N_l = \omega_{B,e} m_e k_n (2l + 1)/(4\pi^2 h)$.

2. One-colour superradiance and inhomogeneous broadening

One-colour superradiance (SR) of femtosecond pulses in a magnetized QW structure at resonance $\Omega \simeq \omega_0$ is realized most easily when $[4, 5]$ $1/T_E \leq \omega_c/2 \leq \omega_{B,e}$ and the SR growth rate of the corresponding initial-value problem reaches its maximum value of the order of half the so-called cooperative frequency of $eh$ pairs:

$$\omega_c = [4\pi d^2 \Gamma \Delta N_l \omega_0/(h\mu^2)]^{1/2}; \quad \Delta N_l \leq 2 \Delta N_{l=0} \leq 4N_0 = \omega_{B,e} m_e k_n/(\pi^2 h). \quad (4)$$

Analysis of stability of the steady state of Eqs. (1)–(3) shows [2, 5, 6] that for the appearance of pulsation regime leading to SR the inequalities $\omega_c^2 (\Delta N_l^0) > 36/(T_2 T_E)$ and $1/T_E > 1/T_1 + 1/T_2$ have to be satisfied. This means that the pumping should exceed some (second) threshold value and low-Q laser cavity must be used. The required cavity length is linearly proportional to the relaxation time of polarization and for $T_2 = 0.2$ pc is of order $10–20 \mu$m. The required magnetic field is inversely proportional to $\Gamma$, so that for $\Gamma = 0.2$ it is about 50 T and decreases as $1/T_2^2$.

Inhomogeneous broadening of the Landau levels can lead to substantial changes of the generation thresholds. Typically it increases the threshold of stationary generation (the first threshold), but it can also decrease the second threshold [6, 7].

To simplify the problem, we will assume that inhomogeneous broadening is defined by discrete fluctuations of a thickness of a QW or barrier that are multiple of the atomic monolayer thickness. In such a discrete model, the neighbouring spectral components (sublevels) of a working transition are separated by some value $\Delta \omega$ which is usually of order several meV. We assume also the simplest distribution of the total pumping $\Delta N_l^0$ between different spectral components $j$, $\Delta N_l^0$, namely a geometric progression law with an exponent $\alpha < 1$: $\Delta N_l^0 / \Delta N_0^0 = \alpha^{j|j|}$.

Qualitative analysis and numerical simulations show that SR pulses under the inhomogeneous broadening have generally larger duration, lower power and repetition rate as compared to SR pulses generated under the condition of homogeneous broadening. The
numerical results for the first \( n_{p1} = \Delta N_{p1}/(2N_0) \) and second \( n_{p2} = \Delta N_{p2}/(4N_0) \) generation thresholds on the interband transition between ground \( e, \hbar \) Landau states are shown in Figs. 1, 2 for typical values of the relaxation times \( T_1 = 10T_2 = 30T_E = 180/\omega_c(2N_0) \) in the case of three sublevels. At a given pumping, the first threshold rises monotonously when \( \Delta \omega \) or \( \alpha \) increases. The second threshold which indicates the onset of SR gets substantially lower with increasing inhomogeneous broadening (growing \( \Delta \omega \) and \( \alpha \)), if the latter is not higher than homogeneous broadening, i.e., if \( \Delta \omega < 1/T_2 \) for \( \alpha \sim 1/2 \). For larger inhomogeneous broadening, when \( 1/T_2 > \Delta \omega > 1/T_E \) and \( \alpha \sim 1/2 \), the second threshold begins growing with increasing sublevel splitting \( \Delta \omega \) due to the weakening of interaction between different spectral components of polarization, because a dephasing timescale is defined by \( 1/\Delta \omega \) and not equal to \( T_2 \). Therefore, the minimum of the ratio of thresholds \( n_{p2}/n_{p1} \) is achieved when \( \Delta \omega \sim 1/T_2 \) and \( \alpha \to 1 \) (see Figs. 1, 2).

3. Two-colour superradiance for adjacent interband Landau transitions

Standard injection or optical pumping of a multilayer heterostructure with magnetized QWs creates carriers with energies considerably higher than the energies of the lower levels of dimensional quantization and Landau sublevels of the lower dimensional quantization level. In this case the intraband scattering provides a cascade pumping of neighbouring

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**Fig. 3.** Two-colour superradiance: radiation intensity, \( I \), and inversion, \( n \), normalized to unity, vs normalized time, \( r = 0.5t_0/(2N_0) \) for the ground, \( l = 0 \), and first, \( l = 1 \), Landau levels at \( n_p = \Delta N_p^2/(4N_0) = 1.55 \) and \( T_1 = 7.5T_{01} = 10T_2 = 30T_E = 180/\omega_c(2N_0) = 9000/\omega_0 = 9450/\omega_1 \).
recombination transitions which can lead, in principle, to a simultaneous generation of corresponding resonant modes.

We have analyzed SR generation of two modes resonant to the two transitions between pairs of neighbouring Landau levels [7] (e.g., between the first and second e, h Landau levels). The rate of intraband relaxation from the upper to lower level was characterized by a phenomenological time of nonradiative cyclotron transition $T_{01}$. Using known expressions for the first and second threshold in the case of an isolated transition [6], we obtain analytical estimates of the threshold pumping in a given 4-level system (see [7]). In fact, each threshold for a system of two incoherently connected transitions corresponds to a minimal pumping required to drive one subsystem out of equilibrium while the state of other subsystem is fixed.

Numerical analysis of corresponding Maxwell–Bloch equations demonstrates the possibility of two-colour SR under continuous pumping. This regime is conserved under substantial variation of the system and pumping parameters (when they are changed by at least tens of per cent). The most important condition for coexistence of SR on the neighbouring recombination transitions is a proximity of non-stationary lasing thresholds for corresponding laser modes and approximate equality of their gain coefficients. An example of two-colour SR is shown in Fig. 3. In this regime the pulse sequences generated on the adjacent recombination transitions can be essentially different (with respect to the repetition rate, power, and duration). However, in any case each pulse is accompanied by changing the inversion sign on the corresponding transition which is unusual for a standard lasing.

4. Summary

In conclusion, our study of threshold conditions and dynamics of femtosecond SR generation in semiconductor QW lasers placed in a quantizing magnetic field shows that a system of cyclotron quantum dots is rather promising for the creation of superradiant laser. In particular, we found that inhomogeneous broadening, until it does not exceed homogeneous one, leads to considerable lowering of a SR threshold. Finally, we demonstrated possibility of two-colour SR under a constant cascade pumping of e, h Landau levels and calculated corresponding thresholds.

References