Impurity potential fluctuations for selectively doped p-Ge/Ge$_{1-x}$Si$_x$

heterostructures in the quantum Hall regime

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Abstract. Two models for random impurity potential (the model with randomly distributed charged centers located within a layer and the model of the system with spacer) are used for the estimation the impurity potential fluctuation parameter’s values: random potential amplitude, the nonlinear screening length in vicinity of integer filling factors $v = 1$ and $v = 2$, the background density of state (DOS). The described models are suitable for the explanation of unusual high value of DOS at $v = 1$ and $v = 2$ in contrast to short-range impurity potential models.

The nature of quantum Hall effect (QHE) is closely linked with a phenomenon of electron localization in a two-dimensional (2D) disorder system in quantizing magnetic field ($B$) [1, 2]. The appearance of quantum plateaux in the $\rho_{\text{xx}}(B)$ dependences with vanishing values of $\rho_{\text{xx}}$ is now commonly accepted to be caused by the existence of disorder-induced mobility gaps in the density of states (DOS) of a 2D-system. When the Fermi level is settled down in the gap the thermally activated behavior of $\rho_{\text{xx}}$ (or $\sigma_{\text{xx}}$) is observed due to the excitation of electrons to the very narrow bands of extended states centered at Landau level (LL) energies $E_N$. The determination of the DOS in the mobility gaps is possible from the data on activation energy $E_A$ as a function of the LL filling factor $v = n/n_B$ ($n$ is the electron density, $n_B = eB/hc$) [3-6]. The filling factor can be tuned by the change of either the carrier density [3] or the magnetic field [4-6].

We have used the method of activated magnetoresistivity for the reconstruction of 2D-hole gas (2DHG) spectrum in quantizing magnetic fields for p-Ge/Ge$_{1-x}$Si$_x$ systems with complex valence band structure [7]. The measurements of longitudinal $\rho_{\text{xx}}$ and Hall $\rho_{\text{xy}}$ resistivities are carried out for multilayer p-Ge/Ge$_{1-x}$Si$_x$ ($x = 0.07$) heterostructures with hole concentration $p = (2.4 \pm 2.6) \cdot 10^{11}$ cm$^{-2}$ and mobilities $\mu_p = (1.1 \pm 1.7) \cdot 10^4$ cm$^2$/Vs in magnetic fields up to 12 T at $T = (1.6 \div 15)$ K.

The heterostructures are consisting of a 20-nm Ge and Ge$_{1-x}$Si$_x$ layers repeated 15–30 times. The 2DHG forms just inside the undoped Ge layer. The regions of Ge$_{1-x}$Si$_x$ barriers doped with boron are separated from the Ge layers by 5-nm undoped Ge$_{1-x}$Si$_x$ spacer layers ($d_s$) (Fig. 1).

The following results for the mobility gap DOS as a function of energy $g(\varepsilon)$ have been obtained. Even in the middle of a gap when the filling factor is close to an integer the density of localized states is found to have a value comparable with or even higher than the DOS of 2DHG without magnetic field ($g_0 \approx 4.5 \cdot 10^{10}$ cm$^{-2}$ meV$^{-1}$). Moreover, $g(\varepsilon)$ remains almost constant in the overwhelming part of the energy intervals between adjacent LL: $g(\varepsilon) \approx g_{c \varepsilon} = (5 \div 7) \cdot 10^{10}$ cm$^{-2}$ meV$^{-1}$ for $v = 1$ and $v = 2$ (Fig. 2). This result is consistent qualitatively with data for the structures with n-type conductivity [3–6]. As for our value of $g_{c \varepsilon}$, it is roughly an order of magnitude higher than those for InGaAs/InP [5]
and for high-mobility AlGaAs/GaAs [4] heterostructures but comparable with those for Si-MOSFET [3] and for intermediate-mobility AlGaAs/GaAs heterostructures [6].

As all short-range impurity potential models lead to an exponential drop in DOS between Landau levels, the clear picture for the DOS in QHE regime may be presented only in terms of the long-range potential fluctuations in combination with the oscillation dependence of DOS on the filling factor. Such an idea has been advanced in early work of Shklovskii and Efros [8] and then developed in series of works of Efros with collaborations (see [9, 10] and references therein). In selectively doped heterostructures the smooth random potential is formed by fluctuations in concentration of remote impurities.

For a random potential $V(r)$ smooth on the scale of magnetic length $l_B$ the localization in QHE regime can be discussed in terms of semiclassical quantization and percolation [11]. In the quasiclassical limit the electron energy in quantizing magnetic field may be presented as

$$E_N(r_0) = \hbar \omega_c \left(N + \frac{1}{2}\right) + V(r_0)$$

with $r_0$ being the oscillator center coordinate. Thus the smooth potential removes the degeneracy on $r_0$ and makes the LL energy dependent on spatial coordinates.

We report here an order of magnitude estimation of spatial scale and amplitude of random potential in p-Ge/Ge$_{1-x}$Si$_x$ heterostructures in QHE regime obtained from an analysis of the mobility gap DOS. Two models for random impurity potential are used.

(i) The model with randomly distributed charged centers located within a thick layer (of a width $b$) close to the 2D electron (hole) gas [8]. For the fluctuation amplitude $F$ of a scale $L$ it is obtained [8]

$$F(L) = \beta \frac{e^2 \sqrt{NL}}{\kappa},$$

where $\beta$ is a numerical coefficient ($\beta \approx 0.1$ [9]), $N$ – the density of charged impurities (per volume) and $\kappa$ – the dielectric constant.

(ii) The model of the system with spacer: a condenser with 2D electron (hole) gas as one plate and randomly distributed charged centers as the other plate, separated by a
It is seen from Eqs. (2) and (3) that without screening the amplitude $F$ diverges at large $L$. When filling factor is close to integer (i) very small concentration of electrons $\delta n \ll n_B$ can be redistributed in space and thus one occurs in conditions of so called nonlinear screening [8–10] (“threshold” screening in terms of [12]). For $\nu = i$ exactly the screening is realized only due to electrons (and holes) induced by an overlap of adjacent fluctuating Landau levels, and so the amplitude of random potential is of the order of corresponding LL gap.

For the investigated heterostructures $N \approx 10^{17} \text{ cm}^{-3}$ ($C = Na \approx 10^{11} \text{ cm}^{-2}$) and the mean distance between impurities ($N^{1/3} \approx 200 \text{Å}$) is comparable both with the width of 2D Ge layer ($d \approx 200 \text{Å}$) and the width of doped part of the sample ($a \approx 100 \text{Å}$).

Thus the described models are not valid precisely but they are suitable to obtain a range of random potential parameter values.

In nonlinear screening regime for the DOS in the middle of mobility gap of width $W$ ($\approx 2 \text{ meV}$) we have [8–10]:

(i) 

$$g \left( \frac{W}{2} \right) = \frac{4\beta e^2 N}{\kappa W^2} \approx 7.5 \cdot 10^{10} \text{ cm}^{-2} \text{ meV}^{-1},$$

(ii) 

$$g \left( \frac{W}{2} \right) = \frac{2\sqrt{C}}{7Wd_s} \approx 9 \cdot 10^{10} \text{ cm}^{-2} \text{ meV}^{-1}.$$
amplitude comparable to the mobility gap, $F \approx W$, we obtain an estimation of the nonlinear screening length $L_c$ (the scale of optimal fluctuation): $L_c \approx 1000 \, \text{Å}$ for model (i) (see Eq. (2)) and $L_c \approx 400 \, \text{Å}$ for model (ii) (see Eq. (3)). As seen in both cases the spatial scale of fluctuations is essentially larger than the magnetic length ($l_B \approx 80 \, \text{Å}$ at $B = 10 \, \text{T}$) and the random potential may be really regarded as the smooth one.

Thus an order of magnitude estimates of random impurity potential parameters for the p-Ge/Ge$_{1-x}$Si$_x$ heterostructures shows that in vicinity of integer filling factors $\nu = 1$ and $\nu = 2$ (i.e. in the regions of plateaux in QHE regime) a sharp broadening of LL takes place. It is reputed that for the filling factors close to half integer (the regions of plateau to plateau transition) the potential fluctuations will be small due to effective (linear) electron screening [8–10].

Acknowledgements
This work is supported in part by RFBR, projects No 99-02-16256 and No 01-02-17685.

References