

## Light localization in a disordered photonic crystal

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**Abstract.** It is known that photonic Bloch states can become strongly localized near the bandedges in a disordered photonic crystal. We show that Bloch states are disrupted and the new localization regime establishes when local fluctuations of the bandedge frequency caused by randomization of refractive index profile becomes as large as the bandgap width.

### Introduction

Light localization in disordered media [1] can be caused when constructive interference of backscattered waves brings transport to a complete halt (strong or Anderson localization). In this regime the transport mean free path  $l$  becomes as short as the wavelength of the wave  $\lambda$  and the Ioffe-Regel criterion for localization (IR)  $kl < 1$  is satisfied [2], where  $k = 2\pi/\lambda$ . During last decades there has been a considerable interest in experimental verification of light localization. We present the results of theoretical analysis aimed at the investigation of the "subtle interplay of order and disorder" in disturbed periodic-on-average 3D photonic crystal [3], which is required for observation of Anderson localization of photons.

### 1 Theory

As a model system we choose synthetic opals [4], which are composed of nearly monodisperse (standard deviation  $\delta$  about 5%) submicron silica spheres, closely packed in a face centered cubic (*fcc*) lattice with a period of  $\approx 200$  nm. For numerical simulations we use routine one-dimensional transfer matrix method [5], in which experimental 3D *fcc* structure is modeled by a refractive index profile, which is periodic only in the [111] direction. In order to incorporate the actual experimental parameters of the system into the calculation scheme (spheres radius  $R$  and volume packing fraction  $\beta$ ) the profile is calculated as  $n(z) = S_{\text{sp}}(z)n_a + (1 - S_{\text{sp}}(z))n_b$ , where  $S_{\text{sp}}(z)$  is relative area cross-section of the spheres in a (111) plane calculated as a function of the distance along the [111]  $z$  direction. Refractive indices of the spheres ( $n_a = 1.37$ ) and surrounding media ( $n_b = 1.47$ ) are chosen close to experimental values [4]. The calculated transmission spectrum of an ideal periodic structure exhibits a gap, centered at reduced frequency  $\nu_0 = 0.60$  (in units of  $c/a$ , where  $c$  is speed of light,  $a$  - *fcc* lattice constant). Its relative width  $\Delta\nu/\nu_0$  about 1% with a midgap value of imaginary wavevector of  $1300 \text{ cm}^{-1}$ .

The disorder is incorporated in the model in analogy with the experimental case of opals by a random distribution of the spheres diameters  $\delta$ , which was chosen to be flat for simplicity. We find that presence of disorder leads to exponential decay of light with thickness not only within the former gap of the periodic structure, but also in the former passbands, thus significantly broadens the gap. The attenuation length is usually defined as

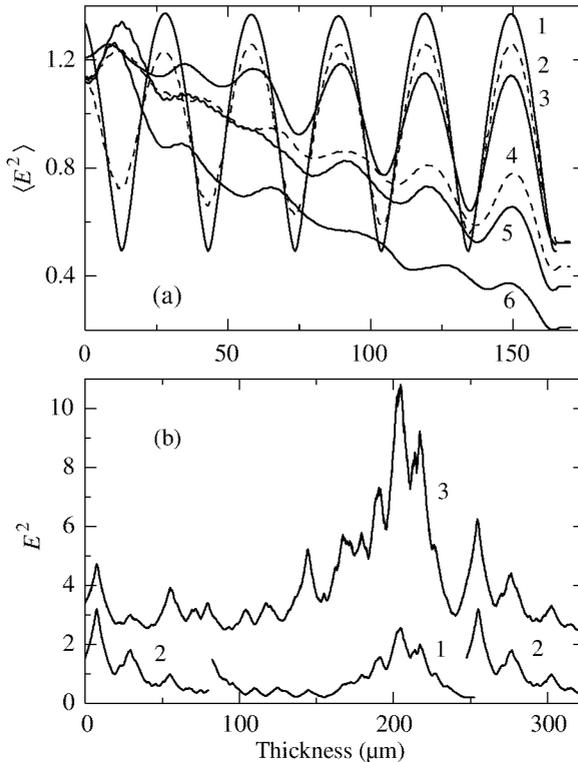
$\xi^{-1} = -\langle \ln T \rangle / L$ , where  $L$  being the sample thickness and brackets denote the ensemble averaging over various (in general infinite) different random configurations [5]. The general tendency for increased transmission at the midgap and decreased at the bandedges, already pointed out in the bibliography [3], is also found in the system studied. It is commonly believed that this counterintuitive effect (increasing transmission with increase of disorder) is the result of the increased photonic DOS due to appearance of strongly localized photonic bandtail states, which fill the gap [3]. In what follows we will present an alternative analysis of light localization in disturbed periodic structures.

## 2 First localization regime, $\delta \ll \Delta\nu/\nu_0$

We believe, that, depending on the amount of disorder  $\delta$ , at least two quite different localization regimes builds up consequently. Indeed in our model system the deviation  $\delta$  of the spheres radius  $R$  results in a local deviation of  $fcc$  lattice constant  $a = 2\sqrt{2} \cdot R$  and in a corresponding local fluctuation of the photonic bandedge frequency. This forms a random profile of refractive index on which the wave is scattered. In the case  $\delta \ll \Delta\nu/\nu_0$  the waves at the bandedges can be well described by the wavevector  $k_{\text{cryst}}$  of the average periodic structure and the modified IR criterion [3]  $k_{\text{cryst}}l < 1$  can be applied for the analysis of localization. Figure 1(a) represents the lower envelope of the calculated electric field profile inside the opal structure for the frequency  $\nu_0 = 0.596$  in the passband near the conduction band edge for different values of disorder. Note that the profiles presented are averaged over 200 random configurations thus reflecting the band structure of the averaged periodic system. It is seen that in a perfect periodic structure ( $\delta = 0$ ), the wave is nearly a standing at a given frequency (curve 1). The corresponding periodic envelope function is defined by  $k_{\text{cryst}} = 1.9 \cdot 10^5 \text{ cm}^{-1}$ . The incorporation of disorder leads to a rapid destruction of coherence (note the vanishing amplitude of the low-frequency periodic modulation in curves 2-5). Low-frequency periodic envelope still exists, however, for some value of disorder, which means that the modified IR criterion [3]  $k_{\text{cryst}}l < 1$  can still be applied. Note, that  $l$  corresponds now to the mean free path in which this coherent Bloch state is disrupted. For example  $\delta = 0.5\%$  for curve 2 and the corresponding  $l$  can be estimated to be  $10^{-3} \text{ cm}$ , which gives  $k_{\text{cryst}}l \approx 100$ . As the frequency approaches the bandedge,  $k_{\text{cryst}}$  approaches zero. Therefore for a 1D crystal there always exists such a frequency region close to the bandedge where  $k_{\text{cryst}}l < 1$ . In order to obtain strong spatial localization in a 3D periodic structure, it is necessary to achieve energy coincidence of such localization regions at the bandedges for all the directions, which is the condition reminiscent of that for opening up of the omnidirectional PBG [3].

## 3 Second regime of localization, $\delta > \Delta\nu/\nu_0$

A qualitatively different regime builds up when  $\delta > \Delta\nu/\nu_0$ . The local fluctuations of the bandedge frequency are so large that they exceed the whole width of the gap. For frequencies in the former passbands, a nonzero probability appears for finding large, sufficiently ordered regions, which act as, even disordered, Bragg mirrors (frequencies fall in the gap). As a result, exponential attenuation of the wave appears in configurationally averaged intensity curves (see curves 4–6 in Fig. 1(a)) even at frequencies in the passbands of periodic structure. For a single configuration, however, sharp resonant modes appear in transmission spectrum, in which the transmission approaches unity. It is the Thouless criterion of localization [6], which we use to examine such states. It requires essentially that the width of energy levels be as small in comparison to the energy spacing between them in order to prevent tunneling between states and to block the transport. We calculate the field profile for a



**Fig. 1.** Electric field intensity profiles for the frequency  $\nu_0 = 0.596$  in the passband. (a) Averaged over 200 random configurations. The standard deviation  $\delta$  of the spheres diameter is 1—0%, 2—0.5%, 3—1%, 4—3%, 5—4%, 6—6%. (b) For a single configuration of disorder of  $\delta = 20\%$ . Curve 3 corresponds to the structure of 330  $\mu\text{m}$  thickness combined from the structure 1 of 165  $\mu\text{m}$  thickness at the center and two structures 2 of 82.5  $\mu\text{m}$  stacked to it from both sides.

single realization of disorder in a  $L = 165 \mu\text{m}$  thickness sample (see curve 1 in Fig. 1(b)). The realization is chosen the same as for spectrum in the inset of Fig. 1 for frequency  $\nu_0 = 0.596$ , for which the resonant state exists. The resulting field profile (see curve 1 in Fig. 1(b)) do exhibit exponential tails, which localize the wave function to a small space volume. Then two identical layers of 82.5  $\mu\text{m}$  thick (see curve 2) are added to this structure on both sides and the resulting field profile is calculated for the whole composite sample of 330  $\mu\text{m}$  (see curve 3). It can be clearly seen from a comparison of curves 1 and 3 that initial state retain completely its initial shape. This indicates that the state 1 “feel” its environment only through exponential tails and is relatively insensitive to the background beyond the localization length, which can be defined now as an exponent in the tail region of the profiles for a single realization of disorder (about 8  $\mu\text{m}$  for curve 1). To obtain complete localization in this sense in a 3D structure, it is necessary to create analogous localized states at the same frequency for all other directions (and polarizations), which spatially overlap. This can be achieved in a disordered 3D photonic crystal with a gaps for different directions, which are energetically overlapping. This condition is also very similar to that for opening up a complete PBG, except that the widths of the perturbed gaps can be significantly larger in disordered crystal, thus facilitating spatial localization.

In conclusion, two different regimes of light localization in a disordered photonic crystal

are identified, which builds up consequently depending on the amount of disorder compared with the normalized width of the gap in corresponding periodic structure. This comparison can be applied to gaps for different directions and, therefore, it remains valid for the photonic crystal of any dimensionality.

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#### **References**

- [1] D. S. Wiersma et al., *Nature* **390**, 671 (1997).
- [2] A. F. Ioffe and A. R. Regel, *Prog. Semicond.* **4**, 237 (1960).
- [3] S. John, *Phys. Rev. Lett.* **58**, 2059 (1987).
- [4] Yu. A. Vlasov et al., *Phys. Rev. B* **55**, 13357 (1997).
- [5] A. R. McGurn et al., *Phys. Rev. B* **47**, 13120 (1993); V. D. Freilikher et al., *Phys. Rev. E* **51**, 6301(1995).
- [6] D. J. Thouless, *Phys. Rep.* **13C**, 93 (1974).