Non-linear Meissner effect in mesoscopic superconductors

P. Singha Deot†, V. A. Schweigert‡, F. M. Peeters† and A. K. Geim§
† Department of Physics, University of Antwerp (UIA), B-2610 Antwerpen, Belgium
‡ Institute of Theoretical and Applied Mechanics, Russian Academy of Sciences, Novosibirsk 630090, Russia
§ High Field Magnet Laboratory, University of Nijmegen, 6525 ED Nijmegen, the Netherlands

Abstract. Magnetization measurements on superconducting bulk samples and large radius cylinders had resulted in the Phenomenological London’s theory that is found to be violated in recent magnetization measurements in superconducting mesoscopic discs that exhibit a non-linear Meissner effect. In this work we show that the Ginzburg-Landau (GL) eqn. can explain this non-linear Meissner effect both in quality and quantity.

Recently Geim et al [1] used sub-micron Hall probes to detect the magnetization of thin (thickness down to $d \sim 0.07 \mu m$) single superconducting Al discs with radius down to $0.3 \mu m$. For such systems the coherence length ($\xi(0) \sim 0.25 \mu m$) is comparable to the size of the disc and finite size effects are very important. With increasing disc radius the magnetization of the system first shows the behavior of a type II superconductor, then of a type I superconductor and by further increasing the disc radius multiple steps are seen in the magnetization which can be dubbed a multi-type I superconducting behavior. This behavior was explained by us [2] where we included the particular geometry of the system in the Ginzburg-Landau (GL) theory for superconductivity which is coupled to the Maxwell equations in order to take into account the bending of the magnetic field lines around the superconducting disc. The rich behavior seen experimentally is due to a competition between surface superconductivity, bulk superconductivity and the geometrical demagnetizing effects.

Another striking effect seen in these systems is the non-linear Meissner effect which we will address here. For example, for the sample of radius $R \sim 0.5 \mu m$ the magnetization for small magnetic fields increases linearly with external field, as expected for a type I superconductor, but increasing the field above about 40 Gauss (for $T = 0.4 K$) the magnetization increases less fast with external field and strong non-linear behavior is observed. This remarkable deviation from London’s theory will be explained here from the GL theory using our previous numerical approach [2, 3].

We fix the radius of the disc to be $0.3 \mu m$, coherence length to be $\xi(0) = 0.25 \mu m$ and penetration length $\lambda(0)$ to be $0.07 \mu m$. We plot the magnetization versus applied magnetic field for 10 different thickness $d$, in Fig. 1. The values of $t = d/\xi(0)$ are shown in the figure. It can be seen that for $t = 0.1, 0.2$ and $0.3$ the disc shows a second order phase transition to the normal state. Whereas for $t = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and $1.0$ we find a first order phase transition to the normal state. The magnetization (multiplied by 0.5) as calculated from the Linearized GL (LGL) theory which is independent of the thickness of the sample is shown by dashed lines for the same $R$ and $\xi(0)$. 

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Fig 1. Numerically obtained magnetization versus the external magnetic field for a superconducting disc for ten values of $t = d / \xi(0)$ shown in the figure. Radius is kept fixed at $a = 0.3 \mu m$, $\xi(0)$ at $0.25 \mu m$ and $\lambda(0)$ at $0.07 \mu m$. The dashed curve is obtained by solving the LGL eqn.

Hence the finite thickness effect explains why a disc can sometimes show a first order and sometimes a second order phase transition to the normal state as is found in the experiment. For example, in Fig. 2, we show that a disc of $R = 0.44 \mu m$, $d = 0.15 \mu m$, $\xi(0) = 0.275 \mu m$ and $\lambda(0) = 0.07 \mu m$ can exhibit a magnetization (in increasing magnetic field) like that of a disc whose magnetization was measured in the experiment and whose $R$ was reported as $0.5 \mu m$, $d$ as between $0.07 \mu m$ and $0.15 \mu m$, $\xi(0)$ as $0.25 \mu m$ and $\lambda(0)$ as $0.07 \mu m$. The large solid dots are the experimental data and the solid curve is our numerical calculation. The dotted curve is a tangent to the experimental data at the origin. So note that the coherence length has been changed by 10% and radius by 12% to reproduce the experimental result. These are well within the errors of their experimental determination. The magnetization has been scaled by $0.626/4\pi$. A detector size larger than the sample size can underestimate the magnitude by a factor of $4\pi$. The field distribution along a radial line starting from the center of the disc is shown in the inset for 11 values of the external applied field. The values of the applied fields is also mentioned on the curves. For the first 10 curves it can be seen that the field is minimum at the center of the disc. It increases drastically with distance from the center and becomes maximum at $0.44 \mu m$ which is precisely the radius of the disc. This means the field is strongly expelled from the center of the disc. The 11th curve (applied field=70.86 Gauss) corresponds to the critical field.

Magnetization measurement on bulk samples and large radius cylinders had shown that in the pure Meissner state, the sample behave as a perfect diamagnet which means
magnetization is proportional to the applied field with a susceptibility of -1. This experimental observation lead to a phenomenological theory well known as London’s theory. It can be seen from the magnetization measurement on discs (see Fig. 2) that London’s theory is not valid for discs because the magnetization is proportional to the applied field initially but above 40 Gauss this linear behavior deviates strongly (i.e., it deviates from the dotted curve). The GL theory can explain this non-linear Meissner effect very well as can be seen from the solid curve in Fig. 2. Before analyzing this effect in detail first we want to point out that as the thickness is varied the critical field \( H_c \) at which the transition to the normal state occurs remains the same. This can be analytically argued in a very simply way. At the critical field the amount of flux contained by the sample is given by the critical field multiplied by the area of the disc. As at the critical field the LGL theory is as good as the GL theory. Hence the flux enclosed by the sample and the upper critical field is independent of the thickness as in the LGL theory.

It can be seen from the inset in Fig. 2 that as the applied field is increased from 0 the field only penetrates near the boundary of the disc (which is due to Meissner effect) but at the critical field it suddenly distributes uniformly over all regions inside the disc. The jump in the magnetization (corresponding to first order transition) occur due to this sudden redistribution of the field. In the LGL theory the field is always uniform at all applied fields and this sudden redistribution of field leading to a jump in the magnetization is absent. So the magnetization gradually goes to zero, resulting in non-linear Meissner effect as can be seen from the dashed curve in Fig. 1. Very thin discs \( t = 0.1, 0.2, 0.3 \) have a similarly uniform distribution of magnetic field (due to large enhancement of penetration length with decreasing thickness) over the whole sample as in the LGL theory, and also show a non-linear Meissner effect resulting in a second order phase transition to the normal state. As we slowly increase the thickness of the disc the field will be expelled from the disc as in a cylinder. Hence beyond a certain thickness the disc will start showing a sudden redistribution of field and a first order transition to the normal state.

Consider the magnetization curve for \( t = 0.4 \) in Fig. 1. Initially the magnetization increases linearly with the magnetic field. Beyond a point P shown in Fig. 1 there is a deviation from the linear behavior and the sample exhibits a non-linear Meissner effect. In discs thinner than \( t = 0.4 \) the magnitude of magnetization decreases, the point of deviation from linear Meissner effect and the peak value shifts to smaller fields. But since the upper critical field remain constant it does not exhibit a jump to 0 like that in LGL theory. On the other hand it can be seen that for thicker discs the non-linear Meissner effect is gradually disappearing. In the limit i.e., for cylinders the peak coincides with the critical field and and the jump occur directly from the maximum value and hence there is no non-linear Meissner effect \([4]\). Only in this regime London’s theory is valid. Hence the non-linear Meissner effect is connected to the nature of phase transition.

From Fig. 2 (solid curve) it can also be seen that even in the linear part of the magnetization curve the susceptibility is less than 1 in magnitude and so the sample is not a perfect diamagnet This is due to the finite size of the sample. Although the magnetic field decreases inside the sample in a finite size sample it never becomes zero. One can see from Fig. 2 (inset) that a large magnetic field is present at the center of the disc and it increases as the applied field is increased.
Fig 2. Our numerical solution (solid curve) and the experimental data (dark circles) for magnetization versus increasing magnetic field at 0.4K. The dotted curve is a tangent to the experimental data at the origin. The parameters used are given in the text. The inset shows magnitude of magnetic field along a radial line starting from the center of the sample, for 11 values of the applied field mentioned on the curves.

This work is supported by the Flemish Science Foundation (FWO-Vl) grant No. G.0232.96, the European INTAS-93-1495-ext project, and the Belgian Inter-University Attraction Poles (IUAP-Vl). One of us (PSD) is supported by a postdoctoral fellowship of FWO and FMP is a Research Director with FWO-Vl.

References

[4] For small radius cylinders there can be a small non-linear Meissner effect. To see non-linear Meissner effect we can take a large radius sample and enhance its penetration length by decreasing its thickness (which is the case of a thin disc) or take a thick sample and decrease its radius to the magnitude of the fixed penetration length (which is the case of a small radius cylinder).