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Relaxed states in plasmas — non-neutral and diamagnetic plasmas

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Abstract. The aim of this paper is to present a unified representation of relaxed states generated in various type of plasma systems. Self-organization of structure is the central them of the recent theory of relaxation phenomena. Invoking fluid mechanical models, many authors have discussed various relaxed states that constitute a special class of macroscopic equilibria (force-balance states). The present theory, however, starts from a kinetic model of relaxed state, which encompasses a variety of structures including both electrostatic and electromagnetic configurations. This model also describes the macroscopic relaxed state characterized by the Beltrami and (generalized) Bernoulli conditions, which gives strong diamagnetism.

I INTRODUCTION

Plasmas, viewed as a fluid, can produce a variety of structures. In the ideal (dissipation-less) model of fluid mechanics, the equilibrium (force-balance) equations may have infinite number of solutions, because the hyperbolic part of the partial differential equation demands Cauchy data that specify the profiles of internal fields. To select a physically realized structure from the total set of ideal equilibrium solutions, we invoke the effect of small dissipations. In approximately collision-less plasmas, it is believed that the effect of dissipation does not modify the equilibrium structure, while it chooses a most preferential profile that can minimize the influence of the dissipation. This stands reason to start from an ideal model, and then search for a relaxed state.

Many authors have discussed various relaxed states that constitute a special class of macroscopic (fluid mechanical) equilibria. The pioneering work in this context was J.B. Taylor's model of the RFP (reversed-field pinch) plasma [1]. The present paper, however, presents a kinetic model of self-consistent relaxed states that are characterized by some rugged constants of motion. Both

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electrostatic and magnetic confinements are discussed on a unified basis. This model also derives the macroscopic relaxed state characterized by the Beltrami and (generalized) Bernoulli conditions, which gives strong diamagnetism.

II KINETIC EQUILIBRIA

A Steady states in kinetic theory and relaxed states

We begin with formulating the equilibrium (steady state) in the phase space of kinetic theory that is spanned by $\boldsymbol{x}$ (coordinate) and $\boldsymbol{p}$ (canonical momentum). Let $f(\boldsymbol{x}, \boldsymbol{p}, t)$ be the distribution function. We consider a collision-less plasma. The evolution of f obeys the Liouville equation (Vlasov equation)

$$\partial_t f + \{H, f\} = 0, \tag{1}$$

where H is the Hamiltonian of a test particle moving in a mean field, and $\{\ ,\ \}$ is the Poisson bracket. The mean field $\boldsymbol{A}$ and ϕ must be consistent to f through Maxwell's equations. We consider a stationary state, we assume that H is independent to t. Then, a steady state for (1) satisfies

$$\{H, f\} = 0. \tag{2}$$

The stationary equation (2) has infinite number of solutions. Let a_j be a time-independent quantity that commutes with H, i.e., $\{H, a_j\} = 0$. The H itself satisfies this condition. Suppose that we know N of such quantities (constants of motion). Using them, we generate a distribution (F is an arbitrary smooth function)

$$f = F(a_1, \cdots, a_N). \tag{3}$$

If N is equal to the degree of freedom, the system is "integrable", and (3) gives the complete solution. Our motivation in this work, however, is to find a special class of solutions that are robust (rugged) against various perturbations destroying the invariance of fragile constants of motion. Only a small number of them are robust in a sense that the ensemble averages (or the total sums) of such quantities are conserved.

The most robust steady state is the Boltzmann distribution

$$f = Z^{-1} e^{-\beta H} \tag{4}$$

where Z (normalization factor) and β (inverse temperature, or a Lagrange multiplier) are positive constants. We obtain this equilibrium by maximizing the entropy over an ensemble that is characterized by a given total energy (i.e., a constant-energy set). If we invoke another constant G (in an ensemble average sense), we must maximize the entropy on the ensemble defined both by H and G. With restricting the totals of H and G, we obtain

$$f = Z^{-1} e^{-\beta H - \gamma G}, \tag{5}$$

where γ is the second Lagrange multiplier. Including some additional constants of motion, we obtain a maximum entropy solution that is more restricted than the Boltzmann equilibrium. Such a solution remains in equilibrium as far as the additional constraints work.

B Equilibrium with momentum conservation

A symmetry of the system warrants the conservation of the corresponding canonical momentum. Suppose that the Lagrangean L is independent of a coordinate x_0, as well as t. Then, the Hamiltonian H and the momentum $p_0 = \partial L / \partial x_0'$ are conserved ($'$ is the time derivative). With an arbitrary constant C, we define

$$\hat{H} = H - C p_0, \tag{6}$$

and consider a distribution [see (5)]

$$f = Z^{-1} e^{-\beta \hat{H}} = Z^{-1} e^{-\beta (H - C p_0)}. \tag{7}$$

This solution of (2) has the following important connotation that provides a physical meaning for the parameter C.

When we discuss a distribution function f, we consider an ensemble of particles, which is characterized by the sum of the Hamiltonian over the all particles. We invoke the conservation of the total energy, but not the energy of each particle. We apply the same framework for the momentum p_0 in (7). We do not assume the conservation of H or p_0 for each particle, while we consider an ensemble determined by the totals of these quantities. Then, the physical meaning of $\hat{H}$ becomes essential. Indeed, we can interpret $\hat{H}$ as the Hamiltonian on a moving frame, and hence, $f = Z^{-1} e^{-\beta \hat{H}}$ is an invariant of the collision operator (the average momentum is unchanged by collisions). This robustness of p_0 warrants the use of p_0 in determining the ensemble.

Let us revisit the change of variables in general inhomogeneous coordinate transform, and see the relation between H and $\hat{H}$. Let $\boldsymbol{U}$ be a certain temporary-constant velocity field. We write the velocity $\boldsymbol{v}$ of the laboratory frame as

$$\boldsymbol{v} = \tilde{\boldsymbol{v}} + \boldsymbol{U}, \tag{8}$$

and set $\tilde{\boldsymbol{x}}' = \tilde{\boldsymbol{v}}$. The Lagrangean of a charged particle (q: charge, m: mass) can be written as

$$L = \frac{m}{2} |\tilde{\boldsymbol{v}} + \boldsymbol{U}|^2 + q(\tilde{\boldsymbol{v}} + \boldsymbol{U}) \cdot \boldsymbol{A} - q\phi.$$

The canonical momentum and the Hamiltonian are

$$\tilde{\boldsymbol{p}} = \frac{\partial L}{\partial \tilde{\boldsymbol{v}}} = m(\tilde{\boldsymbol{v}} + \boldsymbol{U}) + q\boldsymbol{A} = m\tilde{\boldsymbol{v}} + q\tilde{\boldsymbol{A}}, \tag{9}$$

$$\tilde{H} = \frac{1}{2m}\left|\tilde{\boldsymbol{p}} - q\tilde{\boldsymbol{A}}\right|^2 - \frac{m}{2}U^2 + q\tilde{\phi} = \frac{m}{2}\tilde{v}^2 - \frac{m}{2}U^2 + q\tilde{\phi}, \tag{10}$$

where

$$\tilde{\boldsymbol{A}} = \boldsymbol{A} + \frac{m}{q}\boldsymbol{U}, \quad \tilde{\phi} = \phi - \boldsymbol{U}\cdot\boldsymbol{A}. \tag{11}$$

The effective vector potential $\tilde{\boldsymbol{A}}$ includes an additional term that yields the Coriolis force. The scalar potential $\tilde{\phi}$ has received the (nonrelativistic) Lorentz transform. In (10), $-mU^2/2$ is the centrifugal potential. The transform of the Hamiltonian and the momentum can be written as

$$\tilde{H} = H - \boldsymbol{U}\cdot(m\boldsymbol{v} + q\boldsymbol{A}) = H - \boldsymbol{U}\cdot\boldsymbol{p}, \tag{12}$$

$$\tilde{\boldsymbol{p}} = \boldsymbol{p} \equiv m\boldsymbol{v} + q\boldsymbol{A}. \tag{13}$$

Suppose that a component p_0 of the momentum is a constant of motion. Comparing (6) and (12), we observe

$$\tilde{H} = H - Cp_0 = \hat{H}, \tag{14}$$

which implies that $\hat{H}$ is the Hamiltonian in the moving coordinate, and C is the velocity (or the angular velocity if we define $\tilde{\boldsymbol{p}}$ with respect to an angle) of the frame. Hence, the solution (7) represents the "thermal equilibrium" in the moving frame. Let us review some well-known examples. The Hamiltonian H includes the potentials ϕ and $\boldsymbol{A}$ that must be self-consistently determined by the field equation including f.

(1) *Electrostatic equilibrium*

When the magnetic field produced by the internal current is negligibly small in comparison with the externally applied magnetic field, we can use the electrostatic model; $\boldsymbol{A}$ is a prescribed field, while ϕ is determined by the Poisson equation

$$-\Delta\phi = \frac{q}{\epsilon}n. \tag{15}$$

Using (10), the density n is given by

$$n(\boldsymbol{x}) = \int f(\boldsymbol{x},\boldsymbol{v})dv = n_0 e^{-\beta\left(-mU^2/2 - q\boldsymbol{U}\cdot\boldsymbol{A} + q\phi\right)}, \tag{16}$$

where $n_0 = Z^{-1}\int e^{-\beta m v^2/2}\, dv$.

Here, we review the well-known "thermal equilibrium" of a cylindrical single-species plasma confined in a homogeneous magnetic field ($\boldsymbol{B} = B\boldsymbol{e}_z$). Let r-θ-z denote the cylindrical coordinates. By the symmetry $\partial/\partial\theta = 0$, the canonical angular momentum $p_\theta = \partial L/\partial\theta' = mrv_\theta + qrA_\theta$ is conserved. Hence, $\hat{H} = H - \omega p_\theta$ (ω = constant) is a constant of motion. This $\hat{H}$ is the Hamiltonian in a rigid rotation frame. Indeed, setting $\boldsymbol{U} = \omega r\boldsymbol{e}_\theta$ (ω is the angular velocity of the rigid rotation), (10) reads

$$\tilde{H} = \frac{m}{2}\tilde{v}^2 - \frac{m}{2}(r\omega)^2 - qr\omega A_\theta + q\phi = \hat{H}. \tag{17}$$

The equilibrium $f(\hat{H}) = Z^{-1}e^{-\beta\hat{H}}$ represents a drift Maxwellian with a constant angular velocity ω. Here, we seek a solution that has a constant density inside the plasma, i.e.,

$$\hat{H} = \frac{m}{2}\tilde{v}^2 = \frac{1}{2m}\left|\tilde{\boldsymbol{p}} - q\tilde{\boldsymbol{A}}\right|^2. \tag{18}$$

The vector potential for the homogeneous longitudinal magnetic field is $\boldsymbol{A} = (rB/2)\boldsymbol{e}_\theta$. For the distribution function $f(\hat{H}) = Z^{-1}e^{-\beta\hat{H}}$ with the Hamiltonian (18), the density n is constant for the radius $r < a$. Then, the potential is $\phi = -(qn/4\epsilon_0)r^2$ [2]. To satisfy (18), we demand $-(m/2)(r\omega)^2 - qr\omega A_\theta + q\phi = 0$ [see(17)], which reads as the familiar equilibrium condition

$$\omega^2 + \omega_c\omega + \frac{1}{2}\omega_p^2 = 0, \tag{19}$$

where $\omega_c = qB/m$ and $\omega_p^2 = nq^2/m\epsilon_0$.

(2) *Electromagnetic equilibrium*

Next, we discuss magnetic confinement solutions. Let us consider a slab plasma with $\boldsymbol{B} = B_z(x)\boldsymbol{e}_z$. We assume that the system is homogeneous with respect to y ($\partial/\partial y = 0$), so that the canonical momentum $p_y = my' + qA_y(x)$ is conserved. We consider $\hat{H} = H - cp_y$ and the equilibrium distribution $f(\hat{H}) = Z^{-1}e^{-\beta\hat{H}}$, which gives the drift Maxwellian with a constant velocity $\boldsymbol{U} = c\boldsymbol{e}_y$. Here, (10) reads

$$\tilde{H} = \frac{m}{2}\tilde{v}^2 - \frac{m}{2}c^2 - qcA_y + q\phi = \hat{H}.$$

We note that this $\hat{H}$ depends on x. From the distribution function $f(\hat{H})$, we can calculate the density of each species. For appropriate parameters, the charge-neutrality can be achieved, and hence, ϕ can be set zero self-consistently. The drift Maxwellian yields a finite current. The magnetic field produced by this current must be consistent to the vector potential $A_y(x)\boldsymbol{e}_y$ in $\hat{H}$. Setting $B_z = B_0\tanh(x/\ell)$, we can find a self-consistent solution that is called the Harris sheet [3]. The magnetic pressure B_z^2 has a dip around $x = 0$, where the pressure is confined. This equilibrium is based on the conservation of the momentum p_y. Similar calculations in cylindrical coordinates lead to the Bennet pinch equilibrium.

C Equilibrium with adiabatic constants and Beltrami-Bernoulli conditions

As the analysis in the previous section shows, a shear flow is not obtained from the model of momentum conservation. Here we examine an adiabatic invariant model to find an equilibrium with a shear flow. Let us first consider a certain velocity field $\boldsymbol{U}$ and define a moving coordinate by (8). Here we assume that $\boldsymbol{U}$ depends only on x, and

$$\boldsymbol{U} \cdot \nabla x = 0. \tag{20}$$

The transform of coordinates does not bring about temporal variation in any field. The equation of motion associated with the Hamiltonian $\tilde{H}$ of (10) reads

$$\frac{d}{dt}\tilde{x}_j = \frac{\partial}{\partial \tilde{p}_j}\tilde{H} = \frac{1}{m}\left(\tilde{p}_j - q\tilde{A}_j\right), \tag{21}$$

$$\frac{d}{dt}\tilde{p}_j = -\frac{\partial}{\partial \tilde{x}_j}\tilde{H} = \frac{-1}{m}\sum_{\ell}\left(\tilde{p}_j - q\tilde{A}_j\right)\frac{\partial}{\partial \tilde{x}_j}\tilde{A}_\ell - \frac{\partial}{\partial \tilde{x}_j}\left(\frac{-m}{2}U^2 + q\phi\right), \tag{22}$$

or, in the form of Newton's equation (with $\partial\tilde{\boldsymbol{A}}/\partial t = 0$) ,

$$m\frac{d}{dt}\tilde{\boldsymbol{v}} = q\left[-\tilde{\nabla}\tilde{\phi} + \tilde{\boldsymbol{v}} \times (\tilde{\nabla} \times \tilde{\boldsymbol{A}})\right] + \frac{m}{2}\tilde{\nabla}U^2. \tag{23}$$

In (23), the Lorentz force includes the Coriolis force. We define a generalized magnetic field by $\tilde{\boldsymbol{\Omega}} = \tilde{\nabla} \times \tilde{\boldsymbol{A}}$. We assume that $\tilde{\boldsymbol{\Omega}}$ is a smooth and strong field.

Here we introduce an essential assumption

$$\tilde{\boldsymbol{\Omega}} = a\boldsymbol{U}, \tag{24}$$

where a is a certain constant. We denote $\perp$ and $\|$, respectively, the perpendicular and parallel directions with respect to $\tilde{\boldsymbol{\Omega}}$. For "magnetized particles", the generalized magnetic moment $\tilde{\mu} = \tilde{v}_\perp^2/|\tilde{\boldsymbol{\Omega}}|$ is an adiabatic invariant. By (20) and (24), we have $\tilde{\nabla}_\| = 0$ for every scalar field, and hence, the potential force in (23) does not have a parallel component. Therefore, $\tilde{v}_\|$ is a constant of motion, and gyration average of the kinetic energy $m\tilde{v}^2/2$ does not change. Conservation of both $\tilde{v}^2$ and $\tilde{v}_\|$ yields the constancy of $\tilde{v}_\perp^2$, which, together with $\tilde{\mu}$ =constant, imply that $|\tilde{\boldsymbol{\Omega}}|$ is constant (in gyration average) through the motion of particles. By (24), $|\tilde{\boldsymbol{U}}|$ is also constant in gyration average. Combining these adiabatic invariants, we observe that $\boldsymbol{U} \cdot \tilde{\boldsymbol{v}} = \left(v^2 - \tilde{v}^2 - U^2\right)/2$ is an adiabatic invariant.

Using this adiabatic constant, we define

$$\hat{H} = H - m\boldsymbol{U} \cdot \tilde{\boldsymbol{v}} = H - \boldsymbol{U} \cdot (\boldsymbol{p} - \overline{\boldsymbol{p}}), \tag{25}$$

where $\overline{p} = mU + qA$. We may rewrite (25) as

$$\hat{H} = \frac{m}{2}\tilde{v}^2 + \frac{m}{2}U^2 + q\phi = \frac{m}{2}|\boldsymbol{v} - \boldsymbol{U}|^2 + \varphi,$$

where $\varphi = mU^2/2 + q\phi$ is the effective potential. The equilibrium define by $\hat{H}$ reads

$$f(\hat{H}) = Z^{-1}e^{-\beta\hat{H}} = Z^{-1}e^{-\beta(m|\boldsymbol{v}-\boldsymbol{U}|^2/2+\varphi)}, \tag{26}$$

which is a drift Maxwellian with a shear flow velocity $\boldsymbol{U}$. We note that (24) is the determining equation for $\boldsymbol{U}$, which corresponds to the "Beltrami condition" of vortex dynamics systems [4].

The effective potential φ determines the density;

$$n = \int f\, dv = n_0 e^{-\beta\varphi} \quad (n_0 = Z^{-1}\int e^{-\beta m\tilde{v}^2/2}\, dv).$$

The pressure $p = nkT$ $(kT = 1/\beta)$ satisfies the Bernoulli-like condition

$$\nabla p + n\nabla\varphi = \nabla p + n\nabla(mU^2/2 + q\phi) = 0. \tag{27}$$

We note that the Beltrami and Bernoulli conditions recover the fluid mechanical equilibrium model of diamagnetic (high-beta) structures [4,5].

III TOROIDAL EQUILIBRIUM OF NON-NEUTRAL PLASMAS

A Constant-density equilibrium in a straight annular geometry with sheared magnetic field

In this section, we discuss a more specific subject aiming at development of a new-type of non-neutral plasma trap. We consider a toroidal internal ring system that can produce a magnetic shear configuration [6]. To understand the basic property of such a configuration, we start with the analysis of an annular electron plasma column ($a < r < b$) confined by both longitudinal (B_z) and poloidal ($B_\theta = B_p a/r$) magnetic fields (B_z and B_p are constants). The core region of $r < a$ is occupied by a conductor that carries a longitudinal current to produce the poloidal magnetic field. The combination of B_z and B_θ produces a magnetic shear configuration with spiral field lines. The vector potential of the magnetic field is $\boldsymbol{A} = {}^t(0, (B_z/2)r, -B_p a \log r)$. The scalar potential ϕ is given by the Poisson equation (15). Here, we seek for a solution with a constant density. Let us consider a rigid motion of the plasma; $\boldsymbol{U} = {}^t(0, \omega r, U_z)$, where ω is the constant angular frequency of the poloidal rotation and U_z is

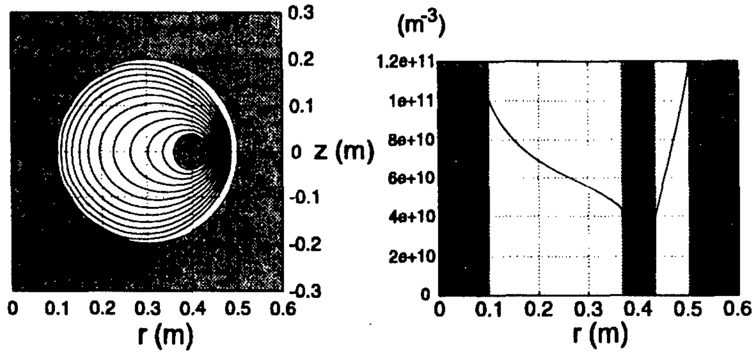

FIGURE 1. Toroidal thermal equilibrium of non-neutral plasma.

the constant longitudinal velocity. To obtain a constant-density equilibrium, we demand $-\frac{m}{2}U^2 - q\boldsymbol{U}\cdot\boldsymbol{A} + q\phi = 0$ [see (10)], which reads

$$\frac{-m}{2}\left(\omega^2 + \frac{qB_z}{m}\omega + \frac{q^2 n_0}{2m\epsilon_0}\right)r^2 + q\left(B_p a U_z + c_1\right)\log r + qc_2 - \frac{m}{2}U_z^2 = 0.$$

The coefficients of r^2 terms yield the conventional equilibrium condition (19), where the ω_c is determined by the longitudinal magnetic field ($\omega_c = qB_z/m$). The coefficients of $\log r$ terms yields $c_1 = -B_p a U_z$.

For this equilibrium, the longitudinal magnetic field B_z plays the principal role to confine the plasma, as in the previous cylindrical equilibrium. The poloidal magnetic field B_θ and the longitudinal flow U_z produces an electric field on the plasma, while it does not work to confine the plasma. Indeed, B_p and U_z determines the coefficient c_1 of the electric field, but they are not related to the density n_0. However, the poloidal magnetic field, producing a magnetic shear, has an important function to stabilize the diocotron (Kelvin-Helmholtz) instability [7].

B Generalized thermal equilibrium in a toroidal geometry with magnetic shear

In a toroidal system, the toroidal symmetry gives the constancy of the toroidal canonical angular momentum. We denote the cylindrical coordinate by (r, ϑ, z). The toroidal angle ϑ parallels the longitudinal direction z in the previous subsection. The poloidal angle is no longer an ignorable coordinate, and hence, we cannot assume the conservation of the poloidal angular momentum (which played the essential role to produce the constant density

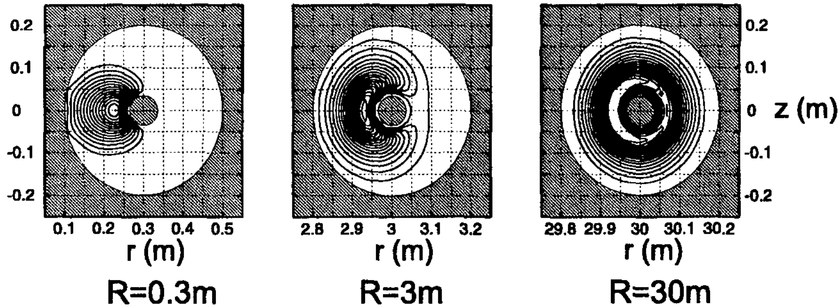

FIGURE 2. Toroidal equilibrium of non-neutral plasma base on the drift model.

equilibria in Secs. II B and III A). Here we consider a more general (non-constant density) solution of the nonlinear Poisson equation (15)-(16) with setting $\boldsymbol{U} = r\Omega \boldsymbol{e}_\vartheta$ (Ω is the angular frequency of the toroidal rotation). Writing the magnetic field as $\boldsymbol{B} = \nabla\psi\times\nabla\vartheta + B_\vartheta r\nabla\vartheta$, the transformed Hamiltonian is

$$\tilde{H} = H - \Omega p_\vartheta = \frac{m}{2}\tilde{v}^2 - \frac{m}{2}(r\Omega)^2 - q\Omega\psi + q\phi.$$

We normalize the variables as $x/\lambda_D = \hat{x}$, and $\beta q\phi = \hat{\phi}$, where λ_D is the Debye length. The nonlinear Poisson equation now reads

$$-\hat{\nabla}^2\hat{\phi} = \exp\{-\hat{\phi} + \beta[m(r\Omega)^2/2 + q\Omega\psi]\}. \tag{28}$$

When $\Omega = 0$, (28) is just the Debye shielding equation. The plasma flow (rotation) has the effect of charge neutralization. To confine a high-density plasma with low temperature ($\lambda_D \ll \ell$), Ω and magnetic field must be carefully chosen, because $e^{-\hat{\phi}}$ becomes a very small number, and the density tends to localize near the boundary, producing a sheath. In Fig. 1, we show an example of thermal equilibrium with $T_e = 10$eV and $n_0 = 10^{11}\mathrm{m}^{-3}$. When we diminish the temperature, equilibrium density becomes strongly localized near the boundaries.

Another model of non-neutral plasma equilibrium considers a drift motion of magnetized particle. For a low density magnetized non-neutral plasma ($\omega_c/\omega_p \gg 1$), we can use the $\mathbf{E}\times\mathbf{B}$ approximation of the flow velocity [8]; $\boldsymbol{u} = -\nabla\phi\times\boldsymbol{B}/B^2$. The macroscopic continuity equation in a steady state reads $\nabla\cdot(n\boldsymbol{u}) = -\left[\nabla\phi\times\nabla(n/B^2)\right]\cdot\boldsymbol{B} = 0$. When the magnetic field has a component perpendicular to $\nabla\phi$, this relation demands $n = B^2F(\phi)$, where F is a certain smooth function. Using this n in the Poisson equation (15), we can find equilibrium solution. In comparison with (16), this model can produce a larger variety of solutions. Figure 2 shows typical equilibrium solutions with different aspect ratios [9]. Because the drift velocity decreases as B increases, the equilibrium density shifts towards a higher field region (paramagnetism).

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