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Flowing Electron Plasmas
as Modified Current Source

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Abstract. Probing of streaming electron plasmas with finite temperature is studied. Due to the fast flow of the electron plasmas, the characteristic curve spreads out significantly and exhibits a long tail. Another significant feature of the characteristic is that it determines a floating potential where the current equals zero, despite there being very few ions in the electron plasma. A high impedance probe, which is popularly used to determine the space potential of electron plasmas, outputs the potential. The method is available only for plasmas with density much smaller than the Brillouin limit.

I INTRODUCTION

There has been considerable interest both experimentally and theoretically in toroidal nonneutral plasmas, which are relevant to heavy ion accelerators [1], sources of highly stripped heavy ions [2], and thermonuclear fusion reactors [3]. These concepts are based totally on the electrostatic confinement of ions in a deep potential cavity. For this purpose, a background cloud of magnetized electrons has been considered to be a candidate to form the negative well. Another significant feature of the background electrons is that a strong electric field $\mathbf{E}$ intrinsically forms due to their space charge. This field can produce strong $\mathbf{E} \times \mathbf{B}$ shear flows if we apply an appropriate magnetic field $\mathbf{B}$. This property may be applied to produce two-fluid high-β plasmas [4] which possess diamagnetic structures formed by the hydrodynamic pressure of the significant shear flows. In fact, sample equilibria of the hydrodynamic (relaxed) states resemble field-reversed configurations [5] and H-mode tokamaks [6]. Thus, studies on flowing plasmas have attracted considerable interest in experimental plasma physics.

One of the key parameters which characterizes nonneutral plasmas is the space potential ϕ_p or, equivalently, number density n. This value is easily

obtained in nonneutral plasmas with cylindrical geometry, which have an axial magnetic field providing radial confinement and applied potentials providing axial confinement. After a variable confinement time, a dump gate is pulsed to ground potential, which allows the remaining electrons to stream out axially along the field lines for collection and analysis. However, in axisymmetric toroidal plasmas, this method cannot be applied because the field line is closed.

In all earlier experiments on toroidal electron plasmas, ϕ_p was determined by a Langmuir probe terminated across a high impedance attenuator. In this method, no current flows in the electric circuit of the probe, i.e. the electron plasma outputs a constant voltage to the probe just like a voltage source in electronics. However, this raises a question. Generally, a neutral plasma behaves as a current source for probe measurements where a current-voltage $(I - V)$ characteristic, consisting of both electron and ion currents, is observed. Moreover, at $V = \phi_p$, I is never zero and the value of I is determined almost completely by electrons. These facts suggest strongly that even in electron plasmas, there should be a current-voltage characteristic which consists entirely of only electrons. Also, I should not be zero at $V = \phi_p$ in electron plasmas, which points out the need for attention to the high impedance method that causes ϕ_p to be overestimated.

This paper answers the question clearly. On experiments in the Proto-RT (Prototype Ring Trap) device [7], current-voltage characteristics of toroidal electron plasmas are measured for the first time. Similar to neutral plasmas, a current-voltage characteristic is established. However, no exponential dependence of I against V appears in the retarding region. Due to the effect of fast flow of electron plasmas, the measured characteristic spreads out significantly, exhibiting a long tail. These properties are interpreted by a calculation of particle flux collected to the probe. With regard to obtaining the value of ϕ_p, an emissive probe which is considered to be more adequate in determining ϕ_p is applied to electron plasmas for the first time. Remarkably, the probe always outputs higher voltage (Φ_E) than the voltage (Φ_H) measured by the same probe with no filament current ($|\Phi_E| < |\Phi_H|$). Here, one notes that for the latter case, the probe works as a high-impedance probe. We show that the disagreement would become relatively small when the flow energy is much smaller than the electrostatic potential energy of electron plasmas, which corresponds to the condition of $n \ll n_B$, where n_B is the Brillouin limit on number density. Another significant feature of the current-voltage characteristic of an electron plasma is that a floating potential ϕ_f, where $I = 0$, is determined, although ideally $\phi_f = -\infty$ because there are no ions. And, $I \neq 0$ at Φ_E, while $I = 0$ at Φ_H as seen in neutral plasmas.

The remainder of this paper is organized as follows. In Sec. II, experimental data of toroidal electron plasmas are presented. Interpretation of the measured current-voltage characteristics and several other phenomena related to this are given in Sec. III and IV. Finally, a summary is given in Sec. V.

II EXPERIMENTAL RESULTS

Experiments are conducted on the Prototype Ring Trap device (Proto-RT). Since the Freon cooling system was completed, we have performed initial experiments to confine energetic electrons in X point, Spherator-like, and dipole-like closed configurations.

A Potential measurements

Since the magnetic surfaces are closed in toroidal plasmas, passive probing is a key method to diagnose the plasmas. To focus attention on it, we have intensively studied pure electron plasmas confined in dipole-like configurations shown in Ref. [7]. The electron gun is fixed on the symmetry plane ($z = 0$), and the radial position of the gun is at $r = 50$ cm. The angle of the injected electrons with respect to the magnetic field is 90 degrees. The electrons circulate in the clockwise direction, which is opposite to the direction of equilibrium rotating flow.

Figure 1 shows time histories of an emissive probe signals at $r = 43$ cm for the case of $I_{ir} = 30$ A with/without a filament current I_{fil}. Substantial difference between the values of Φ_E and Φ_H can clearly be recognized. For the $I_{fil} \sim 1.1$ A case (with emission), the value of Φ_E is -52 V and constant during the discharge. In these experiments, we took at least three data sets at each position to determine the scattering of the data. The accuracy of
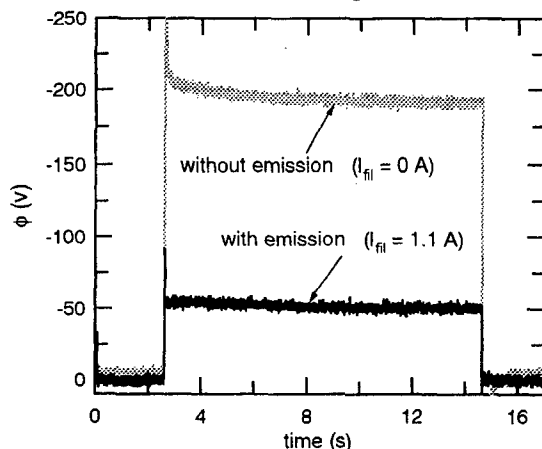
FIGURE 1. Time evolution of measured voltages by an emissive probe with completely the same conditions except for the filament current I_{fil}. Without I_{fil}, the probe works as a high-impedance probe which measures the voltage for $I_p = 0$. The difference between Φ_H and Φ_E is rather large, showing that the high-impedance method does not apply for determining ϕ_p.

the data was good and within 3 %, probably due to the completely static experiments. Also, Φ_E increased little with larger I_{fil}. Thus, we conclude that even in toroidal electron plasmas, emissive probes work like conventional ones in neutral plasmas, and the value of Φ_E gives ϕ_p of toroidal electron plasmas within experimental error.

On the other hand, for the $I_{fil} \sim 0$ A case (without emission), the value of Φ_H is -194 V which is about 3.7 times larger than Φ_E. Also, it gradually increases from -205 (at $t \sim 3$ s) to -190 V (at $t \sim 15$ s). Since the probe in this case inherently acts as a high-impedance probe, this result means that a high-impedance probe cannot measure ϕ_p of Proto-RT electron plasmas, contrary to past experiments on toroidal electron plasmas in which ϕ_p was determined with high-impedance probes.
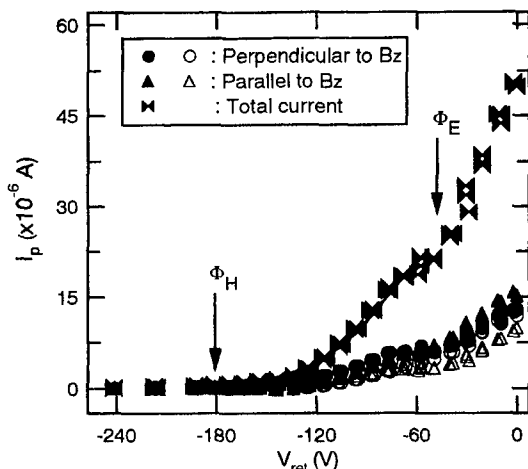
B Current-Voltage characteristics

Since the Φ_H measurement has popularly been used to determine ϕ_p in experiments on toroidal electron plasmas, a question arises on what Φ_H is in Proto-RT. To answer this question, we installed a directional probe to measure a current-voltage characteristic of Langmuir probe and the primary direction of electron flow in the plasmas.

Usually it is very difficult to detect the perpendicular motion of electrons by a probe because the parallel thermal motion of electrons is still faster in strong **B** even if they drift across **B**. However, on Proto-RT the strength of **B** is relatively weak ($B_p \sim 30$ G). Since the strength of the radial electric field E_r can be calculated to be ~ 10 V/cm for the dipole-like configuration, the perpendicular flow velocity v_d ($\sim E_r/B_p$) is thus expected to be $\sim 3 \times 10^5$ m/s. On the other hand, the parallel thermal velocity can be calculated $\sim 10^6$ m/s for $T_e \sim 10$ eV which is comparable to v_d, suggesting that the probe can distinguish the perpendicular flow.

Figure 2 shows typical current-voltage characteristic taken from the directional probe at $r = 43$ cm for the dipole-like configuration. Substantial difference in the collected current can be recognized between the upstream perpendicular- (black circle) and the downstream perpendicular (white circle) direction. This flow direction corresponds to the $-\nabla\phi_p\times\mathbf{B}$ direction, not to the direction of the injected beam from the gun. Thus, this result indicates the existence of equilibrium flow inside the magnetic surfaces.

It may then be considered that the total current plotted in Fig. 2, which is the sum of all the currents of the four independent probes, approximately shows a current-voltage characteristic of a cylindrical probe in toroidal electron plasmas. And, as recognized on the plotted curve, Φ_H indicates the '*floating*' potential ϕ_f where $I_p \sim 0$, although ideally $\phi_f \to -\infty$ because there are no ions in electron plasmas.

FIGURE 2. Current-voltage characteristics measured by a directional probe with four electrodes at $r = 43$ cm for the dipole-like configuration; two of them (denoted by circles) are turned toward the azimuthal direction of machine axis (perpendicular to magnetic field lines), while the others (denoted by triangles) are turned toward the z direction (along magnetic field lines): black circle: θ_-, white circle: θ_+, black triangle: z_+, and white triangle: z_-. The primary direction of perpendicular flow v_f corresponds to the $-\nabla\phi_p \times \mathbf{B}$ direction, strongly suggesting the existence of equilibrium flow. Thus, the total current, which is the sum of all the currents of the four independent electrodes of the directional probe, reflects the characteristic of a cylindrical probe in electron plasmas. As is apparent, Φ_H indicates the *floating potential* V_f, although ideally $V_f = -\infty$ in pure electron plasmas.

Another significant feature of the measured characteristic is that in decreasing the probe potential, I lessens not exponentially but gradually. And the value of I at $V = \Phi_E (\sim \phi_p)$ is finite and never equals 0, as recognized from the plotted data in Fig. 2.

III CALCULATION OF CURRENT-VOLTAGE CHARACTERISTICS OF FLOWING ELECTRON PLASMAS

To study the current collection characteristics of the probe, we consider the sheath configuration around the probe in flowing electron plasmas. As in neutral plasmas, Debye shielding of the electrostatic potential is realized even in pure electron plasmas. However, the shape of the sheath around the probe may be no longer cylindrical due to such a large value of v_f. Although there has been work on electric probes in flowing (neutral) plasmas, no sufficiently precise model is available which can justify empirically or theoretically any

other sheath shape. Thus, as a first-order approximation, a cylindrical sheath is assumed around the probe. We assume a cylindrical sheath in a rectangular coordinate system (x, y, z) which is fixed in the laboratory frame, shown in Ref. [7]. Let v_x, v_y, v_z be the electron velocity components in this frame, and v'_x, v'_y, v'_z be the electron velocity in a rectangular coordinate system in the moving frame with speed v_f and parallel to the laboratory frame. Choosing the y axis along the axis of the cylinder and θ the angle between the $\mathbf{v}_f$ vector and the y axis, then the transformation equations between the velocity components in the two coordinates are

$$v_x = v'_x + v_f \sin\theta\cos\beta, \quad v_y = v'_y + v_f\cos\theta, \quad v_z = v'_z + v_f\sin\theta\sin\beta \qquad (1)$$

where β is the azimuthal angle of $\mathbf{v}_f$ with respect to the x axis.

Assuming that the plasma is in a state of thermal equilibrium on a magnetic surface, then the Boltzmann distribution of the rotating axisymmetric plasmas in the moving frame takes the form

$$f = n_0 \left(\frac{m}{2\pi\kappa T}\right)^{3/2} exp\left(-\frac{H'}{\kappa T}\right) \qquad (2)$$

where n_0, m, κ, and T are number density, electron mass, Boltzmann's constant, and temperature, respectively. The particle energy H' in the moving frame is given by

$$\begin{aligned}
H' &= H - \omega P_\theta \\
&= \frac{1}{2}mv^2 + q\phi - \omega P_\theta \\
&= \frac{1}{2}mv^2 + q\phi - \omega r(mv_\theta + qA_\theta),
\end{aligned} \qquad (3)$$

where q, ϕ, P_θ, ω, and A_θ are charge, electric potential, canonical angular momentum, angular velocity, and the theta component of the vector potential. From eqs. (2) and (3), the distribution function can be rewritten as

$$\begin{aligned}
f &= n_0 \left(\frac{m}{2\pi\kappa T}\right)^{3/2} exp\left[-\frac{1}{\kappa T}\left\{\frac{m}{2}(v - \omega r\hat{\theta})^2 + q\phi - \frac{m}{2}\omega^2 r^2 - qA_\theta\omega r\right\}\right] \\
&= n_0 \left(\frac{m}{2\pi\kappa T}\right)^{3/2} exp\left[-\frac{1}{\kappa T}\left\{\frac{m}{2}(v - \omega r\hat{\theta})^2 + q\phi - C(r)\right\}\right] \\
&\equiv n'\left(\frac{m}{2\pi\kappa T}\right)^{3/2} exp\left[-\frac{1}{\kappa T}\left\{\frac{m}{2}(v - \omega r\hat{\theta})^2 + q\phi\right\}\right],
\end{aligned} \qquad (4)$$

where $n' = n_0 exp(C(r)/\kappa T)$. This distribution function may be modified in a way which is determined by the linear transformations of the velocity components given in eq. (1). Thus, for the laboratory frame, a new distribution $F(v_x, v_y, v_z, \beta)$ at the sheath edge can be written as

$$F = n'(\frac{m}{2\pi\kappa T})^{3/2}exp[-\frac{1}{\kappa T}\{(v_x - v_f sin\theta cos\beta)^2 + (v_y - v_f cos\theta)^2$$
$$+(v_z - v_f sin\theta sin\beta)^2 + q\phi\}]. \qquad (5)$$

Here, we consider an infinitesimal strip of area $Lad\beta$ on the sheath surface which is normal to the x axis, where a is the sheath radius. The particle number of electrons N, with velocity ranges between v_x and $v_x + dv_x$, v_y and $v_y + dv_y$, and v_z and $v_z + dv_z$, which are expected to cross the infinitesimal area per unit time is given by

$$N = v_x F dv_x dv_y dv_z Lad\beta, \qquad (6)$$

where L is the length of the cylinder. Therefore, on multiplying eq. (6) with q and integrating between proper limits, the current collected by the cylindrical probe in electron plasmas is described by

$$I = Laq \int_{\beta=0}^{2\pi} \int_{v_x=0,v_1}^{\infty} \int_{v_y=-\infty}^{\infty} \int_{v_z=-p}^{p} v_x F dv_x dv_y dv_z d\beta. \qquad (7)$$

The lower limit of v_x is zero for $V_{ret} > \phi_p$ and is $v_1 = \sqrt{2qV_{ret}/m}$ for $V_{ret} < \phi_p$. The case where $V_{ret} > \phi_p$ is the 'accelerated current', and the case where $V_{ret} < \phi_p$ is the 'retarded current' in conventional probe theory. The limits of v_y from $-\infty$ to ∞ follow from the assumption that $L \gg r$ for cylindrical probe, where r is the probe radius. For the limits of v_z (tangential component), p must satisfy $p = \sqrt{\gamma^2(v_x^2 + 2qV_{ret}/m)}$ where $\gamma^2 = r^2/(a^2-r^2)$. This is obtained by applying the laws of conservation of energy and angular momentum to the electron trajectory in the sheath. In toroidal electron plasmas, the Debye length λ_D is $1 - 10$ cm due to the low n_0. Accordingly, the value of a ($\geq 4\lambda_D$ at least) is approximately $4 - 40$ cm. On the other hand, r is typically less than 0.5 mm. Hence, $a/r \to \infty$ ($\gamma \to 0$) is a fairly good approximation for electron plasmas, and this condition corresponds to the case of orbital-motion-limited (OML) current. Thus, the final form of eq. (7) for electron plasmas normalized by $I_d = qn'v_{th}S$ (S is the collector area.) is given by

$$I = \frac{2}{\sqrt{\pi}}exp(V_0 - \alpha^2) \sum_{n=0}^{\infty} \frac{\alpha^n}{n!V_0^{n/2}} \Gamma[n + \frac{3}{2}, V_0] J_n(2\alpha V_0^{1/2}) \qquad (V_0 > 0)$$

$$= exp(V_0 - \alpha^2) \sum_{n=0}^{\infty} \frac{(2n + 1)!\alpha^n}{(n!)^2 2^{(2n)}(-V_0)^{n/2}} I_n[2\alpha(-V_0)^{1/2}], \qquad (V_0 < 0) \qquad (8)$$

where $V_0 = q(V_{ret} - \phi_p)/\kappa T$, $\alpha = (v_f/v_{th})sin\theta$, and Γ, J_n, and I_n are the incomplete gamma, Bessel, and modified Bessel functions, respectively.

The $I - V$ characteristics shown in Fig. 3 correspond to typical experiments for the $\theta = 90$ deg. case, in which the values of v_f/v_{th} are 0 (no flow), 1, and 2. Considerable differences can clearly be seen among them. For the case
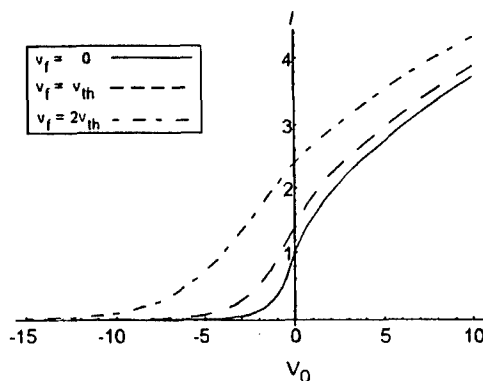
FIGURE 3. The calculated $I - V$ characteristics of electron plasmas in the OML limit for the case of $v_f/v_{th} = 0$, 1, and 2. The value of θ is 90 degrees for this calculation. The horizontal axis denotes $V = V_{ret} - \phi_p$, therefore, ϕ_p is fixed at $V = 0$. On the other hand, the value of I is normalized by the probe current for $v_f/v_{th} = 0$ (no flow) case. As v_f increases, no exponential dependence appears in the retarding region ($V < 0$), although the plasma is assumed to be in a state of thermal equilibrium in the moving frame. This property causes a long tail in the measured profiles observed in the laboratory frame, and moreover, I significantly increases when $v_f \neq 0$ because of the existence of flow flux.

of $v_f/v_{th} = 0$, the value of I in the retarding region ($V_0 < 0$) decreases exponentially, as in stationary plasmas. However, as v_f increases, the decay of I at $V_0 < 0$ changes gradually and is no longer an exponential curve. This brings about a longer tail, as recognized in the characteristic curve, despite T remaining constant. Moreover, the value of I increases for any V_0.

IV APPLICABILITY OF HIGH IMPEDANCE METHOD

As mentioned in the previous section, the value of Φ_H is unstable in pure electron plasmas, which calls for careful attention to the high impedance method. However, practically, the Φ_H measurement may be applied to estimate ϕ_p roughly for electron plasmas in which $\Phi_H/\Phi_E \to 1$ since the error bar of ϕ_p caused by the Φ_H method becomes relatively small for this case. This corresponds to the condition where the thermal energy $mv_{th}^2/2$ is as large as the flow energy $mv_f^2/2$, and the electrostatic energy $q\phi_p$ is much greater than $mv_f^2/2$: $q\phi_p \gg mv_{th}^2/2 \geq mv_f^2/2$.
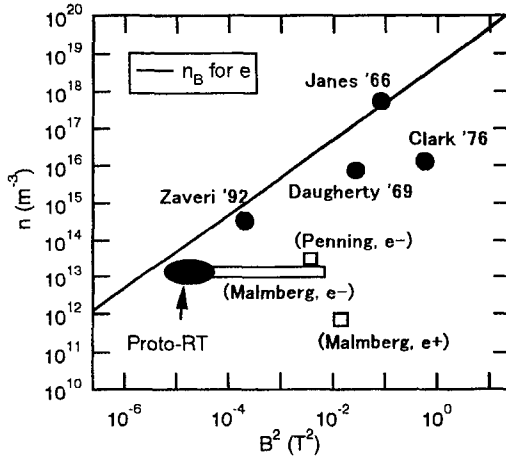
FIGURE 4. Typical plasma parameters (n and B^2) of nonneutral plasma experiments. The black circle shows the region for toroidal experiments on electrons, while the white square does so for linear experiments on electrons, positrons, and ions. The solid line indicates the Brillouin density limit n_B for pure electron plasmas. As recognized, n of toroidal electron plasmas is higher than that of cylindrical plasmas. Some of them are close to n_B, in such cases, the difference between Φ_H and Φ_E clearly appears because $\Lambda \to 1$.

The above condition is satisfied when the plasma density n is much smaller than the Brillouin limit on number density n_B at which the value of ω_p^2/ω_c^2 equals $1/2$, where ω_p and ω_c are the plasma and cyclotron frequencies, respectively. Assuming $v_f \sim E/B$, then the ratio of $mv_f^2/2$ to $q\phi_p$ can be calculated as

$$\Lambda = \frac{mv_f^2/2}{q\phi_p} = \frac{q^2 E^2/2m\omega_c}{q\phi_p} \sim \frac{q}{2m}\frac{\phi_p}{\ell^2}\frac{1}{\omega_c^2} \sim \frac{q}{2m}\frac{qn}{\epsilon_0}\frac{1}{\omega_c^2} = \frac{1}{2}\frac{\omega_p^2}{\omega_c^2}. \tag{9}$$

Hence, if $n \ll n_B$, then $\Lambda \ll 1/4$. Thus, the error bar of ϕ_p caused by the Φ_H method becomes relatively small. Actually, this would hold in most electron plasmas confined in a linear device, although no probe measurements have been applied to them so far.

On the other hand, for the case of $n \sim n_B$, the value of $mv_f^2/2$ becomes comparable to that of $q\phi_p$, which brings about a larger error bar. This situation is easily realized in toroidal electron plasmas. In fact, n of toroidal electron plasmas can locally exceed n_B. Figure 4 shows typical regions of n versus the value of B^2 for past experiments on both toroidal (black circle) and linear (white square) devices (see Ref. [7] for the detail). As can be recognized from the black circle, values of n of most toroidal electron plasmas are close to n_B, and actually the data seem to exceed n_B. This means that the error bar caused by the Φ_H measurement must hardly be small in those experiments.

In fact, on Proto-RT experiments, the error bar is about -150 V which is three times as large as ϕ_p (~ -50 V), as seen in Fig. 4.

V SUMMARY

Current-voltage characteristics of a cylindrical probe are measured in electron plasmas. Similar to a neutral plasma, a current-voltage characteristic is completely established. However, because of fast flow, the characteristic curve significantly spreads out, exhibiting a long tail. This feature can almost be interpreted completely by a calculation of collected currents to the probe. As a first approximation, a cylindrical sheath is assumed to form around the probe even though the probe is immersed in flowing plasmas. The results indicate that the distribution function observed by a probe fixed in the laboratory frame is always a non-Maxwellian even if the rotating plasmas are completely relaxed to a state of thermal equilibrium. This holds even for cold electron plasmas in which a larger value of v_f/v_{th} is attained, causing the characteristic curve to become more broadened.

The space potential ϕ_p of electron plasmas is measured by an emissive probe, which basically works even in flowing electron plasmas. The negative potential Φ_E measured by the emissive probe is always higher than Φ_H: $|\Phi_E| < |\Phi_H|$, which points out the need for attention when the high impedance method is utilized to determine ϕ_p of electron plasmas. However, practically, the Φ_H measurement is still available for plasmas in which $\Phi_H/\Phi_E \to 1$, namely, electrostatic energy ($q\phi_p$) $\gg$ flow energy ($mv_f^2/2$). Since the ratio of $mv_f^2/2$ to $q\phi_p$ can approximately be written as $\omega_p^2/2\omega_c^2$, the condition is satisfied when $n \ll n_B$ ($\Leftrightarrow \omega_p^2/2\omega_c^2 \ll 1/4$) where n_B is the Brillouin limit on number density.

ACKNOWLEDGMENTS

REFERENCES

1. Janes, G. S. *et al.*, Phys. Rev. **145**, 925 (1966).
2. Daugherty, J. D. *et al.*, Phys. Rev. Lett. **20**, 369 (1968).
3. Stix, T. H., Phys. Rev. Lett. **24**, 135 (1970).
4. Himura, H. *et al.*, *accepted for publication in Fusion Energy 2000, 18th IAEA Fusion Energy Conference, Italy*, (IAEA, 2001) IAEA-CN-77-ICP/14.
5. Steinhauer, L. C. and Ishida, A., Phys. Plasmas **5**, 2609 (1998).
6. Mahajan, S. M. and Yoshida, Z., Phys. Plasmas **7**, 635 (2000).
7. Himura, H. *et al.*, Phys. Plasmas **8** (10) (2001).