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Non-Collisional Kinetic Model for Non-Neutral Plasmas in a Penning Trap: General Properties and Stationary Solutions

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Abstract. A non-collisional kinetic model for a non-neutral plasma in a Penning trap is presented. This model describes the evolution of the z-integrated distribution function of the particles, taking into account the three-dimensionality of the problem. The general properties of the model, in particular the conservation laws, are studied. The model is also related to the fluid model proposed by Finn *et al.* and refined by Coppa *et al.* Finally, numerical investigations are presented concerning the equilibrium solutions of the model.

INTRODUCTION

Usually, the dynamics of non-neutral plasmas confined in Penning-Malmberg traps is described by employing a two-dimensional drift-Poisson model, where charged particles are regarded as straight lines (strings) of uniform density, due to the very high value of the axial bouncing frequency. According to this model, the particles exhibit an **E**×**B** drift motion and the electric field is computed self-consistently from the charge density by using the Poisson equation.

This model predicts possible instabilities for the azimuthal modes $m_\theta > 1$. For the $m_\theta = 1$ mode, discrete modes are always stable for any density profiles [1], while the continuum spectrum can only produce an algebraic growth [2]. Nevertheless, experiments show that the mode growth is actually exponential: the contradiction between experimental results and the linear two-dimensional theory is a challenging problem in the theory of non-neutral plasmas.

The present work grounds on recent studies on the evolution of non-neutral plasmas in a Penning-Malmberg trap, pointing out the important role of kinetic effects and of the finite length of the device, in particular for the $m_\theta = 1$ diocotron instability [3-5]. In fact, when a particle approaches the border of the plasma, it feels a confining potential which, in general, depends upon both the radial and the axial coordinates. The radial component of the confining electric field causes an **E**×**B** drift in the azimuthal direction, affecting the rotation frequency. The drift depends upon the axial energy of the particles, which affects the penetration in the confining potential [6].

The aim of the present work is to develop a kinetic theory for non-neutral plasmas taking into account self-consistently all these effects. The kinetic model assumes that

CP606, *Non-Neutral Plasma Physics IV,* edited by F. Anderegg et al.

the frequency are ordered as $\Omega_c >> \Omega_b >> \Omega_E$, being Ω_c the cyclotron frequency, Ω_b the bouncing frequency and Ω_E the **E**×**B** drift rotation, which is comparable to the frequency of the modes of interest. Within this assumption, the planar motion of the electrons is well described by the dynamics of their guiding centers, given by the **E**×**B** drift velocity, and the complete kinetic description of the plasma is then provided by the distribution function $f(r,\theta, z, \xi, t)$, where ξ is the axial energy of the particles. The description is completed by the self-consistent Poisson equation for the electrostatic potential $\phi(r,\theta, z, t)$.

In order to reduce the complexity of the problem, the dimensionality of the model is reduced by integrating along the axial direction. If the electrostatic potential "seen" by a string of electrons varies slowly in time, the ergodic distribution in the phase-space (z, v_z) can be assumed for the particles of the same energy and a kinetic equation for the z-integrated electron distribution, $F(r, \theta, \xi, t)$, can be deduced. The properties of such kinetic equation are the main subject of the present work.

DEDUCTION OF THE KINETIC EQUATION

The starting point in deducting the present model is the kinetic equation for the complete distribution function, $f(r,\theta, z, \xi, t)$,

$$\frac{\partial f}{\partial t}+\nabla_{\perp}\left(\frac{\mathbf{e}_z\times\nabla_{\perp}\phi}{B_0}f\right)+\frac{\partial}{\partial z}\left(v_z f\right)+\frac{\partial}{\partial\xi}\left(\frac{d\xi}{dt}f\right)=0 \tag{1}$$

where the **E**×**B** drift approximation is used and the phase-space variable ξ is the axial energy of the particles

$$\xi=\frac{1}{2}mv_z^2-e\phi(e,\theta,z,t) \tag{2}$$

while the time derivative of ξ is simply given by $d\xi / dt = -e\partial\phi / \partial t$.

The z-integrated distribution $F(r,\theta,\xi,t)$ is introduced by factorizing the distribution function as

$$f(r,\theta,\xi,z,t)=F(r,\theta,\xi,t)g(r,\theta,\xi,z,t) \tag{3}$$

The function g is defined assuming the particles of a given energy ξ to be distributed, in the phase-space (z, v_z), according to the ergodic distribution. Moreover, g is normalized in order that $\int g(r,\theta,\xi, z, t)\, dz = 1$. This leads to

$$g(r,\theta,\xi,z,t)=\frac{\left[e\phi(r,\theta,z,t)-\xi\right]^{-1/2}}{\int\left[e\phi(r,\theta,z,t)-\xi\right]^{-1/2}dz} \tag{4}$$

Within this assumption, the following equation for the function $F(r,\theta, \xi, t)$

$$\frac{\partial F}{\partial t}+\nabla_{\perp}\left(\mathbf{v}_D F\right)+\frac{\partial}{\partial \xi}\left(v_{\xi} F\right)=0 \tag{5}$$

is obtained by integrating Eq. (1) along the axial direction and defining the bounce-averaged streaming velocities as

$$\mathbf{v}_D=\int \frac{\mathbf{e}_z \times \nabla_{\perp} \phi}{B_0} g dz \qquad v_{\xi}=-e \int \frac{\partial \phi}{\partial t} g dz \tag{6}$$

The kinetic model is then completed by the self-consistent Poisson equation in the Penning trap

$$\nabla^2 \phi=\frac{e}{\varepsilon_0} \int F(r, \theta, \xi, t) g(r, \theta, z, \xi, t) d \xi \tag{7}$$

Finally, it must be observed that the kinetic equation, obtained using the adiabatic invariant

$$I(\xi)=\frac{1}{2 \pi} \int \sqrt{2 m(\xi+e \phi)} dz \tag{8}$$

as phase-space variable instead of ξ is simpler, in principle; however, the inversion of Eq. (8), required in the solution of the Poisson equation, is a challenging numerical problem.

GENERAL PROPERTIES

The distribution function F can be related to physical quantities that can be measured experimentally. Denoting with $\sigma(r,\theta, t)$ the z-integrated plasma density (an experimentally-measurable quantity) and with $F_\zeta(r,\theta, \zeta, t)$ the normalized distribution of the kinetic energy of the particles, ζ, the distribution function F can be expressed as $F(r,\theta,\xi,t) = \sigma(r,\theta, t)F_\zeta(r,\theta, \zeta, t)$, with $\zeta = \xi + e\phi_m$, being ϕ_m the potential in the point where the kinetic energy of the particles is measured and $\int F_\zeta(\zeta, r,\theta, t)d\zeta=1$.

The kinetic model presented here satisfies some conservation laws. The canonical angular momentum, $\mathbf{L}_z$, defined as

$$\mathbf{L}_z=-\frac{eB}{2} \int \int \int r^2 F(r, \theta, \xi, t) dr d\theta d\xi \tag{9}$$

and the angular momentum, l_z, proportional to the axial angular momentum of an incompressible fluid,

$$l_z = \int \mathbf{e}_z \cdot \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) d\mathbf{r} \quad (10)$$

are both conserved.

As far as the energy is concerned, the total energy of the plasma, $\mathbf{E}_{tot}$, can be written as the sum of potential and kinetic energies, $\mathbf{E}_{tot} = \mathbf{E}_{pot} + \mathbf{E}_{kin}$, being

$$\begin{aligned} \mathbf{E}_{pot} &= -\frac{1}{2} \int \int e\phi F(r,\theta,\xi,t) g(r,\theta,z,\xi,t) d\mathbf{r} d\xi \\ \mathbf{E}_{kin} &= \frac{1}{2} \int \int m v_z^2 (r,\theta,\xi,t) g(r,\theta,z,\xi,t) d\mathbf{r} d\xi \end{aligned} \quad (11)$$

The calculation of $d\mathbf{E}_{tot} / dt$ leads to the conclusion that the model conserves the total energy, as the variation of the energy of the plasma equals the power provided by the electrodes:

$$\frac{d\mathbf{E}_{tot}}{dt} = \frac{\varepsilon_0}{2} \frac{d}{dt} \int \phi \nabla \phi \cdot \mathbf{n} dS \quad (12)$$

CONNECTION WITH A RECENTLY-PROPOSED FLUID MODEL

The present kinetic model can be related to the fluid model proposed by Finn *et al.* [5] and refined by Coppa *et al.* [4]. The main assumption of that model is that the Maxwell-Boltzmann distribution is reached along the axial direction, so that the particle density $n(r,\theta, z, t)$ can be expressed as

$$n(r,\theta,z,t) = N(r,\theta,t) \exp\left[\frac{e\phi(r,\theta,z,t)}{k_B T}\right] \quad (13)$$

where $N(r,\theta, t)$ is a function that does not depend upon z. Within this assumption, the continuity equation for the z-integrated density, $\sigma(r,\theta, t)$, is written as

$$\frac{\partial \sigma}{\partial t} + \nabla_\perp (\mathbf{V}_\perp \sigma) = 0 \quad (14)$$

where

$$\mathbf{V}_\perp = \frac{1}{B_0} \mathbf{e}_z \times \frac{\int \nabla_\perp \phi \exp[(e\phi)/(k_B T)] dz}{\int \exp[(e\phi)/(k_B T)] dz} \quad (15)$$

The fluid model can be deduced from the present kinetic model by integrating Eq. (5) with respect to ξ, assuming that the function $F(r,\theta,\xi,t)$ can be written as:

$$F(r,\theta,\xi,t) = F(r,\theta,t)\exp\left(-\frac{\xi}{k_B T}\right)\int \frac{dz}{\sqrt{e\phi+\xi}} \qquad (16)$$

This assumption is equivalent to affirm that the electrons are distributed according to the canonical distribution; i.e. the electron density is proportional to $\delta(H-\xi)\exp[-\xi/(k_BT)]$.

NUMERICAL RESULTS

The electrostatic potentials, $\phi(r,z)$, corresponding to axially-symmetric distributions, $F(r,\xi)$, have been calculated by solving numerically the self-consistent Poisson equation [Eq. (7)]. The equation has been discretized in space (using a uniform grid) and energy. Then, suitable quadrature formulas have been used to evaluate the ξ and z-integrals. The Poisson equation has been solved by adding a term $\partial\phi/\partial t$ to the left hand-side and seeking the steady state solution of the diffusion-like equation so obtained.

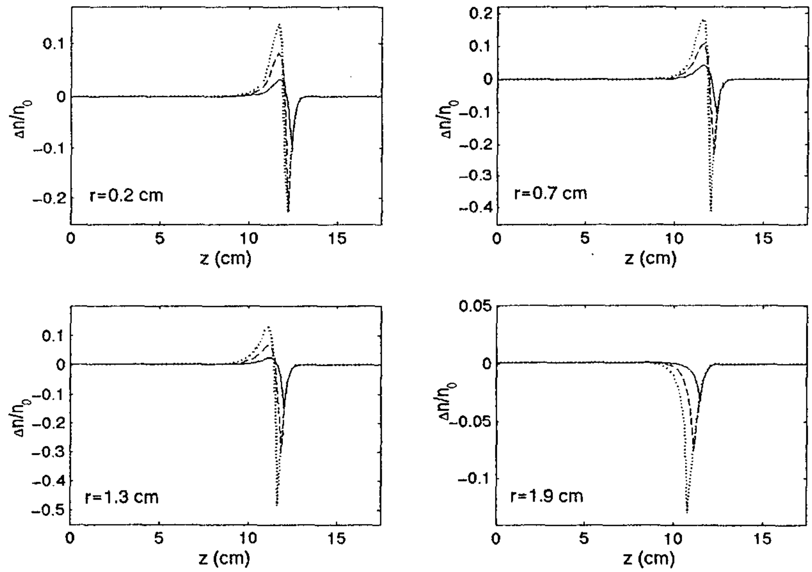

FIGURE 1. Plasma density difference, with respect to the Maxwellian distribution, for the Maxwellian distribution truncated at the energy $I=k_BT/2$ (dotted), $I=k_BT$ (dashed) and $I=2k_BT$ (solid). The confining potential of the trap is V=-50 V, the central electrode length L_c=26 cm, the end electrodes length L_e=4 cm, the wall radius R_w=3.5 cm, the plasma temperature T=1000 K, the magnetic field B_0=1 T and, the parameters of the density profiles introduced by Finn [5] are: $n_0=5\cdot10^{12}$ m^{-3}, μ=5, r_p=2 cm.

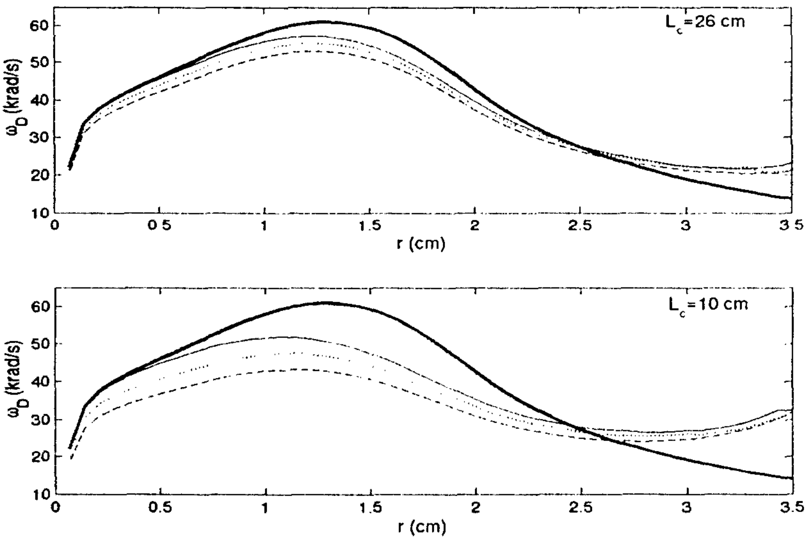

FIGURE 2. Bounce-averaged drift frequency for electrons with Maxwellian distribution at different axial energy: ξ=20.5 eV (solid), ξ=23.3 eV (dotted) and ξ=26 eV (dashed) for two different values of L_c. The predictions of the 2-D theory are shown in thick solid line. The other trap parameters are the same as Fig.1.

The equilibrium solutions have been studied having fixed the plasma density at the center of the trap as the density profiles introduced by Finn *et al.* [5]. The velocity distributions have been chosen to be truncated Maxwellians, as suggested by Hilsabeck *et al.* [3].

Some numerical results are presented on Figs. 1-2. Figure 1 points out the effect of the velocity distribution on the plasma density by comparing different truncated Maxwellians. Figure 2 shows how the rotation drift frequency depends on the particle energy.

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REFERENCES

1. Davidson, R.C., *An Introduction to the Physics of Non-Neutral Plasmas*, Addison-Wesley, Redwood City, 1990.
2. Smith, R.A., and Rosenbluth M.N., *Phys Rev. Lett.* **64**, 649-652 (1990).
3. Hilsabeck, T.J., and O'Neil, T.M., *Phys. Plasmas* **8**, 407-422 (2001).
4. Coppa, G.G.M., d'Angola, A., Delzanno, G.L., and Lapenta, G., *Phys. Plasmas* **8**, 1133-1140 (2001).
5. Finn, J.M., del Castillo-Negrete, D., and Barnes, D.C., *Phys. Plasmas* **6**, 3744-3758 (1999).
6. Peurrung, A.J., and Fajans, J., *Phys. Fluids B* **2**, 693-699 (1990).