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# Phase Noise Estimation for THz Radiation from RTDs and other Solid-State Sources

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**Abstract** - For imaging and remote sensing applications, the phase noise of the THz generator must be minimized. This can be done as part of the optimization process on the basis of a properly chosen figure of merit. For this, the present paper provides an analytical phase noise expression as a function of the device parameters and operation point. The quantum 1/f theory is used to calculate from first principles the 1/f noise present in the device parameters and in the resulting system frequency from resonant tunneling diodes (RTDs), super-electronic lattice devices (SLEDs), Gunn devices (TEDs), and transit time diodes. In general, quantum /f fluctuations in the dissipative elements lead to a  $Q^{-4}$  dependence of the spectral density of fractional frequency fluctuations and of the corresponding phase noise, where  $Q$  is the quality factor.

## I. INTRODUCTION

Fluctuations with a spectral density proportional to  $1/f$  are found in a large number of systems in science, technology and everyday life. These fluctuations are known as  $1/f$  noise in general. They have first been noticed by Johnson in early amplifiers, have limited the performance of vacuum tubes in the thirties and forties, and have later hampered the introduction of semiconductor devices.

The present paper is focused on the general origin of fundamental  $1/f$  noise as a universal form of chaos, and on the cause of its ubiquity. It starts with a *special case* of the general  $1/f$  noise phenomenon, the Quantum  $1/f$  Effect (with its conventional and coherent contributions) which is as fundamental as time and space. The  $1/f$  fluctuations are a necessary consequence of the mathematical homogeneity of the dynamical (or physical) equations describing the motion of an arbitrary chaotic or stochastic nonlinear system. A sufficient criterion was derived by the author. It indicates if an arbitrary system governed by a given system of nonlinear integro-differential equations will exhibit  $1/f$  noise. The criterion was applied to several particular systems, and is used to predict the fundamental quantum  $1/f$  effect as a *special case*.

## II. CONVENTIONAL QUANTUM 1/f EFFECT

This effect<sup>1-6</sup> is present in any cross section or process rate involving charged particles or current carriers. The physical origin of quantum  $1/f$  noise is easy to understand. Consider for example Coulomb scattering of current carriers, e.g., electrons on a center of force. The scattered electrons reaching a detector at a given angle away from the direction of the incident beam are described by DeBroglie waves of a frequency corresponding to their energy. However, some of the electrons have lost energy in the scattering process, due to the emission of bremsstrahlung. Therefore, part of the outgoing DeBroglie waves is shifted to slightly lower frequencies. When we calculate the probability density in the scattered beam, we obtain also cross terms, linear both in the part scattered with and without bremsstrahlung. These cross terms oscillate with the same frequency as the frequency of the emitted bremsstrahlung photons. The emission of photons at all frequencies results therefore in probability density fluctuations at all frequencies. The corresponding current density fluctuations are obtained by multiplying the probability density fluctuations by the velocity of the scattered current carriers. Finally, these current fluctuations present in the scattered beam will be noticed at the detector as low frequency current fluctuations, and will be interpreted as fundamental cross section fluctuations in the scattering cross section of the scatterer. While incoming carriers may have been Poisson distributed, the scattered beam will exhibit super-Poissonian statistics, or bunching, due to this new effect which we may call quantum  $1/f$  effect. The quantum  $1/f$  effect is thus a many-body or collective effect, at least a two-particle effect, best described through the two-particle wave function and two-particle correlation function.

Let us estimate the magnitude of the quantum  $1/f$  effect semiclassically by starting with the classical (Larmor) formula  $2q^2a^2/3c^3$  for the power radiated by a particle of charge  $q$  and acceleration  $a$ . The acceleration can be approximated by a delta function  $a(t) = \Delta v \delta(t)$  whose Fourier transform

$\Delta\mathbf{v}$  is constant and is the change in the velocity vector of the particle during the almost instantaneous scattering process. The one-sided spectral density of the emitted bremsstrahlung power  $4q^2(\Delta\mathbf{v})^2/3c^3$  is therefore also constant. The number  $4q^2(\Delta\mathbf{v})^2/3hfc^3$  of emitted photons per unit frequency interval is obtained by dividing with the energy  $hf$  of one photon. The probability amplitude of photon emission  $[4q(\Delta\mathbf{v})^2/3hfc^3]^{1/2}e^{i\gamma}$  is given by the square root of this photon number spectrum, including also a phase factor  $e^{i\gamma}$ . Let  $\psi$  be a representative Schrödinger catalogue wave function of the scattered outgoing charged particles, which is a single-particle function, normalized to the actual scattered particle concentration. The beat term in the probability density  $\rho=|\psi|^2$  is linear both in this bremsstrahlung amplitude and in the non-bremsstrahlung amplitude. Its spectral density will therefore be given by the product of the squared probability amplitude of photon emission (proportional to  $1/f$ ) with the squared non-bremsstrahlung amplitude which is independent of  $f$ . The resulting spectral density of fractional probability density fluctuations is obtained by dividing with  $|\psi|^4$  and is therefore

$$|\psi|^{-4}S_{|\psi|^2}(f) = 8q^2(\Delta\mathbf{v})^2/3hfc^3 = 2\alpha A/fN = j^2S_j(f), \quad (1)$$

where  $\alpha = e^2/h$ ,  $c = 1/137$  is the fine structure constant and  $\alpha A = 4q^2(\Delta\mathbf{v})^2/3hfc^3$  is known as the infrared exponent in quantum field theory, and is known as the quantum  $1/f$  noise coefficient, or Hooze constant, in electrophysics.

The spectral density of current density fluctuations is obtained by multiplying the probability density fluctuation spectrum with the squared velocity of the outgoing particles. When we calculate the spectral density of fractional fluctuations in the scattered current  $j$ , the outgoing velocity simplifies, and therefore Eq. (1) also gives the spectrum of current fluctuations  $S_j(f)$ , as indicated above. The quantum  $1/f$  noise contribution of each carrier is independent, and therefore the quantum  $1/f$  noise from  $N$  carriers is  $N$  times larger; however, the current  $j$  will also be  $N$  times larger, and therefore in Eq. (1) a factor  $N$  was included in the denominator for the case in which the cross section fluctuation is observed on  $N$  carriers simultaneously.

The fundamental fluctuations of cross sections and process rates are reflected in various kinetic coefficients in condensed matter, such as the mobility  $\mu$  and the diffusion constant  $D$ , the surface and bulk recombination speeds  $s$ , and recombination times  $\tau$ , the rate of tunneling  $j_t$  and the thermal diffusivity in semiconductors. Therefore, the

spectral density of fractional fluctuations in all these coefficients is given also by Eq. (1).

When we apply Eq. (1) to a certain device, we first need to find out which are the cross sections  $\sigma$  or process rates which limit the current  $I$  through the device, or which determine any other device parameter  $P$ , and then we have to determine both the velocity change  $\Delta v$  of the scattered carriers and the number  $N$  of carriers simultaneously used to test each of these cross sections or rates. Then Eq. (1) provides the spectral density of quantum  $1/f$  cross section or rate fluctuations. These spectral densities are multiplied by the squared partial derivative  $(\partial I/\partial \sigma)^2$  of the current, or of the device parameter  $P$  of interest, to obtain the spectral density of fractional device noise contributions from the cross sections and rates considered. After doing this with all cross sections and process rates, we add the results and bring (factor out) the fine structure constant  $\alpha$  as a common factor in front. This yields excellent agreement with the experiment in a large variety of samples, devices and physical systems. Eq. (1) was derived in second quantization, using the commutation rules for boson field operators. For fermions one repeats the calculation replacing in the derivation the commutators of field operators by anticommutators, which yields<sup>1-6</sup>

$$\rho^{-2}S_\rho(f) = j^2S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/f(N-1) \quad (2)$$

This causes no difficulties, since  $N \geq 2$  for particle correlations to be defined, and is practically the same as Eq. (1), since usually  $N \gg 1$ . Eqs. (1) and (2) suggest a new notion of physical cross sections and process rates which contain  $1/f$  noise, and express a fundamental law of physics, important in most high-technology applications.

We turn now to the connection to the coherent Quantum  $1/f$  Effect, essentially caused by the uncertainty of the electron mass, by the coherent state of the field of the electron. The coherent state has an uncertain energy. The coherent state in a conductor or semiconductor sample is the result of the experimental efforts directed towards establishing a steady and constant current, and is therefore the state defined by the collective motion, i.e. by the drift of the current carriers. It is expressed in the Hamiltonian by the magnetic energy  $E_m$ , per unit length, of the current carried by the sample. In very small samples or electronic devices, this magnetic energy

$$E_m = \int (B^2/8\pi) d^3x = [nevS/c]^2 \ln(R/r) \quad (3)$$

is much smaller than the total kinetic energy  $E_k$  of the drift motion of the individual carriers

$$E_k = \sum_i m_i v^2/2 = nSmv^2/2 = E_m/s. \quad (4)$$

Here we have introduced the magnetic field  $B$ , the carrier concentration  $n$ , the cross sectional area  $S$  and radius  $r$  of the cylindrical sample (e.g., a current carrying wire), the radius  $R$  of the electric circuit, and the "coherence ratio"

$$s = E_m/E_k = 2ne^2S/mc^2 \ln(R/r) = 2e^2N'/mc^2, \quad (5)$$

where  $N' = nS$  is the number of carriers per unit length of the sample and the natural logarithm  $\ln(R/r)$  has been approximated by one in the last form. We expect the observed spectral density of the mobility fluctuations to be given by a relation of the form

$$(1/\mu^2)S_\mu(f) = [1/(1+s)][2\alpha A/fN] + [s/(1+s)][2\alpha/\pi fN] \quad (6)$$

which can be interpreted as an expression of the effective Hooge constant if the number  $N$  of carriers in the (homogeneous) sample is brought to the numerator of the left hand side. In this equation  $\alpha A = 2\alpha(\Delta v/c)^2/3\pi$  is the usual nonrelativistic expression of the infrared exponent, present in the familiar form of the conventional quantum 1/f effect<sup>1-6</sup>. This equation is limited to quantum 1/f mobility (or diffusion) fluctuations, and does not include the quantum 1/f noise in the surface and bulk recombination cross sections, in the surface and bulk trapping centers, in tunneling and injection processes, in emission or in transitions between two solids.

Note that the coherence ratio  $s$  introduced here equals the unity for the critical value  $N' = N'' = 2 \cdot 10^{12}/\text{cm}$ , e.g. for a cross section  $S = 2 \cdot 10^{-4} \text{ cm}^2$  of the sample when  $n = 10^{16}$ . For small samples with  $N' \ll N''$  only the first term survives, while for  $N' > N''$  the second term in Eq. (6) is dominant.

### III. DERIVATION OF MOBILITY QUANTUM 1/f NOISE IN $N^+$ -P DIODES

For a diffusion limited  $n^+$ -p junction the current is controlled by diffusion of electrons into the p - region over a distance of the order of the diffusion length  $L = (D_n\tau_n)^{1/2}$  which is shorter than the length  $w_p$  of the p - region in the case of a long diode. If  $N(x)$  is the number of electrons per unit length and  $D_n$  their diffusion constant, the electron current at  $x$  is

$$I_{nd} = -eD_n dN/dx, \quad (7)$$

where we have assumed a planar junction and taken the origin  $x = 0$  in the junction plane. Diffusion constant fluctuations, given by  $kT/e$  times the mobility fluctuations, will lead to local current fluctuations in the interval  $\Delta x$

$$\delta\Delta I_{nd}(x,t) = I_{nd}\Delta x \delta D_n(x,t)/D_n. \quad (8)$$

The normalized weight with which these local fluctuations representative of the interval  $\Delta x$  contribute to the total current  $I_d$  through the diode at  $x = 0$  is determined by the appropriate Green function and can be shown to be  $(1/L)\exp(-x/L)$  for  $w_p/L \gg 1$ . Therefore the contribution of the section  $\Delta x$  is

$$\delta\Delta I_d(x,t) = (\Delta x/L)\exp(-x/L)I_{nd}\delta D_n(x,t)/D_n, \quad (9)$$

with the spectral density

$$S_{\Delta I_d}(x,f) = (\Delta x/L)^2 \exp(-2x/L) I_{nd}^2 S_{D_n}(x,f)/D_n^2 \quad (10)$$

For mobility and diffusion fluctuations the fractional spectral density is given by  $\alpha_{Hnd}/fN\Delta x$ , where  $\alpha_{Hnd}$  is determined from quantum 1/f theory according to Sec II. With Eq. (7) we obtain then

$$S_{\Delta I_d}(x,f) = (\Delta x/L^2)\exp(-2x/L)(eD_n dN/dx)^2 \alpha_{Hnd}/fN \quad (11)$$

The electrons are distributed according to the solution of the diffusion equation, i.e.

$$N(x) = [N(0) - N_p] \exp(-x/L); \\ dN/dx = -\{[N(0) - N_p]/L\} \exp(-x/L). \quad (12)$$

Substituting into Eq. (11) and simply summing over the uncorrelated contributions of all intervals  $\Delta x$ , we obtain

$$S_{I_d}(f) = \alpha_{Hnd}(eD_n/L^2)^2 \int [N_0 - N_p]^2 e^{-4x/L} dx / \{[N(0) - N_p] e^{-x/L} + N_p\} \quad (13)$$

The integral is from 0 to  $w$ . We note that  $eD_n/L^2 = e/\tau_n$ . With the expression of the saturation current  $I_0 = e(D_n/\tau_n)^{1/2}N_p$  and of the current  $I = I_0[\exp(eV/kT) - 1]$ , we can carry out the integration from 0 to 1

$$S_{I_d}(f) = \alpha_{Hnd}(eI/\tau_n) \int du/(au + 1) = \alpha_{Hnd}(eI/\tau_n) F(a). \quad (14)$$

Here we have introduced the notations

$$u = \exp(-x/L), \quad a = \exp(eV/kT) - 1, \\ F(a) = 1/3 - 1/2a + 1/a^2 - (1/a^3) \ln(1+a) \quad (15)$$

Eq. (14) gives the diffusion noise as a function of the quantum 1/f noise parameter  $\alpha_{Hnd}$ . A similar result can be derived for the quantum 1/f fluctuations of the recombination rate  $r$  in the bulk of the p - region, the only difference being the presence of  $\alpha_{Hnr}$  instead of  $\alpha_{Hnd}$  in Eq. (14). The total noise is the given by Eq. (14) with  $\alpha_{Hnd}$  replaced by the sum  $\alpha_{Hnd} + \alpha_{Hnr}$

$$S_{I_d}(f) = (\alpha_{Hnd} + \alpha_{Hnr})(eI/\tau_n) F(a). \quad (16)$$

#### IV. 1/f FLUCTUATIONS IN RTDs

Resonant tunneling diodes have been proposed as generators of THz oscillations and radiation. They consist of two potential barriers enclosing a quantum well. Electrons penetrating the potential barriers by tunneling are controlled by the quasistationary energy levels defined by the potential well. If their energy is close to the first energy level in the well, resonance occurs, and a peak  $I_p$  of the current through the diode occurs. This corresponds to an applied bias voltage  $V_p$ . If, however, the applied voltage increases further, only a negligibly small non-resonant current trickle remains at the voltage  $V=V_v$  and a broad valley is observed in the  $I/V$  characteristic. Scattering processes that reduce the energy of the carriers to a value close to  $eV_p$  will always be present, generating a finite current minimum  $I_v$  at  $V_v$ . Between  $V_p$  and  $V_v$  there is a negative differential conductance

$$G = -(I_p - I_v) / (V_v - V_p) \quad (17)$$

on the  $I/V$  curve, that is used to generate oscillations.

1/f noise in  $I_v$  is given by Eqs. (1)-(2) with

$$(\Delta v/c)^2 = 2eV_v/m. \quad (18)$$

Taking for instance  $V_p = 0.4$  V,  $I_p = 2.5 \cdot 10^8$  A/m<sup>2</sup>,  $V_v = 0.6$  V,  $I_v = 4 \cdot 10^7$  A/m<sup>2</sup>, we obtain with  $m_{eff} = 0.068 m_0$ ,

$$NI_v^{-2} S_{I_v}(f) = 2\alpha A/f = 7.4/0.068 \cdot 10^{-9} = 1.3 \cdot 10^{-7} \quad (19)$$

$N$  is given by

$$N = \tau I_v / e, \quad (20)$$

where  $\tau$  is the life time of the carriers. With a cross-sectional area of  $10^{-6}$  cm<sup>2</sup> and  $\tau = 10^{-10}$ , we obtain  $N = 2.5 \cdot 10^6$ .

Finally, the quantum 1/f frequency fluctuations can be obtained from the formula

$$\omega^2 = -G^3 L_c / C_D^3 (R_L + R_s), \quad (21)$$

which is obtained by equating the RF circuit-limited output power with the DC bias circuit-limited power. Here  $L_c$  is the inductance in the bias circuit,  $C_D$  is the diode capacitance,  $R_L$  the resistance introduced by the RF circuit inductance, and  $R_s$  is the diode series resistance. From Eq. (17) the fluctuations in  $G$  caused by  $I_v$  are calculated and substituted into the linearized form of Eq. (21).

#### V. 1/f FREQUENCY FLUCTUATIONS FROM 1/f NOISE IN RF DISSIPATIVE ELEMENTS

RF resonant circuits can be described as a harmonic

oscillator with losses

$$dx/dt + \gamma dx/dt + \omega_0^2 x = F(t) \quad (22)$$

The quantum 1/f fluctuations are present in the loss coefficient  $\gamma$ . They are given by an expression of the form

$$S_{\delta\gamma/\gamma}(f) = \Lambda/f \quad (23)$$

where  $\Lambda$  is a quantum 1/f coefficient characterizing the elementary loss process.

The resonance frequency is given by

$$\omega_r^2 = \omega_0^2 + \gamma^2 \quad (24)$$

The quantum 1/f fluctuations in the resonance frequency are given by  $\omega_r \delta\omega_r = -2\gamma\delta\gamma$ , or

$$\delta\omega_r/\omega_r = -2(\gamma/\omega_r) \delta\gamma/\gamma = -(1/2Q^2)(\delta\gamma/\gamma) \quad (25)$$

where  $Q = \omega_r/2\gamma$  is the quality factor. Squaring, averaging and particularizing Eq. (25) for the unit frequency interval, we obtain the  $Q^{-4}$  law

$$S_y(f) = \langle (\delta\omega_r/\omega_r)^2 \rangle_f = (1/4Q^4) \langle (\delta\gamma/\gamma)^2 \rangle_f = \Lambda/4fQ^4, \quad (26)$$

where  $y$  is the fractional frequency fluctuation  $\delta\omega_r/\omega_r$ . This law is also applicable to quartz resonators<sup>5</sup>. The examples in the last two sections show how the phase noise is to be found in other solid state sources of THz radiation.

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