Defense Technical Information Center Compilation Part Notice

ADP011678

TITLE: Two-Wave Approximation for Transition Layer of Inhomogeneous Media

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: International Conference on Electromagnetics of Complex Media [8th], Held in Lisbon, Portugal on 27-29 September 2000. Bianisotropics 2000

To order the complete compilation report, use: ADA398724

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:
ADP011588 thru ADP011680

UNCLASSIFIED

Two-Wave Approximation for Transition Layer of Inhomogeneous Media

N. A. Simonov

Institute for Theoretical and Applied Electrodynamics,
Russian Academy of Sciences,
Izhorskaya 13/19, Moscow, 127412 IVTAN, Russia.
Phone (7-095) 485-8322, Fax (7-095) 484-2644, E-mail: simonov@eldyn.msk.ru

Abstract

We regard the transition layer near the bound of inhomogeneous media on the base of the introduced here concept of the generalized wave modes (GWM). We show in present work that previously described one-dimensional nonlocal model of inhomogeneous media [1] is a two-wave approximation for electrodynamic properties of the medium. The second, evanescent, wave in the model represents a sum of wide spectrum of GWM and can be used for approximate description of the transition layer.

1. Introduction

The transition layer is relatively thin area near the bound of medium, where electrodynamic property cannot be described adequately in terms of ε, μ parameters. Usually, inhomogeneous media, such as composite materials, have a depth of the transition layer approximately the same as a characteristic size a of the structure. But one can expect that in special cases this depth can be considerably larger than a, at least for the materials with sufficiently strong interaction between the inclusions.

Taking into account the presence of the transition layer for inhomogeneous materials, one can provide the general conclusion that the measured effective parameters ε^e, μ^e of them must always differ from those of ε_e, μ_e, which one can calculate for the infinite (boundless) medium.

The most transparent example for this conclusion can be the finely stratified medium (the Rytov solution [2]), which contain dielectric and metal layers. Let us consider only TM modes in it. There exist only one propagating (principal) wave mode in such a structure and all higher order modes are evanescent in the quasi-static case. Acher et al [3] used formulas for the ε_e, μ_e parameters, which correspond to the principal wave in such a structure, for calculation the intrinsic permeability of thin metal films. In the limit of strong skin effect those formulas give the result $\varepsilon_e = \varepsilon, \quad \mu_e = 1$, which is not correct, because do not reflects the diamagnetic properties of such a structure. Our theoretical analysis and the experimental examination permit us to found the correct formulas for the considered case:

$$\varepsilon^e = \varepsilon \frac{l}{d} > \varepsilon, \quad \mu^e = \frac{d}{l} < 1 \tag{a}$$

where l is a period of the layers and d is the width of dielectric layers. The difference between the first and the second formulas can be explained by the fact of excitation the wide spectrum of evanescent modes near the plane boundary of this medium. These modes form the transition layer, which depth is close to the skin depth. Therefore, the formulas in [3] are not correct, if the skin effect is noticeable.

2. Determination of the Generalized Wave Modes in Inhomogeneous Medium

Let us consider arbitrary inhomogeneous, but statistically homogeneous and isotropic in **x** and **y** directions medium. It is conveniently to regard the transition layer on the boundary on the base of conception of the generalized wave modes (GWM). Such a mode we determine as a generalized wave process in the medium, when the spatially mean constituents $\mathbf{E}_0$ and $\mathbf{H}_0$ of the electromagnetic field in it spread as a plane wave (factor $\exp(i\omega t)$ is omitted)

$$\mathbf{E}_0(z) = E_0\hat{\mathbf{e}}_0\exp(-ikz),\quad \mathbf{H}_0(z) = H_0\hat{\mathbf{h}}_0\exp(-ikz),\quad \hat{\mathbf{h}}_0 = [\hat{\mathbf{z}}\times\hat{\mathbf{e}}_0] \tag{1}$$

This definition is consistent with the theory of the space dispersion [4] where only spatially means constituents of the field are taken into consideration. In present work we shall take into account the spatially inhomogeneous constituents of the field also. For this purpose we provide decomposition of tangential components of the whole electromagnetic field in arbitrary plane (x, y) in form of finite series of the spatial harmonics (SH). For example, it can be done with help of 2D FFT, but we suppose that each SH is a sum of both elementary plane waves in the (x, y) plane with the same wavenumber. Further we shall apply one-dimension, rather then two-dimensions indexing of SH for the simplicity. Providing transformations of the Maxwell's equations by analogy to the method, have been described in [1], one can obtain the followed system of equations

$$\begin{cases} kE_n = \omega\mu_0\sum\limits_{m=0}^{N-1}\mu_{nm}H_m \\ kH_n = \omega\varepsilon_0\sum\limits_{m=0}^{N-1}\varepsilon_{nm}E_m \end{cases},\qquad n = 0,\ldots,N-1 \tag{2}$$

Here we use SI system, E_n, H_n are the amplitudes of the spatial harmonics, N – the number of the harmonics, $\varepsilon_{nm}, \mu_{nm}$ are complex parameters, which depend on ε, μ distribution in the (x, y) plane and polarization of the SH only. Naturally, the zero index corresponds to the spatially homogeneous constituents of the field. One can show with help of the Maxwell's equations that for the defined polarization of the $\mathbf{E}_0$, $\mathbf{H}_0$ constituents, polarization of the fields of each SH in the defined (x, y) plane do not depend on the wavenumber k of the mode (1). Hence the same is true for the parameters $\varepsilon_{nm}, \mu_{nm}$. Moreover, it seems natural that those parameters do not depend on z for statistically homogeneous media, although we have no proof of it at present time. Nevertheless, if there are some fluctuations of those parameters in z direction, we always can take the mean values of them.

Therefore, the system (2) is a linear system of equations. It can be rewritten in the equivalent and a more convenient form, as a combination of two matrix equations:

$$\begin{aligned} &\underline{\underline{A}}*\underline{E} = n^2\cdot\underline{E},\quad \underline{\underline{A}} = [a_{nm}],\quad a_{nm} = \sum_{k=0}^{N-1}\mu_{nk}\cdot\varepsilon_{km},\quad \underline{E} = (E_0, E_1,\ldots,E_n,\ldots,E_{N-1}) \\ &\underline{\underline{B}}*\underline{H} = n^2\cdot\underline{H},\quad \underline{\underline{B}} = [b_{nm}],\quad b_{nm} = \sum_{k=0}^{N-1}\varepsilon_{nk}\cdot\mu_{km},\quad \underline{H} = (H_0, H_1,\ldots,H_n,\ldots,H_{N-1}) \end{aligned} \tag{3}$$

Here $n \equiv k/k_0$ is a refractive index of the GWM, which can be obtained as a solution to the two equivalent dispersion equation

$$\det\left(\underline{\underline{A}} - n^2\underline{\underline{I}}\right) = 0 \Leftrightarrow \det\left(\underline{\underline{B}} - n^2\underline{\underline{I}}\right) = 0 \tag{4}$$

where $\underline{\underline{I}}$ is a unite matrix. It is clearly that the eigenvectors $\underline{E}, \underline{H}$ of the corresponding matrixes $\underline{\underline{A}}, \underline{\underline{B}}$ describe the spatially spectrum of SH for respective GWM. Therefore, in contradistinction to the conventional concept of wave mode, the GWM, travelling trough the inhomogeneous medium, keeps only the spatial spectrum of transversal distribution of the electromagnetic field.

If we pass to the limit of the continuous spatially spectrum, the characteristic polynomial $\det\left(\underline{\underline{A}} - n^2 \underline{\underline{I}}\right)$ transfer to an entire function [5], which have isolated zeros only. Hence the spectrum of GWM for the medium, which is described by the equation (4), is always discreet. It is clear that similar to the finely stratified medium [2], the only principal GWM is usually propagating and all higher order modes are evanescent in quasi-static case. Now we can conclude that by analogy to the Rytov solution, the transition layer is formed by a series of evanescent generalized modes, which are excited by an incident wave.

3. Two-Wave Approach to Description of the Transition Layer

Let us show that one-dimensional nonlocal model of the inhomogeneous medium [1] corresponds to a two-wave approximation for electrodynamic description of the medium. For the uniformity, we rename the 8 constitutive parameters introduced in [1] to $\varepsilon_{km}, \mu_{km}$. Then we can present 4 equations obtained in [1] in the next form

$$\begin{aligned} k \cdot E_0 &= \omega\mu_0\left(\mu_{00}H_0 + \mu_{01}H_1\right) & \qquad k \cdot H_0 &= \omega\varepsilon_0\left(\varepsilon_{00}E_0 + \varepsilon_{01}E_1\right) \\ k \cdot E_1 &= \omega\mu_0\left(\mu_{10}H_0 + \mu_{11}H_1\right) & \qquad k \cdot H_1 &= \omega\varepsilon_0\left(\varepsilon_{10}E_0 + \varepsilon_{11}E_1\right) \end{aligned} \tag{5}$$

Here E_1 and H_1 are the effective complex amplitudes of the spatially inhomogeneous constituents of the tangential components of the field [1]. These amplitudes can be express in the amplitudes of spatial harmonics (2)

$$E_1 = \sqrt{\sum_{m=1}^{N-1} E_m^2}, \quad H_1 = \sqrt{\sum_{m=1}^{N-1} H_m^2} \tag{6}$$

Parameters $\varepsilon_{km}, \mu_{km}$ can be calculated by formulas, which followed from [1], where functions of distribution of the inhomogeneous constituents of the field correspond to the principal wave in the media.

We can rewrite the system (5) in form of two systems by analogy to eq.(3):

$$\begin{cases} a_{00}E_0 + a_{01}E_1 = n^2 \cdot E_0 \\ a_{10}E_0 + a_{11}E_1 = n^2 \cdot E_1 \end{cases} \qquad \begin{cases} b_{00}H_0 + b_{01}H_1 = n^2 \cdot H_0 \\ b_{10}H_0 + b_{11}H_1 = n^2 \cdot H_1 \end{cases} \tag{7}$$

Here coefficients a_{km} and b_{km} are depended on parameters $\varepsilon_{km}, \mu_{km}$ in similar way to (3):

$$\begin{aligned} a_{00} &= \mu_{00}\varepsilon_{00} + \mu_{01}\varepsilon_{10}, & \qquad a_{01} &= \mu_{00}\varepsilon_{01} + \mu_{01}\varepsilon_{11} \\ a_{10} &= \mu_{10}\varepsilon_{00} + \mu_{11}\varepsilon_{10}, & \qquad a_{11} &= \mu_{10}\varepsilon_{01} + \mu_{11}\varepsilon_{11} \\ b_{00} &= \varepsilon_{00}\mu_{00} + \varepsilon_{01}\mu_{10}, & \qquad b_{01} &= \varepsilon_{00}\mu_{01} + \varepsilon_{01}\mu_{11} \\ b_{10} &= \varepsilon_{10}\mu_{00} + \varepsilon_{11}\mu_{10}, & \qquad b_{11} &= \varepsilon_{10}\mu_{01} + \varepsilon_{11}\mu_{11} \end{aligned} \tag{8}$$

Solution of Eq.(7) can be obtained by analogy to Eq. (4) through dispersion equations

$$\det\left(\underline{\underline{A}}_2 - n^2 \underline{\underline{I}}_2\right) = 0 \Leftrightarrow \det\left(\underline{\underline{B}}_2 - n^2 \underline{\underline{I}}_2\right) = 0 \tag{9}$$

where index 2 corresponds to 2x2 matrixes of Eq.(7). Hence the equations (9) have two solutions for the value n^2. Naturally, the first root of (9) must correspond to the principal wave in the medium. The second solution represents an effective GMW, which is a superposition of all higher evanescent generalized modes in it. Therefore one can expect that the second wave of the one-dimensional model can be used for the approximate description of the transition layer of inhomogeneous media. We mean here the case of plane boundary of the medium and plane wave, which normally fall on it.

The model of medium, where two wave modes can be excited, are well-known from theory of the spatial dispersion [4], although only propagating waves was taken into consideration there. The principal point of this problem is to find the additional bound condition, which is necessary for sewing together tangential components of electromagnetic fields on the boundary for incident wave and both modes. The same problem arises, when we consider the transition layer in two-wave approximation. As an additional boundary condition, we can use in this case the continuity of tangential components of the spatial inhomogeneous constituent on the plane boundary. This requirement is equivalent to the introducing of the complex effective characteristic impedance Z_{01} for the spatial inhomogeneous constituent of the field, which spread outside the bound. Then, if the bound lies at $z = 0$, we can write the additional bound condition in the form of $E_1(0)/H_1(0) = Z_{01}$. The value of $Z_{01} = Z_0\sqrt{\mu_{11}/\varepsilon_{11}}$ can be obtained from the formulas in [1], where one must use values $\varepsilon = 1$, $\mu = 1$ for the free space. We have examined the two-wave approach in particular for the case of the finely stratified medium and get good results. For example, if such a structure contains both dielectric and metal layers, we have the correct result (a) for the limit of strong skin effect in metal layers.

4. Conclusion

The transition layer in inhomogeneous media can be described with help of series of the GWM. But we think that the simple two-wave approach for this problem is quite adequate in many cases. Such an approach is seemed more acceptable, especially in case, when we investigate both theoretically and experimentally the influence of the transition layer on electromagnetic properties of complex composite materials.

Acknowledgement

Author is grateful for A. N. Lagarkov, K. N. Rozanov, and A. P. Vinogradov for the useful discussions. This work was supported by Russian Foundation of Fundamental Investigations Grant no. 99-02-16564-a.

References

[1] N. A. Simonov, "One-dimensional nonlocal model of medium," in *Proc."Bianisotropics'98*, June 1998, Braunschweig, Germany, pp. 281-284.
[2] S. M. Rytov, "Electromagnetic properties of a finely stratified medium," *Sov. Phys. JETP*, vol. 2, pp. 466-475, 1956.
[3] O. Acher *et al.*, "Permeability measurement on ferromagnetic thin films from 50 MHz up to 18 GHz," *Journal of Magnetism and Magnetic Materials*, no. 136, pp. 269 – 278, 1994.
[4] V. M. Agranovich and V. L. Ginzburg, *Spatial Dispersion in Crystal Optics and the Theory of Excitation.* Wiley, New York, 1966.
[5] C. A. Korn and T. M. Korn, *Mathematical Handbook.* New York: McGraw-Hill Book Co., 1968.