UNCLASSIFIED

Defense Technical Information Center Compilation Part Notice

ADP011645

TITLE: A Uniaxial Chiral Slab Backed by Soft and Hard Surfaces Used as a Polarization Transformer

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: International Conference on Electromagnetics of Complex Media [8th], Held in Lisbon, Portugal on 27-29 September 2000. Bianisotropics 2000

To order the complete compilation report, use: ADA398724

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:
ADP011588 thru ADP011680

UNCLASSIFIED

A Uniaxial Chiral Slab Backed by Soft and Hard Surfaces Used as a Polarization Transformer

G. Öğücü and S. Uçkun

Electrical and Electronics Engineering Department, University of Gaziantep,
Gaziantep, 27310, Turkey. email: savas@gantep.edu.tr

Abstract

Plane wave reflection from soft and hard surfaces coated by chiral material has recently been analyzed for normal incidence. In this study, the plane wave reflection from a uniaxial chiral slab backed by soft and hard surfaces is formulated for normal incidence and the polarization properties of the reflected field are investigated.

1. Introduction

The chiral medium is a subclass of bianisotropic media. A special type of bianisotropic chiral media is the axially chiral uniaxial medium. A uniaxial bianisotropic chiral slab can be realized by doping miniature chiral objects (such as wire spirals) into an anisotropic host medium as explained in [1]-[2]. The orientation of the chiral objects must be parallel to a unique preferred direction.

Soft and hard surface (SHS) boundaries are well known from acoustics. They have been also defined for dually polarized electromagnetic waves. Kildal [3] explained the concept of SHS in detail by considering different geometries. A common characteristic of both soft and hard surfaces is that they do not create cross polarization by geometrical optics reflection. For example, a right circularly polarized wave for a hard conducting surface is still right cicularly polarized after reflection and for a smooth conducting surface it is left circularly polarized.

In a previous paper [4], an isotropic chiral slab backed by SHS and its application to polarization transformer have been analyzed. In this study, a plane wave reflected from an infinite uniaxial chiral slab of thickness d, sandwiched between air and SHS, with axis parallel to the interfaces is considered. This characteristic of SHS and easy construction of the uniaxial chiral slab might make the study of this problem worthwhile.

2. Fields at the Interface and Reflection Dyadic

It is known that in the uniaxial chiral medium, the electromagnetic fields satisfy the constitutive relations

$$\mathbf{D} = \overline{\overline{\boldsymbol{\varepsilon}}} \cdot \mathbf{E} - j\sqrt{\mu_o \varepsilon_o}\, \overline{\overline{\boldsymbol{\kappa}}} \cdot \mathbf{H} \tag{1.a}$$

$$\mathbf{B} = \overline{\overline{\boldsymbol{\mu}}} \cdot \mathbf{H} + j\sqrt{\mu_o \varepsilon_o}\, \overline{\overline{\boldsymbol{\kappa}}} \cdot \mathbf{E} \tag{1.b}$$

with the medium parameter dyadics

$$\overline{\overline{\boldsymbol{\varepsilon}}} = \varepsilon_u \hat{\mathbf{u}}\hat{\mathbf{u}} + \varepsilon_t (\hat{\mathbf{v}}\hat{\mathbf{v}} + \hat{\mathbf{z}}\hat{\mathbf{z}}) \tag{2.a}$$

$$\overline{\overline{\boldsymbol{\mu}}} = \mu_u \hat{\mathbf{u}}\hat{\mathbf{u}} + \mu_t (\hat{\mathbf{v}}\hat{\mathbf{v}} + \hat{\mathbf{z}}\hat{\mathbf{z}}) \tag{2.b}$$

$$\overline{\overline{\boldsymbol{\kappa}}} = \kappa \hat{\mathbf{u}}\hat{\mathbf{u}}\,. \tag{2.c}$$

where κ is the chirality parameter and the nonreciprocity parameter χ of [5] is assumed zero. In Eq. (2), $\boldsymbol{u}$ and $\boldsymbol{v}$ are the transverse axes to z-axis with $\hat{\mathbf{u}} = \hat{\mathbf{z}} \times \hat{\mathbf{v}}$. They are chosen such that they are the natural coordinates from the geometrical optics point of view. Moreover, the direction of corrugation on SHS is along v-axis.

To find the reflection coefficient of the plane wave for normal incidence, consider a uniaxial slab which is confined between two infinitely extended planes at $z=-d$ and $z=0$, as shown in Figure 1. The plane wave solutions of the Maxwell's equations for the chiral medium are [2]

$$\mathbf{E}_{\pm}(\mathbf{r}) = E_{\pm} e^{-jk_{\pm}z} \tag{3.a}$$

$$\mathbf{H}_{\pm}(\mathbf{r}) = \frac{k_{\pm}}{k_t \eta_t} \hat{\mathbf{z}} \times \mathbf{E}_{\pm}(\mathbf{r}) \tag{3.b}$$

where + and – indicate the right-hand and left-hand polarization, respectively. In Eq. (3), the corresponding propagation factors are given by

$$k_{\pm} = k_t \sqrt{A_{\pm}} \tag{4.a}$$

where

$$k_t = \omega\sqrt{\mu_t \varepsilon_t}\ ,\ \eta_t = \sqrt{\mu_t/\varepsilon_t}\ ,\ A_{\pm} = \frac{1}{2}\left(\frac{\mu_u}{\mu_t} + \frac{\varepsilon_u}{\varepsilon_t}\right) \pm \sqrt{\frac{1}{4}\left(\frac{\mu_u}{\mu_t} - \frac{\varepsilon_u}{\varepsilon_t}\right)^2 + \frac{\kappa^2 \mu_o \varepsilon_o}{\mu_t \varepsilon_t}} \tag{4.b}$$

After writing the fields inside the uniaxial chiral slab and using boundary conditions at z=-d and z = 0 the following equation can be obtained:

$$\mathbf{E}^r = \overline{\overline{\mathbf{R}}} \cdot \mathbf{E}^{\mathrm{i}} \tag{5}$$

where

$$\overline{\overline{\mathbf{R}}} = \begin{bmatrix} R_{uu} & R_{uv} \\ R_{vu} & R_{vv} \end{bmatrix} = e^{j2k_o d} \begin{bmatrix} \dfrac{U-X}{Y+X} & \dfrac{Z}{Y+X} \\ \dfrac{T}{Y+X} & \dfrac{S-X}{Y+X} \end{bmatrix}. \tag{6}$$

In Eq. (6)

$$S = -AD(k_+ + k_-)^2 + j\frac{k_t\eta_t}{\eta_o}(A+D)(k_+ + k_-)\sin(k_+ + k_-)d + \frac{k_t^2\eta_t^2}{\eta_o^2}\sin^2(k_+ + k_-)d \tag{7.a}$$

$$T = j2(k_- B - k_+ C)\frac{k_t\eta_t}{\eta_o}\sin(k_+ + k_-)d \tag{7.b}$$

$$U = -AD(k_+ + k_-)^2 - j\frac{k_t\eta_t}{\eta_o}(A+D)(k_+ + k_-)\sin(k_+ + k_-)d + \frac{k_t^2\eta_t^2}{\eta_o^2}\sin^2(k_+ + k_-)d \tag{7.c}$$

$$X = (B^2 + C^2)k_+ k_- - BC(k_+^2 + k_-^2) \tag{7.d}$$

$$Y = AD(k_+ + k_-)^2 + j\frac{k_t\eta_t}{\eta_o}(A-D)(k_+ + k_-)\sin(k_+ + k_-)d + \frac{k_t^2\eta_t^2}{\eta_o^2}\sin^2(k_+ + k_-)d \tag{7.e}$$

$$Z = j2(k_+ B - k_- C)\frac{k_t\eta_t}{\eta_o}\sin(k_+ + k_-)d \tag{7.f}$$

where

$$A = \sin k_+ d \sin k_- d\ ,\ B = \sin k_+ d \cos k_- d \tag{8a}$$

$$C = \cos k_+ d \sin k_- d\ ,\ D = \cos k_+ d \cos k_- d \tag{8b}$$

For the isotropic achiral slab, $k_+ = k_- = k_t = k$, and therefore $Z = T = X = 0$ which contributes to the vanishing of the crosspolarized reflection coefficients. The copolarized reflection coefficients are

$$R_{uu} = e^{j2k_o d}\frac{\left(\dfrac{\eta_t^2}{\eta_o^2} - 1\right)\sin 2kd - j2\dfrac{\eta_t}{\eta_o}}{\left(\dfrac{\eta_t^2}{\eta_o^2} + 1\right)\sin 2kd - j\dfrac{\eta_t}{\eta_o}2\cos 2kd} \text{ and } R_{vv} = e^{j2k_o d}\frac{\left(\dfrac{\eta_t^2}{\eta_o^2} - 1\right)\sin 2kd + j2\dfrac{\eta_t}{\eta_o}}{\left(\dfrac{\eta_t^2}{\eta_o^2} + 1\right)\sin 2kd - j\dfrac{\eta_t}{\eta_o}2\cos 2kd} \tag{9}$$

Thus, the incident field does not change polarization after it is reflected, but the phases of the field components are shifted as expected.

For the uniaxial chiral slab there is not always such simple expressions. If we consider a special case for the uniaxial chiral slab with $(k_+ + k_-)d = \pi$ and $(k_+ - k_-)d = \pi/2$, then $R_{uv} = R_{vu} = 0$ and

$$R_{uu} = R_{vv} = -e^{j2k_o d} \frac{2k_+k_- + (k_+ + k_-)^2}{2k_+k_- - (k_+ + k_-)^2} \tag{10}$$

Hence, in this case, as for the isotropic achiral slab, linearly polarized wave is reflected as linearly polarized wave.

3. Polarization Transformer

Let us study the possibility of defining the uniaxial medium backed by SHS as polarization transformer, by appropriate choice of the medium parameter values and thickness of the slab. Assume that the incident field linearly polarized along $\hat{\mathbf{u}}$, reflects right-hand circularly polarized, and that polarized along $\hat{\mathbf{v}}$, reflects left-hand circularly polarized. Then $\overline{\overline{\mathbf{R}}} \cdot \hat{\mathbf{u}}$ will be parallel to $-\hat{\mathbf{u}} + j\hat{\mathbf{v}}$ and $\overline{\overline{\mathbf{R}}} \cdot \hat{\mathbf{v}}$ will be parallel to $-\hat{\mathbf{u}} - j\hat{\mathbf{v}}$. These lead to

$$\mathrm{Re}\left\{-\frac{U-X}{Y+X}\right\} = \mathrm{Re}\left\{\frac{T}{j(Y+X)}\right\}, \text{ and } \mathrm{Re}\left\{\frac{Z}{Y+X}\right\} = \mathrm{Re}\left\{-\frac{S-X}{j(Y+X)}\right\} \tag{11.a}$$

$$\mathrm{Im}\left\{-\frac{U-X}{Y+X}\right\} = \mathrm{Im}\left\{\frac{T}{j(Y+X)}\right\} = 0, \text{ and } \mathrm{Im}\left\{\frac{Z}{Y+X}\right\} = \mathrm{Im}\left\{-\frac{S-X}{j(Y+X)}\right\} = 0 \tag{11.b}$$

to be more explicit these equations give

$$\cos(k_+ + k_-)d = 0 \quad \text{and} \quad \sin(k_+ - k_-)d = \pm\left(\frac{\dfrac{2k_+k_-}{k_t\eta_t/\eta_o} - 2\dfrac{k_t\eta_t}{\eta_o} + k_+ - k_-}{k_+ + k_-} \right) \tag{12}$$

With the conditions in Eq. (12), the reflection dyadic will be

$$\overline{\overline{\mathbf{R}}} = e^{j2k_o d}\left(\frac{1}{\sqrt{2}}(-\hat{\mathbf{u}} + j\hat{\mathbf{v}})\hat{\mathbf{u}} + \frac{1}{\sqrt{2}} j(-\hat{\mathbf{u}} - j\hat{\mathbf{v}})\hat{\mathbf{v}} \right) R_0 \tag{13}$$

where

$$R_0 = \frac{\sqrt{2}\left((k_+k_-)^2 - \left({k_t\eta_t}/{\eta_o} \right)^4 \right)}{(k_+k_-)^2 + 4(k_+k_-)\left({k_t\eta_t}/{\eta_o} \right)^2 + \left({k_t\eta_t}/{\eta_o} \right)^4} \tag{14}$$

As can be deduced from Eq. (13), the incident polarization $\hat{\mathbf{u}}$ gives rise to right-hand circularly polarized, and $\hat{\mathbf{v}}$ to left-hand circularly polarized reflected field.

The ratio $\dfrac{k_+k_-}{(k_t\eta_t/\eta_o)^2}$ in Eq.(12) is not arbitrary. We can impose some conditions on it. From the expression in Eq. (12)

$$-1 \le \frac{\dfrac{2k_+k_-}{k_t\eta_t/\eta_o} - 2\dfrac{k_t\eta_t}{\eta_o} + k_+ - k_-}{k_+ + k_-} \le 1 \tag{15}$$

and hence

$$\left(\frac{\sqrt{5}-1}{2}\right) \leq \frac{k_+ k_-}{\left(k_t \eta_t \big/ \eta_o\right)^2} \leq \left(\frac{\sqrt{5}+5}{2}\right) \tag{16}$$

The graph of the reflection coefficient, $|R_0|$, as a function of Ratio = $\dfrac{k_+ k_-}{(k_t \eta_t / \eta_o)^2}$ is shown in Figure 2. When the Ratio is one, $|R_0|$ vanishes and hence there is no reflection at this point. With the condition in Eq. (16), a highly efficient polarization transformer cannot be achieved by using a uniaxial chiral slab.

4. Conclusion

In this study, the normal incidence of the electromagnetic waves to lossless uniaxial chiral slab backed by SHS is analyzed. The reflection dyadic is derived. It is shown that the slab can be used as polarization transformer if the suitable medium parameters and slab thickness are chosen.

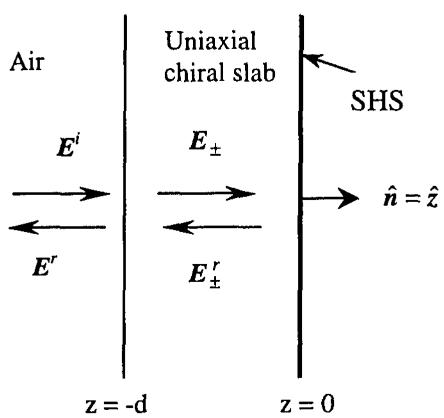

Fig. 1 Geometry of the problem.

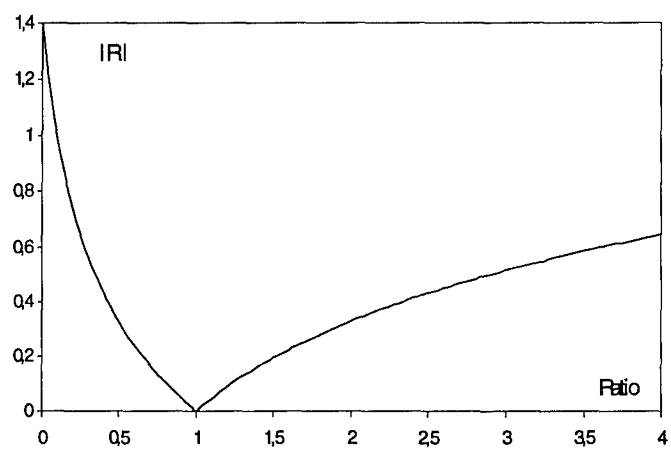

Fig. 2 The reflection coefficient R as a function of the Ratio = $\dfrac{k_+ k_-}{(k_t \eta_t / \eta_o)^2}$.

References

[1] A. J. Viitanen and I. V. Lindell, "Uniaxial chiral quarter-wave polarisation transformer," *Electron. Lett.*, Vol. 29, pp. 1074-1075, June 1993.

[2] I. V. Lindell and A. H. Sihvola, "Plane-wave reflection from uniaxial chiral interface and its application to polarization transformation," *IEEE Trans. Antennas Propagat.*, Vol. 43, pp. 1397-1404,.Dec. 1995.

[3] P.-S. Kildal, "Artificially soft and hard surfaces in electromagnetics," *IEEE Antennas Propagat.*, vol. 38, no. 10, pp. 1537-1544, Oct. 1990.

[4] A. J. Viitanen and P. P. Puska, "Plane wave reflection from a chiral slab backed by soft and hard surfaces with application to polarisation transformers," *IEE Proc.-Microw. Antennas Propagat.*, Vol. 145, no. 4, pp. 299-302, Aug. 1998.

[5] A. J. Viitanen, "Reflected and transmitted fields of uniaxial bianisotropic slab," *Proc. URSI 1995,* Int. Symp. EM Theory, pp. 649-651, 1995.