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NON LINEAR EFFECTS OF APPLIED LOADS AND LARGE DEFORMATIONS ON AIRCRAFT NORMAL MODES

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Summary

Ground Vibration Test (GVT) is the typical way to verify structural dynamic models. The conditions in which the GVT is performed –the aircraft subjected and deformed under gravity loads– are different from the conditions in which the Finite Element Method (FEM) model is usually elaborated (jig shape without loads). They are also different from the in-flight conditions (the aircraft subjected and deformed under inertia and aerodynamic forces). Although in most cases those differences can be negligible, it is not the case of a very large airplane in which the size and flexibility effects are of such nature that updating a FEM model to match GVT results could go in the opposite direction to the actual airplane in-flight. This paper analyses the influence of aircraft deformation (down bending for GVT, jig shape for FEM model, up bending for flight), shape (control surfaces deflections...), and loads (gravity on ground, inertial and aerodynamic forces in flight) on normal modes to have a better insight in GVT and flight test measurements interpretation of a very large airplane. Those effects are significant especially where large concentrated masses (engine-pylon) are present.

Nomenclature

E	Young´s modulus
f	frequency
FEM	Finite Element Method
g	gravity acceleration
GVT	Ground Vibration Test
HTP	Horizontal Tail Plane
I	polar inertia moment of cross section
L	length
m	distributed mass
MTOW	Maximum Take-Off Weight
N	axial force
N_{cr}	Euler buckling load
Ω	natural frequency
p	distributed axial force
q	distributed transversal force
t	time
v	transversal displacement
x	axial co-ordenate

Nomenclature for modal identification

nWB	nth order wing bending mode
nWT	nth order wing torsion mode
nWTX	nth order wing chordwise mode
WTT	wing tip torsion mode
IPP	inboard pylon pitch mode
IPY	inboard pylon yaw
IEY	inboard engine yaw
OPP	outboard pylon pitch
OPY	outboard pylon yaw
OEY	outboard engine yaw
ETL	engine truss lateral
ETZ	engine truss vertical
EYaw	engine yaw
ERoll	engine roll
HB	horizontal tail plane bending
HT	HTP torsion
HTX	HTP chordwise
ERP	elevator rotation in-phase
ERO	elevator rotation out of phase
IEB	inboard elevator bending
OEB	outboard elevator bending
IET	inboard elevator torsion
OET	outboard elevator torsion
EL	elevator lateral

1 Introduction

From the structural dynamics standpoint, a slender arrestor hook during the engagement phase, a rotating helicopter blade or a large flexible solar array of a spacecraft have something peculiar in common. Their dynamic behaviour can only be adequately known if the loads and/or large displacements that are acting on them are also considered.

In a simple structural element like a beam, the presence of large axial loads introduces a new term in the transversal equilibrium equation, thus modifying its solution. One of the most typical example happens when this axial load is constant: under tensile loads the natural frequency of the fundamental transverse bending modes increases while for compression loads diminishes, being zero when the compression load reaches the Euler buckling load.

New terms also appear in the beam equations when very large displacements are involved. Something similar

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happens for other structural elements (shells, plates...) and in turn to the complex structures obtained by assembling these simple elements.

The dynamic solutions of the linear equations are no longer valid and some degree of non-linearity is introduced due to the new terms in the equations. Even in the simplest cases –beams, plates– a general solution of the equation can not be found. Approximate methods or an iterative process using the finite element technique should be used to obtain the correct solution.

This paper is devoted to analyse the effect of the applied loads and large deformations on the modes of a very large –and flexible– airliner. This topic is considered of relevance because the modes of an aircraft constitute the base for the solution of many structural dynamics and aeroelastic problems (flutter, response to gust and continuous turbulence, dynamic landing, etc.) that are necessary to consider in the certification process of an aircraft.

In the aeronautical industry, aircraft modes are computed using the finite element method (FEM). The modal base is typically defined in the jig shape in the assumption that it will span to the in-service modes. The dynamic model is validated through the modes obtained in a full-scale ground vibration test (GVT). If discrepancies arise between model and test, the model is updated to match GVT results.

This paper will show that due to the effect of large loads and displacements, some modes obtained simulating the GVT conditions (1g loads) can exhibit differences with respect to jig shape modes and also with respect to in-service modes. This effect should be taken into account during the model updating process.

Next section will be devoted to a literature survey and a brief description of theoretical background. In subsequent sections, the effect of large applied loads and deformations in the modes will be considered in a set of increasingly complex tasks (clamped wing without pylons, clamped isolated engine-pylon, clamped wing with two engine pylons fully representative of a megaliner wing). Gravity forces will be varied from –1g to 2.5g (the regular load factor envelope in the certification of an aircraft). Aerodynamic loads that equilibrate the aircraft are added to the wing in a second step to have a better insight in these two effects separately.

Large displacements can also be due to regular control surface movements or rotations. This effect is already routinely covered in the case of wing flaps –and will not be repeated herein-. Within the Airbus consortium, CASA has been responsible of the design, analysis, manufacturing and certification of the Horizontal Tailplanes (HTP). In the case of a megaliner, the horizontal tailplane area is well above 200 square meters. The effect of the deflections of the large split elevators on the HTP modes is a new problem that will be briefly described in the last part of the paper.

The paper ends with the conclusions and guidelines learned during this work.

2 Literature survey and theoretical background.

2.1 Introduction

This section is devoted to show a literature survey and a brief theoretical background of the effect of loads and large displacements on normal modes. The section is structured as follows:

- Beams under axial loads.
- Beams subjected to large applied loads and large static deformations.
- Plates under in-plane loading.
- Complex structures.

Although some of these cases are very academic, they will give a good insight into the basic principles underlying the most realistic cases that will be shown in subsequent sections.

2.2 Effect of axial loads on transverse vibration of beams

If the beam is subjected to a time invariant axial loading in the horizontal direction as shown in figure 1, in addition to the lateral loading, the local equilibrium of forces is altered because the internal axial force, N(x) interacts with the lateral displacements to produce an additional term in the moment equilibrium equation.

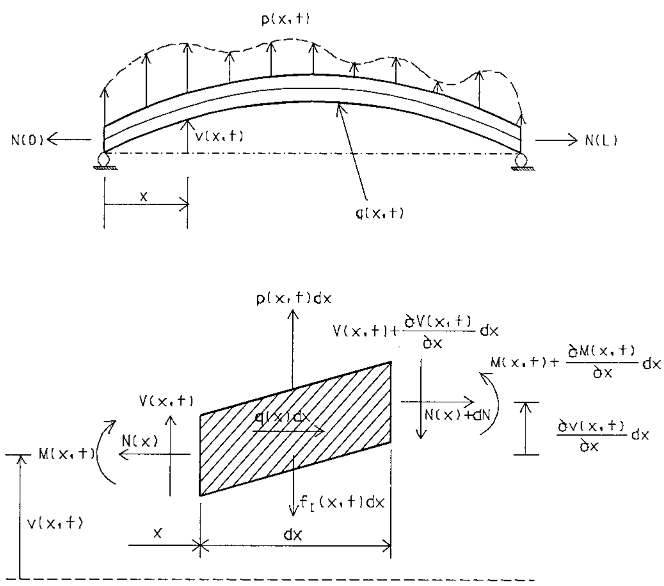

Figure 1. Beam with static axial load and dynamic transverse vibration. Upper: beam deflected. Lower: forces on a differential element.

After some manipulation, the transverse displacement of the beam is stated as:

$$\frac{\partial^2}{\partial x^2}(EI(x)\frac{\partial^2 v(x,t)}{\partial x^2})-\frac{\partial}{\partial x}(N(x)\frac{\partial v(x,t)}{\partial x})+\frac{\partial}{\partial t}(m(x)\frac{\partial v(x,t)}{\partial t})=p(x,t)$$

This equation differs from the traditional one in the second term of the left hand side. As Nihous stated [1], the two last terms of the left-hand side correspond to the vibrating string, i.e. EI=0. Hence, limits of this equation for N>>EI and N=0 must reproduce the results of the vibrating string and a linear Euler beam respectively.

In most of the cases, an exact solution for the modes and frequencies of the system is difficult to obtain. The recourse would be made to one of the approximate solutions such as the Rayleigh-Ritz method, Galerkin's technique or the finite element method.

For uniform beams, the problem becomes simpler when constant axial loads are applied with various types of simple end conditions.

Galef [2] is likely the first to publish a practical formula for the natural frequencies of uniform single-span beams under a constant compressive axial force:

$$\frac{\Omega^2_{compressed}}{\Omega^2_{uncompressed}}=1-\frac{N}{N_{cr}}$$

where Ω stands for the natural frequencies, N is the compressive load, and N_{cr} is the Euler buckling load. The work of Shaker [3] is the first systematic approach to investigate three common problems in aerospace structures: a vibrating beam with arbitrary boundary conditions, a cantilever beam with tip mass under constant axial loads and a cantilever beam with tip mass under axial loads applied on the tip directed to the root. Another value added of the Shaker work is that it extends the analysis also to tension loads.

Continuing the Galef's work, Bokaian [4] establishes the set of boundary condition for which his approximation is correct, and studies the influence of a compressive load on natural frequencies and mode shapes in ten different combinations of boundary conditions. It is interesting to quote the final conclusions of Bokaian: ¨ it is seen that Galef's approximate relationship is valid not only for clamped-clamped, clamped-pinned, pinned-pinned and clamped-free beams upon which this equation is based, but also for sliding-free, clamped-sliding, sliding-pinned and sliding-sliding beams. This is probably because, for this beams, the vibration mode with no axial force and the buckling mode are similar. Galef's expression is not, however valid for pinned-free and free-free beams. ¨ [4].

In a subsequent work, Bokaian [5] extends his study to tension loads.

The arrestor hook behaviour during the engagement phase is one aeronautical problem in which this mentioned effect is present.

CASA is the company responsible of the arrestor hook system within the Eurofighter consortium. In the EF-2000 Typhoon, the arrestor hook is an emergency device which means that it is not regularly used in a normal landing. Therefore, the arrestor hook arm is slender and optimised for a reduced number of engagements. (This is completely different from a naval aircraft in which the arrestor hook is regularly used).

During engagement, the arrestor hook head captures the runway cable. The sudden application of force produces large displacements and deformation on the slender arrestor hook arm. There is an increase in the frequency in the transverse bending mode that is evident in both, the numerical non-linear simulation [6,7] and also in the test-measured results. Figure 2 shows the EF-2000 arrestor hook system and the responses in the engagement phase. A 30 Hz response in the bending moment is evident and it corresponds roughly with the frequency of the arrestor hook under the applied tensile load. This frequency without applied load is in the neighbourhood of 20 Hz.

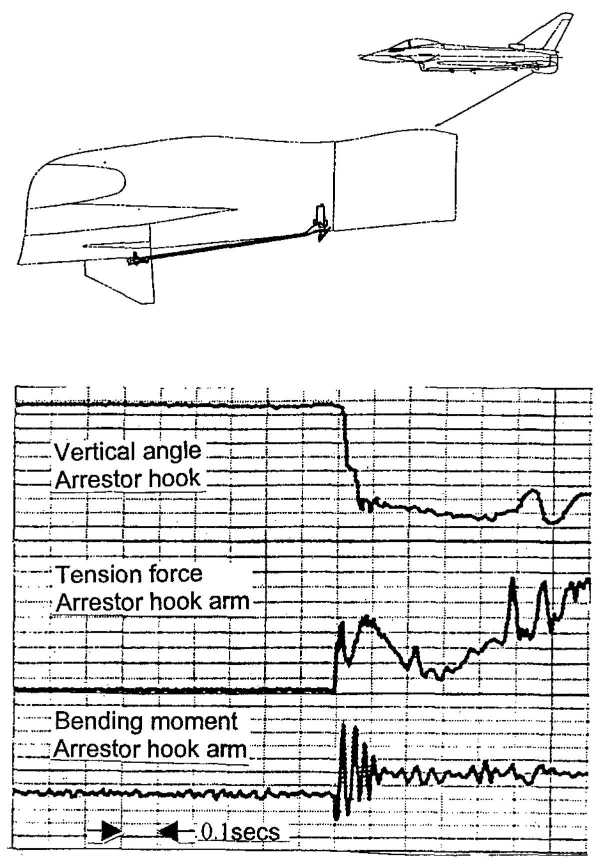

Figure 2. EF-2000 Arrestor hook system. Flight test measured response during engagement showing the increase in bending frequency due to the applied tensile load

2.3 Beams subjected to large applied loads and large static deformations.

Most of the studies in this subject have been promoted by the aeroelastic stability of helicopter rotor blades. Therefore, in many cases, external forces are due to centrifugal effects. The studies consider a beam rotating at constant angular speed about a fixed axis in space by determining first the steady equilibrium position due to centrifugal and gravitational or any other concentrated external load. Afterwards, using a small perturbation technique, the movement around the equilibrium position is studied. The small strain assumption is common in helicopter rotor blades analysis. The flap and lead-lag hinges allow large displacements without a large strain. This fact is used to simplify the stress tensor.

Papers devoted to this subject have been being published since early 40's (Southwell, 1941 [8], Love 1944 [9]) including important contributions like Rose and Friedman (1979, [10]) that analyse the non-linear behaviour of beams with bending-torsion coupling undergoing small strains with rotations.

The experimental studies include at least four references. Dowell, Traybar and Hodges (1977,[11]), Rosen (1983, [12]) Minguet and Dugundji (1990, [13],[14]) and Laulusa (1991, [15]). Natural frequencies of a cantilever beam are reported as function of tip deflection in Dowell & Minguet and the other works deal with static large deflected beams. Their results have been widely used in literature to determine the accuracy of theoretical methods.

The most recent papers that analyse the large loads and deflections on natural frequencies and mode shapes are the five shown in table 1 that shows a relative comparison of what are the contents of each paper.

Figure 3 shows one of the obtained results, quoted from Minguet [13]. This plot presents the changes in natural frequencies for beam with increasing tip deflections. These figures are relevant because they reflect some of the behaviours that have been found in actual airplane modes and will be presented in subsequent sections. Left figure shows the torsion and chordwise modes close coupled in the initial conditions. With increasing tip deflection, torsion mode (1T) changes its mode shape to become a chordwise mode (1F) and decreases significantly its frequency. On the other hand, the torsion mode changes its mode shape to become torsion and its frequency first increases and then decreases with tip deflection.

2.4 Plates under in-plane loading

ESDU 90016 [18] provides a mean of estimating the lower natural frequencies of isotropic or orthotropic, flat rectangular plates under static in-plane loading.

Natural frequencies are obtained by using beam characteristics orthogonal polynomials in the Rayleigh-Ritz method. The effect of in-plane loading is considered in two parts. First, effects of direct in-plane loading only and then the effects of shear loading only.

As in beams, tension in-plane loads will cause an increase of frequency. Compression will produce a decrease of frequency. But one of the differences of the plates subjected to direct in-plane compression loading with respect to beams is that the minimum frequency (reached at the buckling load) is not zero and that in the post-buckling region, the frequency increases with in-plane compression load.

This apparently anti-natural effect has been attributed to an increase of the stiffness of the plate due to curvature and effects associated with initial geometrical imperfections in the plate. The theoretical natural frequencies are evaluated assuming uniform in-plane stresses but in actual plates there is a redistribution of in-plane stress due to the growth of initial geometric imperfection with increasing compressive load.

Plates under shear loading show that natural frequencies are identical for equal positive or negative shear loads although nodal patterns for positive and negative shear loads are mirror images. Some modes can increase frequency with moderate shear loading. As shear loading is increased, the natural frequency of some modes decreases until shear buckling will occur at sufficiently high shear loading.

2.5 Complex structures

Complex structures can only be analysed by the Finite Element Method (FEM) technique.

Some commercial codes are essentially non-linear like the explicit codes used in the simulation of impacts and crashworthiness. Nevertheless, to assess the effect of loads and large displacements on normal modes, an implicit FEM code should be used. The non-linear solution 106 of MSC/NASTRAN in combination with several DMAP alters and in-house CASA software is the procedure adopted to compute the results that will be shown in next sections.

- Stiffness matrix updating.

It is performed using an iterative process. Starting from a converged solution, gradual incremental loads are applied. According to [19] the equilibrium equation in the g-set may be written as:

$$\{P_g\} + \{Q_g\} - \{F_g\} = \{0\}$$

where $\{P_g\}$, $\{Q_g\}$ and $\{F_g\}$ represent vectors of applied loads, constraint forces and element nodal forces, respectively. Since the equilibrium condition is not immediately attained, an iterative scheme such as the Newton-Raphson method is required.

Reference	Minguet,1990 [13][14]	Laulusa, 1991 [15]	Cveticanin 1994 [16]	Sälstrom 1996 [17]
Effects considered	Rotation, Ω. Torsion-bending-extension No shear deformation Translation inertia Small strain Large displacements Moderate rotations Linear material	Rotation, Ω. Torsion-bending (two planes) Warping Translation inertia General formulation Large displacements Large rotations Linear material	Axial-bending coupling No shear deformation Trans & rot. Inertia Small strains Small displacements Linear material	Torsion-bending coupling No shear deformation Translation inertia Small strains Large displacements Large rotations Linear material
Model	Equilibrium in section One dimensional Euler angle	Energetic approach Princ. Virtual Work Bidimensional	Equilibrium in section One dimensional	Equilibrium in section One dimensional Beam kinematics for large rotations
Solution scheme	Small perturbation Finite-difference	Small perturbation Finite elements	Small perturbation Modal expansion	Small perturbation Runge-Kutta integration scheme.
Application	Cantilever beam Composite beams with different lay-ups	Rotating simple-supported-free beam	Simple supported beam	Cantilevered beam
Results	Static deflection Mode shapes and frequencies Influence of tip deflection on frequencies and modes Effects of torsion coupling	Static deflection Influence of displacement and angular speed on natural frequencies.	Influence of load magnitude, rotary inertia and slenderness on the fundamental frequency.	Static deflection Poor dynamic result discussion

Table 1. Summary of recent works about non-linear effects on beam normal modes due to static initial effects

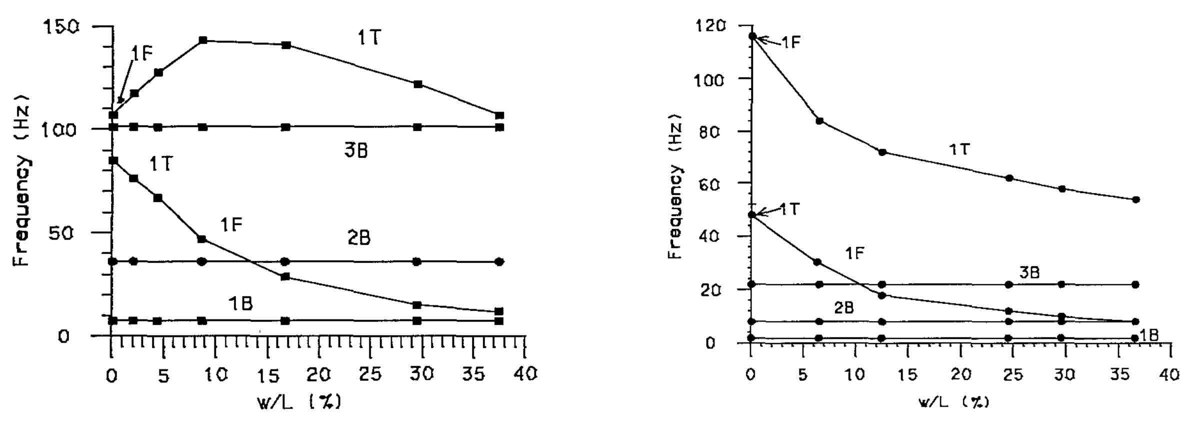

Figure 3 Changes in natural frequencies for beam with increasing tip deflections; a) composite beam, 560 mm in length, [0/90]3s lay-up; b) composite beam, 560 mm in length, [45/0]s lay-up. From Minguet [13], F stands for bending in the stiffest plane (chordwise), B represents modes oscillating in a vertical plane; T is a torsion mode.

Since the error vanishes at constraints points and the constraint forces vanishes at free points, the unbalanced forces acting at nodal points at any iteration step are conveniently defined as an error vector by (dropping the subscript):

$$\{R\}=\{P\}-\{F\}$$

Based on Newton's method, a linearised system of equations is solved for incremental displacements by Gaussian elimination in succession. The Jacobian of the error vector emerges as the tangential stiffness matrix.

New deformations are obtained, continuing the iteration until the residual error –unbalanced load– and the incremental displacements are negligible.

The tangential stiffness consists of the geometric stiffness in addition to the material stiffness.

$$[K_T]=\lfloor K^m \rfloor+\lfloor K^d \rfloor$$

$\lfloor K^m \rfloor$, the material stiffness represents the assembly of elements stiffness without geometric nonlinear effects

$\lfloor K^d \rfloor$ is the additional stiffness due to initial stresses that are included in the incremental process because the initial stresses exist from the second increment.

- Mass matrix updating.

The centres of gravity of the lumped masses have been moved to their corresponding deformed position.

The normal modes of the deformed/loaded structure are computed with the updated stiffness and mass matrices at the final load step.

3 Influence of loads and displacements in normal modes of an actual aircraft

3.1 Introduction

CASA is participating in the Airbus 3E flexible aircraft programme. This programme is dedicated to harmonise and coordinate the work of Airbus Industrie partners in technology acquisition of aeroelasticity with the aim to improve tools and methods, optimise resources and avoid duplication of work to face the development of a megaliner (A3XX) minimising the risk in this technical area. CASA has contributed in the Ground Vibration Test methods package by studying the influence of aircraft shape and loads in the measurements, which in turn can be applied to FEM updating procedures.

Aircraft normal modes are currently performed using a FEM model of the structure. In most of the cases, the FEM model geometry corresponds to the jig shape. Due to the wing size of the A3XX, a significant departure from jig shape can be anticipated for both ground shape and flight shape. This effect in combination with loads (gravity on ground and gravity + aerodynamics in flight) can modify the aircraft normal modes.

Therefore these effects should be analysed in order to have a better insight in GVT measurements and in the way to update FEM models to match test results. Figure 4 shows –at scale– the relative deformations that can be expected in the wing of a megaliner as function of the load factor. At in-flight 2.5g the tip deflection is 15% of the wing span. At in-flight 1g, it is 6%. In GVT conditions -3%.

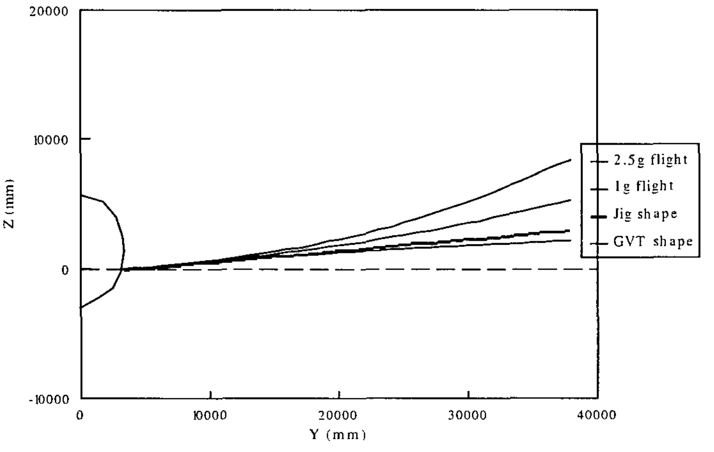

Figure 4.- Megaliner wing deformation in different conditions

In this paper the effect of large loads and displacements on normal modes have been considered in a set of increasingly complex tasks of an actual model of a megaliner structure:

- Wing without engine-pylons.
- Isolated Engine pylons.
- Wing with inboard and outboard engine pylons.

3.2 Analysis conditions

A structural model of the half-complete A3XX status 10c has been used (see figure 5). From this model, the wing and pylons have been extracted for normal modes calculations. Geometry, connectivity, properties, lumped masses, etc. are fully representative of the actual aircraft.

Inertia loads have been considered in the range –1g to 2.5 g (regular load factor envelope for certification). The case of 1g corresponds to GVT conditions. These loads have been applied to the structure in the centre of gravity of each lumped mass.

Aerodynamic loads have been obtained for trimmed flight using a linear method. For this task it has not been necessary to have an accurate information about aerodynamic loads, just a representative set of these loads.

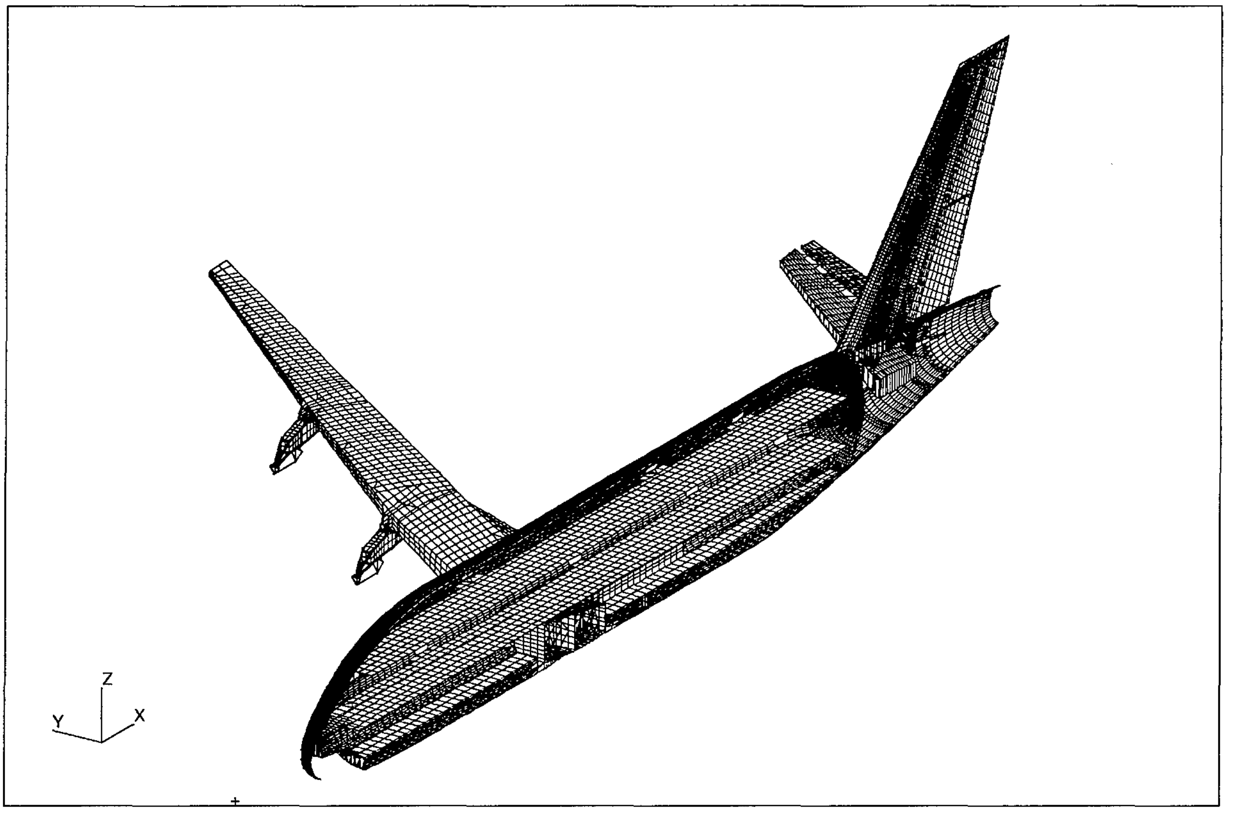

Figure 5.- Megaliner FEM model used to assess the effect of large loads and displacements.

3.3 Clamped wing normal modes results

Figure 6 shows the effect of the inertia loading in the range [-1g, 2.5g]. The case of 0g corresponds with the jig shape modes and has been obtained with the linear solution. The case of 1g corresponds with the GVT simulation.

The first non-linear analyses have been performed with a reduce level of loads (±0.1g) and therefore, they correspond to a reduced level of deformation. These first non-linear analyses have produced very similar results to the linear solution thus giving confidence on the results.

The evolution of modes is quite smooth although some crossings can already be detected, especially with the chordwise modes (WTX) which frequency decreases with increasing load factor and the corresponding increase in deformation. This effect is magnified if two modes of torsion and chordwise are close-coupled at 0g –like 3WT and 3WTX-. There is also a change in mode shape for these two modes: at large load factors the 3WT becomes 3WTX and vice-versa. This effect is very similar to the one reported by Minguet in his paper (see figure 3).

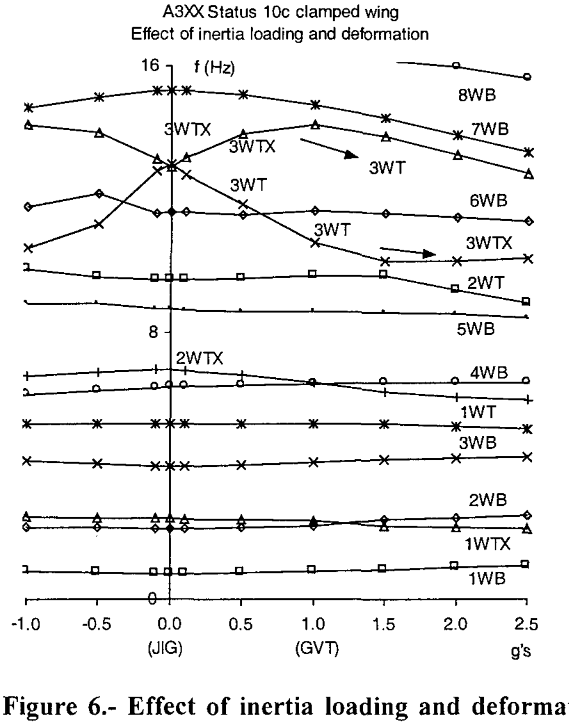

Figure 6.- Effect of inertia loading and deformation on clamped wing normal modes

Note that in the GVT simulation (1g) the 3WT mode is below the 6WB while in the jig-shape simulation (0g) this mode is above. In addition, the 3WT and 3WTX are well separated in GVT simulation while very close in jig-shape modal simulation.

Figure 7 shows the effect of inertia + aerodynamic loading. The modes show a similar pattern that in figure 6 although more pronounced.

Bending modes are not affected. The first torsion and chordwise modes decrease their frequency slightly with load factor. Large order torsion and chordwise modes change more its frequency with load factor.

Comparing figure 6 and 7 can conduct to an interesting remark. According with figure 6, GVT will require modifying the model to increase the frequency of the 3WTX mode with respect to the jig-shape solution. On the other hand, in the in flight condition at 1g, the frequency of this mode is **below** the jig-shape. The same happens with the torsion mode 3WT: the GVT will require modifying the model to decrease this mode from the jig shape solution although in flight at 1g the frequency of this mode is similar to jig-shape. In these two cases, correction of the model to match GVT will produce worse models for comparison with in-flight modes. It is believed that this behaviour happens particularly in these two modes because they are close coupled at 0g (linear solution).

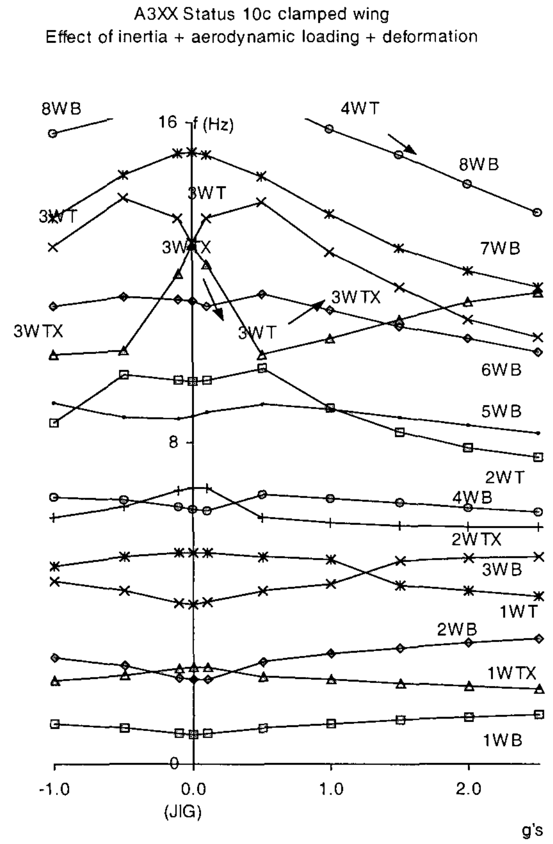

Figure 7.- Effect of inertia + aerodynamic loading + deformation on clamped wing normal modes

3.4 Inboard and outboard engine-pylon normal modes results

The isolated pylons have been also subjected to the inertia loading in the range [-1g, 2.5g].

Low frequency pylon modes found have been: lateral bending, pitch and yaw.

The evolution of frequency of these modes with the load factor has been completely flat in the studied range.

3.5 Clamped wing plus inboard and outboard engine pylons normal modes results.

Figure 8 shows the effect of the inertia loading in the range [-1g, 2.5g]. The case of 0g corresponds with the jig shape modes and has been obtained with the linear solution. The case of 1g corresponds with the GVT simulation.

The evolution of most of the modes is quite smooth. Due to the presence of the pylons, the crossings of modes are more pronounced that in the wing without pylons case. There is one eigenvalue that becomes negative at large load factors. It corresponds to the outboard pylon yaw mode.

Wing bending modes are not affected by load factor. Pylon yaws and wing chordwise modes (especially 3WTX) are the most affected modes. Wing torsion modes show slight variations.

Figure 9 shows the effect of inertia + aerodynamic loading on the clamped with pylons. The modes show a similar pattern that in figure 8 although a lot more pronounced. Even the bending modes change (very mild).

There are two eigenvalues that become negative at certain load factor. Both have in the end significant contribution of engine pylon yaw mode.

Again the modes more affected are the pylon yaw and the wing chordwise. Larger order wing torsion modes like (4WT) are also significantly affected

Due to the presence of pylons, the 3WTX and 3WT are not so coupled at 0g as in the wing without pylons case and their evolution with load factor is more similar between figure 8 and figure 9.

Comparing modes at 1g of figure 8 and figure 9 it can be seen that in two cases the tendency of a mode in GVT is different from in-flight: the O/B pylon yaw and the 2WT. Corrections of these two modes will produce a worse model for in-flight modal comparison.

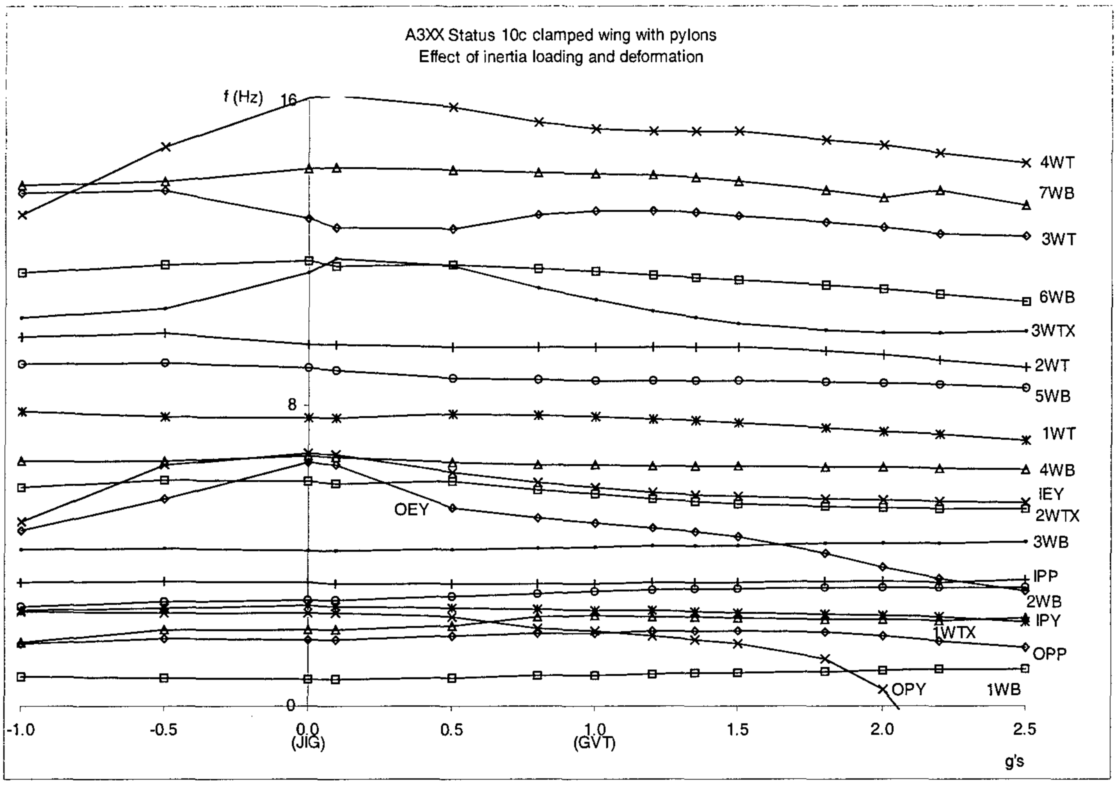

Figure 8.- Effect of inertia + deformation on clamped wing with engine pylons normal modes

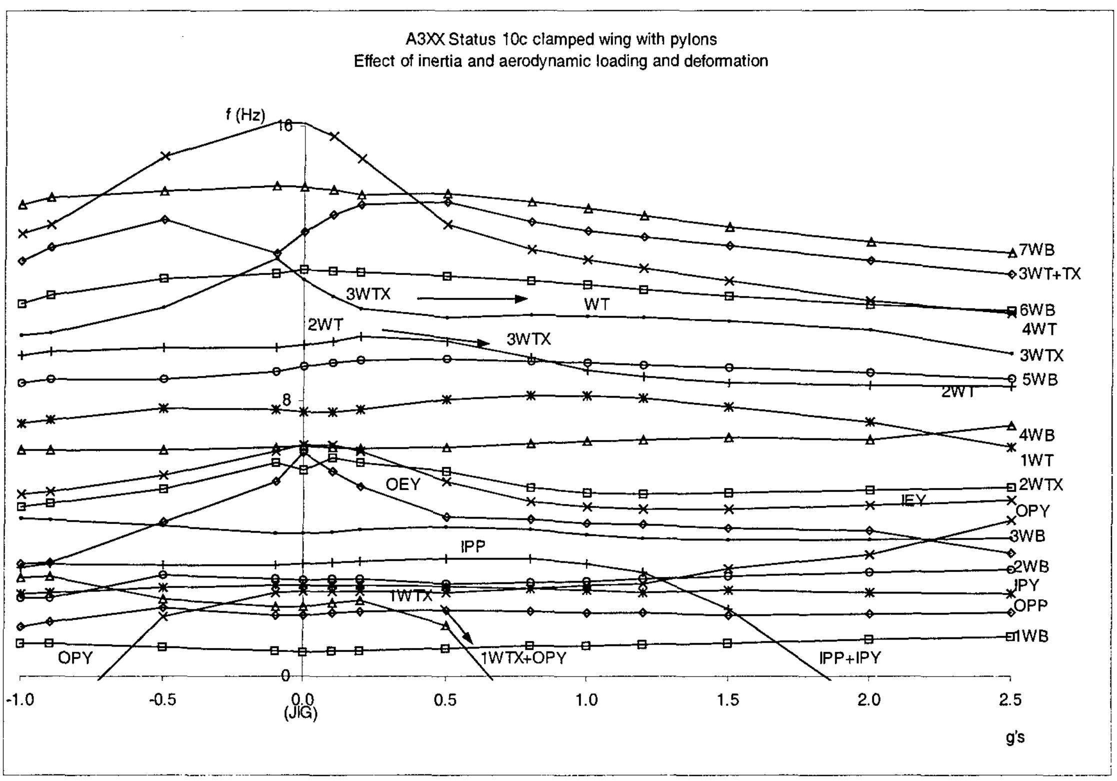

Figure 9.- Effect of inertia + aerodynamic loading + deformation on clamped wing with engine pylons normal modes

3.6 Effect of flexibility

To verify that the results found for a very flexible wing of a megaliner like the A3XX are indeed due to its large flexibility, a verification case has been run using a model of an aircraft significantly more rigid.

The CASA C-295 is a medium size military transport with 21000 kg of MTOW in civil operations and 23200 kg. of MTOW in military operations.

The frequency of the first symmetric wing bending of the C-295 is –roughly– between 4 and 5 times larger than that of the A3XX.

Figure 10 shows the effect of inertia, aerodynamic and displacement on the modes of the CASA C-295 wing at different load factors.

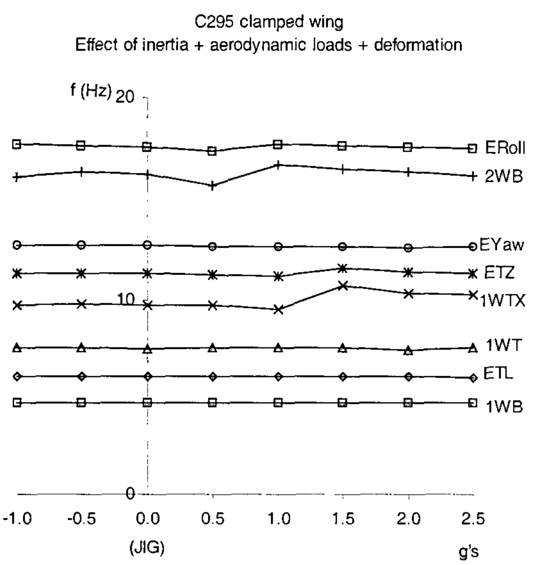

Figure 10.- Effect of large loads and deformations on a relatively rigid aircraft (CASA C-295)

Bending modes are un-affected as in the case of a more flexible wing. But, in addition, the torsion, chordwise modes, engine-truss etc are also basically un-affected showing that the non-linear effect in modes presented in previous sections is due to the large flexibility of the megaliner wing.

4 Effect on HTP modes of elevators deflection angle.

Within the Airbus consortium, CASA has been responsible of the design, analysis, manufacturing and certification of the Horizontal Tailplanes (HTP). In the case of a megaliner, the horizontal tailplane area is well above 200 square meters. The effect of the deflections of the large split elevators on the HTP modes is a new problem briefly described in this section.

The A3XX Horizontal Tailplane FEM model (status 10c) has been used to perform the analysis of the linear normal modes while changing the elevator angle.

Due to its size, the elevator of the A3XX will be split in two spanwise parts. In the present analysis both elevators have been deflected simultaneously from –25° to + 25°.

Figure 11 shows the effect of the elevator deflection in all clamped HTP modes up to 40. Hz

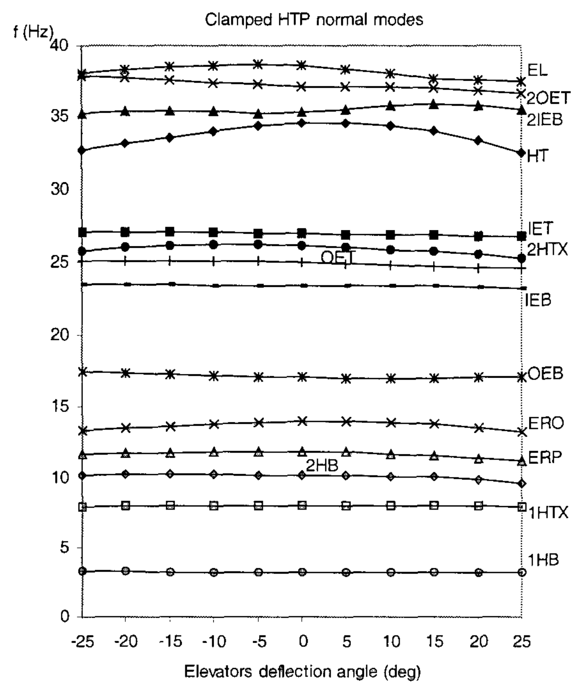

Figure 11.- Effect of elevator deflection angle on a megaliner HTP clamped modes.

Bending modes show no influence of the deflection angle. It means that the elevator chordwise bending contribution even at +25° or –25° is negligible. The most affected modes are:

- 2nd HTP chordwise (2HTX)
- Elevators rotation in-phase (ERP)
- Elevators rotation out of phase (ERO)
- HTP torsion (HT)
- 2nd Inboard elevator bending (2IEB)

The effect can be quantified in roughly 6 % in frequency at maximum deflection angle.

5 Conclusions

The effect of large applied loads and deformation in the normal modes of a very flexible megaliner wing has been presented.

These non-linear effects have been analysed using an iterative process with the finite element technique.

Confidence on the results has been achieved through several means:

- Non-linear solution with low level of loading (0.1g) has been very similar to the linear solution.
- Some modal tendencies found have been similar to the results published in academic papers.
- The larger the flexibility the larger the effect.

Nevertheless, a criticism can be done to the results presented herein. Is the FEM model suitable for the purposes shown? The most likely answer is that the model is not completely suitable especially in the neighbourhood of the large applied loads (engine-pylon fittings). It probably produces more pronounced tendencies that in the reality. For instance, the negative eigenvalues found are numerical results due to bad conditioning of the matrices that will not be present in reality and they will likely be solved using a finer mesh in those areas [19].

But basically the trends shown herein should be completely correct –although likely less pronounced–. Therefore the final message is that the effect of large applied loads and deformations in the normal modes of a very flexible structure –like the wing of a megaliner– must be taken into account.

Failing to do so could produce a misinterpretation of the GVT results and corrections in the FEM model that can go –in some cases– in the opposite sense of the desired one.

This effect should be taken into account. Dedicated FEM model for these purposes should be elaborated and validated. It is believed that the model will only require a finer mesh in certain areas –like the ones in the neighbourhood of large applied masses-.

Further work in this area will include the determination of a flexibility threshold beyond which it is mandatory to include the effect of large loads and displacements. Between the very flexible Airbus A3XX and the relatively rigid CASA C-295, other intermediate aircraft will be considered. Also the effect of the modelling will be studied with mesh size sensitivity analyses.

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7 References

[1] G.C.Nihous. On the Continuity of the Boundary Value Problem for Vibrating Free-Free Straight Beams under Axial Loads. *Journal of Sound and Vibration* Vol. 200 No. 1 (1997) p. 110-119

[2] A. E. Galef. Bending frequencies of compressed beams. *Journal of the Acoustical Society of America* Vol. 44 No. 8 (1968) p. 643

[3] F.J. Shaker. Effect of Axial Load on Mode Shapes and Frequencies of Beams. NASA TN D-8109 1975

[4] A. Bokaian. Natural Frequencies of Beams under Compressive Axial Loads. *Journal of Sound and Vibration* Vol. 126 No. 1 (1988) p. 49-65

[5] A. Bokaian. Natural Frequencies of Beams under Tensile Axial Loads. *Journal of Sound and Vibration* Vol. 142 No. 3 (1990) p. 481-498

[6] H. Climent Análisis Dinámico Gancho de Frenado del EFA 1. CASA C/PD/88-083 (Jul 1988)

[7] J. Burgaz Análisis Dinámico Gancho de Frenado del EFA 2. CASA C/PD/88-113 (Dic 1988)

[8] R.V. Southwell. *An Introduction to the Theory of Elasticity*. Oxford University Press, Second Edition, London, 1941

[9] A.E.H. Love. *A Treatise on the Mathematical Theory of Elasticity*. Dover, Fourth Edition, New York, 1944

[10] A. Rosen, P. Friedmann. The Nonlinear Behavior of Elastic Slender Straight Beams Undergoing Small Strains and Moderate Rotations. *Journal of Applied Mechanics*, Vol. 46 (March 1979), p. 161-168

[11] E.H. Dowell, J. Traybar, D.H. Hodges. An Experimental-Theoretical Correlation Study of Non-Linear Bending and Torsion Deformations of a Cantilever Beam. *Journal of Sound and Vibration* Vol. 50 No.4 (1977) p. 533-544

[12] A. Rosen. Theoretical and Experimental Investigation of the Nonlinear Torsion and Extension of Initially Twisted Bars. *Journal of Applied Mechanics* Vol.50 (Jun 1983) p.321-326

[13] P. Minguet, J. Dugundji. Experiments and Analysis for Composite Blades under Large Deflections. Part I: Static Behavior. *AIAA Journal* Vol. 28 No. 9 (Sep 1990) p. 1573-

[14] P. Minguet, J. Dugundji. Experiments and Analysis for Composite Blades under Large Deflections.

Part II: Dinamic Behavior. *AIAA Journal* Vol. 28 No. 9 (Sep 1990) p. 1580-

[15] A. Laulusa. Theoretical and Experimental Investigation of the Large Deflections of Beams. ONERA TP 1991-35

[16] L.Cveticanin, T. Atanockvic. Non-Linear Vibration of an Extensible Elastic Beam. *Journal of Sound and Vibration* Vol.177 (1994) p.159-171

[17] J.H. Sällström, D.H.L. Poelaert, F.L. Janssens Small Displacements about Equilibrium of a Beam Subjected to Large Static Loads. *AIAA Journal* Vol. 34 No. 11 (Nov 1996) p. 2384-2391

[18] ESDU 90016 Natural Frequencies of Isotropic and Orthotropic Rectangular Plates under Static In-plane Loading (including shear loading) (1993)

[19] MSC/NASTRAN Handbook for Nonlinear Analysis. (Draft) Version 67. Sang H. Lee, Editor. March 31, 1992.