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# Flight Control Law Design and HIL Simulation of a UAV

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## 1 Summary

An adaptive control methodology, merging two known approaches to flight control problem, gain-scheduling and direct eigenspace assignment (DEA), is developed. A gain-scheduled inner (stability) loop structure is shown to minimize the variance of the outer (guidance) loop gains and increase the robustness of the system. The employment of DEA with gain scheduling is observed to decouple the longitudinal and lateral flight modes resulting adequate system stability, enhanced robustness and control surface effectiveness. This methodology is used in the flight control law design of a UAV.

## 2 Introduction

Employment of eigenvalue / eigenstructure assignment techniques has found widespread practical applications that solve a variety of flight control problems. Digital implementation of these applications has evolved to be commonplace by the recent advances in microprocessor technology.

In this paper, two known approaches to the flight control design, gain scheduling and eigenspace assignment are reviewed. In the third and fourth sections, two methods are explained with their formulations, respectively. In section five the flight control problem is stated and developed scheme is presented to solve the problem, to demonstrate synergetic use of these two methods. Section six discusses the results achieved.

## 3 Gain Scheduling

In many situations it is known how the dynamics of a process change with the operating conditions of the process. One source for the change in dynamics may be the nonlinearities that are known. It is then possible to change parameters of the controller by monitoring the operating conditions of the

process. This idea is called gain scheduling. It can be regarded as a special kind of nonlinear open-loop adaptation of regulator parameters in a preprogrammed way. Gain scheduling is easy to implement in computer controlled systems provided that there is support in the available software. This is an ad hoc practice guided by heuristics rule of thumb [lev].

A main problem in the design of systems with gain scheduling is to find suitable scheduling variables. This normally done based on knowledge of the physics of a system. The concept of gain scheduling originated in connection with the development of flight control systems (FCS). In FCS applications, generally the Mach number and the dynamic pressure are measured by air data sensors and used as the scheduling variables. In this paper only the dynamic pressure is used as the scheduling variable since the air vehicle to be controlled has a relatively narrow speed envelope. Figure 1. shows the gain scheduling scheme, which can be viewed as a system with feedback gains adjusted using feedforward compensation.

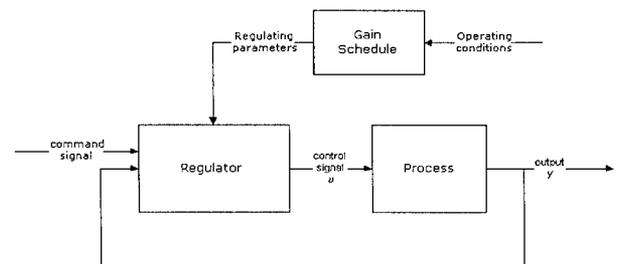


Figure 1. Gain scheduling

## 4 Direct Eigenspace Assignment

### 4.1 Background

Eigenstructure assignment is a useful tool that allows the designer satisfy damping, settling time, and mode decoupling specifications by choosing eigenvalues and eigenvectors. Moore [moo] in 1976 and Shrinathkumar [shri] in 1978 made the first discussions on this approach. Andry et al. [andr] have applied eigenstructure assignment to designing a stability augmentation system for lateral dynamics of L-1011 aircraft in 1983. Sobel and Shapiro [sobel1] used this method to design dynamic compensators for the same aircraft in 1984. Later Sobel et al. [sobe2] proposed a systematic method for choosing elements of the feedback gain matrix which can be suppressed to zero with minimal effect for and F-18 HARV on assignment. Kautsky et al. [kaut] proposed some robustness measures for system eigenvalues to be least sensitive to parameter variations. Direct Eigenspace Assignment (DEA) method was developed in 1986 by Davidson and was used to design lateral-directional control laws for F-18 HARV of NASA in 1992. This method allows designers to shape the closed loop response by direct choice of both desired eigenvalues and eigenvectors. It is called "direct" because unlike some eigenstructure assignment algorithms feedback gains are determined in a single iteration. Davidson et al. [davi1] has shown that during this effort DEA has been demonstrated to be a useful technique for aircraft control law design and issued some guidelines for lateral-directional flying qualities for high performance aircraft [davi2] in 1996.

### 4.2 DEA Formulation

Given the observable, controllable system

$$\dot{x} = Ax + Bu, \quad x \in R^n, \quad u \in R^m \quad (4.1)$$

and

$$y = Cx + Du, \quad y \in R^k \quad (4.2)$$

The total control input is the sum of the augmentation input  $u_c$  and pilot's input  $u_p$ .

$$u = u_p + u_c \quad (4.3)$$

The measurement feedback control law is

$$u_c = Gy \quad (4.4)$$

Solving for  $u$  as a function of the system states and pilot's input yields

$$u = [I_m - GD]^{-1} GCx + [I_m - GD]^{-1} u_p \quad (4.5)$$

The system augmented with the control law is given by

$$\dot{x} = (A + B[I_m - GD]^{-1} GC)x + B[I_m - GD]^{-1} u_p \quad (4.6)$$

The spectral decomposition of the closed loop system is given by

$$(A + B[I_m - GD]^{-1} GC)v_i = \lambda_i v_i \quad (4.7)$$

for  $i=1, \dots, n$  where  $\lambda_i$  is the  $i^{\text{th}}$  system eigenvalue and  $v_i$  is the associated  $i^{\text{th}}$  system eigenvector. Let  $w_i$  be defined by,

$$w_i = [I_m - GD]^{-1} GCv_i \quad (4.8)$$

Substituting this result into equation 4.7 and solving for  $v_i$  one obtains

$$v_i = [\lambda_i I_n - A]^{-1} Bw_i \quad (4.9)$$

This equation describes the achievable  $i^{\text{th}}$  eigenvector of the closed loop system as function of the eigenvalue  $\lambda_i$  and the eigenvector  $w_i$ . By examining this equation one can see that the number of control variables ( $m$ ) determines the dimension of the subspace in which the achievable eigenvectors must reside.

Values of  $w_i$  that yield an achievable eigenspace that is as close as possible in a least squares sense to a desired eigenspace can be determined by defining a cost function values associated with the  $i^{\text{th}}$  mode of the system

$$J_i = \frac{1}{2} [v_{a_i} - v_{d_i}]^H Q_{d_i} [v_{a_i} - v_{d_i}] \quad (4.10)$$

for  $i=1, \dots, n$  where  $v_{a_i}$  is the  $i^{\text{th}}$  achievable eigenvector associated with eigenvalue  $\lambda_i$ ,  $v_{d_i}$  is the  $i^{\text{th}}$  desired eigenvector and  $Q_{d_i}$  is an  $n$ -by- $n$  symmetric positive semi-definite weighing matrix on eigenvalue elements,  $^H$  denotes the complex conjugate transpose operator. This cost function represents the error between the achievable eigenvector and the desired eigenvector weighed by the matrix  $Q_{d_i}$ .

Values of  $w_i$  that minimize  $J_i$  are determined by substituting (4.9) into cost function for  $v_{a_i}$ , taking the gradient of  $J_i$  with respect to  $w_i$ , setting this result equal to zero, and solving for  $w_i$ . This yields

$$w_i = [A_{d_i}^H Q_{d_i} A_{d_i}]^{-1} A_{d_i}^H Q_{d_i} v_{d_i} \quad (4.11)$$

where

$$A_{d_i} = [\lambda_{d_i} I_n - A]^{-1} B \quad (4.12)$$

By concatenating the individual  $w_i$ 's column-wise to form  $W$  and  $v_{d_i}$  column-wise to form  $V_a$  equation can be expressed in matrix form by

$$W = [I_m - GD]^{-1} GCV_a \quad (4.13)$$

The feedback gain matrix that yields the desired closed loop eigenvalues and achievable eigenvectors is given by

$$G = W[CV_a + DW]^{-1} \quad (4.14)$$

### 4.3 DEA Design Algorithm

A feedback gain matrix that yields a desired closed loop eigenspace is determined in the following way [davi1]:

1. Select desired eigenvalues  $\lambda_{d_i}$ , desired eigenvectors  $v_{d_i}$  and desired eigenvector weighing matrices  $Q_{d_i}$ .
2. Calculate  $w_i$ 's using equation 4.11 and concatenate these column-wise to form  $W$ .
3. Calculate achievable eigenvectors  $v_{a_i}$ 's using equation 4.9 and concatenate these column-wise to form  $V_a$ .
4. Calculate feedback gain matrix  $G$  by equation 4.14.

## 5 System Design

### 5.1 Methodology

In Figure 4. system design is depicted. System states for longitudinal control system are  $X = [V, \alpha, q, \theta, h]$ ; airspeed, angle of attack, pitch rate, pitch angle and altitude. Objective is to design both the altitude-hold and the speed-hold autopilots for this system. The related plant inputs are  $U = [\delta_e, \delta_r]$ ; elevator and throttle deflection. Piloted command inputs are desired speed and desired altitude. Thus, theoretically, four gains are required for the elevator inner loop feedback, another four is required for the throttle inner loop feedback.

After determining a particular flight condition, the desired inner loop closed loop eigenvalues and vectors; the DEA algorithm of section 4.3 was utilized in order to calculate the feedback gains required for the inner loop compensators. The DEA algorithm in section 4.3 was coded in X $MATH^{\text{TM}}$  script to enable the designer to repeat this process for all 27 flight conditions, within a few seconds for each condition. In this application dynamic pressure  $Q$  was selected as the scheduling variable, which is  $0.5\rho V^2$ , where  $\rho$  is the air density in  $\text{kg/m}^3$  and  $V$  is the total airspeed in  $\text{m/sec}$ . This automatic gain calculation process was succeeded by plotting each gain versus the dynamic pressure of the corresponding flight condition.

All eight gains were plotted against dynamic pressure and these were fit to polynomials to yield gain formulas as a function of dynamic pressure. Four of these gains are shown in Figure 3-a to 3-d. Plots show that all the longitudinal gains

vary non-linearly and tend to decrease in absolute value with increasing dynamic pressure.

The PI controller gains  $K_{p1}$ ,  $K_{i1}$ ,  $K_{p2}$ ,  $K_{i2}$  were then calculated by the Ziegler-Nichols closed-loop method [astr] and lastly, the rate of climb adjust  $K_{ALT}$ , determined by trial and error for the best rate of climb.

### 5.2 Inner Loop Controller

The microcontroller based controller consists of gain scheduler, which in fact resides as a module inside the flight management software, dynamically calculates the elevator demand in the following scheme:

1. Calculate air density:  $\rho = \rho_0(1 - 0,00002256h)^{4.256}$
2. Calculate dynamic pressure:  $Q = \frac{1}{2}\rho V^2$
3. Using the gain equations that fit the gain plots (Figures 3.a thru 3.d), calculate the gains  $K_v$ ,  $K_\alpha$ ,  $K_q$  and  $K_\theta$ .
4. Calculate Elevator demand:

$$\delta e\text{-demand} = \theta\text{-demand} + K_v \cdot V + K_\alpha \cdot \alpha + K_q \cdot q + K_\theta \cdot \theta$$

The similar effort is performed for the throttle control loop. Altitude and speed gains are calculated, by the controller according to the maximum and minimum rates of climb and forward acceleration specifications respectively.

### 5.3 HIL Simulation

Hardware in the loop simulations were performed on a non-linear aircraft model that covers the complete flight envelope in MATRIX-X/AC-100 $^{\text{TM}}$  environment. Control system of Figure 4. was employed. Sensor were modeled and the control system (the DSP processor) is added into the system loop. A 100 m climb demand was given to the system while the vehicle was in equilibrium, (cruising at 60  $\text{m/s}$ , with trimmed angle of attack and pitch angle of  $1.8^\circ$ , at 1000 m altitude) at 10 $^{\text{th}}$  second. Altitude, pitch angle, angle of attack, speed, throttle and elevator demands were observed. For longitudinal control design an acceptable climb performance (approx. 4  $\text{m/s}$ ) is achieved in the cruise speed of 60  $\text{m/s}$ . Control surfaces move in a moderate speed and within limits. (see Figures 2-a, 2-b, 2-c).

## 6 Conclusion

Direct eigenspace assignment is used guarantee the inner loop stability of the closed loop system. As for longitudinal control, short period and phugoid modes are damped to the same nominal prescribed frequency and damping factor values for all flight conditions so that compensated system shall appear identical poles in all conditions. This gives the designer the opportunity to design the outer loop easier without employing any further gain schedule for the rest of the system. Design philosophy is simply, parametrization of the inner loop gains via dynamic pressure so that the the inner loop dynamics always appear almost the same to the outer guidance controller dynamics.

Gain scheduling had a positive effect on control surface effectiveness and robustness. DEA had positive effects on overall stability of the system. These two methods have been merged successfully to yield a high performance controller that employs the advantage of each method.

## 7 References

- [levi] Levine, W.S., Control Handbook, 1995, CRC Press/IEEE Press.
- [moor] Moore, B.C., "On the flexibility offered by state feedback in multivariable systems beyond close loop eigenvalue assignment", *IEEE Trans. on Automatic Control*, Oct 1976, pp 689-692.
- [shri] Shrinathkumar, S., "Eigenvalue/Eigenvector assignment using output feedback", *IEEE Trans. On Automatic Control*, 1978, AC-23, pp 79-81.
- [andr] Andry, A.N., Shapiro E.Y., Chung, J.C., "Eigenstructure assignment for linear systems", *IEEE Trans Aerospace and Electronic Systems*, 1983, AES-19, pp 711-729.
- [sobe1] Sobel, K.M., Shapiro, E.Y., "Eigenstructure assignment for design of multimode flight control systems", *IEEE Control Systems*, May 1985, pp 9-14.
- [sobe2] Sobel, K.M., Yu, W., Lallman, F.J., "Eigenstructure assignment with gain suppression using eigenvalue/eigenvector derivatives", *Journal of Guidance Control and Dynamics*, 1990, 13, pp 1008-1013.
- [kaut] Kautsky, J., Nichols, N.K., Van Dooren, P., "Robust pole assignment in linear state feedback", *International Journal of Control*, 1985, 41, pp 1129-1155.
- [davi1] Davidson, J.B., Adrisiani, D. "Gain Weighed Eigenspace Assignment", *NASA Tech. Memorandum*, 109130, May 1994.
- [davi2] Davidson J.B., Adrisiani, D., "Lateral-Directional Eigenvector Flying Qualities Guidelines for High Performance Aircraft", *NASA Tech. Memorandum*, 110306 December 1996
- [astr] Astrom, K., Wittenmark B., 1989, Adaptive Control, Addison Wesley.

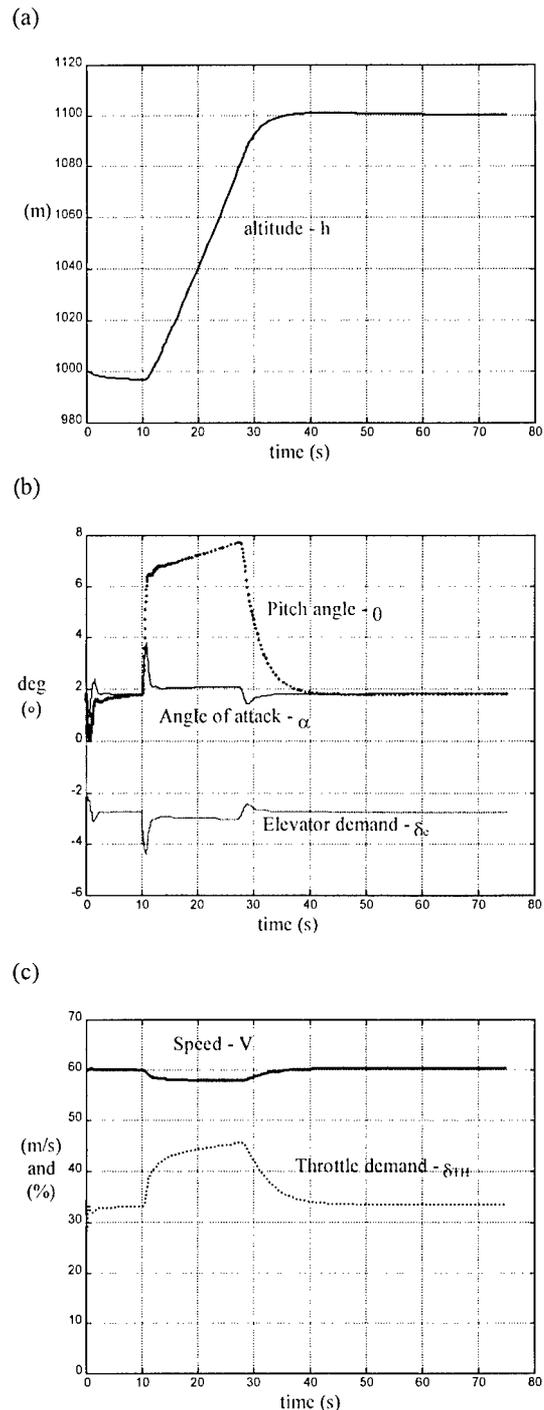


Figure 2. Controller in-the-loop system response to 1000 m climb demand

(both altitude-hold and speed-hold autopilots ON)

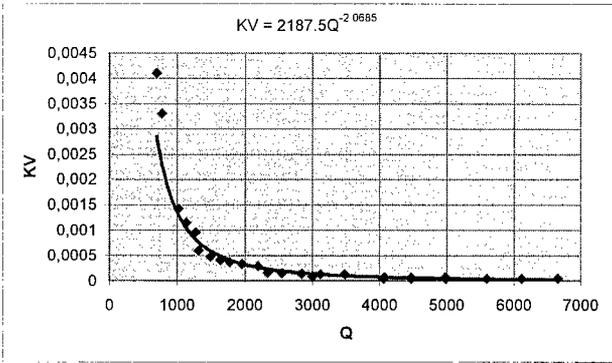


Figure 3-a. Gain vs. dynamic pressure plot and approximate curve fit with its equation for  $K_v$

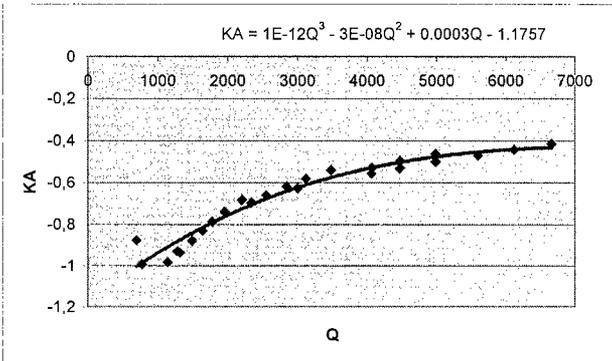


Figure 3-b. Gain vs. dynamic pressure plot and approximate curve fit with its equation for  $K_\alpha$

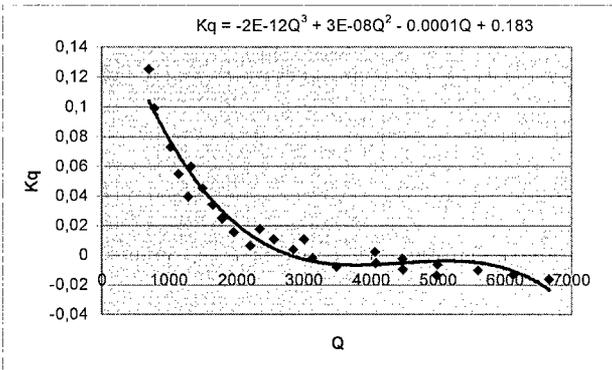


Figure 3-c. Gain vs. dynamic pressure plot and approximate curve fit with its equation for  $K_q$

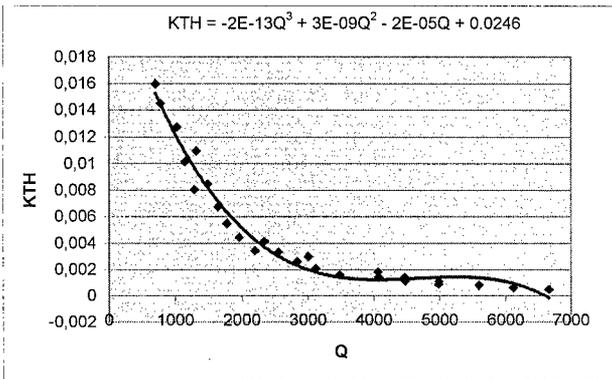


Figure 3-d. Gain vs. dynamic pressure plot and approximate curve fit with its equation for  $K_\theta$

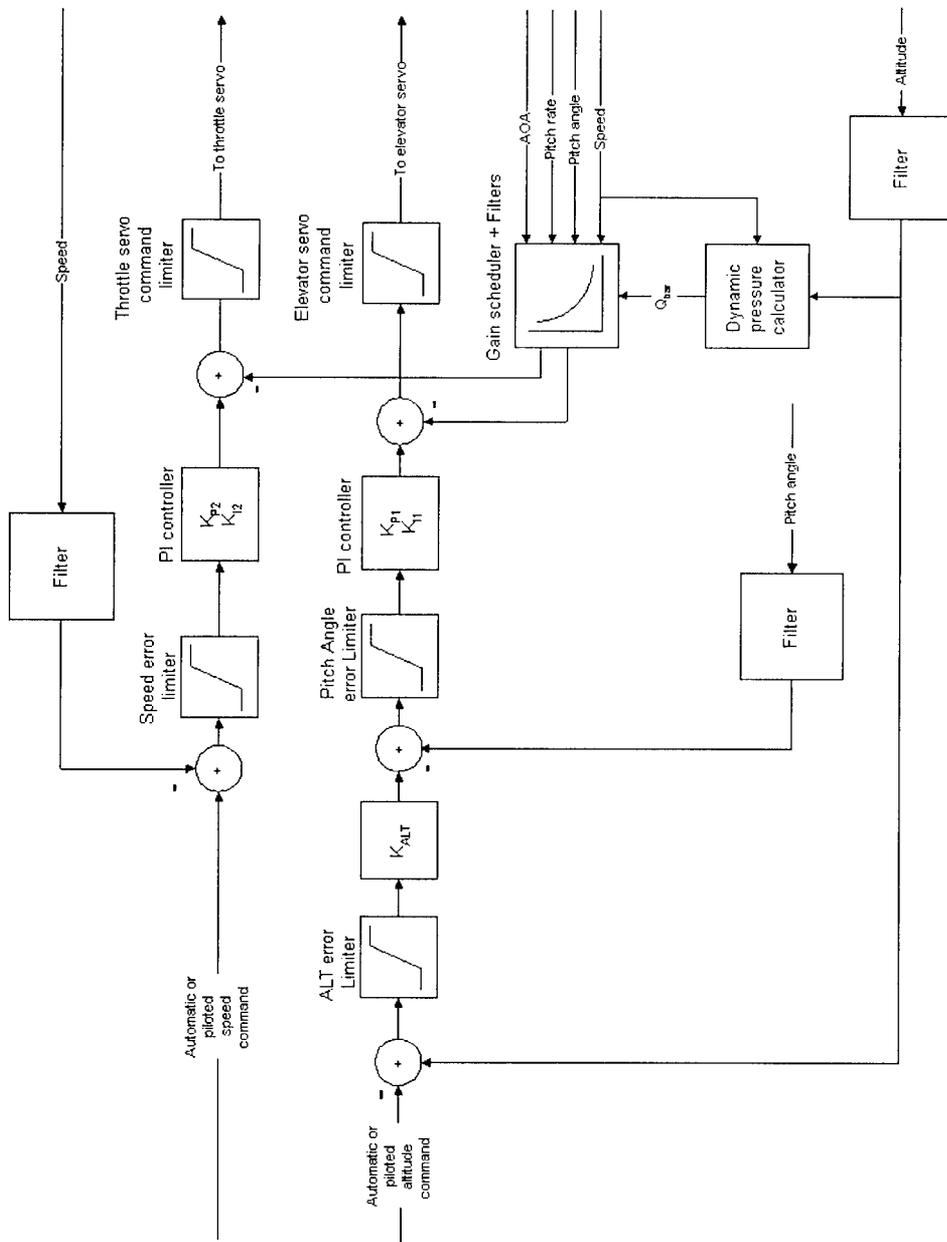


Figure 4. Longitudinal Control System Design