RELIABILITY OF ENGINEERED BASEMENTS AS BLAST SHELTERS

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INTRODUCTION

This paper presents a method for predicting the reliability (probability of nonfailure) of basement shelters when subjected to the blast effects of a single nuclear weapon in its Mach region. The method is described with reference to a reinforced concrete basement shelter whose roof slab is the weakest structural component. This is generally the case in weak-walled conventional buildings when the first floor over the basement is at grade and the peripheral basement walls are not exposed but are in contact with the soil. In such basements, partial or total collapse of the slab results in casualties. Casualties would be produced by debris from the collapsed slab, the building above, and by pressure build-up within when the shelter envelope is breached. The objective then is to determine the probability of roof slab collapse and on this basis to determine the probability of people survival. The paper presents the method of analysis and illustrates its application by means of an example problem.

STRUCTURAL ANALYSIS

The form of structural analysis performed is described in Reference (1). The reinforced concrete slab is modeled as a single degree of freedom system whose flexural resistance is a piecewise linear function. The resistance function, see Figure 1, relates the flexural slab resistance to the deflection at its midpoint. Since shear is a possible mode of failure, the analysis is also concerned with peak dynamic reactions distributed along the edges of the slab.

The blast load is approximated by a function having an instantaneous rise to peak overpressure, followed by an exponential decay, see Figure 2. It has the following form (Reference 2).

\[
F(t) = F_1(1 - t/t_d)e^{-t/t_d}
\]

where \( F_1 \) = peak overpressure

\( t_d \) = positive phase duration or the overpressure

The spatial distribution of the blast load is assumed to be uniform over the surface of the slab.

Since both the loading and resistance are complex functions, it was

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necessary to use a numerical procedure to obtain the peak midpoint deflection and the peak dynamic reactions. The primary equations used in the analysis are the following.

\[ K_{LM} \ddot{y} + R(y) = F(t) \]  \hspace{1cm} (2)

where \( K_{LM} \) = the load-mass factor (Reference 1),
\( M_t \) = the total mass of the slab
\( R(y) \) = flexural resistance
\( F(t) \) = load-time history, see Equation (1)
\( V(t) = C_1 R(y) + C_2 F(t) \)  \hspace{1cm} (3)

where \( V(t) \) = the dynamic reaction along the given edge (a or b) of the slab, see Figure 3
\( C_1, C_2 \) = constants whose values depend on the aspect ratio of the slab

**PROBABILITY OF FAILURE**

In the case of two failure modes, the probability of slab failure, \( P(F) \), is (Reference 3)

\[ P(F) = 1 - [1 - P(F_b)][1 - P(F_v)] \]  \hspace{1cm} (4)

when the modes are independent, and

\[ P(F) = \max [P(F_b), P(F_v)] \]  \hspace{1cm} (5)

when the modes are highly correlated. In Eqs. (4) and (5), \( P(F_b) \) is the probability of failure due to flexure, and \( P(F_v) \) is the probability of failure due to shear. The actual probability of failure is between these two bounds.

Probabilities of failure due to flexure and shear were each computed using the following expression

\[ P(F) = 1 - \phi\left(\frac{\ln(\tilde{r}/\tilde{s})}{\sqrt{\ln(1 - \Omega_R^2)(1 - \Omega_S^2)}}\right) = 1 - \phi\left(\frac{\ln\tilde{r}}{\zeta_0}\right) \]  \hspace{1cm} (6)

where \( \phi(\ ) \) = the cumulative density function of the standard normal distribution
\( \tilde{r} \) = the median value of the resistance parameter in flexure or shear
\( \tilde{s} \) = the median value of the load parameter in flexure or shear
\( \Omega_R, \Omega_S \) = coefficients of variation of the resistance and load parameters respectively
\[ \sigma = \text{median safety factor} \]
\[ u = \text{total degree of dispersion of the safety factor} \]

For the case of flexural response the median safety factor \( \nu \) is taken as the ratio of \( y_m/y_p \), where \( y_m \) is the ultimate (collapse) midpoint deflection of the slab, see Figure 1, and \( y_p \) is the midpoint deflection at a given load. The value of \( y_m \) is taken as \( 0.15a \), where "a" is the short span dimension of the slab (Reference 4).

For the case of shear response, the median safety factor is taken as the ratio of \( v_m/v_p \), where \( v_m \) is the ultimate unit shear capacity of the slab and \( v_p \) is the corresponding shear stress at a given load. The shear stress is computed at the periphery of the slab by the use of dynamic reactions mentioned earlier. The ultimate unit shear capacity of the slab is based on the following formula which is the standard ACI (Reference 5) formula modified as suggested in Reference 6.

\[ v_m = 1.5(2 f'_c') \]  

where \( f'_c' = 1.25f'_c \) = the ultimate compressive strength of concrete increased to account for the increase in strength due to dynamic loading conditions (Reference 1).

SAMPLE APPLICATION

Figure 3 shows the plan view of a reinforced concrete slab whose reinforcing steel extends over and beyond the supports. Supports are continuous along the edges of the slab. The reinforcement in the short direction is 0.27(in)\(^2\)/ft (572mm\(^2\)/m) and in the long direction is 0.19(in)\(^2\)/ft (402mm\(^2\)/m). The slab is 9-in (228.6mm) thick. The compressive strength of concrete, \( f'_c = 3000 \text{ psi} \) (20.7 MPa) and the yield strength of reinforcing steel, \( f_y = 60,000 \text{ psi} \) (414 MPa).

In performing the analysis, the following parameters were treated as random variables, i.e., \( F_i, t_d, f'_c, f_y, A_s \) (cross-sectional area of reinforcing rods), \( d \) (effective depth of the slab).

Coefficients of variation of the basic parameters were obtained from available experimental data (Ref. 4, 7, 8). Corresponding coefficients of variation of slab resistance, peak deflection and peak shear stress were determined on the basis of a first order approximation, Reference 3.

This slab was analyzed when subjected to a series of blast loads of increasing intensity with durations corresponding to a 1-MT surface burst. Results of the analysis are shown in Figure 4 and Figure 5. Figure 4 shows the probabilities of failure in flexure and shear taken separately and determined on the basis of Eq. (6). Figure 5 shows the bounds on the probability of failure computed on the basis of (4) and (5).
SUMMARY AND CONCLUSIONS

A method for predicting the probability of failure of structures by considering multiple failure modes was formulated. It was applied to the analysis of a reinforced concrete slab when subjected to a uniformly distributed blast load over its surface. Currently available criteria for failure due to flexure and shear (Ref. 4 and 5) were used in predicting the probability of failure.

This method is capable of considering all major components of a structure, the respective failure modes of each component, and of predicting the probability of failure of the structure as a whole.

ACKNOWLEDGMENT

This study was supported by the Federal Emergency Management Agency under Contract EMW-C-0374.

REFERENCES


**Fig. 1** Resistance Function

**Fig. 2** Load Function

**Fig. 3** Slab Dimensions

\[ R_1, y_1 = \text{Resistance and deflection at the end of the elastic range} \]

\[ R_2, y_2 = \text{Resistance and deflection at the end of the elasto-plastic range} \]
Fig. 4 Probabilities of Failure Due to Flexure and Shear

Fig. 5 Bounds on the Probability of Slab Failure