DESIGN OF PERIODICALLY CORRUGATED DIELECTRIC ANTENNAS FOR MILLIMETER-WAVE APPLICATIONS

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Periodic dielectric waveguide structures were extensively analyzed and were understood in the context of integrated-optics theory. Recently, it has been also demonstrated experimentally that this class of structures offer many advantages for use as millimeter-wave antennas. A considerable effort is currently under way to fully develop its potential. Making use of the available analytic methods previously devised, we have carried out an extensive and systematic analysis of periodically corrugated dielectric antennas. The results are exhibited in a way that is most pertinent to millimeter-wave applications. On the basis of the obtained numerical data, a set of design guidelines for the corrugated dielectric antennas is established herein.
1. Introduction

A dielectric waveguide with a periodic perturbation, or simply a periodic dielectric waveguide, has been shown to hold substantial promise as a leaky-wave antenna for millimeter applications\(^1\)-\(^5\). Such an antenna structure may be conveniently fabricated on a uniform dielectric waveguide to form a completely integrated mm-wave system. In addition, the dielectric leaky-wave antennas offer the advantage of electronic beam steering\(^1\),\(^2\). Therefore, in recent years, a considerable effort has been made for a better understanding of this particular class of antennas in order to explore its applications to mm-wave systems.

In this paper, we present a systematic analysis of the periodically corrugated dielectric waveguide for use as a leaky-wave antenna for mm-wave applications. We recognize that this class of structures has been extensively analyzed for integrated-optics applications\(^7\) such as beam-to-surface-wave couplers, distributed feedback reflectors, and filters. In fact, leaky-wave antennas and optical periodic couplers are based on the same physical principle\(^4\),\(^7\) and a great deal of information on the basic wave characteristics of the optical devices can simply be carried over for the understanding of the mm-wave antennas. However, because of the substantial difference in the permittivity of the materials commonly used in the two frequency ranges and because of different processes by which these devices are
constructed, the design procedures for mm-wave antennas differ from that for optical couplers in many respects. The main purpose of this paper is to make use of the existing theoretical tools for developing design guidelines for the particular class of corrugated dielectric leaky-wave antennas, appropriately taking into account special requirements for mm-wave applications.

For mm-wave applications, a corrugated dielectric antenna has, at most, a few wavelengths in width, in contrast to thousands of wavelengths for most optical couplers. Therefore, an optical periodic coupler can be formulated as a two dimensional boundary value problem that may support independent TE or TM modes propagating normally to the grooves of the corrugation. It has been shown that to investigate the effect of finite antenna width, the most basic problem to be solved is the guiding of waves propagating at an oblique angle in an infinitely wide periodic structure. This is a three dimensional electromagnetic boundary value problem that supports only hybrid modes, e.g., it requires the coupling between TE and TM modes. Such a vector boundary value problem has been formulated and the propagation characteristics for the general case of oblique guidance have been subsequently analyzed. The effect of the finite antenna width on the performance of the corrugated dielectric antenna has been reported. It has been shown that for an antenna structure of finite width, the longitudinal phase constant is determined by the
unperturbed uniform waveguide and the decay constant does not differ from that of the two-dimensional case, as long as the width of the antenna is not very small. In this work, we shall assume that the antenna width is large enough so that the leakage constant can be obtained from the simpler normal guidance case.

2. Design of the periodically corrugated dielectric antenna

The configuration of a dielectric antenna structure is shown in Fig.1. Such an electromagnetic structure has been previously analyzed in the context of optical periodic couplers and many radiation characteristics of the structure have been known. The antenna structure is characterized by four parameters: the thickness of the uniform region, \( t_f \), the thickness of the corrugation region, \( t_g \), the period of the corrugation, \( d \), and the aspect ratio \( a = d_1/d \). Since the lengths can always be normalized to the free-space wavelength, it is not necessary to consider the frequency of the source as an independent parameter. On the other hand, since a corrugated dielectric antenna is expected to be fabricated from an originally uniform waveguide by machining, it is more appropriate to use the original waveguide height \( h = t_g + t_f \) as a parameter. This means that when the waveguide is machined, both \( t_g \) and \( t_f \) will change. Therefore, it is necessary to investigate the combined effect of change in \( t_f \) and \( t_g \) on the propagation characteristics of the antenna structure.
Because of the periodicity of the leaky-wave antenna, the electromagnetic fields everywhere must consist of all the space harmonics. For TM modes, we have:

\[
H_y(x,z) = \sum_{n=-\infty}^{\infty} I_n(z) \exp(ik_{xn}x) \tag{1}
\]

where \( I_n \) is the \( n \)-th harmonic amplitude of the magnetic field and \( k_{xn} \) is the \( x \)-component of the propagation constant of the \( n \)-th harmonic and is related to that of the fundamental harmonic by:

\[
k_{xn} = k_{xo} + 2n\pi/d = \beta - j\alpha + 2n\pi/d = \beta_n - j\alpha \tag{2}
\]

where \( \beta \) and \( \alpha \) are the propagation and attenuation constants of the fundamental harmonic, respectively. It is noted that the propagation constant of the \( n \)-th harmonic differs from that of any other harmonic but the attenuation constant, \( \alpha \), is the same for all harmonics. By solving the boundary value problem, it has been shown that \( k_{xo} \) and the harmonic amplitudes, \([I_n, n = 0, \pm 1, \pm 2, \ldots]\), can be determined in terms of the eigenvalues and eigenvectors of a coupling matrix characterizing the corrugation region. For the given antenna structure on a ground plane of infinite extend, because the energy can only radiate into the air region, it is unnecessary to determine explicitly the harmonic amplitudes, if we are interested only in the case of single beam radiation. In the present study, therefore, we are left with only the determination of the dispersion root \( k_{xo} \) of the antenna. For simplicity, \( k_{xo} \) will be referred to as the leaky wave constant. Since \( \alpha \) may be used as a measure of
the rate of energy leakage, it will also be called the leakage or radiation constant, in contrast to the phase constant $\beta$.

In the air region where the medium is uniform, each harmonic propagates independently as a plane wave. The transverse propagation constant of the $n$-th harmonic is given by:

$$k_{zn} = (k_0^2 - k_{xn}^2)^{1/2}$$  \hspace{1cm} (3)

For a very shallow corrugation or $t_0$ very small, it is intuitively expected that $k_{xo} = \beta_{sw} = k_0n_{eff}$, where $\beta_{sw}$ is the longitudinal propagation constant of the unperturbed structure, as shown in Fig.2. Under such an approximation and invoking (2), we obtain, from (3):

$$k_{zn} = k_0[1 - (n_{eff} + n\lambda/d)^2]^{1/2}$$  \hspace{1cm} (4)

For only the $n = -1$ harmonic to radiate, we must have $k_{z,-1}$ real and all $k_{zn}$ imaginary for $n \neq -1$. These conditions can alternatively be expressed as:

$$\frac{\lambda}{n_{eff} + 1} < d < \frac{\lambda}{n_{eff} - 1}, \hspace{1cm} \text{for } n_{eff} > 1$$  \hspace{1cm} (5)

$$\frac{\lambda}{n_{eff} + 1} < d < \frac{2\lambda}{n_{eff} + 1}, \hspace{1cm} \text{for } n_{eff} < 1$$  \hspace{1cm} (6)

Under these conditions, the $n = -1$ harmonic radiates into the air region at an angle (with respect to the $z$-axis):
\[ \theta_{\text{rad}} = \sin^{-1}\left(\frac{\beta_{-1}}{k_0}\right) = \sin^{-1}\left(n_{\text{eff}} - \frac{\lambda}{d}\right) \quad (7) \]

The sign of \( \theta_{\text{rad}} \) determines the forward or backward radiation. Although the above design formulas are derived under the assumption that \( t_g \) is very small, they also hold for \( t_g \) large, if the normalized phase constant or the effective index of refraction, \( n_{\text{eff}} \), is accurately obtained for a given \( t_g \).

It is noted that \( n_{\text{eff}} \) of a periodic dielectric waveguide does not depend appreciably on the period \( d \) of the structure. Eq. (7) shows that the choice of the period \( d \) is simply determined by the radiation angle. Therefore, the effect of the period \( d \) on the antenna performance will not be further elaborated.

2.1. Effect of the corrugation thickness \( t_g \) on the radiation angle

As a uniform slab of the thickness \( h \) is machined, the dielectric material is scooped out. As a result, the effective thickness of the corrugated slab is reduced, and so is the effective dielectric constant. It has been well known\(^4,7\) that the effective dielectric constant of the corrugated slab may be simply determined by a uniform double layer structure with the volume average of the dielectric constant for the periodic corrugation region. As an example, Fig.2 shows the effective dielectric constant as a function of the corrugation thickness for a periodic silicon slab with the aspect ratio \( d_1/d = 0.5 \). In Fig.2, the solid
curves are for fixed thickness of the original slab, \( h = t_f + t_g \) as a parameter, whereas the dashed curves are for fixed thickness of the uniform portion of the corrugated structure. For example, in the case of an originally uniform slab of thickness \( h = 0.2\lambda \), a corrugation of the thickness \( t_g / \lambda = 0.05 \) and an aspect ratio \( a = d_1 / d = 0.5 \) will result in an antenna structure with an effective dielectric constant \( \varepsilon_{\text{eff}} = 10 \), as marked by the cross labelled by A on the curve for \( h / \lambda = 0.2 \) in Fig. 2. The same effective dielectric constant can be achieved by other combinations of \( h \) and \( t_g \) values, such as \( h / \lambda = 0.24 \) and \( t_g / \lambda = 0.1 \), as marked by the cross labelled by B in Fig. 2. It is noted that the dashed curves are useful for the design of other types of structures, and will not be further elaborated here. After \( \varepsilon_{\text{eff}} = n_{\text{eff}}^2 \) is determined for a given antenna structure, the radiation angle is determined according to (7). Thus, the set of solid curves provides the necessary information for determining the dependence of the radiation angle on the corrugation thickness.

2.2. Effect of the corrugation thickness on the radiation constant

It has been known\(^1\) that the radiation constant \( \alpha \) is proportional to \( t_g^2 / t_g \) for \( t_g \) small, and reaches a saturation value for \( t_g \) large, if the guided-wave field is evanescent in the corrugation region. For the aspect ratio \( d_1 / d = 0.5 \), the average dielectric constant of the corrugation region is
\[ \varepsilon_{\text{ave}} = 6.5, \] and the guided-wave field is indeed evanescent in the corrugation region. The radiation constant as a function of the corrugation thickness is shown in Fig.3. Evidently, the radiation constant \( \alpha \) varies with the corrugation thickness \( t_g \) in the fashion expected.

On the other hand, the fabrication of a periodically corrugated dielectric antenna by machining changes not only the thickness of the corrugation region, but also that of the uniform region. Therefore, it is necessary to consider the combined effect of the changes in both the thicknesses of the corrugated and uniform regions. When a uniform dielectric slab is cut to produce the corrugation, the thickness of the remaining uniform region decreases at the same rate as the depth of the grooves increases. Based on Fig.3, it is clear that the leakage or radiation constant will increase initially as the grooves are cut deeper and deeper. However, because of the changing thickness of the remaining uniform portion that affects the basic surface wave, it is not clear at this point that if the radiation constant is still too small, any further cut can be helpful for achieving a larger radiation constant.

Fig.4 shows the variations of the radiation constant and the radiation angle \( \theta_{\text{rad}} \) as the thickness of the corrugation region \( t_g \) is increased, for the case of the original thickness \( h = 0.3\lambda \), the period \( d = 0.25\lambda \), and the aspect ratio \( d_1/d = 0.5 \). When the grooves are shallow (\( t_g \) small)
the solid curve shows that the radiation constant $\alpha$ increases with the groove depth, as expected. However, $\alpha$ reaches a maximum value at $t_g \approx 0.1\lambda$ and then decreases as $t_g$ is further increased. Such a decrease in $\alpha$ can be explained as follows: As $t_g$ is increased, the remaining uniform portion of the structure becomes thinner and thinner and, as a result, the phase constant of the guided wave becomes smaller and smaller. This phenomenon is exhibited by the dashed curve for the radiation angle in Fig. 4. More specifically, the radiation angle increases in the backward direction with increasing $t_g$. Eventually, the antenna structure will cease to radiate in the backward end-fire direction and the guided wave becomes totally bounded.

Fig. 5 shows the results for a case of a thinner original uniform slab, $h = 0.25\lambda$, while all other structure parameters are kept unchanged from the proceeding case. However, it is interesting to observe that the maximum value of $\alpha$ occurs at the same thickness of the remaining uniform slab $t_f = 0.2\lambda$ in both cases. In fact, this result holds for any structure with $d = 0.25\lambda$ and $d_1/d = 0.5$, as long as the thickness of the original uniform slab is sufficiently large.

2.3. Effect of the aspect ratio

The numerical results presented in the preceding section are all for the case of equal width for the teeth and grooves or the aspect ratio $d_1/d = 0.5$. In the fabrication
process, the aspect ratio can be easily controlled, if... For comparison purposes, the radiation constant $\alpha$ as a function of the groove depth $t_g$ is shown in Fig. 6 for an original unit cell of film thickness $h = 0.25\lambda$ and the period $d = 1.25\lambda$ as in Fig. 5, but with wider teeth ($d_1/d = 0.8$) or narrower grooves. With such a new aspect ratio, the attainable maximum value of $\alpha$ is increased by a factor of about 4 in the case of $d_1/d = 0.8$, but the grooves have to be cut deeper ($t_g \approx 0.1\lambda$).

In order to fully understand the effect of the aspect ratio, the radiation constant as a function of the aspect ratio is investigated and the results are plotted in Fig. 7, for the structure and parameters indicated. In the two limiting cases of the aspect ratio: $d_1/d = 0$ and 1, the periodic structure becomes uniform and it is expected that no radiation can occur. For optical structures with a relatively low dielectric constant, it is well known$^7$ that the maximum radiation occurs at the aspect ratio $d_1/d = 0.5$. For mm-wave antennas, however, the dielectric constant is relatively high and the field variation in the corrugation region depends strongly on the aspect ratio, and so does the radiation constant. Fig. 6 shows that the value of $\alpha$ peaks around the aspect ratio $d_1/d = 0.7$, instead of $d_1/d = 0.5$ as usually expected. Such a shift in the aspect ratio can be explained qualitatively as follows: For the given structure, the fields in the corrugation region are evanescent for the aspect ratio $d_1/d = 0.5$; therefore, the surface wave...
is only weakly perturbed by the corrugations and the radiation constant is small. When the aspect ratio is increased, more surface-wave energy is shifted to the corrugation region, resulting in a larger perturbation of the surface wave and a larger radiation constant. Thus, we may conclude that for a larger radiation constant, the aspect ratio should be relatively large \((d_i/d > 0.5)\) or the grooves should be relatively narrow.

3. Discussions and conclusions

The radiation characteristics of thick corrugated dielectric antennas are relatively insensitive to the change of structure parameters. From the practical viewpoint, this property will afford a larger tolerance in a mass production of the antennas. On the basis of the numerical results presented above, we may now establish the following general guidelines for the design of the corrugated dielectric antenna structures:

(1) The thickness of the original uniform slab and the period of the corrugation are determined by the desired radiation angle according to (7).

(2) For a uniform dielectric slab, there exists a groove depth \(t_g\) that yields a maximum value of radiation constant; a desired radiation constant
smaller than the maximum value may be realized
by a larger or smaller value of $t_g$, depending on
the radiation angle desired.

(3) Within the constraint of the radiation angle, the
larger the thickness of the original uniform slab,
the larger the maximum value of the radiation
constant $\alpha$ achievable.

(4) There exists a groove width or an aspect ratio
that yields a maximum value of the radiation
constant $\alpha$ and a smaller value of $\alpha$ may be
realized by adjusting the aspect ratio.

(5) For a uniform dielectric slab of larger thickness
and high dielectric constant, the maximum value
of $\alpha$ occurs with the groove width smaller than
the tooth width. In other words, cutting the groove
with a larger width may not help increase the
radiation constant.

In addition to those presented above, we have carried
out extensive numerical results for many different
situations. In particular, for the case of relatively thin antenna structures, the radiation characteristics are generally much more sensitive to the change of structure parameters, and each structure has to be quantitatively evaluated individually. Therefore, general design guidelines for thin corrugated dielectric antennas are difficult to develop.

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Reference


Fig. 1. Geometrical configuration of corrugated dielectric antenna.

Fig. 2. Variations of effective dielectric constant vs corrugation depth $t_g$ for uniform double-layer waveguide.
Fig. 3. Variations of radiation constant vs. corrugation depth $t_g$ for fixed thickness of the uniform region.
Fig. 4. Variations of radiation constant vs. corrugation depth for $t_f + t_g = 0.3 \lambda$, $d = 0.25 \lambda$ and $d_1 = 0.5d$. 

$t_f/\lambda$ $t_g/\lambda$

$\alpha \lambda$ $-\theta_{\text{rad}}$

$t_g$ $d$ $d_1$

$t_f$

$t_f + t_g = 0.3 \lambda$

$d = 0.25 \lambda$

$d_1 = 0.5d$
Fig. 5. Variations of radiation constant vs. corrugation depth for $t_f + t_g = 0.25 \lambda$, $d = 0.25 \lambda$ and $d_1 = 0.3d$. 

$\alpha \lambda$ vs. $t_g/\lambda$ for $t_f/\lambda$ and $\theta_{rad}$.
Fig. 6. Variations of radiation constant vs. corrugation depth for $t_f + t_g = 0.25\lambda$, $d = 0.25\lambda$ and $d_1 = 0.8d$. 
Fig. 7. Variations of radiation constant vs. aspect ratio $d_1/d$ for $t_f + t_g = 0.25 \lambda$, $t_g = 0.05 \lambda$, and $d_1 = 0.5d$. 