SIMULTANEOUS NULLING IN THE SUM AND DIFFERENCE PATTERNS OF A MONOPULSE RADAR

RANDY L. HAUPT
Electromagnetic Sciences Division
Rome Air Development Center
Hanscom AFB, MA 01731

Best
Available
Copy

Abstract

Adaptive nulling in a monopulse antenna requires consideration of both the sum and difference channels. This paper describes a phase only nulling technique which simultaneously places nulls in the far field sum and difference patterns using one set of phase shifters.

1. Introduction

In the past few years, considerable research and development has been done in the field of adaptive antennas. Communication and sonar systems have reaped some of the benefits of adaptive antenna technology, while radars lag behind. Some of the reasons for this dichotomy are adaptive techniques are not well suited for microwave frequencies; radars have large antennas, hence more adaptive loops; and a radar has tight time constraints due to target searching. As a result, only a handfull of radars incorporating sidelobe cancelling techniques exist today. Fully adaptive radar antennas with many degrees of freedom are not practical to implement at this time.

Monopulse radars present an even more difficult adaptive antenna problem. A monopulse antenna uses two antenna patterns simultaneously: 1) a sum pattern to detect and range a target and 2) a difference pattern to determine the angular location of the target. Most adaptive antenna research has ignored the difference pattern, even though both patterns must have a null in the direction of the interference to enhance the radar's performance. Placing a null in the sum pattern will not automatically place a null in the difference pattern. Consequently, most system requirements have assumed that the sum channel requires separate adaptive weights and control from the difference channel.

This paper has a dual purpose. First, it shows that a null can theoretically be placed in the sum and difference channels of a monopulse antenna using one set of adaptive weights. An adaptive technique incorporating this theory would greatly reduce the hardware and software requirements for a monopulse antenna. A second reason for writing this paper is to emphasize the need for adapting in the difference channel. I know of monopulse antennas being designed for adaptive circuitry in the sum channel only. In order to maintain tracking performance, the difference pattern must be adapted as well.

2. Nulling in Antenna Patterns

This section of the paper shows that a null synthesized in the sum pattern will not necessarily result in a null in the difference pattern and visa versa. An equally spaced linear array of isotropic elements is used in the analysis (Fig. 1). The output of each element passes through a phase shifter which steers the mainbeam as well as provides the adaptive cancellation. Next the signal is split into a sum channel signal and a difference channel signal. Each channel has an amplitude weighting, designed to give a certain sidelobe level. All the sum channel signals are added together in phase and the resultant signal goes to a receiver. One half of the array's difference channel signals receive a 180° phase shift before being added together with the other half of the difference channel signals. Phase only nulling in the sum channel can be accomplished using a phase only beam space algorithm[1,2]. The algorithm generates a cancellation beam in the direction of interference, then subtracts the beam from the quiescent pattern to get a resultant pattern with a null in the direction of interference.

The phase and amplitude weights for the sum channel are

$$W_n = a_n e^{j\theta n} \tag{1}$$

where θ_n is the adapted phase setting and a_n the amplitude weight. For low sidelobe antennas W_n may be approximated by

$$W_n = a_n (1+j\theta_n) \tag{2}$$

The far field pattern of this weight is

$$S(u) = \sum_{n=1}^{N} a_n(1+j\theta_n)e^{jkd_n u} \tag{3}$$

k = propagation constant = $2\pi / \lambda$

λ = wavelength

$d_n = d_o$

$u = \sin\theta$

θ = angle from boresight

$$= \sum_{n=1}^{N} a_n e^{jkd_n u} + j \sum_{n=1}^{N} \theta_n a_n e^{jkd_n u} \qquad (4)$$

The jammers are known to be at the angles θ_m and m ranges from 1 to M, the number of jammers.

The first summation in equation 4 is the far field antenna pattern of the quiescent weights. The second summation is the cancellation beams generated by the adaptive weights. At each jammer angle θ_m, the quiescent pattern and cancellation beam match in amplitude, but are 180° out of phase.

$$j \sum_{n=1}^{N} a_n \, \theta_n e^{jks_n u_m} = -\sum_{n=1}^{N} a_n e^{jkd_n u_m} \quad m=1,2,\ldots,M \qquad (5)$$

Use Euler's formula to put the exponent into real and imaginary form

$$j \sum_{n=1}^{N} a_n \, \theta_n \, (\cos(kd_n um) + j\sin(Kd_n um)) \qquad (6)$$

$$= -\sum_{n=1}^{N} a_n(\cos(kd_n u_m) + j\sin(kd_n u_m))$$

Next, equate the real and imaginary parts

$$\sum_{n=1}^{N} a_n\ \theta_n \cos(kd_n u_m) = \sum_{n=1}^{N} a_n \sin(kd_n u_m) \qquad (7)$$

$$\sum_{n=1}^{N} a_n\ \theta_n \sin(kd_n u_m) = \sum_{n=1}^{N} a_n \cos(kd_n u_m) \qquad (8)$$

Because $a_n \sin kd_n u_m$ is an odd function, it equals zero when summed from 1 to N. Thus, equation 7 equals zero. The second equation does not equal zero as long as θ_n is an odd function. Equation 8 can be put into the matrix form

$$Ax = B$$

where

$$A = \begin{bmatrix} a_1\sin(kd_1u_1) & a_2\sin(kd_2u_1)\ldots & a_N\sin(kd_Nu_1) \\ a_1\sin(kd_1u_2) & a_2\sin(kd_2u_2)\ldots & a_N\sin(kd_Nu_2) \\ \vdots & \vdots & \vdots \\ a_1\sin(kd_1u_M) & a_2\sin(kd_2u_M)\ldots & a_N\sin(kd_Nu_M) \end{bmatrix}$$

$$X = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \cdot \\ \cdot \\ \cdot \\ \theta_N \end{bmatrix}$$

$$B = \begin{bmatrix} \sum_{n=1}^{N} a_n \cos(kd_n u_1) \\ \cdot \\ \cdot \\ \cdot \\ \sum_{n=1}^{N} a_n \cos(kd_n u_m) \end{bmatrix}$$

This equation has more unknowns than equations. It can be solved using the method of least squares.

$$x = A^T(AA^T)^{-1}B \qquad (9)$$

The vector x contains the adapted weights θ_n that give M nulls in the direction of the jammers.

Figure 2 shows the far field pattern of a 20 element array with a 35 dB Taylor distribution $\bar{n} = 6$. The next figure shows the cancellation beams used to place a null in the pattern at 22° and 59°. In phase only nulling, a cancelling beam in the θ_m direction has a corresponding beam at $-\theta_m$. When these two patterns are added together the pattern in Figure 4 is obtained. This pattern has nulls in the desired directions. At $-\theta_m$ the cancellation patterns and quiescent pattern add in phase to raise the sidelobes of the resultant pattern in those directions.

Applying these phase shifts to the array in Figure 1 puts nulls in the sum pattern. These phase shifters are shared by both the sum and difference channels. A 35 dB, $\bar{n} = 6$ Bayliss amplitude distribution on a 20 element array has a far field pattern shown in Figure 5. The phase shifters, θ_n, change this pattern into the one in Figure 6. Nulls are not formed at the angles θ_m. In fact, the difference pattern has worse characteristics after the adapting. A similar analysis can be done for the difference pattern.

$$W_n = b_n e^{j\theta_n} \; ; \; b_n = \text{difference amplitude weights} \tag{10}$$

$$\approx b_n (1+j\theta_n) \tag{11}$$

The difference far field pattern is given by

$$D(U) = \sum_{n=1}^{N} b_n(1+j\theta_n) e^{jkd_n u} \tag{12}$$

At the angles θ_n, $D(u_m)$ is zero

$$j \sum_{n=1}^{N} b_n \theta_n \left(\cos(kd_n u_m) + j\sin(kd_n u_m)\right)$$

$$= \sum_{n=1}^{N} b_n\left(\cos(kd_n u_m) + j\sin(kd_n u_m)\right) \qquad (13)$$

Equating the real and imaginary parts gives

$$\sum_{n=1}^{N} b_n \theta_n \cos(kd_n u_m)$$

$$= \sum_{n=1}^{N} b_n \sin(kd_n u_m) \qquad (14)$$

$$\sum_{n=1}^{N} b_n \theta_n \sin(kd_n u_m) = \sum_{n=1}^{N} b_n \cos(kd_n u_m) \qquad (15)$$

Unlike the sum amplitude distribution, the difference amplitude weights are an odd function. Instead of equation 14 going to zero, equation 15 equals zero. Likewise, this equation may be put into matrix form and solved for the adaptive weights, θ_n. Some results are shown in Figures 7 and 8. Figure 9 shows the difference adapted weights applied to the sum pattern. Again, the desired nulls do not appear.

In order to simultaneously place nulls in the sum and difference patterns, one set of adaptive weights could be placed in the sum channel, while another is placed in the difference channel. This method calls for an extensive duplication of hardware. In addition, phased arrays are normally built with one set of phase shifters that are shared by both channels. This technique could not be readily implemented on existing antennas. These problems can be overcome by using a special technique that simultaneously places nulls in the sum and difference patterns using the one set of phase shifters shared by both channels. Such a technique is described in the following section.

3. Simultaneous Nulling in Sum and Difference Patterns

Equations 8 and 14 hold true for placing nulls in the sum and difference patterns. Rather than solving these two systems of equations separately, they are combined into one system of equations. The resulting matrix equation Ax = B has the components

$$A = \begin{bmatrix} a_1 \sin(kd_1u_1) \ldots & a_N \sin(kd_Nu_1) \\ \vdots & \vdots \\ a_1 \sin(kd_nu_m) \ldots & a_N \sin(kd_Nu_m) \\ b_1 \cos(kd_1u_m) \ldots & b_N \cos(kd_Nu_1) \\ \vdots & \vdots \\ b_1 \cos(kd_1u_m) \ldots & b_N \cos(kd_Nu_m) \end{bmatrix}$$

$$x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

$$B = \begin{bmatrix} \sum_{n=1}^{N} a_n \cos(kd_n u_1) \\ \vdots \\ \sum_{n=1}^{N} a_n \cos(kd_n u_m) \\ -\sum_{n=1}^{N} b_n \sin(kd_n u_1) \\ \vdots \\ \sum_{n=1}^{N} b_n \sin(kd_n u_m) \end{bmatrix}$$

The least mean square solution to this equation yields a θ_n which has nulls in both the sum and difference patterns. The previous cases run for the sum and difference patterns were tried again for the new technique. The results appear in Figures 10 and 11. These patterns were obtained by placing a phase shift of θ_n on the phase shifters of the array in Figure 1.

[illegible] Conclusion

The technique described in this paper is only theoretical and not meant for direct implementation. However, it does draw attention to the need for simultaneous nulling in the sum and difference channels of a monopulse antenna. Nulling only in the sum channel is not adequate. Also, the techique developed shows that it is theoretically possible to simultaneously null in both the sum and difference patterns using one set of adaptive weights. Even though this method of nulling is theoretical, it has potential for practical implementation. For instance, an adaptive loop could be used to adjust the height of the cancellation beams for a non ideal pattern. In this way the nulls are adaptively formed rather than synthesized.

References

1. Baird, Charles A. and Rassweiller, George G. "Adaptive Sidelobe Nulling Using Digitally Controlled Phase Shifters". IEEE Trans. Ant. and Prop. Vol. AP-24, No. 5, Sep 1976, pp 638-649.

2. Shore, Robert A. "Nulling in Linear Array Patterns with Minimization of Weight Perturbations." RADC report to be published.

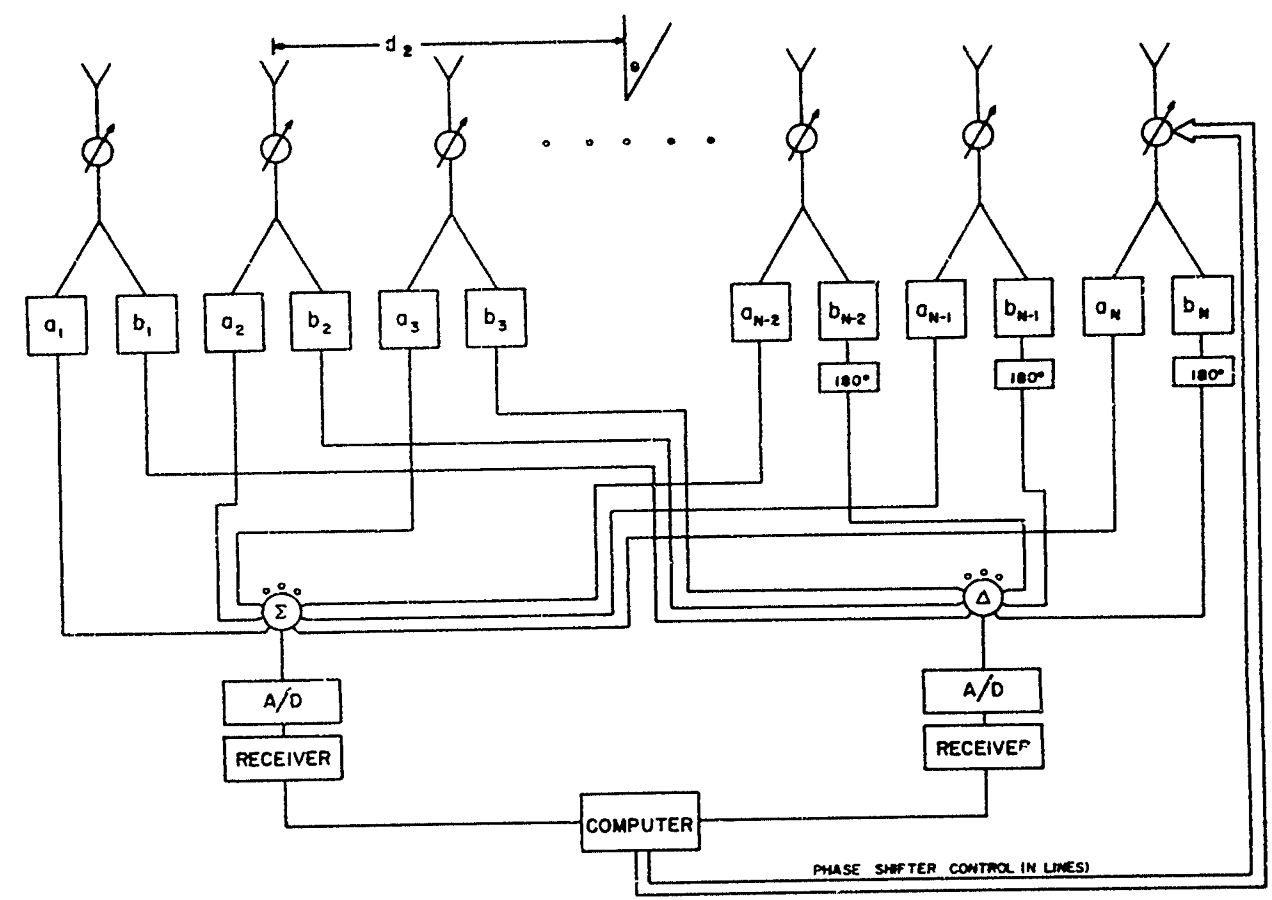

Fig. 1 Monopulse Phased Array

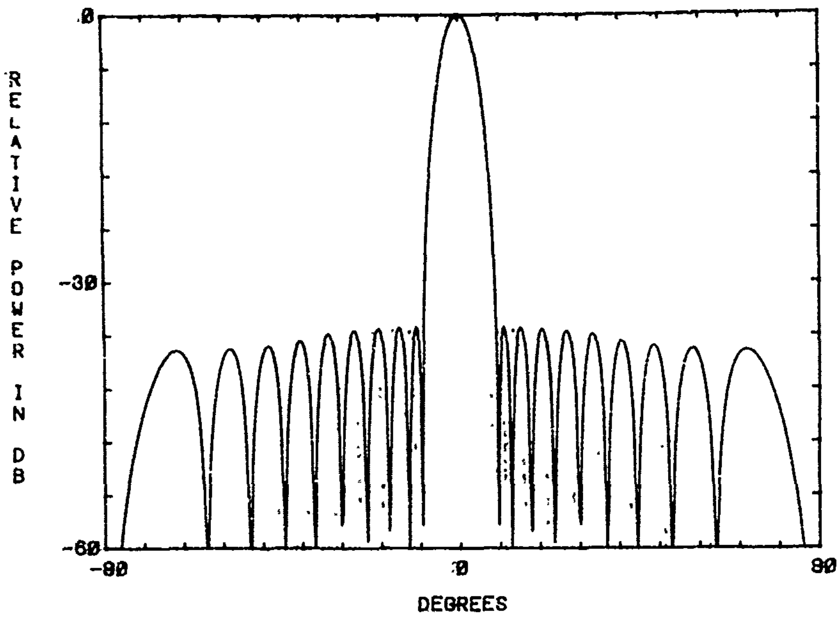

Fig. 2 Quiescent Far Field Sum Pattern

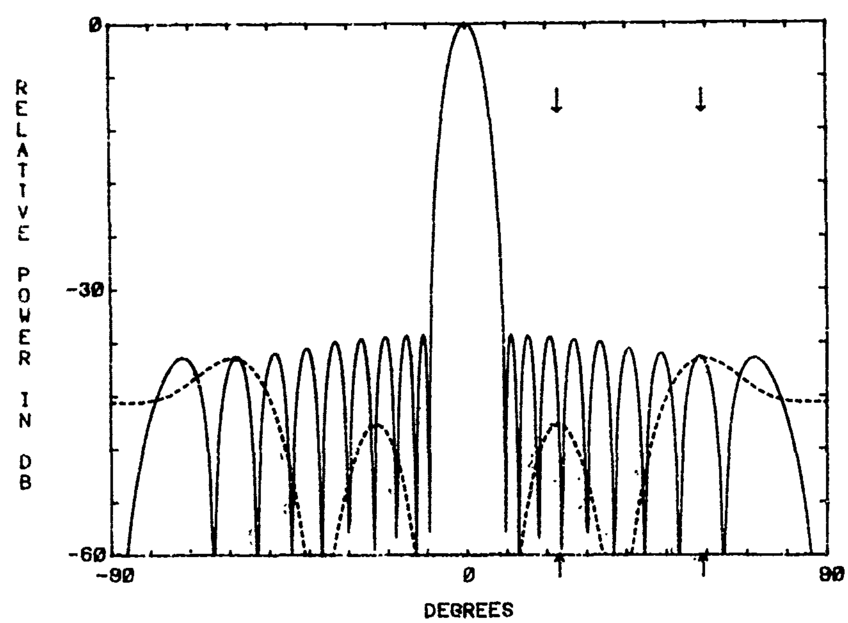

Fig. 3 Sum Pattern and Its Cancellation Beams

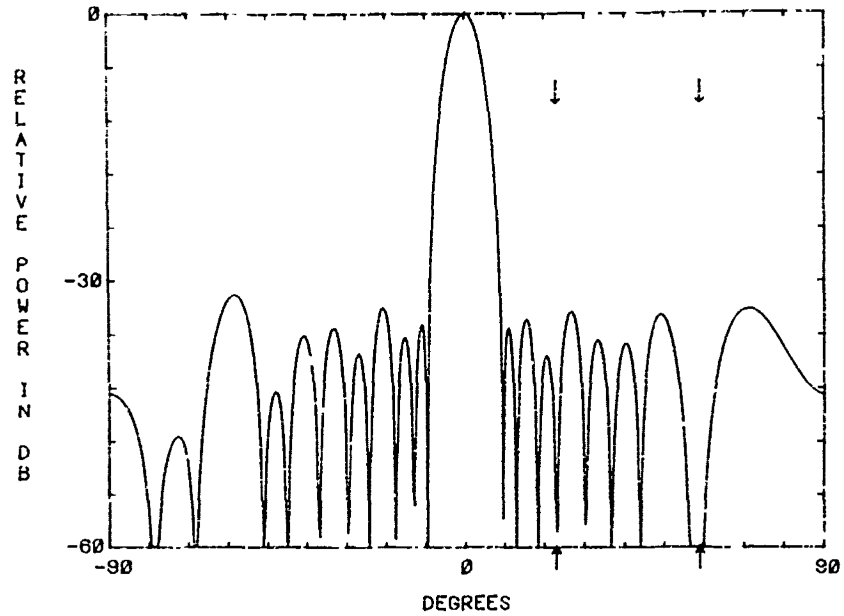

Fig. 4 Result of Adding Quiescent Pattern and Cancellation Beams

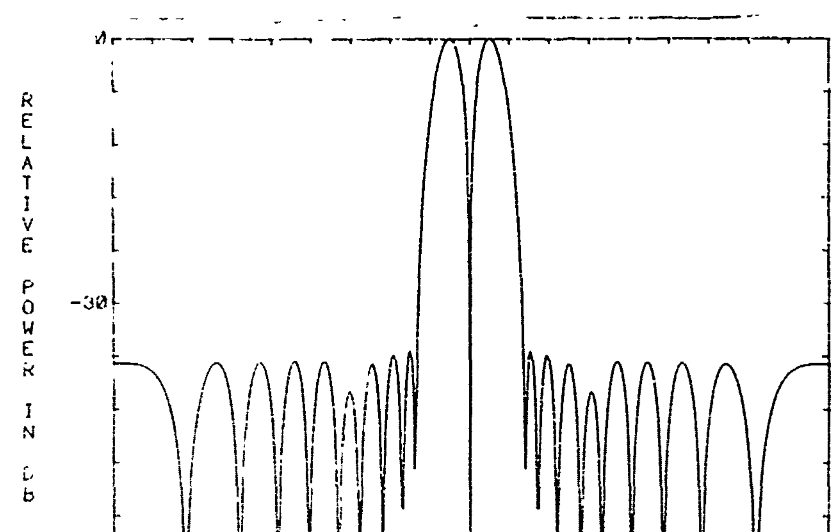

Fig. 5 Quiescent Far Field Difference Pattern

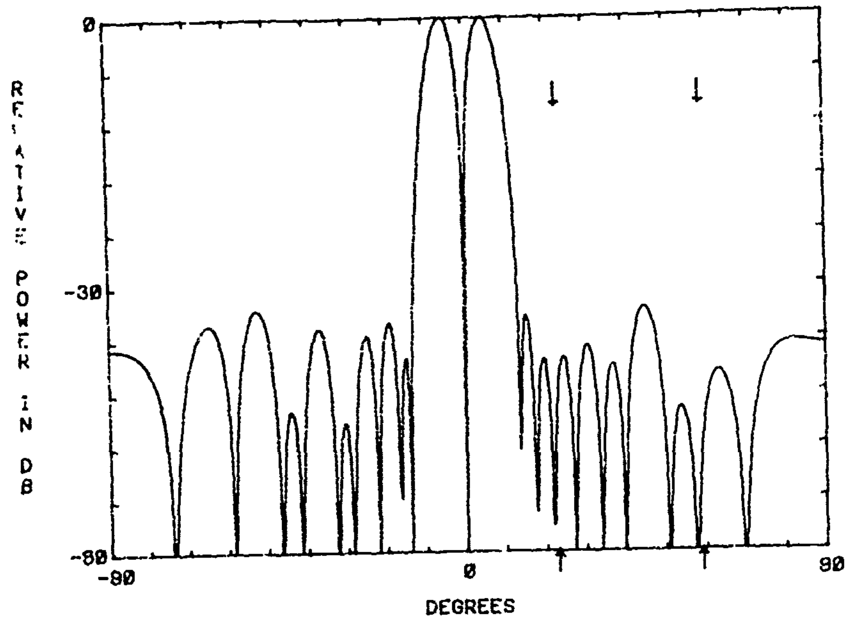

Fig. 6 Far Field Difference Pattern
When Phase Shifters are
Adjusted for Nulls in the
Sum Channel

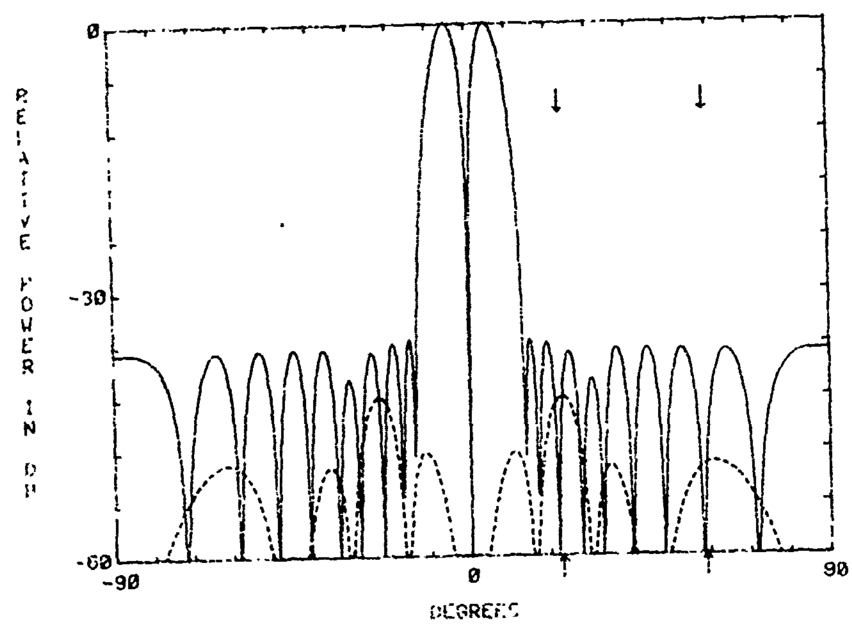

Fig. 7 Difference Pattern and Its
Cancellation Beams

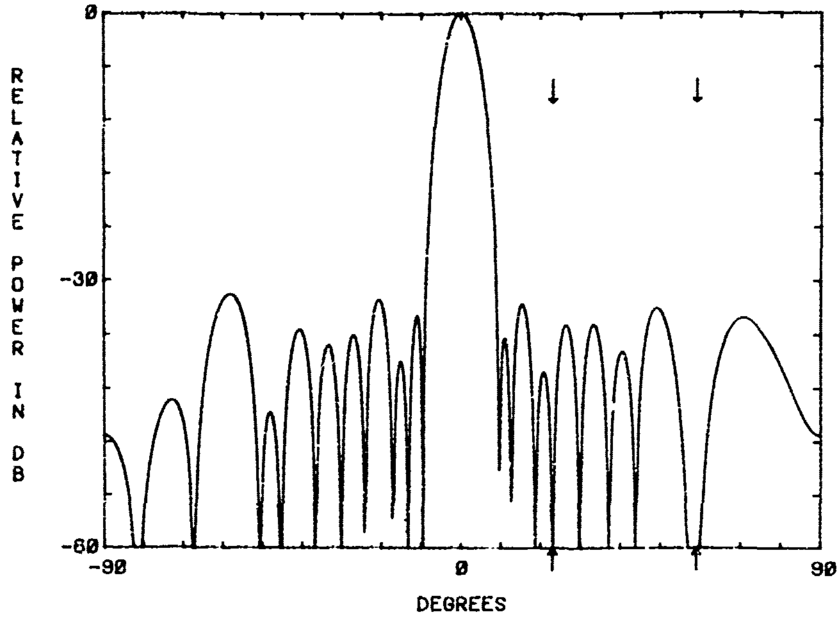

Fig. 8 Result of Adding Quiescent Difference Pattern and Difference Cancellation Beams

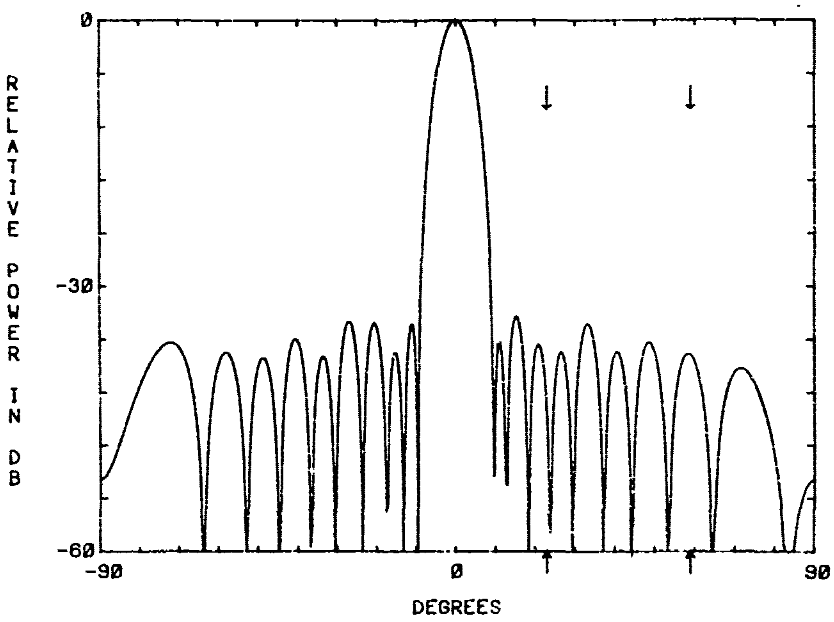

Fig. 9 Far Field Sum Pattern When Phase Shifters Are Adjusted for Nulls in the Difference Channel

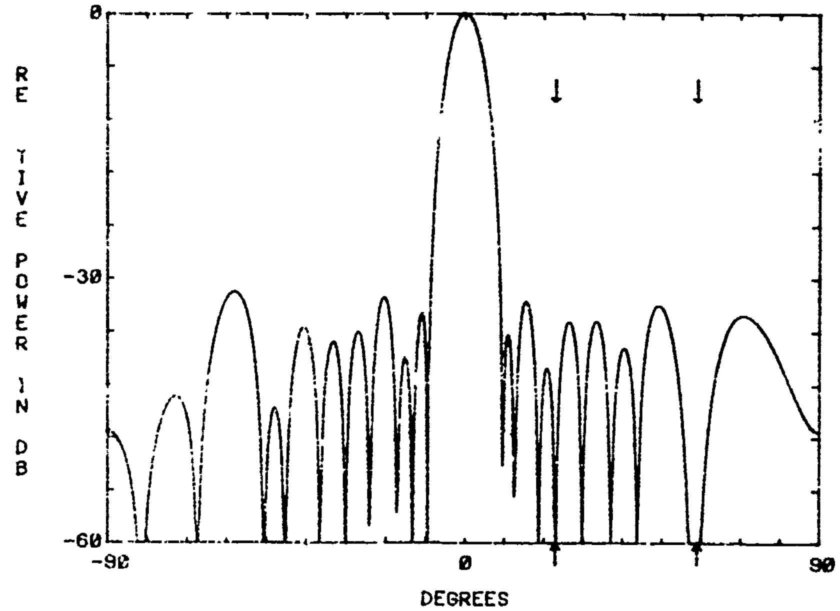

Fig. 10 Far Field Sum Pattern
Resulting from Simultaneous
Nulling

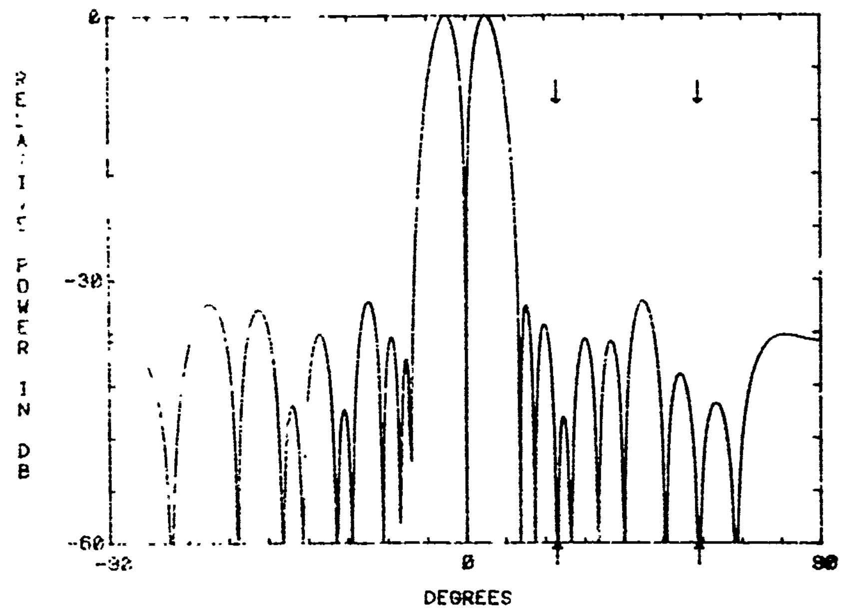

Fig. 11 Far Field Difference Pattern
Resulting from Simultaneous
Nulling