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AVOIDANCE OF SONAR ARRAY AMBIGUITY [R]

BY

D. E. WESTON

JULY 1976

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The response of an electrically steered linear array of omnidirectional sensors shows two ambiguous beams. Cross-fixing with two such arrays can lead to a multiple choice with up to four possible target positions, so we try here to find the best relative orientation for the arrays. Consideration of the areas of unambiguous cover clearly shows this best orientation to be a right angle, with a steady deterioration as the parallel arrangement is approached. Building up the right-angle forms into crosses shows that cover is good for the so-called Potent Cross, Fylfot and Swastika but poor for the Greek Cross. With larger fields of arrays the Fylfot field is very well behaved indeed, but the Potent Greek is undesirable. Very near a coast some tracks are implausible, and the best arrangement for a pair now involves that array lying next to the coastline being parallel to it, with the second array pointing at the first.

With a curved array the directional ambiguity is reduced, the effectiveness in removing it varying as the array width measured in wavelengths. Instead there is sometimes a focussing ambiguity, and it pays to put the concave side of the array facing the direction of greatest interest.
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INTRODUCTION

A sensor array in a wave field can be used to estimate the direction of a noise source. But such an array may have a high response in more than one direction. If two arrays are used to locate a target by cross-fixing, this multiple response effect may lead to ambiguity in the estimate of position. We are thinking here of passive sonar arrays, perhaps used for surveillance, though this is not necessarily the only application. This paper tackles only one part of the problem, the formal or geometrical part. It takes some account of the influence of topography but little of such practical matters as location accuracy. It concentrates on the ambiguity fields of patterns of straight-line arrays, though a connection with the theory of curved arrays is also explored.

This paper is one of four treating successive steps in the conceptual design of an array system -

(a) The decision on scale: should there be a few large arrays or many small arrays [1]

(b) The detailed spatial arrangement of the array positions [2]

(c) The relative orientations of the arrays to avoid ambiguity, as discussed here

(d) The relative orientations of the arrays allowing also for location accuracy [3].

Of course these steps are not entirely independent, the answers have to be modified to allow for the topography of any area of interest, and there are many further stages in the full design.

2 RELATIVE ORIENTATION OF TWO STRAIGHT ARRAYS

Consider the response of a single array with omnidirectional sensors or hydrophones arranged along a straight line. For a source, nominally at infinity, the array can be steered to give a full response by introducing electrical delays so that the arrivals at all the sensors are in-phase. But the same response will then be obtained over a cone of angles centred on the physical axis of the array. We will deal here with the two-dimensional case, when the cone reduces to a pair of bearing lines. Our array obviously cannot tell left from right, the standard comparison is with a learner driver. We will ignore the possibility of high responses at further angles, which can occur with equally-spaced sensors and single high frequencies.

One way to help is to introduce a second array. The example in figure 1(a) shows that this can be completely successful, not only can each array sort out its real beam from its image beam, but between them they can fix the precise position of the target. Figure 1(b) shows that it does not always work like that. With a different geometry there are now four beam intersections or possible target locations. In fact in the general case there may be 0, 1, 2, 3 or 4 beam intersections for given sets of steering angles.
In this section we wish to tackle the problem of how best to choose the two array orientations in order to maximise the area in which a target may lie without ambiguity arising, and this involves finding a good criterion to describe the ambiguity. Note that there is a two-parameter family of orientations, since each array pair may be defined by a centre spacing plus the two array angles given relative to the direction of the line joining the array centres. If we were to decide to look with 15° steps this would imply 12 orientations, since 180° rotation does not affect the arrays. The full family contains 144 cases, but if we reject mirror image and upside-down repeats the number of truly different cases is 43. This is a sufficiently large number to frighten us off, and instead we list in table 1 a number of cases with special features. The first five entries have one special feature which defines a whole class of cases. The remaining four are the only ones which combine two or more special features, and occur only with specific orientations. The table also lists for each geometry the possible numbers of beam intersections. Some of the beam intersection numbers have been enclosed in brackets to show that they are limiting cases, on which there is further discussion below.
The figure for the highest possible number of beam intersections suggests itself as one possible criterion, and this points to classes A and D as best. But in the light of our further discussion this is not considered a good criterion, and the choices not particular good. This is because there is no indication of how often the different numbers of intersections occur. In addition it is thought that the main distinction should lie between one unambiguous intersection and more than one intersection, the differences among 2, 3 and 4 crossings having less significance.

The next step was therefore to prepare some plots showing the area where there is just one intersection, i.e. no ambiguity. These have been arranged in figures 2 to 6 according to the various Table 1 classes, so that although there are only eleven different plots some appear in more than one figure. These plots constitute one of the main results of this paper. A swift glance at these figures shows a surprising degree of complication, as exemplified in the following notes on their construction.

(i) The boundary between the ambiguous and unambiguous regions is sometimes marked by a straight line, corresponding to a given beam or its image beam, when the given beam passes through the centre of the other array.

(ii) The boundary is otherwise formed by the curved locus of the intersection point, with the condition that the image point is formed at infinity. This implies that two of the beams are parallel. The curved forms can be sketched quite quickly using simple geometrical constructions, and the resulting shapes are often quite familiar. One plot may show a curved boundary with several branches.

(iii) As a limiting case the unambiguous area can shrink to a mere line, though the central line for the 90°/90° tram-line arrays (figure 2, etc) is the only example found.

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<td>Parallel</td>
<td>0 (1) 2</td>
</tr>
<tr>
<td>B</td>
<td>Symmetrical</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>C</td>
<td>Right-angle</td>
<td>0 1 2 4</td>
</tr>
<tr>
<td>D</td>
<td>One 0° array</td>
<td>0 1 2</td>
</tr>
<tr>
<td>E</td>
<td>One 90° array</td>
<td>0 1 2 3 4</td>
</tr>
<tr>
<td>ABD</td>
<td>In-line arrays, 0°/0°</td>
<td>0 2</td>
</tr>
<tr>
<td>ABE</td>
<td>Tram-line arrays, 90°/90°</td>
<td>0 (1) 2</td>
</tr>
<tr>
<td>BC</td>
<td>Right symmetrical, 45°/135°</td>
<td>0 1 2 4</td>
</tr>
<tr>
<td>CDE</td>
<td>Tee arrays, 90°/0°</td>
<td>0 1 2</td>
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Table 1. Special cases with two straight-line arrays
FIG. 2 AMBIGUITY PLOTS FOR CLASS A OR PARALLEL PAIRS OF ARRAYS
FIG. 3 AMBIGUITY PLOTS FOR CLASS B OR SYMMETRICAL PAIRS OF ARRAYS

ARRAY, SHOWING ORIENTATION

LINE JOINING ARRAY CENTRES.
REFERENCE DIRECTION

AREA OF NO AMBIGUITY
BOUNDARY OF AMBIGUITY AREA
ASYMPTOTE FOR BOUNDARY

POINT OF NO AMBIGUITY
LINE OF NO AMBIGUITY

90° ± 90°
57.5° ± 111.2°
35.4° ± 135.4°
22.5° ± 22.5°
0° ± 22.5°
Fig. 4 Ambiguity plots for class C or right-angle pairs of arrays.

- Line of no ambiguity.
- Point of no ambiguity.
- Area of no ambiguity.
- Boundary of ambiguity area.
- Asymmetry for boundary.

Array showing orientation.

Line joining array centres, reference direction.
FIG. 5 AMBIGUITY PLOTS FOR CLASS D PAIRS ONE ARRAY AT 0°
FIG. 6 AMBIGUITY PLOTS FOR CLASS E PAIRS. ONE ARRAY AT 90°

ARRAY SHOWING ORIENTATION
LINES JOINING ARRAY CENTRES.
REFERENCE DIRECTION

LINE OF NO AMBIGUITY
POINT OF NO AMBIGUITY

AREA OF NO AMBIGUITY.
BOUNDARY OF AMBIGUITY AREA
ASYMPTOTE FOR BOUNDARY

90°/90°
45°/90°
0°/90°
(iv) As one approaches the end-fire condition the two beams will coalesce, and two of the intersections will also coalesce. We can get another sort of line of no ambiguity, of which there are many examples.

(v) As a further specialisation of (iv), if we take the end-fire condition for both arrays we get a point of no ambiguity, with several examples.

(vi) If the target is on the line passing through both array centres we may be able to determine that it is indeed on the line, or a given part of the line, but unable because of the angle of cut to locate it accurately. One can argue about whether this inability is due to ambiguity or to lack of accuracy, in any case we have made no special marking.

We can base our second criterion on a qualitative examination of the unambiguous areas in figures 2 to 6. In figure 2 the parallel class all agree in showing no unambiguous area, a clearcut but not very encouraging result. In figure 3 the symmetrical class show a steady improvement as one goes from one extreme of parallel geometry up to the right-angle formation, and then a decline down to the other extreme parallel geometry. For the right-angle class in figure 4 the plots differ considerably in character, but there is always some symmetry between the ambiguous and unambiguous areas, i.e. 50% unambiguous. The symmetry in the general right-angle case, represented by the 30°/120° plot, provides proof of this 50% relation. In figure 5 with one array at 0° the unambiguous area increases as the second array angle approaches 90°. Note for this class that the uncertainties near the line passing through the arrays (vi above) will be worsened because the 0° array will then be steered to endfire. In figure 6 with one array at 90° the unambiguous area increases as the second array approaches 0°.

These figures all tell a consistent story, that the highest attainable proportion of unambiguous area is one half, occurring for the class C or right-angle pairs. This good behaviour is not critical, we need not insist on a precise right angle, but there is a steady deterioration as we approach the parallel condition. It is difficult to describe this deterioration by any mathematical law. If we deal in terms of area it is the area at long ranges which controls the result, and this depends on the boundary asymptotes. For example in figure 3 the non-ambiguous area is 0% for the limiting case 0°/0°, jumps to 50% for a small departure from 0°/0° and stays at 50% till the array angles reach 60°/120°, then falls linearly with array angle till one reaches 0% at 90°/90°. In figure 6 the area falls linearly from 50% at 0°/90° to 37½% at 45°/90°, and then with a steeper linear slope to 0% at 90°/90°. But we should obviously give different weightings to the different areas depending on their distance from the arrays.

3. EFFECT OF A NEARBY COASTLINE

The particular pair that will do best depends on the application, but it seems reasonable to think first of the class C or right-angle pairs and look at the two extreme examples in figure 4. The 0°/90° tee array configuration has the general attraction of a very simple ambiguity plot. The symmetrical 45°/135° pair seems to be an obvious choice for installation near a coastline, since the whole area in front (i.e. to the right) of the line joining the
arrays is unambiguous, and at long ranges this area opens out in a $90^\circ$ angle. Close in the angles of cut tend to be good, just where the bearings are close to the natural acoustic axes.

This $45^\circ/135^\circ$ choice could be a good one if the important area is that in front of the arrays, and there is an area of significant size but lesser importance between the arrays and the coastline. However if the arrays are very close to the coast the presence of the land will rule out some possible target locations, and may change some of the ambiguity plots so that other array pairs become a better choice. This effect is not restricted to coastlines, it can arise if there are patches of water so shallow that a target could not or would not go there, or if the presence of targets is ruled out by other information. Coastline effects will be modelled here as straight-line boundaries, constrained to pass through one or both arrays.

One simple case to consider is the boundary passing through both arrays of the pair, ie appearing as a vertical line in figures 2-6. This case divides into two, with the area of interest on either the right or the left. In another simple case the boundary is orthogonal to the above, appearing as a horizontal line in the figures. If this boundary goes through the upper array the area of interest has to lie beneath it and embrace the lower array, if it goes through the lower array the relevant area embraces the upper array. Thus there are four special cases, and these have been used for sample studies on figures 2-6. The effects on the plots can be determined almost by inspection. The presence of the coastline obviously cannot reduce the unambiguous area: it is found that sometimes there is no change, sometimes there is a partial improvement, and sometimes the whole of the area of interest becomes unambiguous.

Let us start with the vertical boundary and go back to our $0^\circ/90^\circ$ and $45^\circ/135^\circ$ examples. We find there is no change looking either to the right or left, though it should be noted that for the $45^\circ/135^\circ$ pair the unambiguous cover is more than 50% looking right and less than 50% looking left. Again there is no change for the $90^\circ/90^\circ$ pair (figure 2, 3 or 6), but here the whole area remains ambiguous. As an example of a partial improvement take the $45^\circ/45^\circ$ pair of figure 2, where the change on either side is from 0% unambiguous to rather less than 50% unambiguous. For the $0^\circ/0^\circ$ pair (figure 2, 3 or 5) the unambiguous area changes from 0% to 100%. The above examples are sufficient to demonstrate that the right-angle pairs are not the best, and that the class A or parallel pairs are no longer uniformly bad.

We will take only one example for a horizontal coastline, that through the lower or $90^\circ$ array of the $0^\circ/90^\circ$ pair. In the area of interest embracing the upper or $0^\circ$ array the partial unambiguous cover improves to 100% unambiguous cover. This case gives us the final clue needed in order to write out the recipe for 100% unambiguous area.

First we must put one array next to the boundary and parallel to it so that one of its two ambiguous beams always points to the land and may be disregarded. Second the other array may be placed anywhere in the field but must point at the first array; the two beams of this other array pass on either side of the first array and only one of these beams can result in an intersection. There is admittedly a special case if the target lies on the line joining the two arrays, or on its extension, but this does not affect the figure of 100% for the unambiguous area. The above specification appears to be both necessary and sufficient. Note that it is a definition
of the class D pairs with one array at $0^\circ$ (figure 5), where the boundary must go through and parallel the lower array so that it may appear sloping in the figure.

As practical choices the $45^\circ/135^\circ$ pair still looks good if one is reasonably near a coastline, and either the $0^\circ/0^\circ$ or $0^\circ/90^\circ$ pair if very near the coastline. Considerations of cabling and location accuracy do not show a clear general superiority for one or other of the last two. The $0^\circ/0^\circ$ pair with both arrays near the coast seems the more obvious one, and there may be greater security for the arrays. But the possibilities of the less conventional $0^\circ/90^\circ$ arrangement should not be overlooked, laying the second array away from the coast might put it in water with better sound transmission characteristics.

The coastline effect has introduced the idea of track possibility, which can be extended to the idea of track plausibility and used as the basis for a third criterion. Apart from any special topographic effects, if an intersection is at such a long range that any target is unlikely to be detected it can virtually be ruled out. If we are able to observe over a period we can track, and the real track must correspond to a realistic speed, and often to a steady course and speed. Tracks with very high speeds may be identified as ghost tracks and disregarded; and tracks with curvature or varying speed will be suspect, especially if they are in competition with steady tracks. Note that for ghost tracks the range and speed are linked, a large apparent distance will usually correspond to a high apparent speed.

Analysis of these points must also allow for the finite bearing accuracy of the arrays, which gets worse as one approaches end-fire. Note in passing that the rejection of long-range targets and the allowance for finite bearing accuracy both mean that the lines and points of no ambiguity in figures 2 to 6 will grow a little, both in area and in importance.

No systematic work has been done on these aspects of track plausibility, but we note for example that with the $0^\circ/0^\circ$ in-line geometry (figure 2 etc) the two possible tracks will be exact images of one another, and the idea does not help at all. It would be highly desirable not only to pursue the second criterion (ambiguity plot) for each of the 43 cases mentioned earlier, but to extend this to the third criterion (plausibility). Consideration of tracks has introduced more parameters into the problem and greatly increased the potential labour. For this and more general reasons it would regrettably be necessary in this next stage to hand things over to the tender care of a computer.

4. Patterns with Several Arrays

We will move on now to consider the ambiguity plots for patterns of arrays far from coastlines, initially for small numbers of arrays and specifically for four. We will continue, at least for a while, to take the arrays in nearest-neighbour pairs as regards cross-fixing. The pairs considered, as at the end of section 2, are the $45^\circ/135^\circ$ and the $0^\circ/90^\circ$. With four arrays we can construct four different regular cross formations, as named and illustrated in figure 7. There is of course a degree of arbitrariness about the names, and for example the Fylfot may also be called the swastika, anticlockwise swastika, gammadion, gammadion or Buddhist Cross. The absolute orientation is not important here.
<table>
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<th>NAME</th>
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<th>GREEK CROSS</th>
<th>FYLFOT</th>
<th>SWASTIKA</th>
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<td></td>
<td></td>
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**FIG. 7 PATTERNS WITH FOUR ARRAYS**
Before proceeding further we need to simplify the ambiguity plots (see eg figure 4), and in particular to show the effective unambiguous cross-fix area as limited by the 50% detection range. This range must be comparable to the array separation: if it is much less there is little chance of cross-fixing, and if it much more the system design has been uneconomic [2]. Figure 8 shows the cross-fix areas, before any ambiguity considerations are introduced, for two different detection range assumptions.

**Assumption (a)**
Detection range = separation

**Assumption (b)**
Detection range = \( 1\frac{1}{2} \times \) separation

**FIG 8 NOMINAL CROSS-FIX AREAS**

The nominal areas have then been squared up, as shown. This is not really too outrageous a procedure, because the 50% detection range is only a statistical figure defining one point on a gently-sloping curve of detection probability versus range, there is no sharp cut-off. Note that the 50% increase in range has increased the cross-fix area by a nominal factor of 4.
1 and 2 refer to the number of unambiguous cross-fix pairings.
Figure 9 shows the unambiguous cross-fix areas for both figure 8 assumptions applied to the 45°/135° right symmetrical pair, and how the unambiguous cross-fix cover builds up for the Potent and Greek Cross patterns. Unambiguous cross-fixes may sometimes be possible from more than one pair of arrays, as indicated, it is not the same thing as the number of arrays detecting. Figure 10 is the equivalent for the 0°/90° tee arrays, giving the Fylfot and Swastika patterns. It is interesting to see how the Fylfot ambiguity plot mimics the Swastika, and vice-versa. On assumption (b) the plot for the Greek Cross turns it into a quite passable Maltese Cross. The assumption (b) plots tend to be larger-scale versions of those with assumption (a), i.e. their important characteristics are generally similar and they can be lumped together for much of the following discussion.

The main object in preparing figures 9 and 10 was to compare the ambiguity plots for the different cross patterns, qualitatively the cover gives us a fourth criterion. The Potent Cross is certainly the best as regards the area within the cross or square of arrays, which would often be the area of most importance. It is poor outside the cross, especially near the sides. The Greek Cross pattern tends to be complementary, completely lacking in the centre and along the diagonals, but good along the side. This ambiguity pattern does not have obvious applications, though it could perhaps be useful for arrays spaced round an island. The Fylfot and Swastika plots are of course mirror images of another, their great advantage being a relatively even distribution of cover.

Versions of these cross patterns could obviously be invented for other regular polygonal forms, such as triangles and pentagons. One approach preserves symmetry - with all arrays tangent to the circumscribed circle, or all pointing to the centre, or all pointing to the next one round. Another approach keeps the right-angle relation between neighbours, possible with an even number of arrays as in the hexagon. As one moves round such a hexagon the character changes from Potent Cross to Fylfot to Greek Cross, it is three-in-one, truly a chimera. One can cope with small irregularities in the polygons in similar ways.

We can go still further and build the cross patterns up into array fields of indefinite extension. Thus figures 11 and 12 show samples taken from much larger fields. The result is a square lattice, which ref 2 shows to be superior to a triangular lattice as regards cross-fixing. Note that to get full symmetry in such a square lattice one has to start with right-angle pairs. The assumed detection ranges and cross-fix areas are as in part (a) of figures 8, 9 and 10. For any individual square there are 4 pairs of arrays which may give cover, for the class C right-angle pairs half the cover is unambiguous, so the average number of unambiguous cross-fix pairings should be 2. As a fifth criterion we may look to see how even the cover is.

If we start in one corner of figure 11 with a Potent Cross we find that this automatically ensures that there is a Greek Cross next to it, in fact they alternate. We do not have Potent fields and Greek fields, only Potent Greek fields. Unfortunately the unambiguous area for the Greek Cross is such that it does not look after its own centre, but merely reinforces the cover within the neighbouring Potent Crosses. The result is a chessboard pattern in which the unambiguous pairing alternates between 0 and 4. Plainly the Potent Greek is undesirable.
FIG. 11 POTENT GREEK ARRAY FIELD

FIG. 12 FYLFOT ARRAY FIELD
In figure 12 we start with the anticlockwise or Fylfot form, leading to the clockwise or Swastika form next door, and then continuing to alternate. The covers for these forms dovetail together to give us 2 unambiguous pairings throughout. This remarkable result makes the fylfot fields particularly suitable for large areas.

Let us now look at assumption (b), with longer detection ranges. The ambiguous areas in figure 11 are still there, though they shrink somewhat to a roughly circular shape. The cover in figure 12 is still uniform, though now there are 8 unambiguous cross-fix pairings throughout. The conclusions on the superiority of Fylfot fields are unchanged.

Note that regular fields of the above types cannot be built using symmetrical array patterns in triangles, pentagons or hexagons - only squares or crosses are suitable. The asymmetrical pattern of the chimaera hexagon is also suitable.

So far the arrays have been considered in pairs, which has been convenient for the present analysis. But in any practical location system one would consider together the information from all the arrays successful in making detection, and either physically or in effect plot bearing lines from all these arrays to see where all the lines intersect. With three or more arrays there would normally be no ambiguity. The present nearest-neighbour analytic approach is valid for the lower ranges (compare assumption a) where there would often be only two detections, this is for our present purposes the "worst case" and therefore the one to which we should pay most attention. Remember now that the 50% detection range describes a probability rather than specifying a cut-off range, and unfortunately this means that the answers on the number of detections etc can never be clear-cut. Thus although the present analysis is thought valuable it would certainly be worthwhile to set up a further analysis leading to a sixth criterion based on multiple detections - though this could prove to be quite complicated.

Note that this paper and the various criteria discussed are basically concerned with choosing the best design for a system, rather than the organisation for making the best use of a given system. Many important factors have not been brought in, eg knowledge of track history can make up for an uneven distribution of non-ambiguous cover.

5. CURVED ARRAYS

Ambiguity may also be removed by complicating the array, ie giving it some width. There are various ways possible of achieving this, but we attempt a discussion here without considering either the general design or the engineering difficulties. We will not even discuss the pros and cons of trying to do it at all. However it is convenient to treat the geometry of the simple curved array, which array has the advantage of requiring no complication in the sensors and probably no increase in their number. The main reason for the discussion here is a link with the ideas on linear arrays, and in addition we shall see how directional ambiguity for the linear array is turned into a focussing ambiguity for the curved array.

Let us start with an example that demonstrates the effects for a case
where the geometry is exact. Figure 13 shows a parabolic array receiving sound from a distant target, lying to the left on the centre line of the parabola. In order to add the sensor signals coherently we must introduce electrical delays, the relative electrical advance increasing roughly as the square of the departure from the vertex. But these same electrical delays will also work for a target at the focus of the parabola. There is an obvious optical analogy here, the light from a distant source on the right would be brought to a focus at the same point by a similar parabolic mirror. There is a continual recurrence of this analogy with mirrors and lenses for light, or indeed for other forms of radiation including acoustics.

The focussing may be explained in either of two equivalent ways. First we may recall that it is a property of the parabola that the length of the light path from the distant right-hand source to the focus is a constant, independent of the path and the point of reflection. Similarly for our array, the greater path lengths to the focus from the more extreme points are balanced precisely by the greater electrical advances. Second we may concentrate on a small element of the parabola, when the equality of the incidence and reflection angles ensures that the light ray will pass through the focus. Similarly a small element of array may be treated as linear and having simple directional steering, the equality of angles for the two ambiguous beams again makes one beam pass through the focus. This is the link, referred to earlier, with the subject matter of the previous sections.
To examine the general case we will now abandon precision, assume that the array lengths are small compared to their radius of curvature, and make no distinction between parabolae and arcs of circles. The ideas should still work for beams steered well away from the natural acoustic axis, since within limits beam steering and focussing are independent operations, compare the optical case and some specific array analysis by Welsby [4].

Our calculations can be mainly in terms of curvatures. Let the physical curvature of the array be $C_p$, taken as positive when concave to the right (as in figure 13). When acting alone this will produce quite different effects to the right and left. The quadratic variation of electrical delay may be thought of as introducing an effective curvature $C_e$. This is taken as positive when the delay decreases as one moves away from the array centre (as for the application in figure 13). When acting alone this will cause a convergence or focussing, the same to both right and left.

If both curvatures are present the total on the right is $(C_e + C_p)$ and the focus distance if the total curvature is positive is simply

$$U = (C_e + C_p)^{-1}. \quad (1)$$

Similarly on the left the focus distance

$$V = (C_e - C_p)^{-1}. \quad (2)$$

Combining these gives the familiar optical formula

$$\frac{1}{U} + \frac{1}{V} = \frac{2}{R_e}. \quad (3)$$

Note that it is the radius of curvature $R_e$ associated with the electrical delays that appears in this formula, not the physical radius $R_p$. Thus equation (3) is interesting but perhaps not very useful, since, with target movement, $R_p$ is of course unchanged but $R_e$ will be altered.

The general case has three branches. If $C_e > |C_p|$ there are two foci, as shown in figure 14. Here the curved solid line shows the physical run of the array, and the dashed banana lines show the effective electrical modifications. If $|C_e| < |C_p|$ only one of the total curvatures is positive and there is a focus on only one side. On the other side the beams from the separate elements of the array diverge. If $C_e < -|C_p|$ there are no foci.

There are three special cases, all touched on already. If $C_e = 0$ we will get one focus due to $C_p$ on the concave side of the array, and in this case the geometry is exact for an array consisting of a circular arc. If $C_p = 0$ the $C_e$ curvature, depending on its sign, will produce either two foci or none. If $C_e = C_p$ one of the foci will move to infinity, and it is
important which one moves. For the case shown in figure 13 \( C_e \) is defined as positive, the infinity is on the convex side of the array, and we have the extra focus on the concave side. But if \( C_e \) is negative the infinity comes on the concave side, and there is no extra focus.

For a practical surveillance system these last points are of importance. If the convex side of the array lies towards the real target at infinity, the extra focus could be a nuisance, even though its range of \( R_e/2 \) or \( R_p/2 \) might be low enough for it to be recognized as false. As the real target approaches the array so \( C_e \) must increase, the false position will also approach closer, and there is slightly less chance of confusion. If the concave side of the array lies towards the real target at infinity there is, very satisfactorily, no ambiguity. As the real target approaches the array so \( C_e \) must increase, until at a real range of \( R_p/2 \) a false focus position appears at infinity. As the real target continues to approach so will the range to the false focus decrease, though one hopes that the strength of signal from the real target would indicate the real position. It is concluded that any curved array should if possible be arranged with its concave side towards the direction of greatest interest.

The degree of importance of all the matters discussed above depends on the numerical values in the particular application, and we must now introduce the wavelength \( \lambda \). It is especially significant how the likely target ranges compare with the Fresnel distance, which has order of magnitude \( L^2/\lambda \) where
L is array length. The Fresnel distance is a measure of the range where the acoustic near field gives way to the far field, the necessary electrical delays for this distance differing only a little from those for infinity. For target ranges much shorter than the Fresnel distance it is necessary to focus as well as to steer. The Fresnel distance is also a measure of the limit beyond which range cannot easily be estimated from wavefront curvature. We will assert that practical target ranges often do lie well within the Fresnel distance, for applications on a wide range of scales.

At the beginning of this section it was stated that curvature in the array could remove ambiguity, implying removal of directional ambiguity, i.e. for beams at infinity. In fact the ambiguous beam is smeared out, rather than removed. We have not yet enquired into the effectiveness of this, one measure of effectiveness being the ratio of the responses, which in turn equals the inverse ratio of the beamwidths. A general beamwidth formula can be found from a stationary phase approach, each small element of the array contributing a beam element of negligible width in its appropriate steered direction. In the high-frequency limit the total beamwidth $B$ is simply $|CL|$ where $C$ is the total effective curvature, combining the $C_p$ and $C_e$ effects. This result holds whether one is on the convex side of the total curvature so that there is an immediate divergence, or on the concave side where there is convergence to a focus before the divergence begins. The ratio of beamwidths in the general case may alternatively be expressed as the inverse ratio of the focus or image distances $[V/U]$, true even if the calculated values for $U$ or $V$ or both are negative.

This ratio formula breaks down if one focus is at infinity. The effective beamwidth for this focus is then set by wave theory considerations, near the acoustic axis of the array it is approximately $\lambda/L$. The formula for the beam on the other side of the array becomes $|2C_pL|$. Now the sagittal width of the curved array is

$$t = \left| \frac{C_p L^2}{B} \right|$$

(4)

where $t$ is the separation of the two closest parallel lines that can enclose the array, implying that one line is tangent to the array centre. In an alternative definition of width one of the parallel lines is tangent to the array end and the separation $t' = 4t$. The other-side or ambiguous beamwidth becomes

$$B = \left| 2C_p L \right| = 16t/L = 4t'/L.$$  (5)

The effectiveness in smearing or in removing ambiguity is the beamwidth ratio

$$S = \left| 2C_p L^2/\lambda \right| = 16t/\lambda = 4t'/\lambda.$$  (6)

Thus $S$ is proportional to array width measured in wavelengths, and in any practical design the curvature must be chosen to make $S$ sufficiently large. Note that $S$ also has the order of magnitude of the Fresnel distance measured in units of radius of curvature.
The mention above of the principle of stationary phase leads us into a warning on the use of line arrays with higher orders of curvature and more complicated shapes. The more complicated arrays may well help as regards the false foci, but by degrading them rather than destroying them, ie the focal point will turn into a focal line. But the response in a given direction is in the high frequency limit directly proportional to the local radius of curvature at the appropriate array element, so that false response peaks may appear, especially if there are points of inflexion. The magnitudes of the higher spatial derivatives describing the line also need to be chosen in the light of the acoustic frequencies to be monitored.

6. CONCLUSIONS

(a) Cross-fixing with a pair of linear arrays can give up to four possible target positions, depending on the arrangement of the arrays etc, but the number possible is found not to be a particularly good criterion in judging the various arrangements.

(b) A second criterion based on plots of the unambiguous cover shows that when far from coasts it is best if the arrays are oriented at a right angle relative to one another, a very clear-cut result.

(c) Very near a coast some of the ambiguous tracks are ruled out, it is then best for the array lying next to the coastline to be parallel to it with the second array pointing at the first. This leads to the idea of track plausibility, which could also take account of unusual range or suspicious speed of track, and be the basis for a third criterion.

(d) Building up from right-angle array pairs to crosses of four arrays, distant from coastlines, a fourth criterion allows us to accept the so-called Potent Cross, the Fylfot and the Swastika; but to downgrade the Greek Cross for its lack of unambiguous cover in the centre.

(e) A fifth criterion concerning the unambiguous cover when we build up still further into large fields of arrays shows that the Fylfot field is very well behaved indeed, but the Potent Greek is undesirable.

(f) It would be desirable to extend to a sixth criterion, where multiple (more than two) target detections would be considered together and in addition allowance would be made for the practical shape of the curve of detection probability versus range.

(g) The effectiveness of a curved array in removing the directional ambiguity varies as its sagittal width measured in wavelengths.

(h) The focussing ambiguity of a curved array is conveniently treated in terms of its physical curvature and electrical delay curvature. The concave side of the array should face the direction of greatest interest.

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