UNCLASSIFIED

| AD NUMBER |
| --- |
| **ADB805893** |
| LIMITATION CHANGES |
| TO:<br>**Approved for public release; distribution is unlimited.** |
| FROM:<br>**Distribution authorized to DoD only; Administrative/Operational Use; JUL 1943. Other requests shall be referred to National Aeronautics and Space Administration, Washington, DC. Pre-dates formal DoD distribution statements. Treat as DoD only.** |
| AUTHORITY |
| **NASA TR Server website** |

THIS PAGE IS UNCLASSIFIED

JAN 30 1947



NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

# WARTIME REPORT

ORIGINALLY ISSUED

July 1943 as
Advance Restricted Report 3G07

## PORPOISING

## A COMPARISON OF THEORY WITH EXPERIMENT

By Kenneth S. M. Davidson
with
F. W. S. Locke, Jr., and Anthony Suarez
Stevens Institute of Technology

N A C A LIBRARY
LANGLEY MEMORIAL AERONAUTICAL
LABORATORY
Langley Field, Va.

NACA

WASHINGTON

NACA WARTIME REPORTS are reprints of papers originally issued to provide rapid distribution of advance research results to an authorized group requiring them for the war effort. They were previously held under a security status but are now unclassified. Some of these reports were not technically edited. All have been reproduced without change in order to expedite general distribution.

NASA Technical Library
3 1176 01403 4756

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

ADVANCE RESTRICTED REPORT

# PORPOISING

## A COMPARISON OF THEORY WITH EXPERIMENT

By Kenneth S. M. Davidson
with
F. W. S. Locke, Jr., and Anthony Suarez

## SUMMARY

This report makes a direct comparison between the observed and the calculated longitudinal dynamic stability of a particular dynamic model of a flying boat moving on the water.

Good agreement was obtained between experiment and theory (using Glauert's statement of the theory) for trim angles in the vicinity of the lower limit, at one speed a little above the hump. The agreement is shown in the following tabulation, which gives the experimental and theoretical values of the aerodynamic component of the pitch-damping derivative $M_q$ required for stability at various trim angles, at the speed in question.

Required Aerodynamic $M_q$ for Stability at Various Steady-Motion Trim Angles at One Speed

| Steady-motion trim angle (deg) | Required aerodynamic $M_q$ | |
|---|---|---|
| | Experiment | Theory |
| 5.8 | -21.1 | -21.5 |
| 6.4 | -17.6 | -16.2 |
| 7.1 | -12.0 | -11.3 |
| 8.3 | -5.8 | -5.3 |
| 11.0 | -.5 | -2.0 |

Data are given in an appendix from which similar trials of the correspondence between experiment and theory might be made for trim angles in the vicinity of the upper limit at the same speed, and for a higher speed. The calculations have not been completed for these cases, however, because it became apparent that the calculated stability could not be expected to be as reliable for upper-limit trim angles as for lower-limit trim angles, and because the theory had not particularly recommended itself in the initial trial.

The theoretical method in its present form is laborious; it depends for its application upon experimental constants which are not more easily determined than a direct experimental determination of stability, and it has not yet shown itself capable of pointing out design trends tending to reduce instability to any greater extent than the direct experimental method.

## INTRODUCTION

Previous attempts to determine the reliability of the classical method of calculating the longitudinal dynamic stability of flying boats moving on the water by comparing observed and calculated stabilities in specific cases have generally suffered from a lack of adequate data. This is particularly true of the work described in reference 1 and 2, where neither the observed stability nor the test information required for the stability calculation was known to better than a first approximation for the cases in question. It is much less true of the work described in reference 3, but an exact quantitative comparison was not attempted in that instance.

The study here considered was restricted to a particular dynamic model for which all three of the necessary components, namely,

- the observed stability,
- the basic hydrodynamic derivatives needed for the calculation, and
- the basic aerodynamic derivatives needed for the calculation,

could be determined with sufficient exactness to insure a reliable comparison between experiment and theory. This

restriction eliminates all questions regarding the relationship between flying boat and model in respect to form and particulars, the correspondence of speed, applied moment, or pure scale effects. It reduces the problem to comparing two results (observed and calculated), both of which are based upon accurate determinations for precisely the same combinations of model, forces, and moments.

The study was further restricted to one speed - both because of the labor involved in the actual testing and computations necessary to deduce the calculated stability, and because the study was intended primarily to be exploratory.

The model selected represents an actual flying boat in its original experimental form. Porpoising had been experienced in the full-size flying boat over a range of speeds intermediate between the hump and the get-away. In particular, the following approximate information was supplied in advance by the manufacturers:

| | (mph) |
|---|---|
| Planing | 35 |
| Porpoising | 43-62 |
| Take-off | 78 |

The speed selected for the comparison was about 48 miles per hour. (See p. 4.)

The tests for observed stability were made by the method developed at this Tank for experimental investigations of porpoising, in which predetermined aerodynamic forces and moments, and their derivatives, are applied mechanically to a dynamic model of the hull alone. This method is described in reference 4. Steady-motion tests to provide the necessary data for computing the hydrodynamic derivatives were made by ordinary towing-tank methods.

The particulars and specifications used are given on pages 11 and 12, which show also the aerodynamic characteristics of the hydrofoil which was substituted for the wing in the porpoising tests.

This investigation, conducted at Stevens Institute of Technology, was sponsored by and conducted with financial assistance from the National Advisory Committee for Aeronautics.

## TESTS

Porpoising tests were first made to determine the stability limits for the entire range of speeds between the hump and the get-away. The results of these tests are shown in figure 1, from which it will be seen that the range of speeds within which the lower limit lies above the free-to-trim track is in generally good agreement with the approximate range within which the builder reported porpoising in the actual flying boat; it may therefore be inferred that the actual porpoising was of the lower-limit type. The speed selected for the comparison of observed and calculated stabilities - 15.89 feet per second (corresponding to $C_V = 4.30$, or about 48 miles per hour in full size) - lies within this range.

The next step was to extend the porpoising tests at the selected speed to provide a broader basis of observed stability for comparison with the calculated stability. The principal requirement in this respect was a wider range of values of the aerodynamic pitch-damping derivative $M_q$. The results, covering values of $M_q$ from 0 to -20.0 (the latter value resulting in stability at the lowest steady-motion trim angle considered, 5.8°), are shown in figure 2. The values given correspond to the aerodynamic $M_q$ applied by the tail only. The total $M_q$ present was greater by the amount contributed by the model hull itself. A separate measurement of the latter, in air, gave -0.55.

Finally, the model was tested in steady motion, at the same speed by the ordinary towing tank procedure, to determine the relationships in steady motion between water-borne load, moment, heave, and trim, as a basis for deducing the hydrodynamic derivatives needed in the stability calculations. The results, covering fairly wide ranges of the variables, are shown in the form of a grid in figure 3.

## CALCULATIONS

That which is here referred to as the "classical" method of calculating the longitudinal dynamic stability of a flying boat moving on the water was first proposed by Perring and Glauert (reference 1).* This method may be said to involve two

*Hereafter referred to as Glauert.

essentially separable parts: 1) the purely theoretical equations of motion leading to the conditions for stability, and 2) the evaluation of the required derivatives. The first part is straightforward rigid dynamics (which was applied some years ago to the parallel problem of the stability of an airplane in flight) with some reconsideration of the relative importance of the various derivatives. The second part is inherently less simple than for the airplane in flight because the required derivatives represent the summation of aerodynamic and hydrodynamic components and, although the aerodynamic components follow the lead of the older method, the hydrodynamic components are more troublesome.

The eight hydrodynamic derivatives which have to be considered may be grouped under two headings:

Displacement derivatives

$Z_z$ rate change of $\Delta$ with respect to H, $\tau$ constant, per unit mass

$Z_\theta$ rate change of $\Delta$ with respect to $\tau$, H constant, per unit mass

$M_z$ rate change of M with respect to H, $\tau$ constant, per unit moment of inertia

$M_\theta$ rate change of M with respect to $\tau$, H constant, per unit moment of inertia

Velocity derivatives

$Z_q$ rate change of $\Delta$ with respect to q, per unit mass

$Z_w$ rate change of $\Delta$ with respect to w, per unit mass

$M_q$ rate change of M with respect to q, per unit mass

$M_w$ rate change of M with respect to w, per unit mass

Glauert derived tentative algebraic expressions for all of these, with empirical constants, from steady-motion test data on planing surfaces.

Later, Klemin, Pierson, and Storer (reference 2)* and others (in unpublished reports) noted: 1) that the displacement

*Hereafter referred to as Klemin.

derivatives could be deduced directly from a "general" tank test of the hull, and 2) that in Glauert's expressions the velocity derivatives were dependent on the displacement derivatives. They therefore substituted the experimentally determined displacement derivatives - retaining, in principle, Glauert's transformations to the velocity derivatives.

Since the calculations considered in this report were based on displacement derivatives determined from a "general" test, the detailed steps were first carried through in accordance with Klemin's analysis (reference 2). Subsequently, in comparing Klemin's and Glauert's analyses, a difference was noted in the expressions for $Z_w$ which, upon examination, was found to result from a difference in the definitions of $Z_\tau$ and $M_\tau$. (See p. 14.) Since this difference evidently might affect the results materially, all necessary steps in the calculations were repeated using Glauert's analysis (reference 1). Both sets of results are included here.

Considering the calculations in detail:

1. Five steady-motion trim angles were covered, embracing a range from well below the lower limit of stability to well above it. Figure 4 is a chart of all the steady-motion conditions, which are spotted also on other pertinent charts. The five trim angles corresponded to substantially equal differences of applied moment.

2. The first step in carrying out the calculations was to determine the hydrodynamic displacement derivatives from the steady-motion test data. These were read from various cross plots of the data shown in the general grid in figure 3. The derivatives are shown in figure 5 and in the tabulation on page 15.

3. The hydrodynamic velocity derivatives were then computed, the aerodynamic derivatives deduced in the ordinary way and, finally, the resultant values of all derivatives added up and tabulated for ready reference in computing Routh's discriminants.

4. All computations are given in detail on pages 15-27, from which it will be seen that three values of the aerodynamic $M_q$ were considered, for both Klemin's and Glauert's analyses. These were, respectively, 0, -4.39 (the normal value according to the table of particulars), and -20.0 (the amount determined experimentally as necessary to cause stability at $\tau = 5.8^\circ$).

5. A summary of all Routh's discriminants is given on page 28.

COMPARISON OF OBSERVED AND CALCULATED STABILITY

A. Following is a comparison of the limiting trim angles for stability at fixed values of $M_q$ as determined

1) from the limit curve in figure 2 in the case of the observed,*

2) by interpolation from the summary of Routh's discriminants on page 28 in the case of the calculated.

Limiting Trim Angles for Stability at Fixed Damping

| Total aerodynamic $M_q$ | Limiting trim angle | | |
|---|---|---|---|
| | Observed** | Calculated | |
| | | Klemin | Glauert |
| 0 | ← Indeterminate → | | |
| -4.39 | 8.6 | 9.1 | 8.7 |
| -20.0 | 6.0 | 7.4 | 6.0 |

Apart from the indeterminate (and inconsequential) result for $M_q = 0$, this comparison shows

1) a reasonably satisfactory general agreement between theory and experiment,

2) a decided preference for Glauert's analysis over that of Klemin.

*The limit curve on this figure is for a sweep of 0° in trim angle for the porpoising cycle, as opposed to the sweep of 2° previously used on figure 1.

**Corrected for contribution of model hull to total aerodynamic $M_q$.

B. Another, and somewhat more adequate, procedure is to compare the limiting values of $M_q$ for stability at given values of trim instead of the limiting trim angles for stability at fixed values of $M_q$.

Required Damping for Stability at Given Trim Angles

| Steady-motion trim angle (deg) | Required aerodynamic $M_q$ | | |
|---|---|---|---|
| | Observed* | Calculated | |
| | | Klemin | Glauert |
| 5.8 | -21.1 | + (?) | -21.5 |
| 6.4 | -17.6 | + (?) | -16.2 |
| 7.1 | -12.0 | + (?) | -11.3 |
| 8.3 | -5.8 | -7.5 | -5.3 |
| 11.0 | -.5 | -2.0 | -2.0 |

In this form the comparison emphasizes much more strongly the good agreement between theory and experiment, provided that Glauert's analysis is employed. It emphasizes also the apparent inadequacy of Klemin's analysis; for the three lower trim angles (see p. 28) the results by Klemin's analysis indicate increasing instability as the damping is increased, so that positive $M_q$ would apparently be necessary to cause stability.

## DISCUSSION

The difference between Klemin's and Glauert's analyses arising, as noted on page 6, through a difference in the definitions of $Z_T$ and $M_T$, brings out clearly an essential difficulty with the method of calculation as it now stands. There is nothing in Klemin's account to indicate that he purposely departed from Glauert's analysis. Whether the departure was intentional or accidental is of small moment because, in fact, Glauert's definitions of $Z_T$ and $M_T$ cannot easily be shown to be fundamentally more correct than Klemin's; both

*Corrected for contribution of model hull to total aerodynamic $M_q$.

are approximations open to some question. Hence, the fact that Glauert's definitions were more successful in the present limited instance cannot be given too much weight.

## NOTATION

The following symbols are used:

$\Delta_o$ static displacement, pounds

$Z$, $\Delta$ load on the water, pounds

$\theta$, $\tau$ trim angle, angle between forebody keel and undisturbed water surface

$H$ heave of center of gravity, referred to static displacement and zero trim, foot

$z$ height of center of gravity above free water surface, foot

$M$ applied moment, pound-foot

$m$ mass in vertical oscillation, slugs (corresponding to static displacement)

$I_p$ pitching moment of inertia about center of gravity, slug-foot square

$v$ forward velocity of model, foot per second

$w$ vertical velocity of model, foot per second

$q$ angular velocity of model, radians per second

$p$ height of center of gravity above keel at main step, foot

$r$ distance of center of gravity forward of main step, foot

$\zeta$ draft of keel at main step below free water surface, foot

$R_o$ resistance in steady motion, pounds

See also figure 6.

Stevens Institute of Technology,
Hoboken, N. J., August 1941.

## REFERENCES

1. Porring, W. G. A., and Glauert, H.: The Stability on the Water of a Seaplane in the Planing Condition. R. & M. No. 1493, British A.R.C., 1933.

2. Klemin, Alexander, Pierson, John D., and Storer, Edmund M.: An Introduction to Seaplane Porpoising. Jour. Aero. Sci., vol. 6, no. 8, June 1939, pp. 311-318.

3. Benson, James M., and Lina, Lindsay J.: The Effect of Dead Rise upon the Low-Angle Type of Porpoising. NACA A.R.R., Oct. 1942.

4. Davidson, Kenneth S. M., and Locke, F.W.S., Jr.: Some Systematic Model Experiments on the Porpoising Characteristics of Flying-Boat Hulls. NACA A.R.R., June 1943.

## PARTICULARS

The following particulars were used:

| | Full-size | Model |
|---|---|---|
| Drawing | | (See fig. 7) |
| Stevens Model No. | | 294-9 |
| Scale | 1 | 1/20 |
| **Dimensions** | | |
| Beam at main step, in. | 102 | 5.10 |
| Angle between forebody keel and base line, deg | 0 | 0 |
| Angle between afterbody keel and base line, deg | 7.5 | 7.5 |
| Height of main step at keel, in. | 4.00 | 0.20 |
| Center of gravity forward of main step, in. | 71.1 | 3.55 |
| Center of gravity above base line, in. | 112 | 5.62 |
| Gross weight, $\Delta_o$, lb | 40,000 (sea water) | 5.00 (fresh water) |
| Load coefficient, $C_\Delta$ (sea water) | 1.02 | |
| Moment of inertia in pitch, slug-ft$^2$ | $1.813 \times 10^5$ | |
| lb in.$^2$ | $8.40 \times 10^8$ | 262 |
| Wing span, ft | 118 | 5.90 |
| Wing area, S, sq ft | 1405 | 3.51 |
| Mean aerodynamic chord (M.A.C.), in. | 154 | 7.70 |
| Aspect ratio (geometric) | 9.91 | 9.91 |
| Horizontal tail area, sq ft | 216 | 0.541 |
| Elevator area, sq ft | 63.6 | 0.159 |
| Distance, center of gravity to 35 percent M.A.C. horizontal tail (tail length), ft | 43.8 | 2.19 |
| Thrust line, above base line at main step, in. | 172.8 | 8.64 |
| Thrust line, inclined upward to base line, deg | 1.0 | 1.0 |

Note - No allowances made for difference between fresh and sea water densities.

Ratios, $\frac{\text{full-size}}{\text{model}}$

| | |
|---|---|
| of speed, $\lambda^{1/2}$ | 4.472 |
| of linear dimension, $\lambda$ | 20 |
| of area, $\lambda^2$ | 400 |
| of volume, $\lambda^3$ | 8,000 |
| of moment, $\lambda^4$ | 160,000 |
| of moment of inertia, $\lambda^5$ | 3,200,000 |

| Aerodynamic characteristics | Full size | Model |
|---|---|---|
| $C_L$ at $\tau = 5^\circ$ (relative to base line) | 1.337 | 1.337 |
| L at $\tau = 5^\circ$ | $2.235\ v_s^2$ | $5.59 \times 10^{-3} v_m^2$ |
| $dC_L/d\tau$ | 0.0975 | 0.0975 |
| $dL/d\tau\ (dZ/d\theta)$, lb/deg | $0.163\ v_s^2$ | $0.407 \times 10^{-3} v_m^2$ |
| $dL/dw\ (dZ/dw)$, lb sec/ft $\left(\frac{dL}{d\alpha} \times \frac{1}{v}\right)$ | $0.163\ v_s$ | $0.407 \times 10^{-3} v_m$ |
| $dC_{M_{c.g.}}/d\alpha_{b.l.} = dC_{M_{c.g.}}/d\tau$ (av.) | 0.0229 | 0.0229 |
| $dM_{c.g.}/d\tau\ (dM/d\theta)$, lb ft/deg (av.) | $0.491\ v_s^2$ | $0.614 \times 10^{-4} v_m^2$ |
| $dM/dq$, lb ft sec/rad | $2500\ v_s$ | $1.562 \times 10^{-2} v_m$ |
| $dM/dw$, lb sec (av.) | $26.7\ v_s$ | $0.333 \times 10^{-2} v_m$ |
| $dM/dq \div (dM/dw)$ ft/rad | 93.6 | 4.69 |
| $dM/dq \div (dM/dw)$/tail length, 1/rad | 2.14 | 2.14 |

Hydrofoil characteristics

| | |
|---|---|
| Area, sq ft | 0.00942 |
| Aspect ratio (circular plan form) | 1.27 |
| $dC_L/d\alpha$ (av.) | 0.0350 |
| $dL/d\alpha$, lb/deg (av., fresh water) | $0.320 \times 10^{-3} v_m^2$ |
| $d\alpha/d\tau$ | 1.27 |
| $dL/d\tau\ (dZ/d\theta)$ $(= dL/d\alpha \times d\alpha/d\tau)$, lb/deg | $0.406 \times 10^{-3} v_m^2$ |
| $dL/dw\ (dZ/dw)$, lb sec/ft $\left(= \frac{dL}{d\alpha} \times \frac{1}{v} + \text{apparatus}\right)$ | $0.406 \times 10^{-3} v_m$ |

# FORMAL STATEMENT OF CALCULATIONS

1. For stability, Routh's discriminant and A, B, C, D, and E must be positive.

$$R = B\,C\,D - A\,D^2 - B^2\,E$$

where

$R$ = Routh's discriminant

$A = 1$

$B = -(Z_w + M_q)$

$C = -(Z_z + M_\theta - Z_w M_q + Z_q M_w)$

$D = Z_z M_q - Z_q M_z + Z_w M_\theta - Z_\theta M_w$

$E = Z_z M_\theta - Z_\theta M_z$

2. Derivatives in basic form

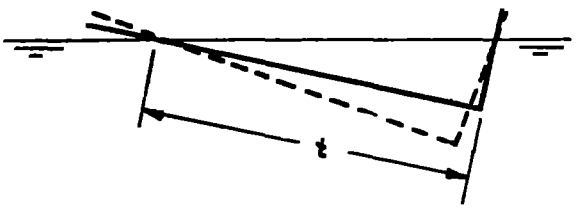
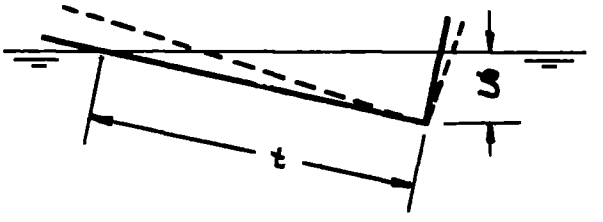
| Aerodynamic | | Hydrodynamic | |
|---|---|---|---|
| **Displacement derivatives** | | | |
| | | $Z_z$ | from test data |
| $Z_\theta$ | predetermined | $Z_\theta$ | from test data $\begin{cases} Z_\tau \frac{d\tau}{d\theta} + Z_t \frac{dt}{d\theta} & \text{(reference 1)} \\ Z_\tau \frac{d\tau}{d\theta} + Z_\zeta \frac{d\zeta}{d\theta} & \text{(reference 2)} \end{cases}$ |
| | | $M_z$ | from test data |
| $M_\theta$ | predetermined | $M_\theta$ | from test data $\begin{cases} M_\tau \frac{d\tau}{d\theta} + M_t \frac{dt}{d\theta} & \text{(reference 1)} \\ M_\tau \frac{d\tau}{d\theta} + M_\zeta \frac{d\zeta}{d\theta} & \text{(reference 2)} \end{cases}$ |
| **Velocity derivatives** | | | |
| $Z_w = Z_\theta \frac{d\theta}{dw}$ | | $Z_w = Z_\tau \frac{d\tau}{dw}$ | |
| $Z_q$ | negligible | $Z_q = \frac{\partial Z}{\partial u}\frac{du}{dq} + \frac{\partial Z}{\partial w}\frac{dw}{dq}$ | |
| $M_w$ | negligible | $M_w = M_\tau \frac{d\tau}{dw}$ | |
| $M_q$ | predetermined | $M_q = \frac{\partial M}{\partial u}\frac{du}{dq} + \frac{\partial M}{\partial w}\frac{dw}{dq}$ | |

Note - $\tau$ and $\theta$ are the same angle. $\tau$ is used here to indicate a less complete derivative. In the calculations (p. 15) $\tau$ is used for the trim angle in degrees; $\theta$ for the same angle in radians.

## 3. Derivatives Reduced for Computation

| Aerodynamic | Hydrodynamic |
|---|---|
| **Displacement derivatives** | |
| | $Z_z$ from test data |
| $Z_\theta$ predetermined | $Z_\theta$ from test data |
| | $M_z$ from test data |
| $M_\theta$ predetermined | $M_\theta$ from test data |
| **Velocity derivatives** | |
| $Z_w = \frac{1}{v} Z_\theta$ | $Z_w = \begin{cases} \frac{1}{v} Z_\theta - Z_z \left(- p\,\theta + r - \frac{s}{\theta}\right) & \text{Ref. (1)} \\ \frac{1}{v} Z_\theta - Z_z \left(- p\,\theta + r\right) & \text{Ref. (2)} \end{cases}$ |
| | $Z_q = \frac{2 Z_\Omega}{v} \left(p - [s - r]\,\theta\right) - Z_w \left(p\,\theta + [s - r]\right)$ |
| | $M_w = \begin{cases} \frac{1}{v} M_\theta - M_z \left(- p\,\theta + r - \frac{s}{\theta}\right) & \text{Ref. (1)} \\ \frac{1}{v} M_\theta - M_z \left(- p\,\theta + r\right) & \text{Ref. (2)} \end{cases}$ |
| $M_q$ predetermined | $M_q = \frac{2 M_\Omega}{v} \left(p - [s - r]\,\theta\right) - Z_w \left(p\,\theta + [s - r]\right)$ |

## 4. Difference between $Z_\tau$ and $M_\tau$ in References (1) and (2)

| Reference (1) (Glauert) | Reference (2) (Klemin) |
|---|---|
|  |  |
| $Z_\tau = \left(\frac{\partial Z}{\partial \tau}\right)_{t=\text{Const.}}$ | $Z_\tau = \left(\frac{\partial Z}{\partial \tau}\right)_{s=\text{Const.}}$ |
| $M_\tau = \left(\frac{\partial M}{\partial \tau}\right)_{t=\text{Const.}}$ | $M_\tau = \left(\frac{\partial M}{\partial \tau}\right)_{s=\text{Const.}}$ |

## BASIC DISPLACEMENT DERIVATIVES FOR ALL CALCULATIONS (See fig. 39)

**Model No. 294-9**

$v_m = 15.89$ f.p.s.

$$m = \frac{5.00}{32.2} = 0.1554 \qquad \frac{1}{m} = \frac{32.2}{5.00} = 6.44$$

$$mk^2 = \frac{262}{32.2 \times 144} = 0.0566 \qquad \frac{1}{mk^2} = \frac{32.2 \times 144}{262} = 17.68$$

| | | | | | |
|---|---|---|---|---|---|
| $\zeta$ deg. | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
| $M_o$ lb.ft. | +0.512 | +0.271 | +0.031 | -0.202 | -0.412 |

$$Z_z = \left(\frac{\partial \Delta}{\partial H}\right)_{\zeta=k} \times 12 \times 6.44 = \left(\frac{\partial \Delta}{\partial H}\right)_{\zeta=k} \times 77.28$$

| | | | | | |
|---|---|---|---|---|---|
| $Z_z$ | -305.0 | -320.0 | -346.0 | -406.0 | -614.0 |

$$Z_\theta = \left(\frac{\partial \Delta}{\partial \zeta}\right)_{H=k} \times 57.3 \times 6.44 = \left(\frac{\partial \Delta}{\partial \zeta}\right)_{H=k} \times 369$$

| | | | | | |
|---|---|---|---|---|---|
| $Z_\theta$ | -234.0 | -221.0 | -208.0 | -190.0 | -162.0 |

$$M_z = \left(\frac{\partial M}{\partial H}\right)_{\zeta=k} \times 17.68$$

| | | | | | |
|---|---|---|---|---|---|
| $M_z$ | +532 | +430 | +316 | +144 | -149 |

$$M_\theta = \left(\frac{\partial M}{\partial \zeta}\right)_{H=k} \times \frac{1}{12} \times 57.3 \times 17.68 = 84.4 \times \left(\frac{\partial M}{\partial \zeta}\right)_{H=k}$$

| | | | | | |
|---|---|---|---|---|---|
| $M_\theta$ | -72.9 | -65.0 | -59.9 | -59.9 | -166 |

## CALCULATION OF VELOCITY DERIVATIVES ACCORDING TO KLEMIN, REFERENCE (2)

**Model No. 294-9**

$v_m = 15.89$ f.p.s.

$$Z_w = \frac{1}{v}\left(Z_\theta - Z_z\left[-p\,\theta + r\right]\right)$$

$$p = \frac{5.62}{12} = 0.468 \qquad r = \frac{3.55}{12} = 0.296$$

| | | | | | |
|---|---|---|---|---|---|
| $\theta = \zeta/57.3$ | +0.101 | +0.112 | +0.124 | +0.145 | +0.192 |
| $-p\,\theta$ | -0.047 | -0.052 | -0.058 | -0.068 | -0.090 |
| $[-p\,\theta + r]$ | +0.249 | +0.244 | +0.238 | +0.228 | +0.206 |
| $Z_\theta$ | -234.0 | -221.0 | -208.0 | -190.0 | -162.0 |
| $Z_z[-p\,\theta + r]$ | -75.9 | -78.1 | -82.3 | -92.6 | -126.5 |
| Diff. | -158.1 | -142.9 | -125.7 | -97.4 | -35.5 |
| $Z_w$ | -9.95 | -8.99 | -7.91 | -6.13 | -2.23 |

(Continued)

| $\gamma$ deg. | | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
|---|---|---|---|---|---|---|
| | | $M_w = \frac{1}{v}(M_\theta - M_z[-p\,\theta + r])$ | | | | |
| $M_\theta$ | | -72.9 | -65.0 | -59.9 | -59.9 | -166 |
| $M_z[-p\,\theta + r]$ | | +132.0 | +105.0 | +75.2 | +32.8 | -30.7 |
| | Diff. | -204.9 | -170.0 | -135.1 | -92.7 | -135.3 |
| $M_w$ | | -12.89 | -10.70 | -8.50 | -5.83 | -8.51 |

Calculation of Center of Pressure Position

$$M_o = -Z_o\,(p\,\theta + [s - r]) + R_o\,(p - [s - r]\,\theta)$$

$$[s - r] = \frac{M_o + Z_o\,p\,\theta - R_o\,p}{-Z_o - R_o\,\theta}$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Delta = Z_o$, lbs. | | -3.51 | -3.45 | -3.37 | -3.25 | -2.97 |
| $R_o$, lbs. | | -0.57 | -0.55 | -0.54 | -0.53 | -0.60 |
| $M_o$ | | +0.512 | +0.271 | +0.031 | -0.202 | -0.412 |
| $(Z_o\,p\,\theta)$ | | -0.165 | -0.179 | -0.195 | -0.221 | -0.267 |
| | | +0.347 | +0.092 | -0.164 | -0.423 | -0.679 |
| $(R_o\,p)$ | | -0.267 | -0.257 | -0.253 | -0.248 | -0.281 |
| | (a) | +0.614 | +0.349 | +0.089 | -0.175 | -0.397 |
| $-Z_o$ | | +3.51 | +3.45 | +3.37 | +3.25 | +2.97 |
| $-(R_o\,\theta)$ | | +0.058 | +0.062 | +0.067 | +0.077 | +0.115 |
| | (b) | +3.568 | +3.512 | +3.437 | +3.327 | +3.085 |
| $[s - r] = (a)/(b)$ | | +0.172 | +0.0994 | +0.0259 | -0.0528 | -0.1287 |

| | | | | | |
|---|---|---|---|---|---|
| $Z_q = \frac{2\,Z_o}{v}(p - [s - r]\,\theta) - Z_w\,(p\,\theta + [s - r])$ | | | | | |
| $\frac{2\,Z_o}{v} \times 6.44$ | -2.845 | -2.797 | -2.732 | -2.634 | -2.407 |
| $(p - [s - r]\theta)$ | +0.451 | +0.457 | +0.465 | +0.476 | +0.495 |
| $(\frac{2Z_o}{v} \times 6.44)(p - [s - r]\theta)$ | -1.283 | -1.278 | -1.270 | -1.254 | -1.187 |
| $Z_w$ | -9.95 | -8.99 | -7.91 | -6.13 | -2.23 |
| $(p\,\theta + [s - r])$ | +0.219 | +0.151 | +0.084 | +0.015 | -0.039 |
| $-Z_w\,(p\,\theta + [s - r])$ | +2.179 | +1.357 | +0.664 | +0.092 | -0.087 |
| $Z_q$ | +0.896 | +0.079 | -0.606 | -1.162 | -1.274 |

| $\tau$ deg. | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
|---|---|---|---|---|---|
| $M_q = \frac{2M_o}{V}(p - [s - r]\theta) - M_w(p\theta + [s - r]\theta)$ | | | | | |
| $\frac{2M_o}{V} \times 17.68$ | +1.139 | +0.603 | +0.069 | -0.449 | -0.917 |
| $\left(\frac{2M_o}{V} \times 17.68\right)\left(p - [s - r]\theta\right)$ | +.514 | +.276 | +.032 | -.214 | -.452 |
| $M_w$ | -12.89 | -10.70 | -8.50 | -5.83 | -7.89 |
| $-M_w(p\theta + [s - r])$ | +2.823 | +1.616 | +.714 | -.087 | -.308 |
| $M_q$ | +3.337 | +1.892 | +.746 | -.301 | -.760 |

**Aerodynamic Derivatives**

$Z_\theta = \frac{dL}{d\tau} \times 57.3 \times 6.44$ -37.92

$M_\theta = \frac{dM}{d\tau} \times 57.3 \times 17.68$ -15.71

$Z_w = \frac{1}{V} Z_\theta$ -2.386

$M_q = \frac{dM}{dq} \times 17.68$ -14.388

## SUMMARY OF DERIVATIVES

(KLEMIN)

| | J deg. | 5.8 | | 6.4 | | 7.1 | | 8.3 | | 11.0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Aero | Hydro | Σ | Hydro | Σ | Hydro | Σ | Hydro | Σ | Hydro | Σ |
| $Z_z$ | 0.0 | -305 | -305 | -320 | -320 | -346 | -346 | -406 | -406 | -614 | -614 |
| $Z_\theta$ | -37.92 | -234 | -271.9 | -221 | -258.9 | -208 | -245.9 | -190 | -227.9 | -162 | -199.9 |
| $Z_w$ | -2.386 | -9.95 | -12.34 | -8.99 | -11.38 | -7.91 | -10.30 | -6.13 | -8.516 | -2.23 | -4.616 |
| $Z_q$ | 0.0 | +0.896 | +0.896 | +0.079 | +0.079 | +0.606 | -0.606 | -1.162 | -1.162 | -1.274 | -1.274 |
| $M_z$ | 0.0 | +532 | +532 | +430 | +430 | +316 | +316 | +144 | +144 | -149 | -149 |
| $M_\theta$ | -15.71 | -72.9 | -88.6 | -65.0 | -80.7 | -59.9 | -75.6 | -59.9 | -75.6 | -166 | -181.7 |
| $M_w$ | 0.0 | -12.89 | -12.89 | -10.70 | -10.70 | -8.50 | -8.50 | -5.83 | -5.83 | -7.89 | -7.89 |
| $M_q$ | -4.388 | +3.337 | -1.051 | +1.892 | -2.496 | +0.746 | -3.642 | -0.301 | -4.689 | -0.760 | -5.148 |

# CALCULATION OF ROUTH DISCRIMINANT

## (KLEMIN)

Design Aerodynamic $M_q = -4.388$

| | τ deg | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
|---|---|---|---|---|---|---|
| A | | +1 | +1 | +1 | +1 | +1 |
| | $Z_w$ | -12.34 | -11.38 | -10.30 | -8.516 | -4.616 |
| | $M_q$ | -1.051 | -2.496 | -3.642 | -4.689 | -5.148 |
| B | | +13.391 | +13.876 | +13.942 | +13.205 | +9.764 |
| | $Z_z$ | -305 | -320 | -346 | -406 | -614 |
| | $M_\theta$ | -88.6 | -80.7 | -75.6 | -75.6 | -181.7 |
| | $Z_q M_w$ | -11.5 | -0.8 | +5.2 | +6.8 | +10.1 |
| | $-Z_w M_q$ | -13.0 | -28.4 | -37.5 | -39.9 | -23.8 |
| C | | +418.1 | +429.9 | +453.9 | +514.7 | +809.4 |
| | $Z_z M_q$ | +320 | +799 | +1260 | +1904 | +3161 |
| | $-Z_q M_z$ | -477 | -34 | +191 | +167 | -190 |
| | $+Z_w M_\theta$ | +1093 | +918 | +779 | +644 | +839 |
| | $-Z_\theta M_w$ | -3505 | -2770 | -2090 | -1329 | -1577 |
| D | | -2569 | -1087 | +140 | +1386 | +2233 |
| | $Z_z M_\theta$ | +27,020 | +25,820 | +26,160 | +30,690 | +111,560 |
| | $-Z_\theta M_z$ | +144,650 | +111,330 | +77,700 | +32,820 | -29,790 |
| E | | +171,670 | +137,150 | +103,860 | +63,510 | +81,770 |
| BCD | | -14,383,000 | -6,484,000 | +885,960 | +9,424,000 | +17,647,000 |
| $-AD^2$ | | -6,599,800 | -1,182,000 | -19,600 | -1,921,000 | -4,986,000 |
| $-B^2E$ | | -30,784,000 | -26,407,000 | -20,188,000 | -11,074,000 | -7,796,000 |
| R ($10^6$ x) | | -51.77 | -34.07 | -19.32 | -3.57 | +4.87 |

## CALCULATION OF ROUTH DISCRIMINANT
## (KLEMIN)
### Aerodynamic $M_q = 0$

| $\gamma$ deg. | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
|---|---|---|---|---|---|
| A | +1 | +1 | +1 | +1 | +1 |
| $Z_w$ | -12.34 | -11.38 | -10.30 | -8.516 | -4.616 |
| $M_q$ | + 3.337 | + 1.892 | +0.746 | - 0.301 | -0.760 |
| B | + 9.00 | + 9.488 | + 9.554 | +8.817 | +5.376 |
| $Z_z$ | -305 | -320 | -346 | -406 | -614 |
| $M_\theta$ | - 88.6 | - 80.7 | - 75.6 | - 75.6 | -181.7 |
| $Z_q M_w$ | - 11.5 | - 0.8 | + 5.2 | + 6.8 | + 10.1 |
| $-Z_w M_q$ | + 41.2 | + 21.53 | + 7.683 | - 2.563 | - 3.6 |
| | +365.9 | +379.97 | +408.72 | +477.36 | +789.1 |
| $Z_z M_q$ | -1018 | -605 | -258 | +122 | +466.6 |
| $- Z_q M_z$ | - 477 | - 34 | +191 | +167 | -190 |
| $+ Z_w M_\theta$ | +1093 | +918 | +779 | +644 | +839 |
| $-Z_\theta M_w$ | -3505 | -2770 | -2090 | -1329 | -1577 |
| D | -3907 | -2491 | -1378 | - 396 | -461.4 |
| $Z_z M_\theta$ | +27020 | +25820 | +26160 | +30690 | +111560 |
| $-Z_\theta M_z$ | +144650 | +111330 | +77700 | +32820 | - 29790 |
| E | +171670 | +137150 | +103860 | +63510 | + 81770 |
| BCD | -12,796,000 | -8,980,000 | -5,381,000 | -1,667,000 | -1,954,000 |
| $-AD^2$ | -15,265,000 | -6,205,000 | -1,899,000 | - 157,000 | - 223,000 |
| $-B^2E$ | -13,905,000 | -12,347,000 | -9,480,000 | - 494,000 | - 509,000 |
| R ($10^6$ x) | -41.97 | -27.53 | -16.76 | -2.32 | -2.69 |

## CALCULATION OF ROUTH DISCRIMINANT

(KLEMIN)

Aerodynamic $M_q$ = -20.0

| J deg. | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
|---|---|---|---|---|---|
| A | +1 | +1 | +1 | +1 | +1 |
| $Z_w$ | -12.34 | -11.38 | -10.30 | -8.516 | -4.616 |
| $M_q$ | -16.66 | -18.11 | -19.25 | -20.301 | -20.76 |
| B | +30.00 | +29.49 | +29.55 | +28.817 | +25.376 |
| $Z_z$ | -305 | -320 | -346 | -406 | -614 |
| $M_\theta$ | - 88.6 | - 80.7 | - 75.6 | - 75.6 | -181.7 |
| $Z_q M_w$ | - 11.6 | - 0.8 | + 5.2 | + 6.8 | + 10.1 |
| $-Z_w M_q$ | -205.6 | -206.1 | -198.3 | -172.9 | - 95.8 |
| C | +610.7 | +607.6 | +614.7 | +647.7 | +881.4 |
| $Z_z M_q$ | +5081 | +5795 | +6661 | +8242 | +12746 |
| $-Z_q M_z$ | - 477 | - 34 | + 191 | + 167 | - 190 |
| $+Z_w M_\theta$ | +1093 | + 918 | + 779 | + 644 | + 839 |
| $-Z_\theta M_w$ | -3505 | -2770 | -2090 | -1329 | - 1577 |
| D | +2192 | +3909 | +5541 | +7724 | +11818 |
| $Z_z M_\theta$ | +27020 | +25820 | +26160 | +30690 | +111560 |
| $-Z_\theta M_z$ | +144650 | +111330 | +77700 | +32820 | - 29790 |
| E | +171670 | +137150 | +103860 | +63510 | 81770 |
| BCD | +40,160,000 | +70,042,000 | +100,650,000 | +144,166,000 | +264,326,000 |
| $-AD^2$ | - 4,805,000 | -15,280,000 | - 30,703,000 | - 59,660,000 | -139,665,000 |
| $-B^2E$ | -154,505,000 | -119,274,000 | - 90,691,000 | - 52,740,000 | - 52,655,000 |
| R ($10^6$x) | -119.15 | -64.51 | -20.74 | +31.77 | +72.01 |

## CALCULATION OF VELOCITY DERIVATIVES ACCORDING TO GLAUERT, REFERENCE (1)

Model No. 294-θ

$v_m$ = 15.89 f.p.s.

(Displacement derivatives same as before)

$$Z_w = \frac{1}{v}\left(Z_\theta - Z_z\left[-p\theta + r - \zeta/\theta\right]\right)$$

p = 0.468 r = 0.296

| ϒ deg. | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
|---|---|---|---|---|---|
| θ | +0.101 | +0.112 | +0.124 | +0.145 | +0.192 |
| ζ | +0.0858 | +0.0808 | +0.0747 | +0.0669 | +0.0545 |
| ζ/θ | +0.850 | +0.721 | +0.602 | +0.461 | +0.284 |
| pθ | +0.0473 | +0.0524 | +0.0580 | +0.0679 | +0.0899 |
| -pθ + r - ζ/θ | -0.601 | -0.477 | -0.364 | -0.233 | -0.078 |
| $Z_\theta$ | -234 | -221 | -208 | -190 | -162 |
| $Z_z[-p\theta + r - \zeta/\theta]$ | +183.3 | +152.6 | +125.9 | + 94.6 | + 47.9 |
| Diff. | -417.3 | -373.6 | -333.9 | -284.6 | -209.9 |

v = 15.89 f.p.s.

| | | | | | |
|---|---|---|---|---|---|
| $Z_w$ | -26.26 | -23.51 | -21.01 | -17.91 | -13.21 |

$$M_w = \frac{1}{v}\left(M_\theta - M_z\left[-p\theta + r - \zeta/\theta\right]\right)$$

| | | | | | |
|---|---|---|---|---|---|
| $M_\theta$ | -72.9 | -65.0 | -59.9 | -59.9 | -166 |
| $M_z[-p\theta + r - \zeta/\theta]$ | -319.7 | -205.1 | -115.0 | -33.6 | + 11.6 |
| Diff. | +246.8 | +140.1 | + 55.1 | -26.3 | -177.6 |

v = 15.89 f.p.s. .

| | | | | | |
|---|---|---|---|---|---|
| $M_w$ | +15.53 | +8.82 | +3.47 | -1.66 | -11.18 |

$$Z_q = \frac{2\,Z_o}{v}\left(p - [s-r]\,\theta\right) - Z_w\left(p\,\theta + [s-r]\right)$$

| | | | | | |
|---|---|---|---|---|---|
| $\frac{2\,Z_o \times 6.44}{v}$ | -2.845 | -2.797 | -2.732 | -2.634 | -2.407 |
| $(p - [s-r]\,\theta)$ | +0.4506 | +0.4569 | +0.4648 | +0.4756 | +0.4927 |
| $\frac{2\,Z_o \times 6.44}{v}(p-[s-r]\theta)$ | -1.282 | -1.278 | -1.270 | -1.253 | -1.186 |
| $Z_w$ | -26.26 | -23.51 | -21.01 | -17.91 | -13.21 |
| $(p\theta + [s-r])$ | + 0.2193 | + 0.1518 | + 0.0839 | + 0.0153 | - 0.0388 |
| $-Z_w\,(p\theta + [s-r])$ | + 5.759 | + 3.569 | + 1.763 | + 0.274 | - 0.513 |
| $Z_q$ Sum of Products | + 4.477 | +2.291 | + 0.493 | - 0.979 | - 1.699 |

| $\tau$ deg. | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
|---|---|---|---|---|---|
| $M_q = \frac{2M_o}{v}(p - [s - r]\,\theta) - M_w\,(p\,\theta + [s - r])$ | | | | | |
| $\frac{2M_o}{v} \times 17.68$ | +1.139 | +0.6030 | +0.069 | -0.4495 | -0.9167 |
| $\frac{2M_o}{v} \times 1.68\,(p - [s - r]\,\theta)$ | +.513 | +.276 | +.032 | -.214 | -.452 |
| $M_w$ | +15.53 | +8.82 | +3.47 | -1.66 | -11.18 |
| $M_w\,(p\,\theta + [s - r])$ | +3.406 | +1.339 | +.291 | -.025 | -.43 |
| $M_q$ diff. of products | -2.89 | -1.06 | -.259 | -.189 | -.862 |

Aerodynamic Derivatives

$Z_\theta = \frac{dL}{d\tau} \times 57.3 \times 6.44$ -37.92

$M_\theta = \frac{dM}{d\tau} \times 57.3 \times 17.68$ -15.71

$Z_w = \frac{1}{v} Z_\theta$ -2.386

$M_q = \frac{dM}{dq} \times 17.68$ -4.388

## SUMMARY OF DERIVATIVES (GLAUERT)

| $\delta$ deg. | | 5.8 | | 6.4 | | 7.1 | | 8.3 | | 11.0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Aero | Hydro | $\Sigma$ | Hydro | $\Sigma$ | Hydro | $\Sigma$ | Hydro | $\Sigma$ | Hydro | $\Sigma$ |
| $Z_z$ | 0.0 | -305 | -305 | -320 | -320 | -346 | -346 | -406 | -406 | -614 | -614 |
| $Z_\theta$ | -37.92 | -234 | -271.9 | -221 | -258.9 | -208 | -245.9 | -190 | -227.9 | -162 | -199.9 |
| $Z_w$ | -2.386 | -26.26 | -28.65 | -23.51 | -25.90 | -21.01 | -23.40 | -17.91 | -20.30 | -13.21 | -15.60 |
| $Z_q$ | 0.0 | +4.477 | +4.477 | +2.291 | +2.291 | +0.493 | +0.493 | -0.979 | -0.979 | -1.699 | -1.699 |
| $M_z$ | 0.0 | +532 | +532 | +430 | +430 | +316 | +316 | +144 | +144 | -149 | -149 |
| $M_\theta$ | -15.71 | -72.9 | -88.6 | -65.0 | -80.7 | -59.9 | -75.6 | -59.9 | -75.6 | -166 | -181.7 |
| $M_w$ | 0.0 | +15.53 | +15.53 | +8.82 | +8.82 | +3.47 | +3.47 | -1.66 | -1.66 | -11.18 | -11.18 |
| $M_q$ | -4.388 | -2.89 | -7.278 | -1.06 | -5.448 | -0.259 | -4.647 | -0.189 | -4.577 | -0.882 | -5.270 |

## CALCULATION OF ROUTH DISCRIMINANT

(GLAUERT)

Design Aerodynamic $M_q$ = -4.388

| J deg. | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
|---|---|---|---|---|---|
| A | +1 | +1 | +1 | +1 | +1 |
| $Z_w$ | -28.65 | -26.90 | -23.40 | -20.30 | -16.60 |
| $M_q$ | - 7.278 | - 5.448 | - 4.347 | -4.577 | - 5.27 |
| B | +35.93 | +31.35 | +28.05 | +24.88 | +20.87 |
| $+Z_z$ | -305 | -320 | -346 | -406 | -614 |
| $+M_\theta$ | - 88.6 | -80.7 | - 75.6 | - 75.6 | -181.7 |
| $+Z_q M_w$ | + 69.53 | + 20.21 | + 1.71 | + 1.63 | + 19.0 |
| $-Z_w M_q$ | -208.5 | -141.1 | -108.7 | - 92.9 | - 82.2 |
| C | +532.6 | +521.6 | +528.6 | +572.9 | +858.9 |
| $+Z_z M_q$ | +2219.8 | +1743.3 | +1607.9 | +1858.3 | +3235.8 |
| $-Z_q M_z$ | -2381.8 | - 985.1 | - 155.8 | + 141.0 | - 253.2 |
| $+Z_w M_\theta$ | +2538.4 | +2090.1 | +1769.0 | +1534.7 | +2834.5 |
| $-Z_\theta M_w$ | +4222.6 | +2283.5 | + 853.3 | - 378.3 | -2234.9 |
| D | +6599.0 | +5132.0 | +4074.0 | +3156.0 | +3582.2 |
| $+Z_z M_\theta$ | + 27023 | + 25824 | + 26158 | + 30694 | +111564 |
| $-Z_\theta M_z$ | +144651 | +111327 | + 77704 | + 32818 | - 29785 |
| E | +171670 | +137150 | +103860 | + 63512 | + 81780 |
| +BCD | +126,278,000 | +83,919,000 | +60,406,000 | +44,985,000 | +64,212,000 |
| $-AD^2$ | - 43,547,000 | -26,337,000 | -16,597,000 | - 9,960,000 | -12,832,000 |
| $-B^2E$ | -221,626,000 | -134,794,000 | -81,717,000 | -39,315,000 | -35,620,000 |
| R ($10^6$ x) | -138.90 | -77.21 | -37.91 | -4.29 | +15.76 |

## CALCULATION OF ROUTH DISCRIMINANT

(GLAUERT)

Aerodynamic $M_q = 0$

| | | | | | | |
|---|---|---|---|---|---|---|
| J | deg. | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
| A | | +1 | +1 | +1 | +1 | +1 |
| | $Z_w$ | -28.65 | -25.90 | -23.40 | -20.30 | -15.60 |
| | $M_q$ | - 2.89 | - 1.06 | -0.259 | - 0.189 | - 0.88 |
| B | | +31.54 | +26.96 | +23.66 | +20.49 | +16.48 |
| | $+Z_z$ | -305. | -320. | -346. | -406. | -614. |
| | $+M_\theta$ | - 88.6 | - 80.7 | - 75.6 | - 75.6 | -181.7 |
| | $+Z_q M_w$ | + 69.53 | + 20.2 | + 1.7 | + 1.6 | + 19.0 |
| | $-Z_w M_q$ | - 82.8 | - 27.5 | - 5.9 | - 3.8 | - 13.7 |
| C | | +406.9 | +408.0 | +426.0 | +483.8 | +790.4 |
| | $+Z_z M_q$ | +881.5 | +339.2 | + 89.6 | + 76.7 | +540.3 |
| | $-Z_q M_z$ | -2381.8 | -985.1 | -155.8 | +141.0 | -253.2 |
| | $+Z_w M_\theta$ | +2538.4 | +2090.1 | +1769.0 | +1534.7 | +2834.5 |
| | $-Z_\theta M_w$ | +4222.6 | +2283.5 | + 853.3 | - 378.3 | -2234.9 |
| D | | +5260.7 | +3727.7 | +2556.1 | +1374.1 | + 886.7 |
| | $+Z_z M_\theta$ | + 27023 | + 25824 | + 26158 | + 30694 | +111564 |
| | $-Z_\theta M_z$ | +144651 | +111327 | + 77704 | + 32818 | - 29785 |
| E | | +171670 | +137150 | +103860 | + 63512 | + 81780 |

| | | | | | |
|---|---|---|---|---|---|
| +BCD | +67,514,000 | +40,993,000 | +25,763,000 | +13,622,000 | +11,550,000 |
| $-AD^2$ | -27,675,000 | -13,896,000 | - 6,534,000 | - 1,888,000 | - 786,000 |
| $-B^2E$ | -170,772,000 | -99,686,000 | -58,140,000 | -26,665,000 | -22,211,000 |
| R ($10^6$ x) | -130.93 | -72.59 | -38.91 | -14.93 | -11.45 |

## CALCULATION OF ROUTH DISCRIMINANT

(GLAUERT)

Aerodynamic $M_q$ = -20.0

| $\gamma$ deg. | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
|---|---|---|---|---|---|
| A | +1 | +1 | +1 | +1 | +1 |
| $Z_w$ | -28.65 | -25.90 | -23.40 | -20.30 | -15.60 |
| $M_q$ | -22.89 | -21.06 | -20.26 | -20.19 | -20.88 |
| B | +51.54 | +46.96 | +43.66 | +40.49 | +36.48 |
| $+Z_z$ | -305.0 | -320.0 | -346.0 | -406.0 | -614.0 |
| $+M_\theta$ | - 88.6 | - 80.7 | - 75.6 | - 75.6 | -181.7 |
| $+Z_q M_w$ | + 69.53 | + 20.2 | + 1.7 | + 1.8 | + 19.0 |
| $-Z_w M_q$ | -655.8 | -545.5 | -474.1 | -409.9 | -325.7 |
| C | +979.9 | +926.0 | +894.0 | +889.9 | +1102.4 |
| $+Z_z M_q$ | +6981.5 | +6739.2 | +7010.0 | +8197.1 | +12820.0 |
| $-Z_z M_z$ | -2381.8 | - 985.1 | - 155.8 | + 141.0 | - 253.2 |
| $+Z_w M_\theta$ | +2538.4 | +2090.1 | +1769.0 | +1534.7 | +2834.5 |
| $-Z_\theta M_w$ | +4222.6 | +2283.5 | + 853.3 | - 378.3 | -2234.9 |
| D | +11360.7 | +10127.7 | +9476.5 | +9494.5 | +13166.4 |
| $+Z_z M_\theta$ | + 27023 | + 25824 | + 26158 | + 30694 | + 111564 |
| $-Z_\theta M_z$ | +144651 | +111327 | + 77704 | +32818 | - 29785 |
| E | +171670 | +137150 | +103860 | +63512 | + 81780 |
| +BCD | +573,761,000 | -440,403,000 | +369,887,000 | +342,107,000 | +529,495,000 |
| $-AD^2$ | -129,066,000 | -102,570,000 | - 89,804,000 | - 90,146,000 | -173,354,000 |
| $-B^2E$ | -455,956,000 | -302,449,000 | -197,978,000 | -104,124,000 | -108,832,000 |
| R ($10^6$ x) | -11.26 | +35.38 | +82.11 | +147.84 | +247.31 |

## SUMMARY OF ROUTH DISCRIMINANTS

| $\tau$ deg. | 5.8 | 6.4 | 7.1 | 8.3 | 11.0 |
|---|---|---|---|---|---|
| $M_q$ (KLEMIN) | | | | | |
| 0 | $-41.97\times10^6$ | $-27.53\times10^6$ | $-16.76\times10^6$ | $-2.32\times10^6$ | $-2.69\times10^6$ |
| -4.388 | $-51.767\times10^6$ | $-34.073\times10^6$ | $-19.322\times10^6$ | $-3.486\times10^6$ | $+4.865\times10^6$ |
| -20.00 | $-119.15\times10^6$ | $-64.51\times10^6$ | $-20.74\times10^6$ | $+31.77\times10^6$ | $+72.01\times10^6$ |
| (GLAUERT) | | | | | |
| 0 | $-130.933\times10^6$ | $-72.589\times10^6$ | $-38.911\times10^6$ | $-14.93\times10^6$ | $-11.45\times10^6$ |
| -4.388 | $-138.895\times10^6$ | $-77.212\times10^6$ | $-37.908\times10^6$ | $-4.290\times10^6$ | $15.760\times10^6$ |
| -20.00 | $-11.261\times10^6$ | $+35.384\times10^6$ | $+82.105\times10^6$ | $+147.837\times10^6$ | $+247.309\times10^6$ |

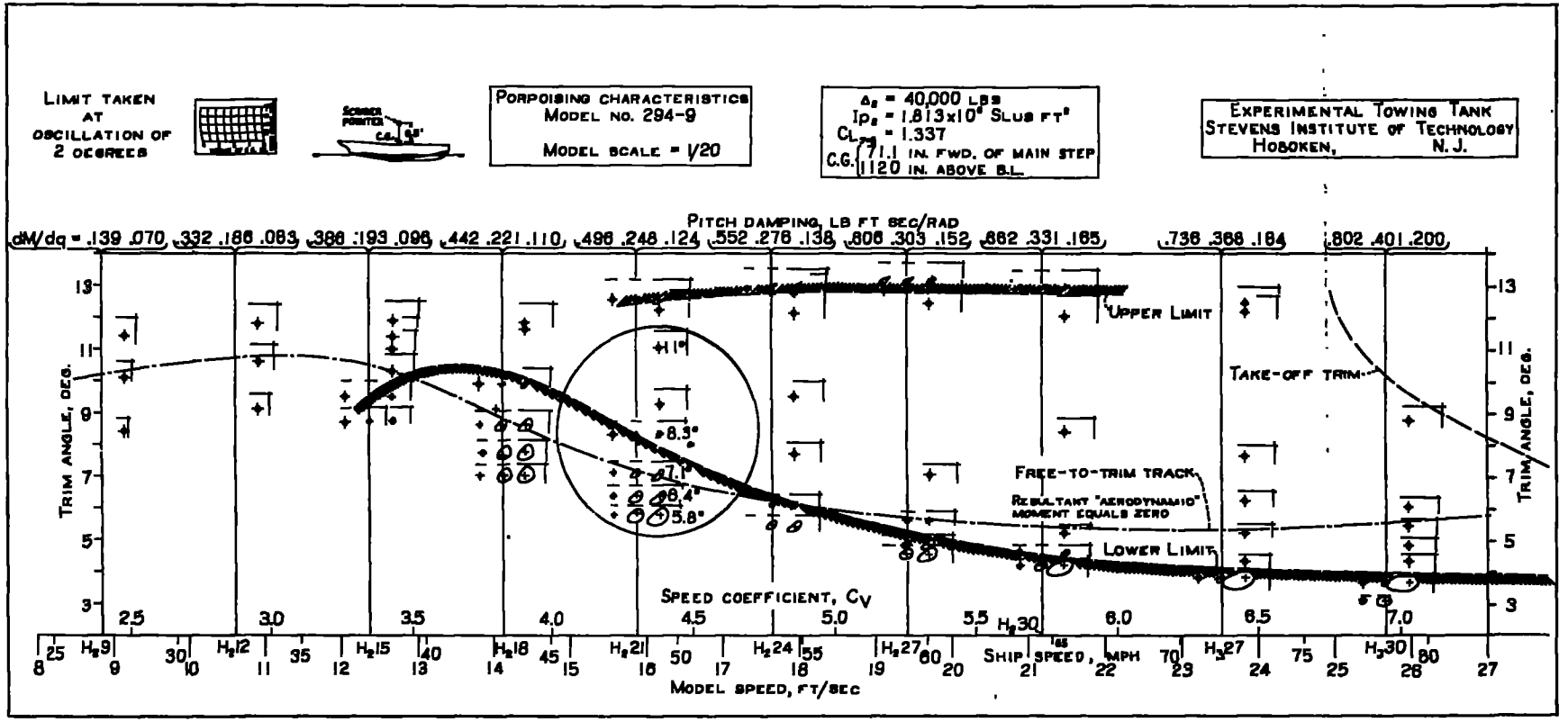



Figure 1. - Stability limits and free-to-trim track for model used in investigation, showing the graphical records of the porpoising cycles. The region for which the calculations were made is enclosed by a heavy circle.

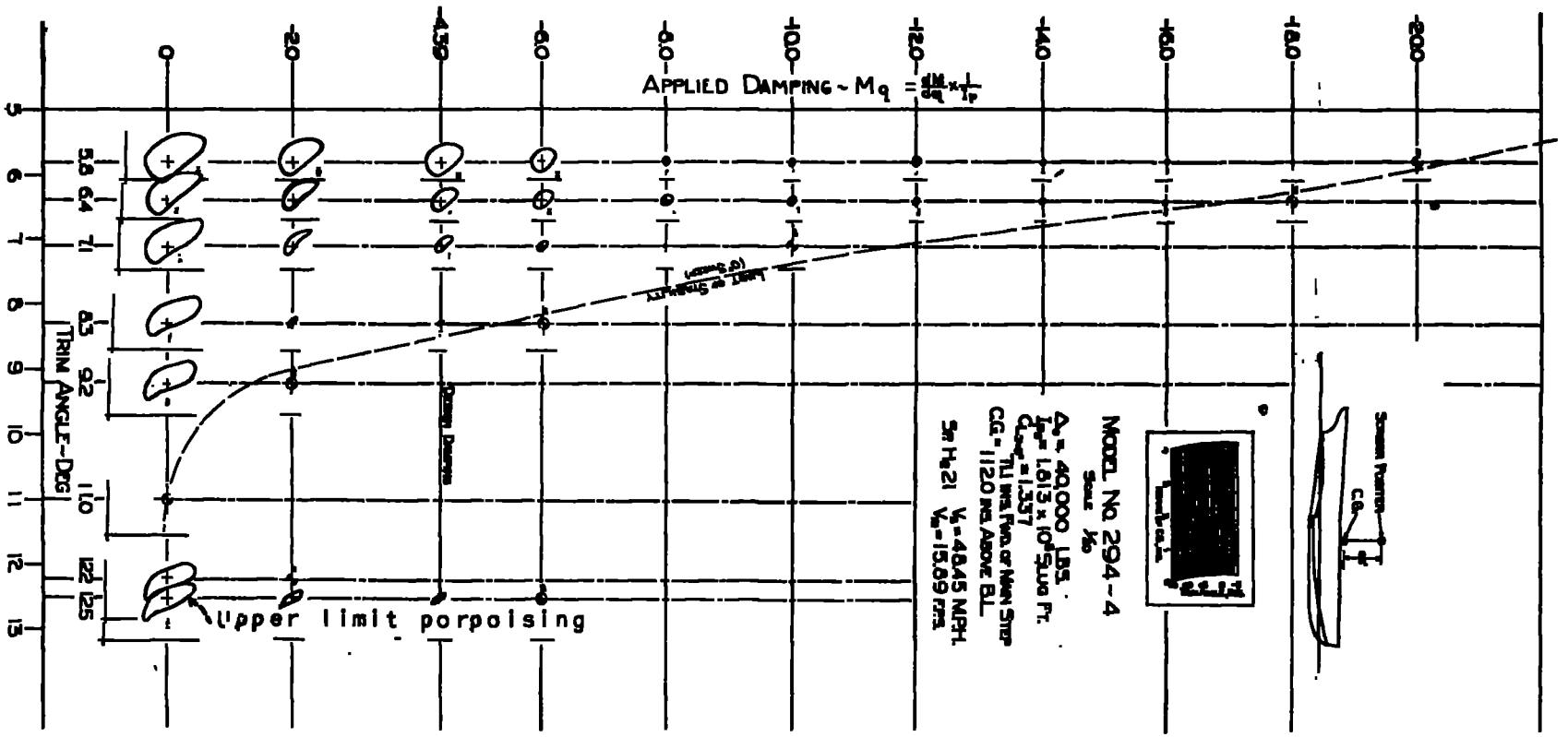



Figure 2. - The lower stability limit, at the speed investigated in the calculations, showing required tail damping as a function of trim angle, with the graphical records of the porpoising cycles. This is a more detailed investigation of the region inclosed by a circle in figure 1.

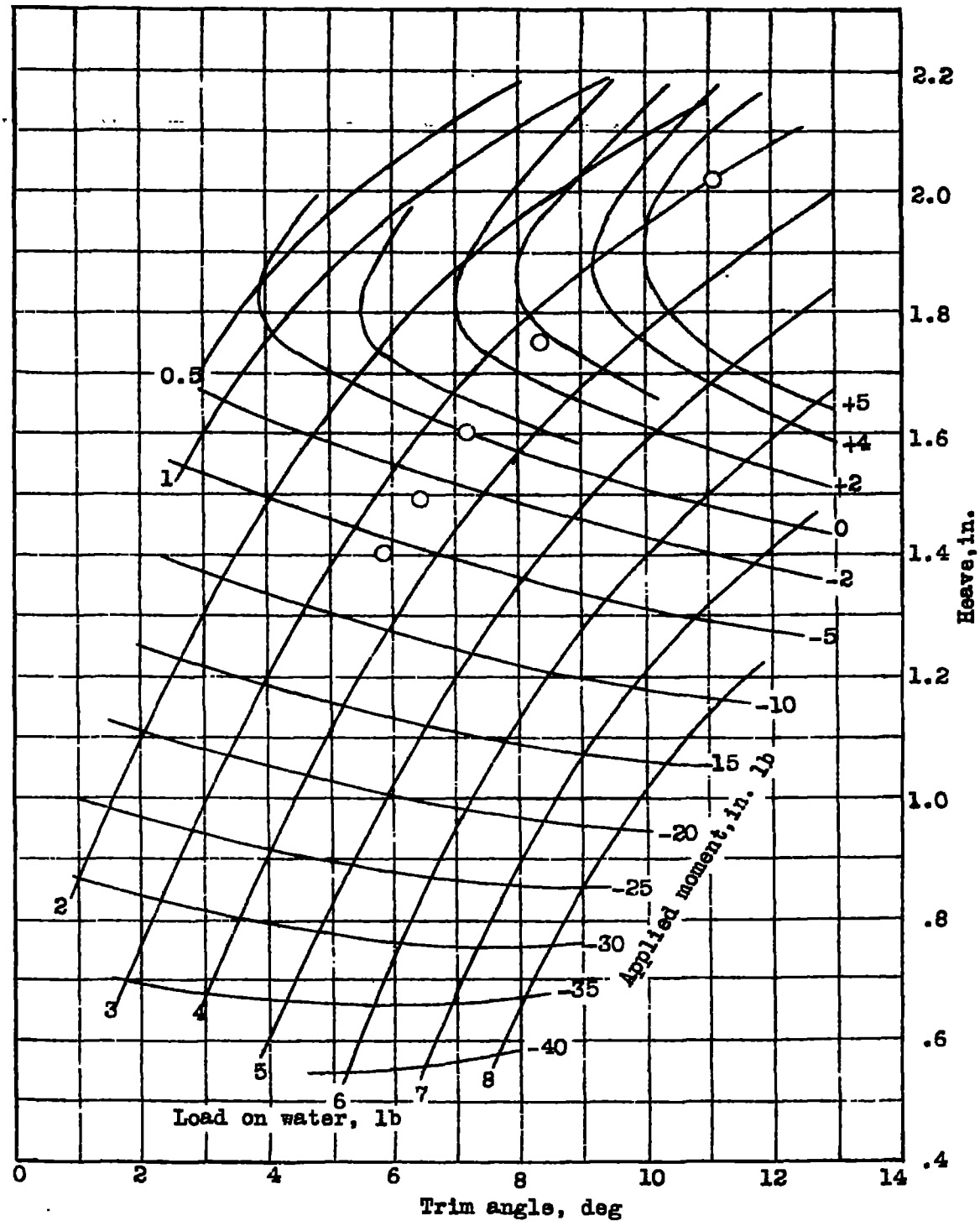



Figure 3.- Steady-motion test data Model No. 204-9. $V_m$ = 15.89 fps

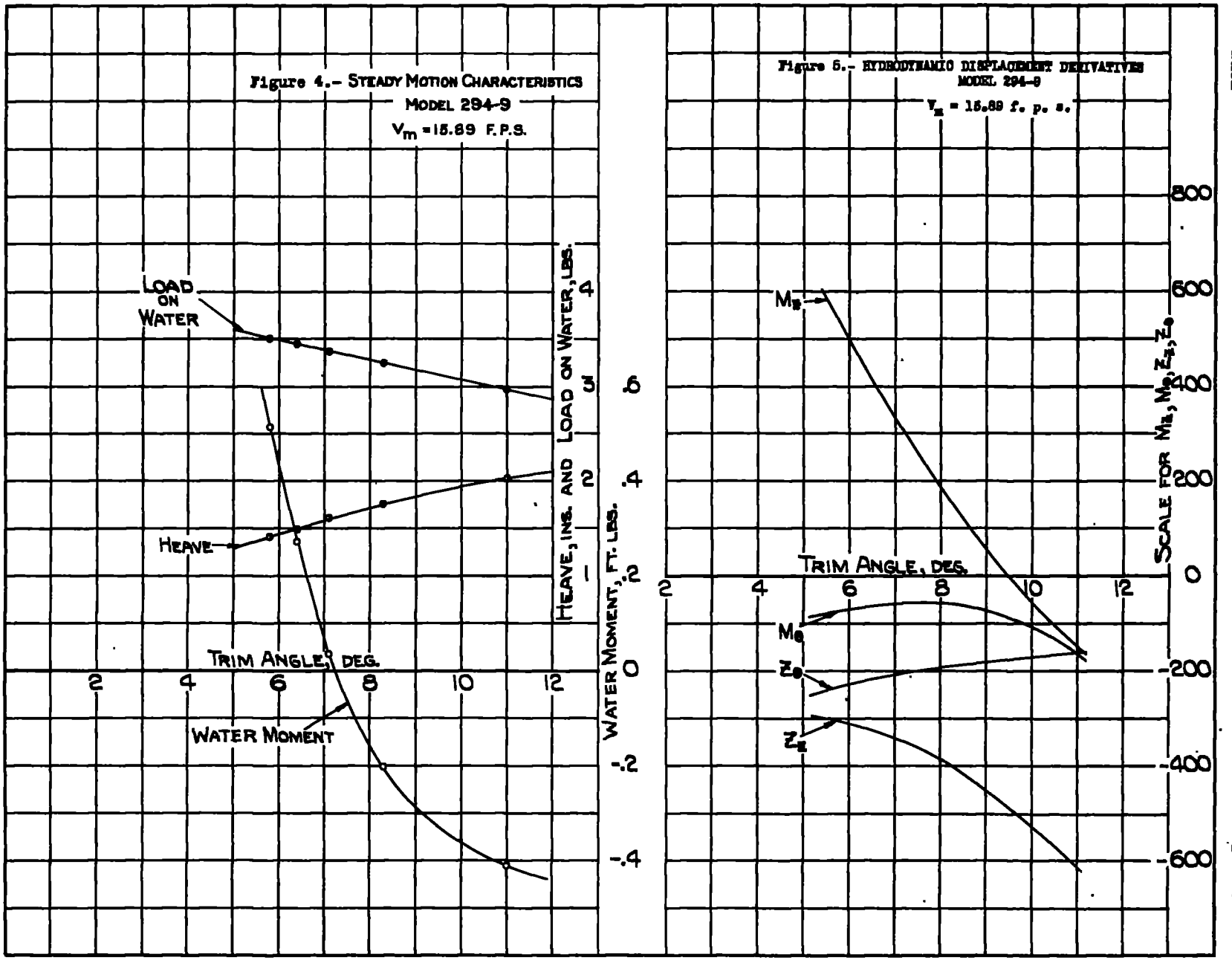



Figure 4.- STEADY MOTION CHARACTERISTICS
MODEL 294-9
$V_m$ = 15.89 F.P.S.

Figure 5.- HYDRODYNAMIC DISPLACEMENT DERIVATIVES
MODEL 294-9
$V_m$ = 15.89 f. p. s.

(1 block = 10 divisions on 1/30 Engr. scale)

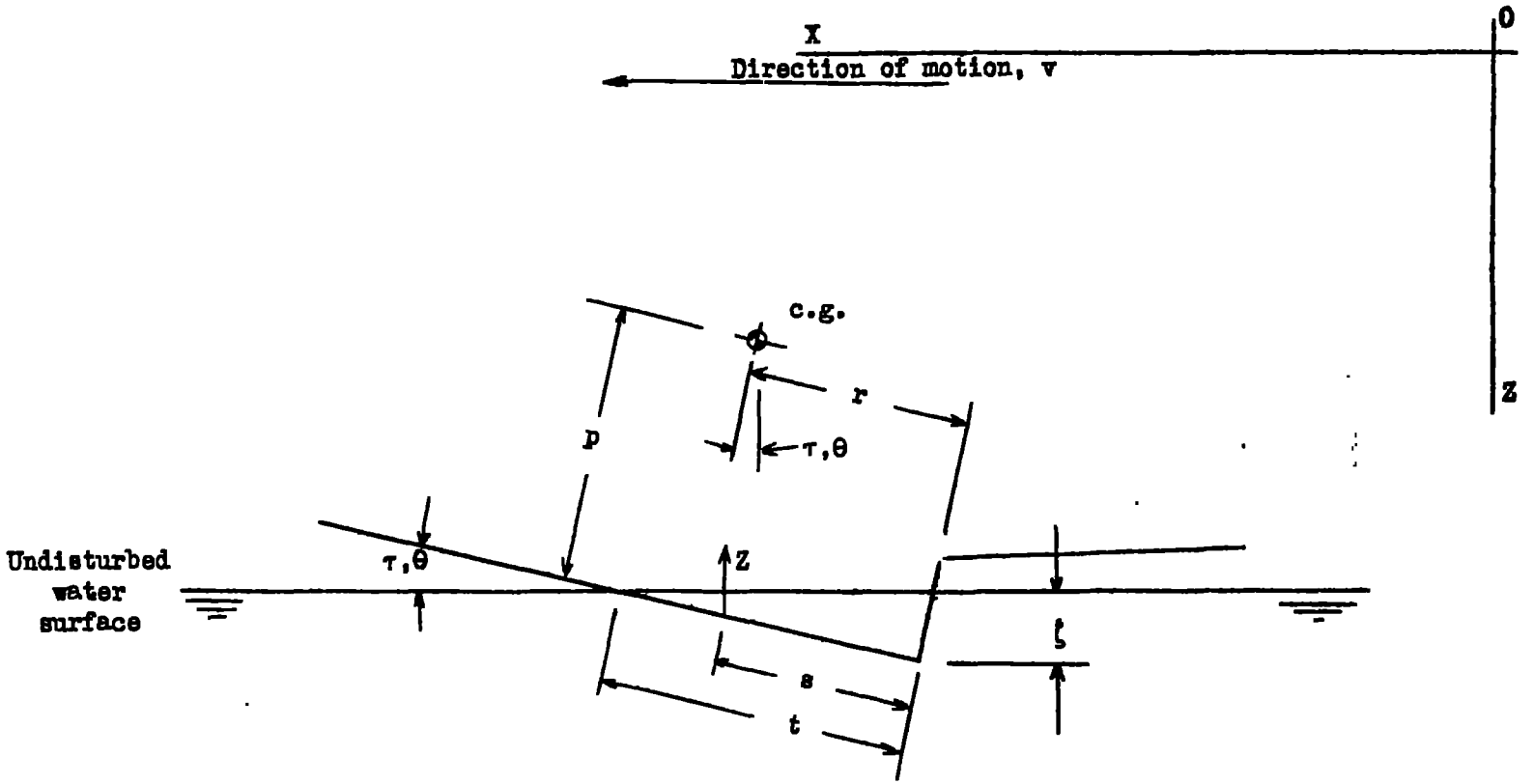
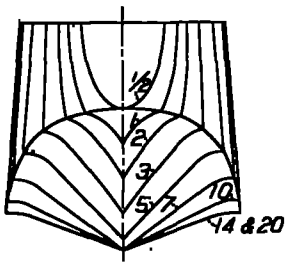
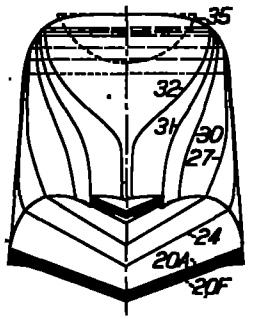



Figure 6.- System of axes employed in stability calculations.




Front view

Scale: 1/80 full size




Rear view

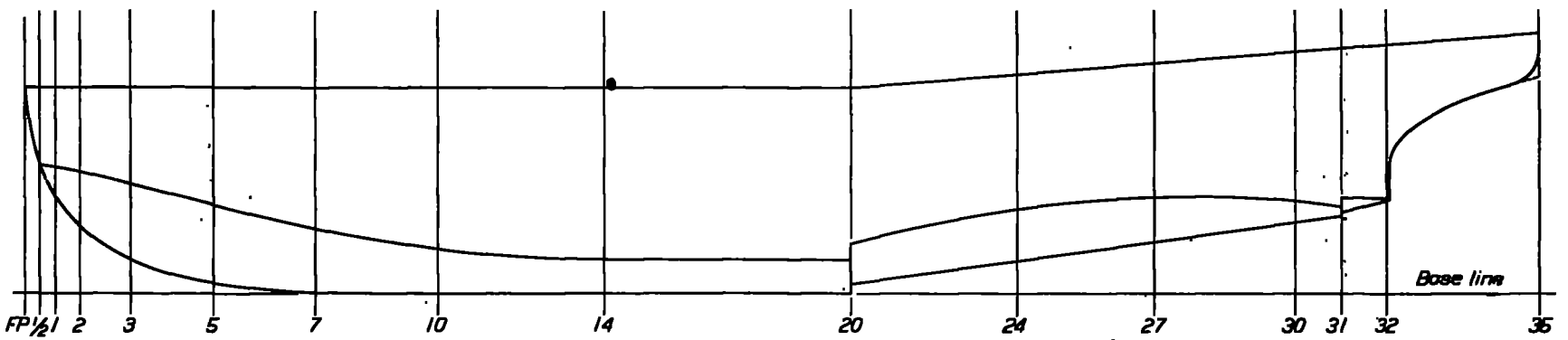



Figure 7.- Model No. 294-9.

## APPENDIX

This appendix presents additional steady-motion test data for higher trim angles at the original speed and for a higher speed.

The data were obtained after the work described in the body of the report had been completed, and with the expectation of extending the number of comparisons between experiment and theory, particularly for trim angles in the vicinity of the upper limit. This plan was abandoned when it was found that the sharp curvatures in the moment curves, occurring at trim angles in the region of upper-limit porpoising, made it extremely difficult to deduce accurate values for the displacement derivatives in this region. If the displacement derivatives were inaccurately defined, it was obvious that the velocity derivatives depending on them would be inaccurate also and that the final values of Routh's discriminant would certainly be open to question. Thus it appeared that reliable comparisons between experiment and theory would be difficult to obtain. The data are presented in toto (figs. 8 to 19), however, so that any one desiring to do so may carry through the calculations.

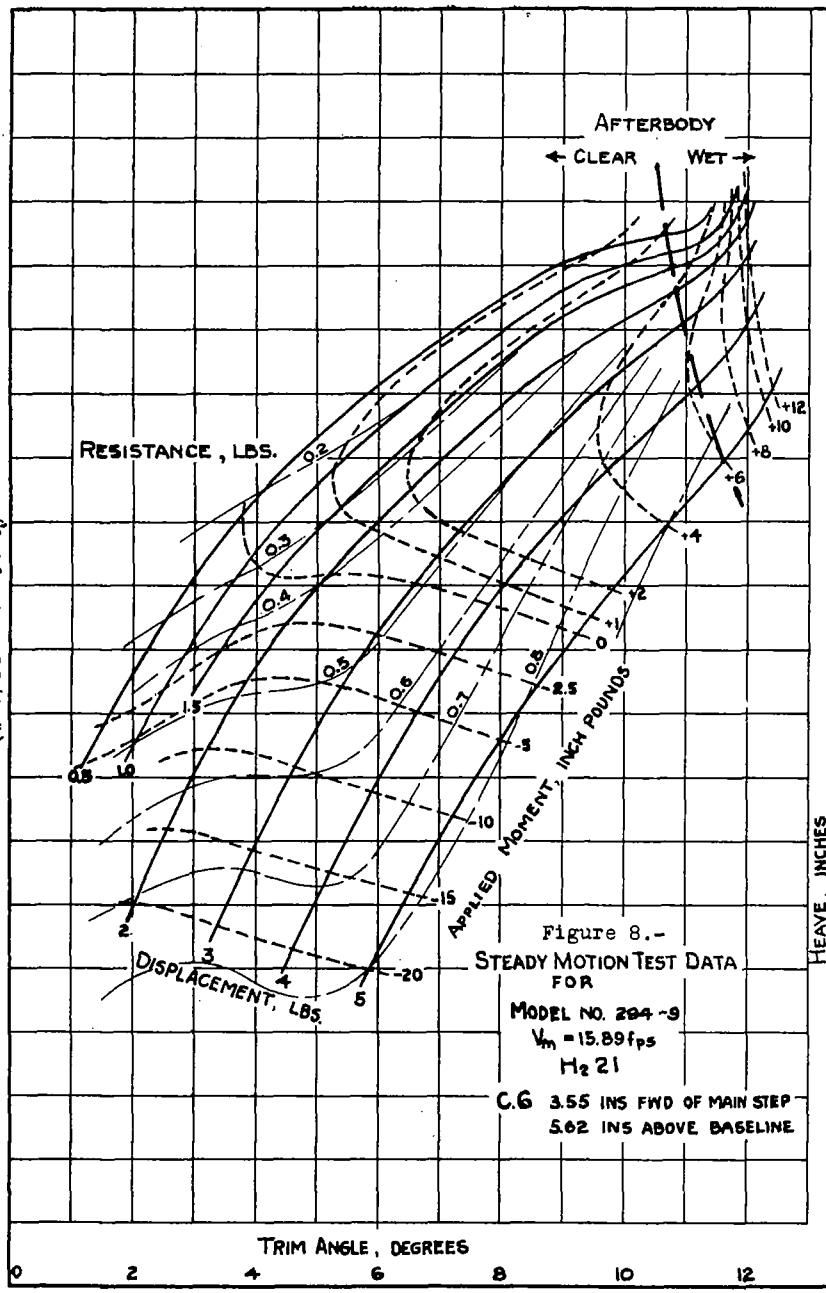



Figure 8.-
STEADY MOTION TEST DATA
FOR
MODEL NO. 294-9
$V_m$ = 15.89 fps
$H_2$ 21
C.G. 3.55 INS FWD OF MAIN STEP
5.62 INS ABOVE BASELINE

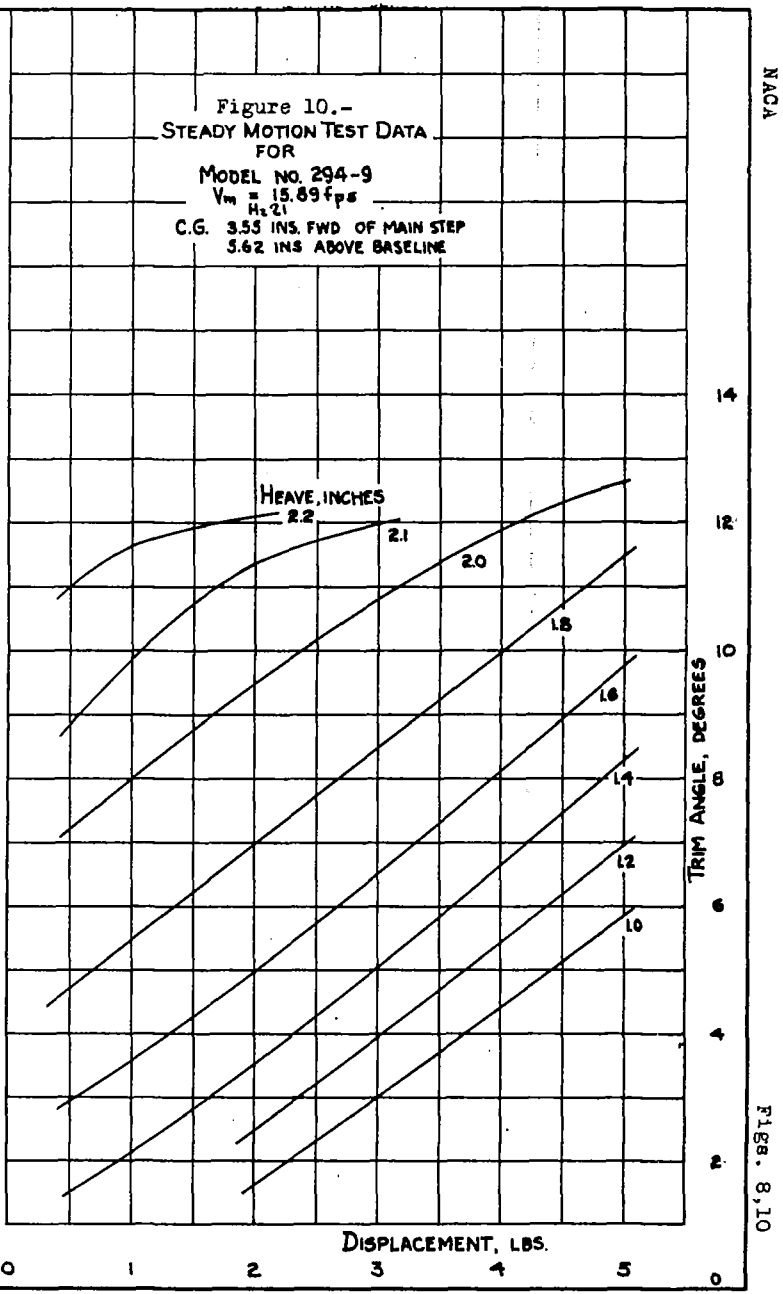



Figure 10.-
STEADY MOTION TEST DATA
FOR
MODEL NO. 294-9
$V_m$ = 15.89 fps
$H_2$ 21
C.G. 3.55 INS. FWD OF MAIN STEP
5.62 INS ABOVE BASELINE

(1 block = 10/30")

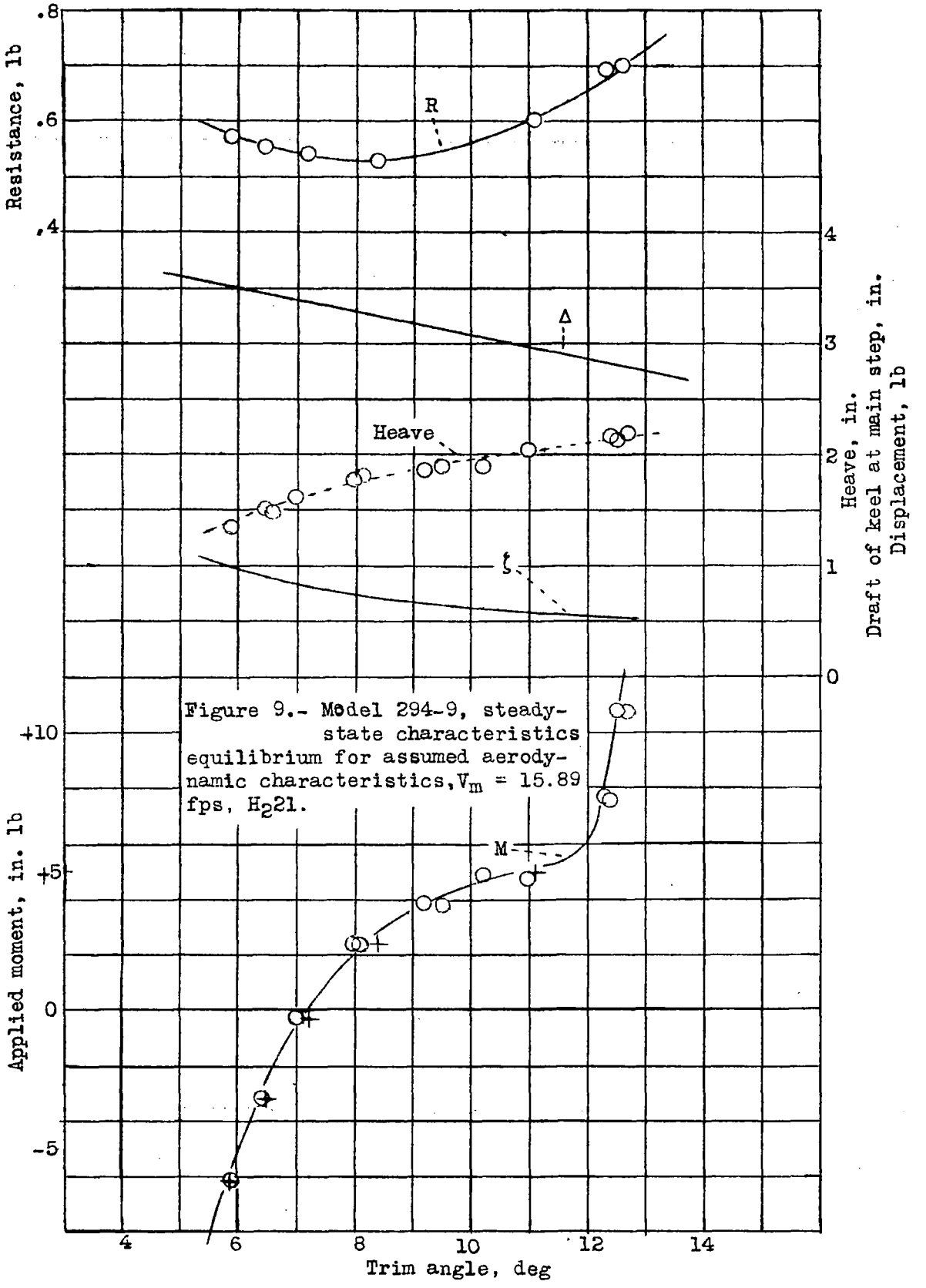



Figure 9.- Model 294-9, steady-state characteristics equilibrium for assumed aerodynamic characteristics, $V_m$ = 15.89 fps, $H_2$21.

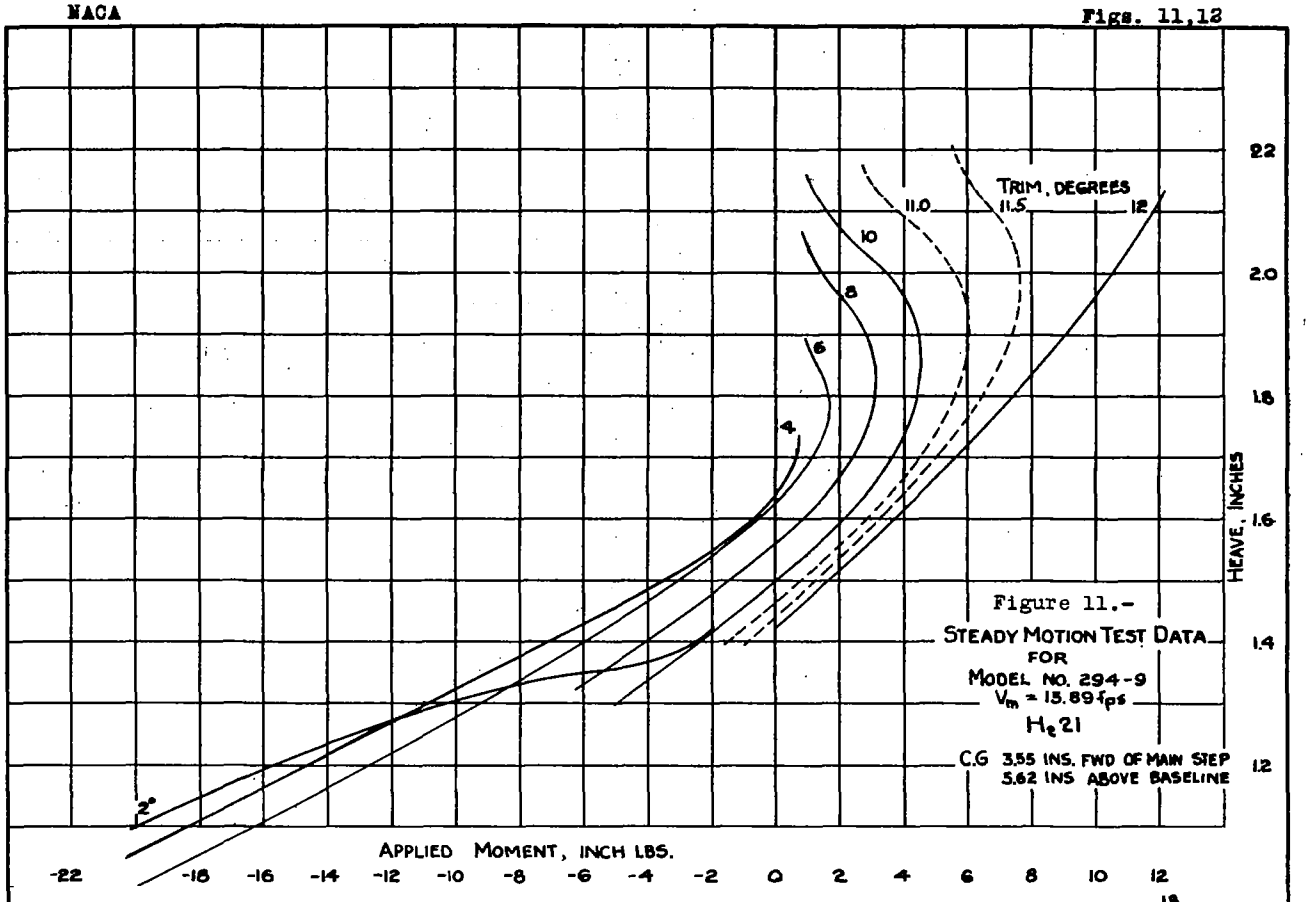



Figure 11.- STEADY MOTION TEST DATA FOR MODEL NO. 294-9 $V_m = 15.89$ fps $H_2$ 21 C.G 3.55 INS. FWD OF MAIN STEP 5.62 INS ABOVE BASELINE

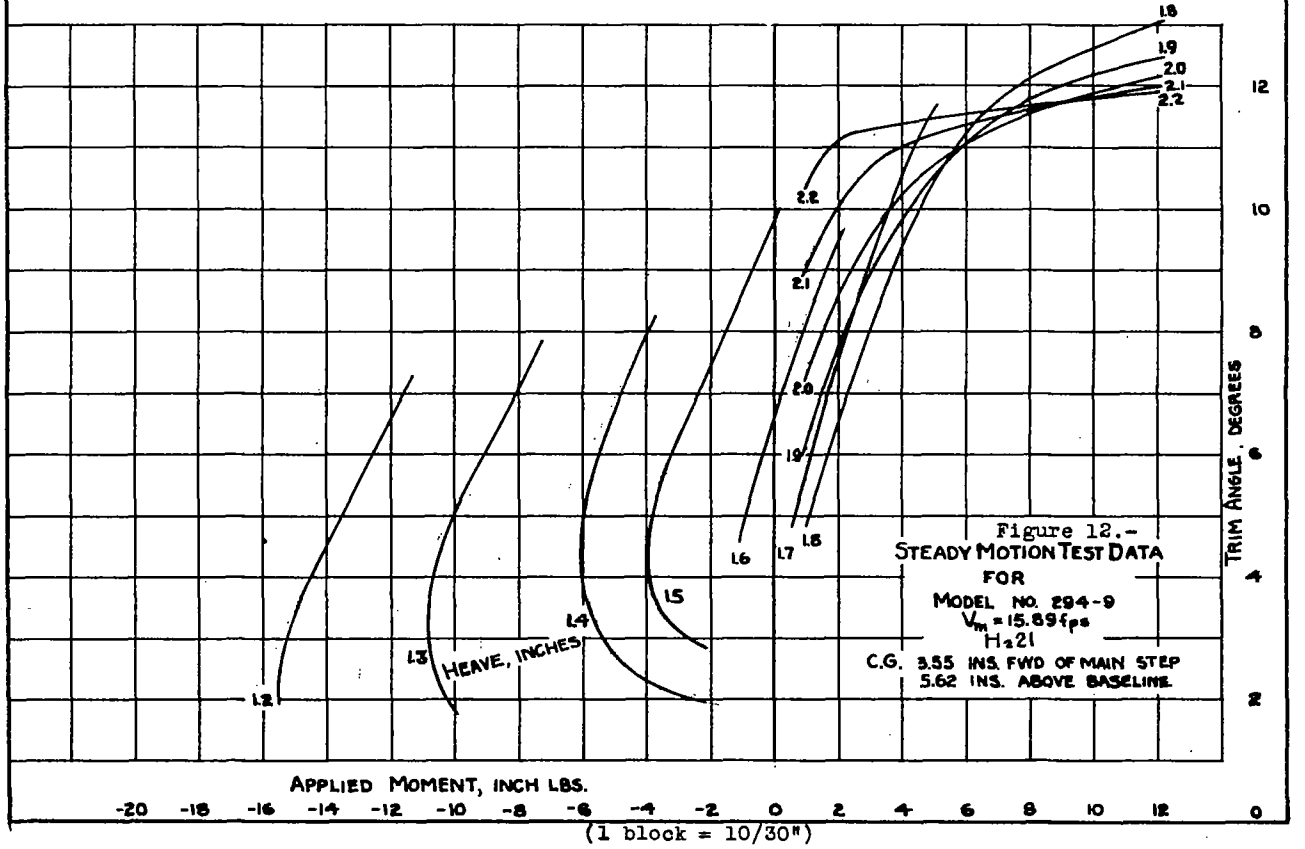



Figure 12.- STEADY MOTION TEST DATA FOR MODEL NO. 294-9 $V_m = 15.89$ fps $H_2$ 21 C.G. 3.55 INS. FWD OF MAIN STEP 5.62 INS. ABOVE BASELINE

(1 block = 10/30")

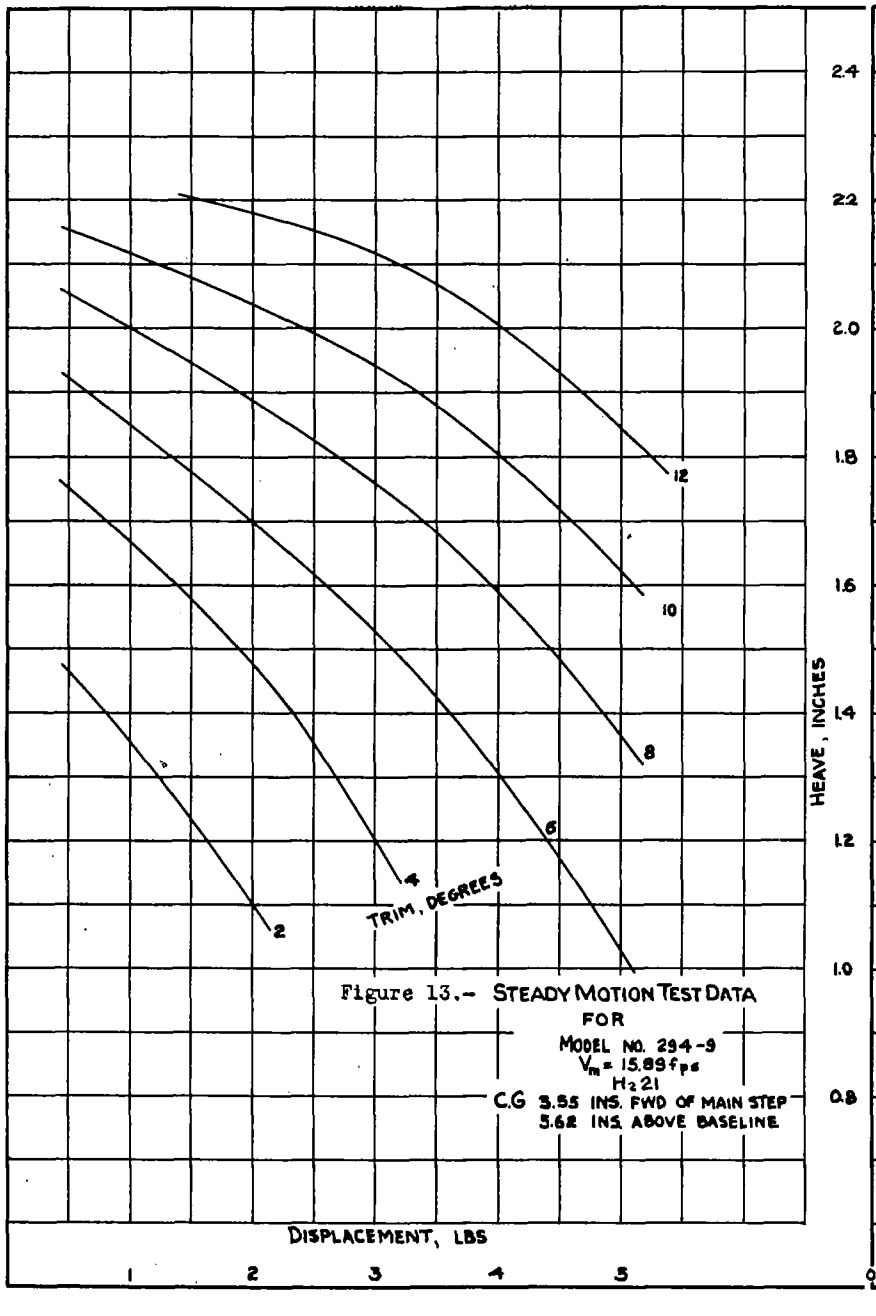



Figure 13.- STEADY MOTION TEST DATA FOR MODEL NO. 294-9
$V_m$ = 15.89 fps
H = 21
C.G 3.55 INS. FWD OF MAIN STEP
5.62 INS. ABOVE BASELINE

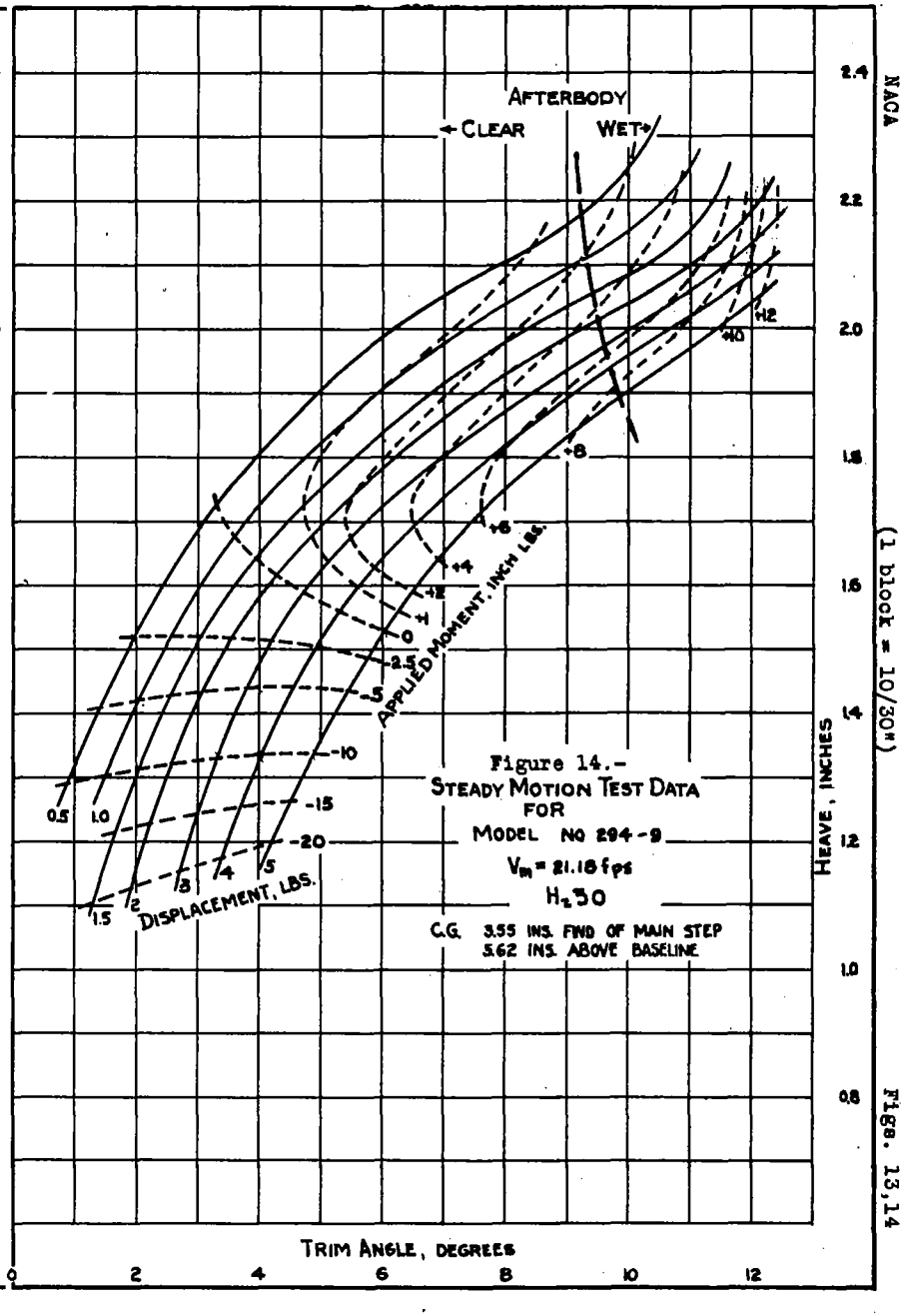



Figure 14.- STEADY MOTION TEST DATA FOR MODEL NO 294-9
$V_m$ = 21.18 fps
H = 30
C.G. 3.55 INS. FWD OF MAIN STEP
5.62 INS. ABOVE BASELINE

(1 block = 10/30")

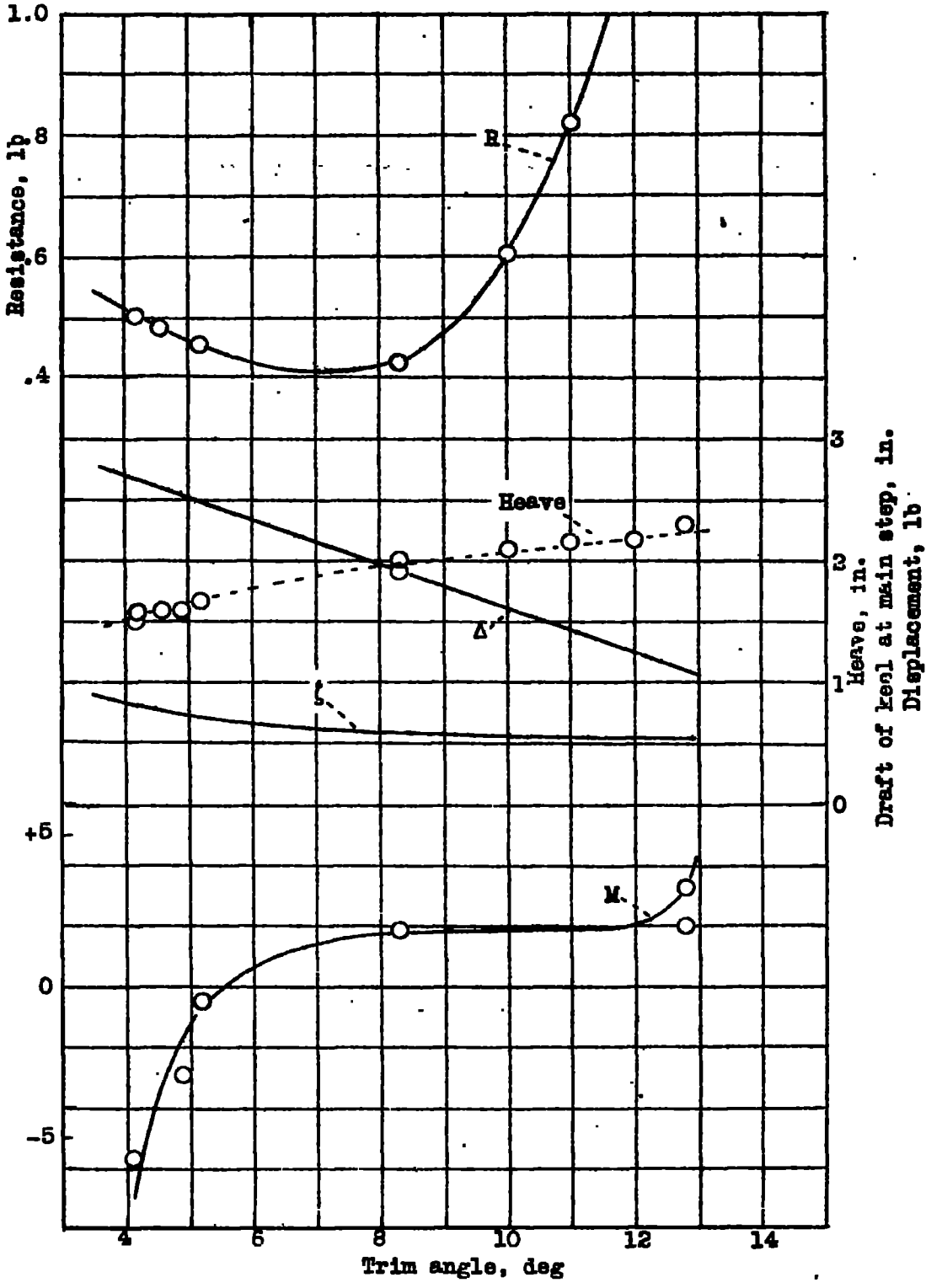



Figure 15.- Model 294-9, steady-state characteristics, equilibrium for assumed aerodynamic characteristics. $V_m$ = 21.18 fps, $H_2$30.

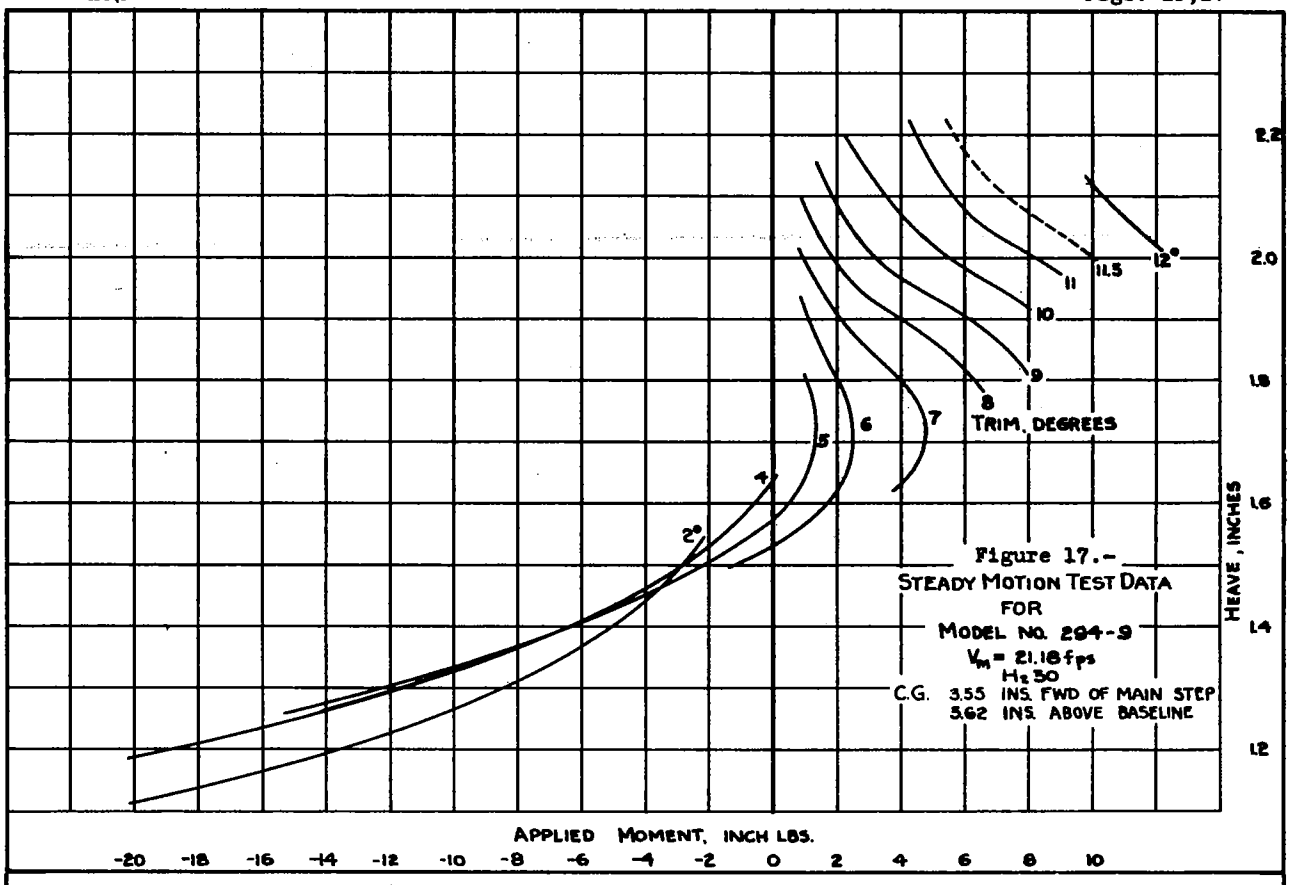



Figure 17.- STEADY MOTION TEST DATA FOR MODEL NO. 294-9 $V_m$ = 21.18 fps $H_2$ 50

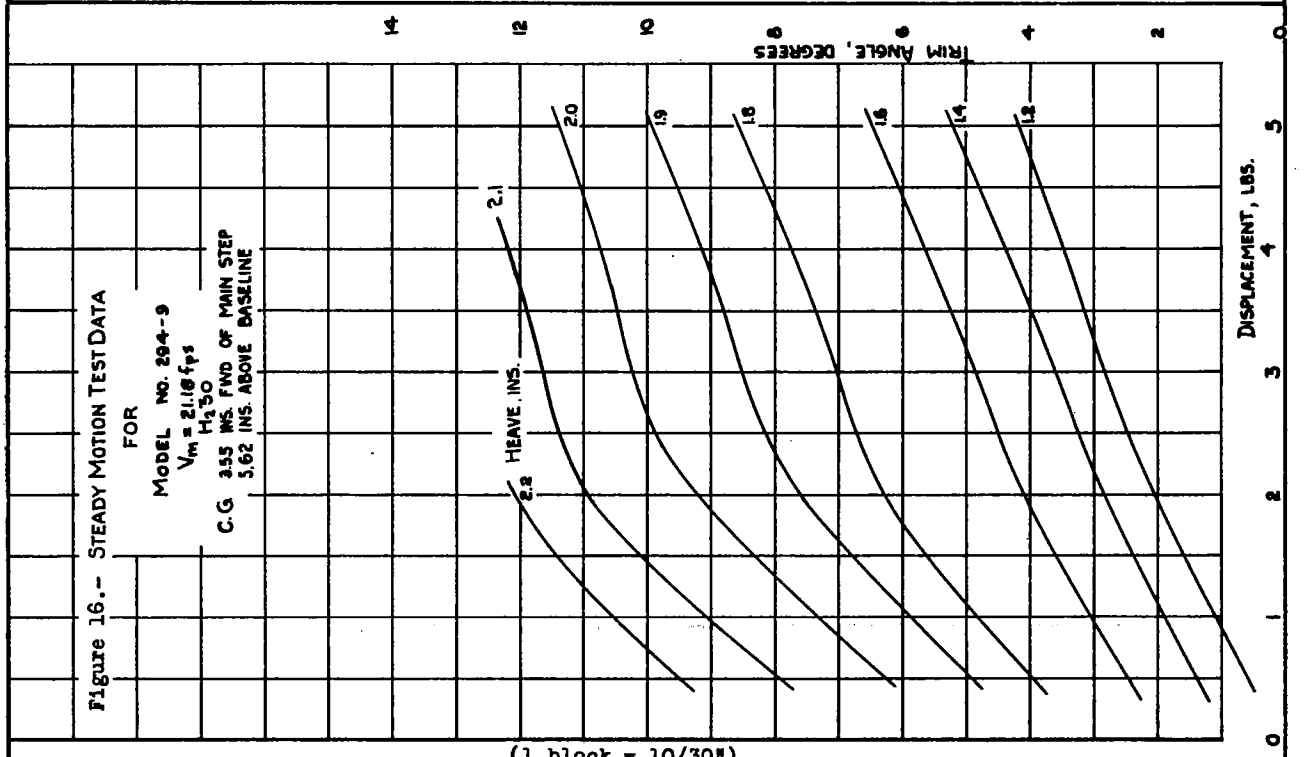
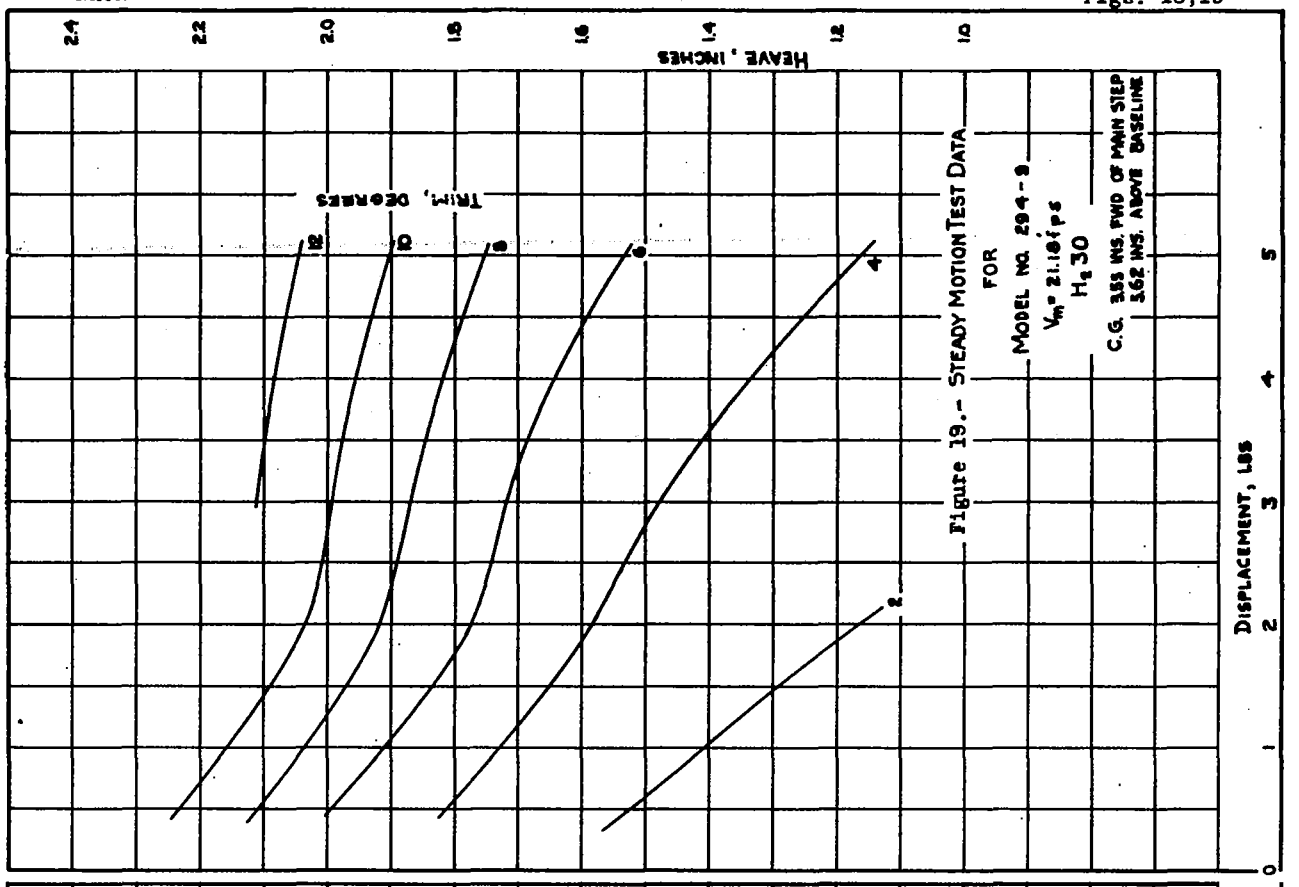
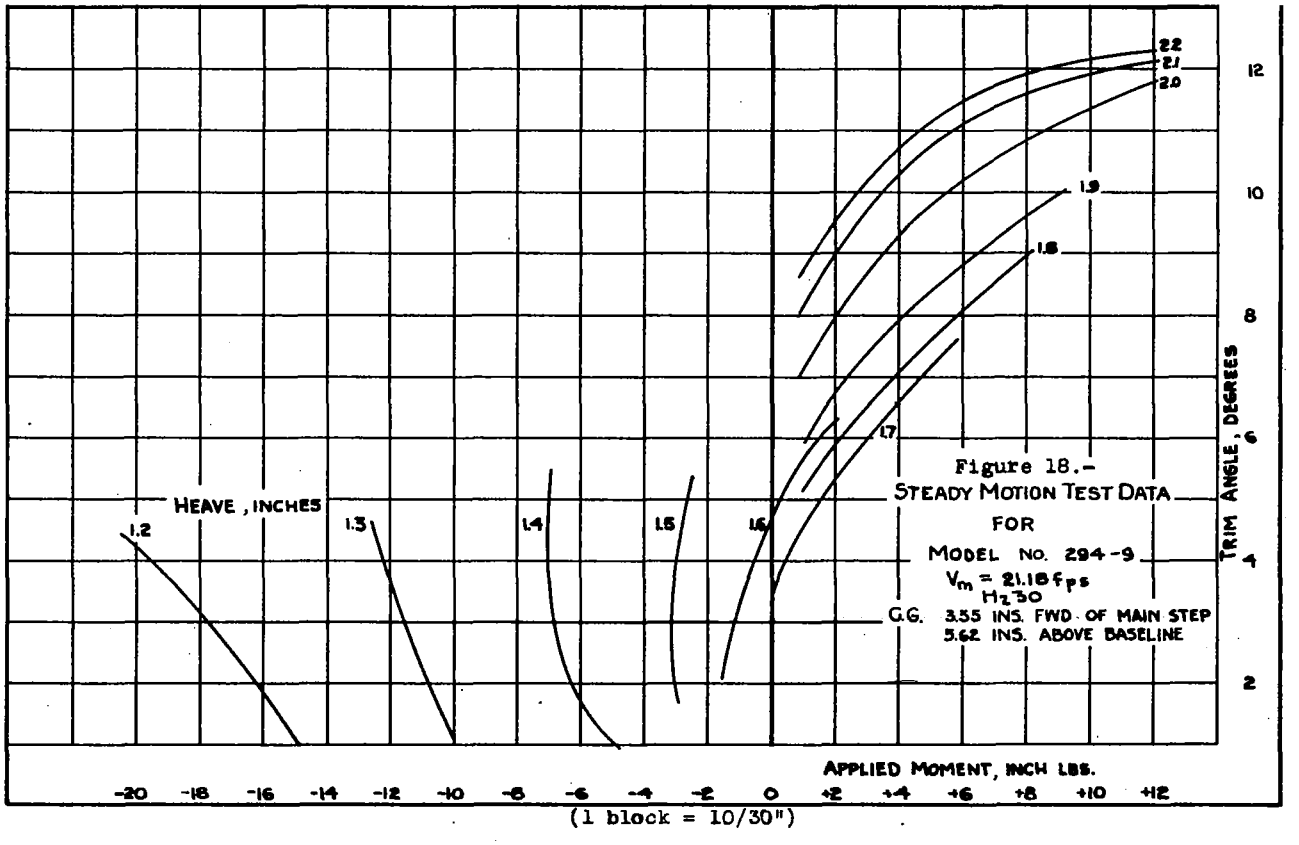



Figure 16.- STEADY MOTION TEST DATA FOR MODEL NO. 294-9 $V_m$ = 21.18 fps $H_2$ 50

(1 block = 10/30")


HEAVE, INCHES
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
TRIM, DEGREES
12
10
8
6
4
2
DISPLACEMENT, LBS
0
1
2
3
4
5
Figure 19.- STEADY MOTION TEST DATA
FOR
MODEL NO. 294-9
$V_m$ = 21.18 fps
$H_2$ 30
C.G. 3.55 INS. FWD OF MAIN STEP
5.62 INS. ABOVE BASELINE


TRIM ANGLE, DEGREES
2
4
6
8
10
12
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
HEAVE, INCHES
Figure 18.-
STEADY MOTION TEST DATA
FOR
MODEL NO. 294-9
$V_m$ = 21.18 fps
$H_2$ 30
C.G. 3.55 INS. FWD OF MAIN STEP
5.62 INS. ABOVE BASELINE
APPLIED MOMENT, INCH LBS.
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
+2
+4
+6
+8
+10
+12

(1 block = 10/30")

TITLE: Porpotsing - Comparison of Theory with Expertment

AUTHOR(S): Davidson,Kenneth S. M.; Locke, F. W. S.; and others
ORIGINATING AGENCY: National Advisory Committee for Aeronautics, Washington, D. C.
PUBLISHED BY: (Same)

ATI- B839
REVISION (None)
ORIG. AGENCY NO. ARR-3G07
PUBLISHING AGENCY NO.

| DATE | DOC. CLASS. | COUNTRY | LANGUAGE | PAGES | ILLUSTRATIONS |
|---|---|---|---|---|---|
| July '43 | Unclass. | U.S. | Eng. | 42 | tables, graphs, drwgs |


ABSTRACT:

Compartson is made of observed and calculated tongitudinal dynamtc stabilitty of a parttcular dynamic model of a flying boat moving over water. Good agreement was obtained between experiment and theory, using Glauert's statement of theory, for trim angles in vicinity of lower limit at one speed. Tabulation gives experimental and theoretical values of the aerodynamtc derivatives required for stability at vartous trim angles at the speed in question. Data are appended from which similar comparison may be made for trtm angles in the vicinity of upper limits at the same or higher speeds.


DISTRIBUTION: Request coptes of this report only from Originating Agency

DIVISION: Aerodynamtcs (2)
SECTION: Stability and Control (1)

SUBJECT HEADINGS: Seaplanes - Longitudinal stability (B40F2.5)

ATI SHEET NO.: R-2-1-32


AD-B805 893

LOS ANGELES PUBLIC LIBRARY
NACA TECHNICAL LIBRARY
MARCH 31 DECEMBER 1947

P1/3, P1/1

Fed 23

★ Seaplanes

Aerodynamic stability

TITLE: Porpoising - Comparison of Theory with Experiment

AUTHOR(S): Davidson, Kenneth S. M.; Locke, F. W. S.; and others
ORIGINATING AGENCY: National Advisory Committee for Aeronautics, Washington, D. C.
PUBLISHED BY: (Same)

ATI- 8839
REVISION (None)
ORIG. AGENCY NO. ARR-3G07
PUBLISHING AGENCY NO.

| DATE | DOC. CLASS. | COUNTRY | LANGUAGE | PAGES | ILLUSTRATIONS |
|---|---|---|---|---|---|
| July '43 | Unclass. | U.S. | Eng. | 42 | tables, graphs, drwgs |


ABSTRACT:

Comparison is made of observed and calculated longitudinal dynamic stability of a particular dynamic model of a flying boat moving over water. Good agreement was obtained between experiment and theory, using Glauert's statement of theory, for trim angles in vicinity of lower limit at one speed. Tabulation gives experimental and theoretical values of the aerodynamic derivatives required for stability at various trim angles at the speed in question. Data are appended from which similar comparison may be made for trim angles in the vicinity of upper limits at the same or higher speeds.


DISTRIBUTION: Request copies of this report only from Originating Agency

DIVISION: Aerodynamics (2)
SECTION: Stability and Control (1)

SUBJECT HEADINGS: Seaplanes - Longitudinal stability (840F2.5)

ATI SHEET NO.: R-2-1-32