UNCLASSIFIED

AD NUMBER	
ADB805391	
CLASSIFICATION CHANGES	
TO:	**unclassified**
FROM:	**restricted**
LIMITATION CHANGES	
TO: **Approved for public release; distribution is unlimited.**	
FROM: **Distribution authorized to DoD only; Administrative/Operational Use; JAN 1943. Other requests shall be referred to National Aeronautics and Space Administration, Washington, DC. Pre-dates formal DoD distribution statements. Treat as DoD only.**	
AUTHORITY	
NACA list dtd 28 Sep 1945; NASA TR Server website	

THIS PAGE IS UNCLASSIFIED

NACA
CLASSIFICATION CANCELLED

TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 884

LARGE-DEFLECTION THEORY FOR END COMPRESSION OF LONG RECTANGULAR PLATES RIGIDLY CLAMPED ALONG TWO EDGES

By Samuel Levy and Philip Krupen
National Bureau of Standards

CLASSIFIED DOCUMENT

This document contains classified information affecting the National Defense of the United States within the meaning of the Espionage Act, USC 50:31 and 32. Its transmission or the revelation of its contents in any manner to an unauthorized person is prohibited by law. Information so classified may be imparted only to persons in the military and naval Services of the United States, appropriate civilian officers and employees of the Federal Government who have a legitimate interest therein, and to United States citizens of known loyalty and discretion who of necessity must be informed thereof.

Washington
January 1943

NASA Technical Library
3 1176 01433 8017

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 884

LARGE-DEFLECTION THEORY FOR END COMPRESSION OF LONG RECTANGULAR PLATES RIGIDLY CLAMPED ALONG TWO EDGES

By Samuel Levy and Philip Krupen

SUMMARY

The von Kármán equations for flat plates are solved beyond the buckling load up to edge strains equal to eight times the buckling strain, for the extreme case of rigid clamping along the edges parallel to the load. Deflections, bending stresses, and membrane stresses are given as a function of end compressive load. The theoretical values of effective width are compared with the values derived for simple support along the edges parallel to the load. The increase in effective width due to rigid clamping drops from about 20 percent near the buckling strain to about 8 percent at an edge strain equal to eight times the buckling strain. Experimental values of effective width in the elastic range reported in NACA Technical Note No. 684 are between the theoretical curves for the extremes of simple support and rigid clamping.

INTRODUCTION

The stress distribution in rectangular plates which have buckled under compressive loads applied to the ends is important for estimating the load carried by the sheet in sheet-stringer construction. The compression flange of a monocoque box beam is an excellent example of such construction. After the sheet buckles, its effectiveness in supporting the load is reduced so that the "effective width" of sheet between stringers will be less than the stringer spacing by an amount which will depend on the dimensions of the sheet, the amount by which the buckling load has been exceeded, and the restraint provided by the stringers.

For convenience, the analysis of a rectangular plate under end compressive loads with elastic restraint against rotation along the two unloaded edges may be separated into

two problems: the determination of the buckling load of the plate; and the determination of the effective width, deflections, and stress distribution for loads greater than the buckling load.

Solutions for the buckling load have already been derived by Lundquist and Stowell (reference 1) and by Dunn (reference 2). Lundquist and Stowell obtained a solution by an energy method using a small correction term based on an exact solution. Dunn solved the second of von Kármán's equations for plates with large deflection on the assumption that the deflection along the direction of loading can be expressed as a single sine function. Dunn's assumption is valid for small deflections and the results represent exact solutions of the equations.

In order to determine exactly the load after buckling, the large deflection theory of plates must be applied. Such a solution is known only for a square plate having simply supported edges on all four sides. (See reference 3.)

Cox obtained an approximate solution for plates having the unloaded edges clamped (reference 4). In this derivation, Cox approximated the transverse section of the buckled surface of the plate by a combination of a squared sine function and a straight line and he assumed that the strain is uniform along the whole length of a narrow element of the panel. In view of the questions that may be raised concerning Cox's assumptions it was decided to derive an exact solution of the problem for plates having the unloaded edges clamped and to compare the results with Cox's approximate solution and with the experimental data that are available.

SYMBOLS

For an initially flat rectangular plate of uniform thickness (fig. 1) the following symbols may be applied:

a plate length

b plate width

h plate thickness

E Young's modulus

μ Poisson's ratio

D	flexural rigidity $\frac{Eh^3}{12(1-\mu^2)}$
x, y	coordinate axes lying along the edges of the plate, x axis in direction of load
m_x	edge bending moment per unit length about the x axis
$\overline{p}_x$	average compressive stress in x-direction
$\overline{\epsilon}_x$	average compressive strain in x-direction
σ_x, σ_y	extreme fiber stresses in directions of axes
σ_x', σ_y'	median fiber stresses in directions of axes
σ_x'', σ_y''	extreme fiber bending stresses in directions of axes
τ_{xy}	shearing stress
w	normal displacement of points at the midthickness
$w_{m,n}$	deflection coefficients
F	stress function
$b_{m,n}$	stress coefficients
$p\ (x,y)$	pressure replacing edge moments, m_x
$p_{m,n}$	coefficient in Fourier series for $p(x,y)$
k_m	moment coefficient
c	moment arm of substitute pressure, $p(x,y)$
m,n	subscripts
ϵ_x, ϵ_y, γ_{xy}	strains
p_{cr}	critical value of $\overline{p}_x$

ANALYSIS

Expressions for Stresses and Strains

The median stresses at the midthickness of the plate are related to the stress function F by

$$\left.\begin{aligned} \sigma_x' &= \frac{\partial^2 F}{\partial y^2} \\ \sigma_y' &= \frac{\partial^2 F}{\partial x^2} \\ \tau_{xy}' &= -\frac{\partial^2 F}{\partial x \partial y} \end{aligned}\right\} \text{(Reference 5)} \qquad (1)$$

the extreme-fiber bending stresses in the plate are related to the deflections by

$$\left.\begin{aligned} \sigma_x'' &= -\frac{Eh}{2(1-\mu^2)}\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) \\ \sigma_y'' &= -\frac{Eh}{2(1-\mu^2)}\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right) \\ \tau_{xy}'' &= -\frac{Eh}{2(1+\mu)}\left(\frac{\partial^2 w}{\partial x \partial y}\right) \end{aligned}\right\} \text{(Reference 5)} \qquad (2)$$

and the extreme fiber bending stresses at the rigidly clamped edges of the plate (y=0, y=b) are related to the edge bending moment per unit length by

$$\left.\begin{aligned} \sigma_x'' &= \mu\frac{6m_x}{h^2} \\ \sigma_y'' &= \frac{6m_x}{h^2} \end{aligned}\right\} \begin{array}{c}(y=0,\ y=b)\\ \text{(See reference 5)}\end{array} \qquad (3)$$

The strains at the midthickness of the plate are, from equation (1),

$$\left.\begin{aligned}
\epsilon_x' &= \frac{1}{E}\left(\sigma_x' - \mu\,\sigma_y'\right) = \frac{1}{E}\left(\frac{\partial^2 F}{\partial y^2} - \mu\,\frac{\partial^2 F}{\partial x^2}\right) \\
\epsilon_y' &= \frac{1}{E}\left(\sigma_y' - \mu\,\sigma_x'\right) = \frac{1}{E}\left(\frac{\partial^2 F}{\partial x^2} - \mu\,\frac{\partial^2 F}{\partial y^2}\right) \\
\gamma_{xy}' &= \frac{2(1+\mu)}{E}\,\tau_{xy}' = -\frac{2(1+\mu)}{E}\,\frac{\partial^2 F}{\partial x \partial y}
\end{aligned}\right\} \tag{4}$$

Relation between Edge Moment and Equivalent Normal Pressure

The edge moments at the rigidly clamped edges ($y=0$, $y=b$) (fig. 2 (a)) can be expressed in terms of a substitute pressure distribution near the edges of the plate as shown in figure 2(b). When this pressure distribution is expressed by a Fourier series (see reference 6) and the value of the moment arm c approaches zero, the substitute pressure becomes

$$p(x,y) = \sum_{n=1,3,5,\ldots} n\,\frac{4\pi m_x}{b^2}\,\sin\frac{n\pi y}{b} \tag{5}$$

The bending moment per unit length, m_x, at the restrained edges ($y=0$, $y=b$), is as yet unknown. It may be respresented as a sine series with fundamental wave length $2a$ because the plate in figure 2 may be regarded as a portion of an infinitely long plate with alternating inward and outward buckles of length a. Let

$$m_x = \frac{b^2}{4\pi}\sum_{m=1,2,3,\ldots} k_m\,\sin\frac{m\pi x}{a} \tag{6}$$

Combining equation (6) and equation (5) gives

$$p(x,y) = \sum_{m=1,2,3,\cdots} \sum_{n=1,3,5,\cdots} p_{m,n} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad (7)$$

where

$$p_{m,n} = nk_m \qquad (8)$$

Relation between Stress Function, Lateral Deflection, and Pressure Coefficients

Because the edge moments m_x have been replaced by an auxiliary pressure function $p(x,y)$, equation (7), the general solution for the simply-supported rectangular plate (reference 3) may be applied. This solution was derived in terms of Fourier's series, from von Kármán's equations given on page 323 of reference 5.

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right]$$

$$\frac{\partial^4 w}{\partial x^2} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D} + \frac{h}{D}\left[\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}\right] \qquad (9)$$

In order to meet the conditions of symmetry, Fourier's series for the deflection must have the form

$$w = \sum_{m=1,2,3,\cdots} \sum_{n=1,3,5,\cdots} w_{m,n} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad (10)$$

The normal pressure is described by Fourier's series, equation (7). The stress function is, according to reference 3,

$$F = -\frac{\overline{p}_x y^2}{2} + \sum_{m=0,2,4,\ldots} \sum_{n=0,2,4,\ldots} b_{m,n} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \qquad (11)$$

and the displacement of the edges x=0, x=a toward each other is:

$$a\overline{\epsilon}_x = \frac{\overline{p}_x a}{E} + \frac{\pi^2}{8a} \sum_{m=1,2,3,\ldots} \sum_{n=1,3,5,\ldots} m^2 w_{m,n}^2 \qquad (12)$$

where $\overline{\epsilon}_x$ is the average compressive strain in the x-direction and $\overline{p}_x$ is the average compressive stress in the x-direction.

The solution for large deflections has been carried out (reference 3) for the extreme case of a square plate (or an infinitely long plate) with simply supported edges, using $\mu = 0.316$. In the present paper, the solution for large deflections is carried out for the other extreme case of the unloaded edges rigidly clamped, using a ratio of buckle width to buckle length of 1.5 and $\mu = 0.316$. Figures 3 and 6 of reference 1 show that a ratio of buckle width to buckle length of 1.5 approaches the buckle spacing for an infinitely long plate having rigidly clamped edges.

The general solution of reference 3 gives equations for calculating the coefficients $b_{m,n}$ in the stress function F (equation (11)), in terms of the deflection coefficients $w_{m,n}$ in equation (10). For the special case (b = 1.5a), these equations reduce to the form given in table 1. Equations for the first 26 stress coefficients $b_{m,n}$ are given in table 1.

The general solution of reference 3 also gives the family of equations relating the pressure coefficients $p_{m,n}$ (equation (8)), the deflection coefficients $w_{m,n}$, and the average compressive stress in the x-direction $\overline{p}_x$.

For the special case ($b=1.5a$ and $\mu = 0.316$), these equations can, with the aid of table 1, be reduced to the form given in table 2. For these equations, only the first 59 terms have been retained. The error introduced by limiting the solution in this way will be considered subsequently.

As an example of the use of table 2, the first five terms of the first equation are

$$0 = -\frac{b^4 k_1}{\pi^4 E h^4} + 0.978\,\frac{w_{1,1}}{h} - 0.228\,\frac{\overline{p}_x\, b^2}{E\, h^2}\,\frac{w_{1,1}}{h}$$

$$+ 0.3791\left(\frac{w_{1,1}}{h}\right)^3 - 0.950\left(\frac{w_{1,1}}{h}\right)^2\left(\frac{w_{1,3}}{h}\right) - \cdots \quad (13)$$

The values of the bending moment coefficients k_m are given by the condition that the slope is zero at the edges of the plate ($y=0$, $y=b$)

$$\left(\frac{\partial w}{\partial y}\right)_{y=0} = 0; \qquad \left(\frac{\partial w}{\partial y}\right)_{y=b} = 0$$

Substituting equation (10) into these equations gives

$$0 = \sum_{m=1,2,3\ldots}^{\infty}\ \sum_{n=1,3,5,\ldots}^{\infty} \frac{n\pi}{b}\, w_{m,n} \sin\frac{m\pi x}{a} \qquad (14)$$

This equation is equivalent to the family of equations

$$\left.\begin{aligned}
0 &= w_{1,1}+3w_{1,3}+5w_{1,5}+7w_{1,7}+\cdots \\
0 &= w_{3,1}+3w_{3,3}+5w_{3,5}+7w_{3,7}+\cdots \\
0 &= w_{5,1}+3w_{5,3}+5w_{5,5}+7w_{5,7}+\cdots \\
&\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}\right\} \qquad (15)$$

The equations in table 2 were solved for the linear-deflection-coefficient term in terms of the remaining terms and these values were substituted in equations (15). The resulting equations, after terms are combined, are given in table 3.

As an example of the use of table 3, the first few terms of the first equation are

$$0 = -3.010 \frac{b^4 k_1}{\pi^4 E h^4} - 0.233 \frac{\overline{p}_x b^2}{E h^2} \frac{w_{1,1}}{h}$$

$$- .058 \frac{\overline{p}_x b^2}{E h^2} \frac{w_{1,3}}{h} - \ldots + 0.307 \left(\frac{w_{1,1}}{h}\right)^3$$

$$- .453 \left(\frac{w_{1,1}}{h}\right)^2 \left(\frac{w_{1,3}}{h}\right) - \ldots \qquad (16)$$

The equations given in tables 2 and 3 involve the known average compressive stress in the x-direction $\overline{p}_x$, the unknown deflection coefficients $w_{m,n}$, and the unknown moment coefficients k_m. For any value of $\overline{p}_x$, there are a sufficient number of equations in tables 2 and 3 to determine each of the unknown deflection coefficients and unknown moment coefficients.

It will be noted that the equations in tables 2 and 3 are cubics and their solution therefore gives three values for each of the deflection coefficients $w_{m,n}$. Some of these values correspond to stable equilibrium, whereas the remaining values are either imaginary or correspond to unstable equilibrium. Fortunately, if the equations in tables 2 and 3 are solved by a method of successive approximation, the successive approximations approach values corresponding to stable equilibrium.

In the solution of the equations in tables 2 and 3, values of the principal deflection coefficient $w_{1,1}/h$ were first chosen. Successive approximations were then used to determine the values of $\overline{p}_x b^2/Eh^2$, the first 21 deflection coefficients $w_{m,n}/h$, and the first 3 moment coefficients k_m corresponding to the chosen values of $w_{1,1}/h$.

The work of computation was reduced by starting with a good approximation to the unknown coefficients, which was obtained by extrapolation from the known coefficients for smaller values of $w_{1,1}/h$. The results for 14 values of $w_{1,1}/h$, increasing by small increments from 0 to 3.00, are given in tables 4 and 5 and figure 3.

The membrane stress coefficients were computed from table 1 and table 4 and are given in table 6. The membrane stresses for the corner of the plate, the midpoints of the edges, and the center of the plate were then computed from equations (1) and (11) and table 6 with the results given in tables 7 to 10 and figure 4. At the maximum computed load, the membrane stresses at the corner (σ_x' at $x=0$, $y=0$) and at the midpoint of the restrained edge (σ_x' at $x=a/2$, $y=0$) are more than twice the average compressive stress $\overline{p}_x$, while the membrane stress at the center of the loaded edge (σ_y' at $x=0$, $y=b/2$) is less than half of $\overline{p}_x$.

The bending stresses were computed for the center, the corners of the plate, and the midpoint of the loaded edge from equations (2) and (10) and from table 4 with the results given in tables 7, 8, 10 and figure 5. The bending stresses were computed for the midpoint of the restrained edge from equations (3) and (6) and table 5 with the results given in table 9 and figure 5. Equations (3) were used in this instance instead of equations (2) since equations (3) represent the limit value of equations (2) as the edge is approached. At the maximum computed load, the transverse extreme-fiber bending stress at the midpoint of the restrained edge (σ_y'' at $x = a/2$, $y=0$) is more than 4 times the average compressive stress $\overline{p}_x$, while the axial extreme fiber bending stress at the center of the plate (σ_x'' at $x = a/2$, $y = b/2$) is about $1\frac{1}{2}$ times the average compressive stress $\overline{p}_x$.

Inasmuch as the shearing stresses at the points considered are zero, the principal stresses in the extreme fibers are equal to the sums of the membrane stresses and the extreme-fiber bending stresses. The values are given in tables 7 to 10 and in figure 6. At the maximum computed load, the largest extreme-fiber stress (σ_y on the inside of the buckle at $x = a/2$, $y=0$) becomes about 5 times as great as the average compressive stress $\overline{p}_x$.

The deflection of the center of the plate was computed from equation (10) and the results are given in table 10 and figure 3.

The ratio of effective width to initial width (defined as the ratio of the actual load carried by the plate to the load the plate would have carried if the stress had been uniform and equal to the Young's modulus times the average edge strain) was computed from equation (12) and table 4 with the results given in table 11 and figure 7. At the maximum computed load the average edge strain ratio is 48.66 while the ratio of effective width to initial width is 0.478. In figure 7 is also plotted the effective-width curve for the simply supported plate (reference 3). It can be seen that the difference in effective widths between the extreme cases of simply supported and rigidly clamped plates is about 20 percent near the buckling loads and decreases to about 8 percent at the highest computed loads.

Convergence of Solution

The exactness of the solution increases with the number of terms in the equations in tables 2 and 3. In the present solution, the first 56 cubic terms were retained in tables 2 and 3. The effect of limiting the number of terms is brought out by the comparison in table 12 of solutions in which 1, 10, and 56 cubic terms were retained. When only one cubic term was kept it was the cube of $w_{1,1}/h$; when 10 were kept, they were cubic products of $w_{1,1}/h$, $w_{1,3}/h$, and $w_{3,1}/h$; and when 56 were kept they were cubic products of $w_{1,1}/h$, $w_{1,3}/h$, $w_{1,5}/h$, $w_{3,1}/h$, $w_{3,3}/h$ and $w_{5,1}/h$. It is evident from table 12 that the convergence is rapid for effective width.

COMPARISON WITH RESULTS PREVIOUSLY OBTAINED BY H. L. COX

The effective width for rigid clamping is compared in figure 8 with that obtained by Cox (reference 4) on the assumption that the strain is uniform along the whole length of a narrow element of the plate. It is evident that, even though Cox has only roughly approximated the actual deflections and strains in the plate, his results are in excellent agreement with the "exact" results obtained in the present paper for edge strains less than 3 times the critical edge strain. For larger edge strains, his results are low by as much as 6 percent.

COMPARISON WITH EXPERIMENTAL RESULTS

The relations between edge-strain ratios and experimental values of effective width for a sheet-stringer panel of 0.070 inch 24S-T alclad aluminum alloy and a panel of 0.025 inch 24S-T aluminum alloy (reference 8) are shown in figure 9. It is evident that in the case of the alclad specimen, in which the stringers approximated simple support, the agreement is excellent up to $\bar{\epsilon}_x \frac{b^2}{h^2} = 8.2$, corresponding to a strain at the edge of 0.0025, for which yielding due to the combined bending and membrane stresses was probably taking place in the aluminum coating. In the case of the aluminum alloy specimen the observed effective width fell midway between that calculated for the extreme cases of simply supported and rigidly clamped edges up to $\bar{\epsilon}_x \frac{b^2}{h^2} = 26$, corresponding (see tables 10 and 11 and fig. 6) to stresses at the midpoint of the restrained edge of about 27,000 pounds per square inch. Beyond this point the observed values closely approach the curve for simply supported edges; this result may be explained by a release of the bending stresses at the restrained edge due to yielding of the material.

National Bureau of Standards
Washington, D. C., May 20, 1942

REFERENCES

1. Lundquist, Eugene E., and Stowell, Elbridge Z.: Critical Compressive Stress for Flat Rectangular Plates Supported along All Edges and Elastically Restrained Against Rotation along the Unloaded Edges. Rep. No. 733, NACA, 1942.

2. Dunn, Louis G.: An Investigation of Sheet-Stiffener Panels Subjected to Compression Loads with Particular Reference to Torsionally Weak Stiffeners. T.N. No. 752, NACA, 1940.

3. Levy, Samuel: Bending of Rectangular Plates with Large Deflection. T.N. No. 846, NACA, 1942.

4. Cox, H. L.: The Buckling of Thin Plates in Compression. R. & M. No. 1554, British A.R.C., 1933.

5. Timoshenko, S.: Theory of Elastic Stability. McGraw-Hill Book Co., Inc., 1936.

6. Woods, Frederick S.: Advanced Calculus. Ginn and Co., Boston, 1926, pp. 295-301.

7. Lundquist, Eugene E., and Stowell, Elbridge Z.: Restraint Provided a Flat Rectangular Plate by a Sturdy Stiffener Along an Edge of the Plate. Rep. No. 735, NACA, 1942.

8. Ramberg, Walter, McPherson, Albert E., and Levy, Sam: Experimental Study of Deformation and of Effective Width in Axially Loaded Sheet-Stringer Panels. T.N. No. 684, NACA, 1939.

TABLE 1.- STRESS COEFFICIENTS IN TERMS OF DEFLECTION COEFFICIENTS [b/a = 1.5]

$$b_{0,2} = \frac{E}{28.44}\,(2w_{1,1}^2 - 4w_{1,1}\,w_{1,3} - 4w_{1,3}\,w_{1,5} - 4w_{1,5}\,w_{1,7} - 4w_{1,7}\,w_{1,9} - 4w_{1,9}\,w_{1,11} - 4w_{1,11}\,w_{1,13} + 18w_{3,1}^2 - 36w_{3,1}\,w_{3,3} - 36w_{3,3}\,w_{3,5} + 50w_{5,1}^2 + 98w_{7,1}^2 \cdots)$$

$$b_{0,4} = \frac{E}{455.1}\,(16w_{1,1}\,w_{1,3} - 16w_{1,1}\,w_{1,5} - 16w_{1,3}\,w_{1,7} - 16w_{1,5}\,w_{1,9} - 16w_{1,7}\,w_{1,11} - 16w_{1,9}\,w_{1,13} - 16w_{1,11}\,w_{1,15} + 144w_{3,1}\,w_{3,3} - 144w_{3,1}\,w_{3,5} - 144w_{3,3}\,w_{3,7} \cdots)$$

$$b_{0,6} = \frac{E}{2304}\,(36w_{1,1}\,w_{1,5} - 36w_{1,1}\,w_{1,7} + 18w_{1,3}^2 - 36w_{1,3}\,w_{1,9} - 36w_{1,5}\,w_{1,11} - 36w_{1,7}\,w_{1,13} - 36w_{1,9}\,w_{1,15} + 324w_{3,1}\,w_{3,5} - 324w_{3,1}\,w_{3,7} + 162w_{3,3}^2 - 324w_{3,3}\,w_{3,9} \cdots$$

$$b_{0,8} = \frac{E}{7282}\,(64w_{1,1}\,w_{1,7} - 64w_{1,1}\,w_{1,9} + 64w_{1,3}\,w_{1,5} - 64w_{1,3}\,w_{1,11} - 64w_{1,5}\,w_{1,13} - 64w_{1,7}\,w_{1,15} - 576w_{3,1}\,w_{3,9} \cdots)$$

$$b_{0,10} = \frac{E}{17778}\,(100w_{1,1}\,w_{1,9} - 100w_{1,1}\,w_{1,11} + 100w_{1,3}\,w_{1,7} - 100w_{1,3}\,w_{1,13} + 50w_{1,5}^2 - 100w_{1,5}\,w_{1,15} + 900w_{3,1}\,w_{3,9} - 900w_{3,1}\,w_{3,11} + 900w_{3,3}\,w_{3,7} \cdots)$$

TABLE 1.- Continued.

$$b_{0,12} = \frac{E}{36864}\,(144w_{1,1}\,w_{1,11} - 144w_{1,1}\,w_{1,13} + 144w_{1,3}\,w_{1,9} - 144w_{1,3}\,w_{1,15} + 144w_{1,5}\,w_{1,7} + 1296w_{3,1}\,w_{3,11} - 1296w_{3,1}\,w_{3,13} + 1296w_{3,3}\,w_{3,9} + 1296w_{3,5}\,w_{3,7} \cdots)$$

$$b_{2,0} = \frac{E}{144.0}\,(2w_{1,1}^2 - 4w_{1,1}\,w_{3,1} + 18w_{1,3}^2 - 36w_{1,3}\,w_{3,3} + 50w_{1,5}^2 - 100w_{1,5}\,w_{3,5} + 98w_{1,7}^2 - 196w_{1,7}\,w_{3,7} + 162w_{1,9}^2 + 242w_{1,11}^2 + 338w_{1,13}^2 - 4w_{3,1}\,w_{5,1} \cdots)$$

$$b_{2,2} = \frac{E}{300.4}\,(16w_{1,1}\,w_{1,3} + 16w_{1,1}\,w_{3,1} + 64w_{1,3}\,w_{1,5} - 64w_{1,3}\,w_{3,1} - 16w_{1,3}\,w_{3,5} + 144w_{1,5}\,w_{1,7} - 144w_{1,5}\,w_{3,3} - 64w_{1,5}\,w_{3,7} + 256w_{1,7}\,w_{1,9} - 256w_{1,7}\,w_{3,5} + 400w_{1,9}\,w_{1,11} - 400w_{1,9}\,w_{3,7} + 576w_{1,11}\,w_{1,13} + 64w_{3,1}\,w_{5,1} - 16w_{3,1}\,w_{5,3} \cdots)$$

$$b_{2,4} = \frac{E}{1111}\,(-4w_{1,1}\,w_{1,3} + 36w_{1,1}\,w_{1,5} + 36w_{1,1}\,w_{3,3} - 4w_{1,1}\,w_{3,5} + 100w_{1,3}\,w_{1,7} + 100w_{1,3}\,w_{3,1} - 4w_{1,3}\,w_{3,7} + 196w_{1,5}\,w_{1,9} - 196w_{1,5}\,w_{3,1} - 36w_{1,5}\,w_{3,9} + 324w_{1,7}\,w_{1,11} - 324w_{1,7}\,w_{3,3} + 484w_{1,9}\,w_{1,13} - 484w_{1,9}\,w_{3,5} + 676w_{1,11}\,w_{1,15} + 196w_{3,1}\,w_{5,3} \cdots)$$

TABLE 1.- Continued.

$$b_{2,6} = \frac{E}{3600}\,(-16w_{1,1}\,w_{1,5}+64w_{1,1}\,w_{1,7}+64w_{1,1}\,w_{3,5}-16w_{1,1}\,w_{3,7}$$
$$+144w_{1,3}\,w_{1,9}+144w_{1,3}\,w_{3,3}+256w_{1,5}\,w_{1,11}$$
$$+256w_{1,5}\,w_{3,1}-16w_{1,5}\,w_{3,11}+400w_{1,7}\,w_{1,13}$$
$$-400w_{1,7}\,w_{3,1}+576w_{1,9}\,w_{1,15}-576w_{1,9}\,w_{3,3}$$
$$-784w_{1,11}\,w_{3,5}\cdots)$$

$$b_{2,8} = \frac{E}{9474}\,(-36w_{1,1}\,w_{1,7}+100w_{1,1}\,w_{1,9}+100w_{1,1}\,w_{3,7}$$
$$-36w_{1,1}\,w_{3,9}-4w_{1,3}\,w_{1,5}+196w_{1,3}\,w_{1,11}+196w_{1,3}\,w_{3,5}$$
$$-4w_{1,3}\,w_{3,11}+324w_{1,5}\,w_{1,13}+324w_{1,5}\,w_{3,3}$$
$$-4w_{1,5}\,w_{3,13}+484w_{1,7}\,w_{3,1}+484w_{1,7}\,w_{1,15}$$
$$-676w_{1,9}\,w_{3,1}-900w_{1,11}\,w_{3,3}\cdots)$$

$$b_{2,10} = \frac{E}{21120}\,(-64w_{1,1}\,w_{1,9}+144w_{1,1}\,w_{1,11}+144w_{1,1}\,w_{3,9}$$
$$-64w_{1,1}\,w_{3,11}-16w_{1,3}\,w_{1,7}+256w_{1,3}\,w_{1,13}$$
$$+256w_{1,3}\,w_{3,7}-16w_{1,3}\,w_{3,13}+400w_{1,5}\,w_{1,15}$$
$$+400w_{1,5}\,w_{3,5}+576w_{1,7}\,w_{3,3}+784w_{1,9}\,w_{3,1}$$
$$-1024w_{1,11}\,w_{3,1}-1296w_{1,13}\,w_{3,3}-1600w_{1,15}\,w_{3,5}\cdots)$$

TABLE 1.- Continued.

$$b_{2,12} = \frac{E}{41616}\,(-100w_{1,1}\,w_{1,11}+196w_{1,1}\,w_{1,13}+196w_{1,1}\,w_{3,11}$$
$$-100w_{1,1}\,w_{3,13}-36w_{1,3}\,w_{1,9}+324w_{1,3}\,w_{1,15}$$
$$+324w_{1,3}w_{3,9}-4w_{1,5}\,w_{1,7}+484w_{1,5}\,w_{3,7}$$
$$+676w_{1,7}\,w_{3,5}+900w_{1,9}\,w_{3,3}+1156w_{1,11}\,w_{3,1}$$
$$-1444w_{1,13}\,w_{3,1}-1764w_{1,15}w_{3,3}\cdots)$$

$$b_{4,0} = \frac{E}{2304}\,(16w_{1,1}\,w_{3,1}-16w_{1,1}\,w_{5,1}+144w_{1,3}\,w_{3,3}-144w_{1,3}w_{5,3}$$
$$+400w_{1,5}\,w_{3,5}+784w_{1,7}\,w_{3,7}\cdots)$$

$$b_{4,2} = \frac{E}{2844}\,(-4w_{1,1}w_{3,1}+36w_{1,1}\,w_{3,3}+36w_{1,1}\,w_{5,1}-4w_{1,1}\,w_{5,3}$$
$$+100w_{1,3}\,w_{3,1}+196w_{1,3}\,w_{3,5}-196w_{1,3}\,w_{5,1}$$
$$+324w_{1,5}\,w_{3,3}+484w_{1,5}\,w_{3,7}-484w_{1,5}\,w_{5,3}$$
$$+676w_{1,7}\,w_{3,5}+1156w_{1,9}\,w_{3,7}\cdots)$$

$$b_{4,4} = \frac{E}{4807}\,(64w_{1,1}\,w_{3,5}+64w_{1,1}\,w_{5,3}-64w_{1,3}\,w_{3,1}$$
$$+256w_{1,3}\,w_{3,7}+256w_{1,3}\,w_{5,1}+256w_{1,5}\,w_{3,1}$$
$$+576w_{1,5}\,w_{3,9}-576w_{1,5}\,w_{5,1}+576w_{1,7}\,w_{3,3}$$
$$+1024w_{1,9}\,w_{3,5}\cdots)$$

TABLE 1.- Concluded.

$$b_{4,6} = \frac{E}{9216}\,(-4w_{1,1}\,w_{3,5}+100w_{1,1}\,w_{3,7}+100w_{1,1}w_{5,5}-36w_{1,3}w_{3,3}$$
$$+324w_{1,3}\,w_{3,9}+324w_{1,3}\,w_{5,3}-196w_{1,5}\,w_{3,1}$$
$$+676w_{1,5}\,w_{3,11}+676w_{1,5}\,w_{5,1}+484w_{1,7}\,w_{3,1}$$
$$+900w_{1,9}\,w_{3,3}+1444w_{1,11}\,w_{3,5}\cdots)$$

$$b_{4,8} = \frac{E}{17778}\,(-16w_{1,1}\,w_{3,7}+144w_{1,1}\,w_{3,9}-16w_{1,3}\,w_{3,5}$$
$$+400w_{1,3}\,w_{3,11}-144w_{1,5}\,w_{3,3}+784w_{1,5}\,w_{3,13}$$
$$+784w_{1,5}\,w_{5,3}-400w_{1,7}\,w_{3,1}+784w_{1,9}\,w_{3,1}$$
$$+1296w_{1,11}\,w_{3,3}\cdots)$$

$$b_{6,0} = \frac{E}{11664}\,(36w_{1,1}\,w_{5,1}+324w_{1,3}\,w_{5,3}+18w_{3,1}^2+162w_{3,3}^2\cdots)$$

$$b_{6,2} = \frac{E}{12844}\,(-16w_{1,1}\,w_{5,1}+64w_{1,1}\,w_{5,3}+256w_{1,3}\,w_{5,1}$$
$$+784w_{1,5}\,w_{5,3}+144w_{3,1}\,w_{3,3}+576w_{3,3}\,w_{3,5}\cdots)$$

$$b_{6,4} = \frac{E}{16727}\,(-4w_{1,1}\,w_{5,3}+100w_{1,1}\cdot w_{5,5}-196w_{1,3}\,w_{5,1}$$
$$+676w_{1,5}\,w_{5,1}-36w_{3,1}\,w_{3,3}+324w_{3,1}\,w_{3,5}$$
$$+900w_{3,3}\,w_{3,7}\cdots)$$

$$b_{6,6} = \frac{E}{24354}\,(-144w_{1,3}\,w_{5,3}-576w_{1,5}\,w_{5,1}-144w_{3,1}\,w_{3,5}$$
$$+576w_{3,1}\,w_{3,7}+1296w_{3,3}\,w_{3,9}\cdots)$$

$$b_{8,0} = \frac{E}{36864}\,(64w_{3,1}\,w_{5,1}\cdots)$$

$$b_{8,2} = \frac{E}{38940}\,(-4w_{3,1}\,w_{5,1}+196w_{3,1}\,w_{5,3}+324w_{3,3}\,w_{5,1}\cdots)$$

$$b_{8,4} = \frac{E}{45511}\,(-16w_{3,1}\,w_{5,3}-144w_{3,3}\,w_{5,1}\cdots)$$

$$b_{10,0} = \frac{E}{90000}\,(50w_{5,1}^2\cdots)$$

TABLE 2A (TOP).- RELATION BETWEEN THE MOMENT COEFFICIENTS, THE DEFLECTION COEFFICIENTS, AND THE AVERAGE COMPRESSIVE STRESS IN THE x-DIRECTION

$\left[\frac{b}{a} = 1.5;\ \mu = 0.316\right]$

	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =
$\frac{b^4}{\pi^4 D}$	$-k_1$	$-k_3$	$-3k_1$	$-3k_3$	$-5k_1$	$-k_5$	$-7k_1$	$-9k_1$	$-11k_1$	$-13k_1$	$-15k_1$
1	$.978\frac{w_{1,1}}{h}$	$41.8\frac{w_{3,1}}{h}$	$11.72\frac{w_{1,3}}{h}$	$79.1\frac{w_{3,3}}{h}$	$68.7\frac{w_{1,5}}{h}$	$303.5\frac{w_{5,1}}{h}$	$243\frac{w_{1,7}}{h}$	$641\frac{w_{1,9}}{h}$	$1408\frac{w_{1,11}}{h}$	$2715\frac{w_{1,13}}{h}$	$4780\frac{w_{1,15}}{h}$
$\frac{\bar{p}_x b^2}{Eh^2}$	$-.228\frac{w_{1,1}}{h}$	$-2.05\frac{w_{3,1}}{h}$	$-.228\frac{w_{1,3}}{h}$	$-2.05\frac{w_{3,3}}{h}$	$-.228\frac{w_{1,5}}{h}$	$-5.70\frac{w_{5,1}}{h}$	$-.228\frac{w_{1,7}}{h}$	$-.228\frac{w_{1,9}}{h}$	$-.228\frac{w_{1,11}}{h}$	$-.228\frac{w_{1,13}}{h}$	$-.228\frac{w_{1,15}}{h}$
$\left(\frac{w_{1,1}}{h}\right)^3$	.3791	-.0625	-.3166	0	0	0	0	0	0	0	0
$\left(\frac{w_{1,1}}{h}\right)^2 \frac{w_{1,3}}{h}$	-.950	4.80	2.316	-.636	-1.023	0	0	0	0	0	0
$\left(\frac{w_{1,1}}{h}\right)^2 \frac{w_{3,1}}{h}$	-.1875	3.583	.480	-2.88	0	-.216	0	0	0	0	0
$\left(\frac{w_{1,1}}{h}\right)^2 \frac{w_{3,3}}{h}$	0	-2.879	-.635	.915	.658	.256	0	0	0	0	0
$\left(\frac{w_{1,1}}{h}\right)^2 \frac{w_{1,5}}{h}$	0	0	-1.023	.658	3.528	0	-1.109	0	0	0	0
$\left(\frac{w_{1,1}}{h}\right)^2 \frac{w_{5,1}}{h}$	0	-.2160	0	.257	0	8.437	0	0	0	0	0
$\frac{w_{1,1}}{h} \left(\frac{w_{1,3}}{h}\right)^2$	2.318	-2.686	0	0	2.870	0	-1.152	0	0	0	0
$\frac{w_{1,1}}{h} \frac{w_{1,3}}{h} \frac{w_{3,1}}{h}$	.960	-9.70	-5.371	16.19	4.15	2.79	0	0	0	0	0
$\frac{w_{1,1}}{h} \frac{w_{1,3}}{h} \frac{w_{3,3}}{h}$	-2.2714	16.19	0	0	-4.68	-7.50	3.919	0	0	0	0
$\frac{w_{1,1}}{h} \frac{w_{1,3}}{h} \frac{w_{1,5}}{h}$	-2.045	4.15	5.739	-4.69	0	0	8.05	-2.680	0	0	0
$\frac{w_{1,1}}{h} \frac{w_{1,3}}{h} \frac{w_{5,1}}{h}$	0	2.787	0	-7.50	0	-19.01	0	0	0	0	0
$\frac{w_{1,1}}{h} \left(\frac{w_{3,1}}{h}\right)^2$	3.583	0	-4.85	0	0	2.11	0	0	0	0	0

TABLE 2A (BOTTOM).- RELATION BETWEEN THE MOMENT COEFFICIENTS, THE DEFLECTION COEFFICIENTS, AND THE AVERAGE COMPRESSIVE STRESS IN THE x-DIRECTION

$\left[\frac{b}{a} = 1.5;\ \mu = 0.316\right]$

	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	Rest 0 =
$\frac{b^4}{\pi^4 E h^4}$	$+5k_3$	$-7k_3$	$-9k_3$	$-11k_3$	$-13k_3$	$-3k_5$	$-5k_5$	$-7k_5$	$-9k_5$	$-11k_5$	$-13k_5$	$-nk_m$
1	$189.8\frac{w_{3,5}}{h}$	$444\frac{w_{3,7}}{h}$	$948\frac{w_{3,9}}{h}$	$1849\frac{w_{3,11}}{h}$	$3315\frac{w_{3,13}}{h}$	$394\frac{w_{5,3}}{h}$	$611\frac{w_{5,5}}{h}$	$1028\frac{w_{5,7}}{h}$	$1746\frac{w_{5,9}}{h}$	$2910\frac{w_{5,11}}{h}$	$4699\frac{w_{5,13}}{h}$	$\frac{(2.25m^2+n^2)^2}{10.8}\frac{w_{m,n}}{h}$
$\frac{\bar{p}_x b^2}{Eh^2}$	$-2.05\frac{w_{3,5}}{h}$	$-2.05\frac{w_{3,7}}{h}$	$-2.05\frac{w_{3,9}}{h}$	$-2.05\frac{w_{3,11}}{h}$	$-2.05\frac{w_{3,13}}{h}$	$-5.70\frac{w_{5,3}}{h}$	$-5.70\frac{w_{5,5}}{h}$	$-5.70\frac{w_{5,7}}{h}$	$-5.70\frac{w_{5,9}}{h}$	$-5.70\frac{w_{5,11}}{h}$	$-5.70\frac{w_{5,13}}{h}$	$-.228m^2\frac{w_{m,n}}{h}$
$\left(\frac{w_{1,1}}{h}\right)^3$	0	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{1,1}}{h}\right)^2 \frac{w_{1,3}}{h}$	.00811	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{1,1}}{h}\right)^2 \frac{w_{3,1}}{h}$	0	0	0	0	0	.00317	0	0	0	0	0	0
$\left(\frac{w_{1,1}}{h}\right)^2 \frac{w_{3,3}}{h}$	-2.92	0	0	0	0	-.590	0	0	0	0	0	0
$\left(\frac{w_{1,1}}{h}\right)^2 \frac{w_{1,5}}{h}$	-1.795	.0400	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{3,1}}{h}\right)^2 \frac{w_{5,1}}{h}$	0	0	0	0	0	-7.98	0	0	0	0	0	0
$\frac{w_{1,1}}{h}\left(\frac{w_{1,3}}{h}\right)^2$	-.479	.00810	0	0	0	0	0	0	0	0	0	0
$\frac{w_{1,1}}{h}\frac{w_{1,3}}{h}\frac{w_{3,1}}{h}$	-7.02	0	0	0	0	-2.562	.2816	0	0	0	0	0
$\frac{w_{1,1}}{h}\frac{w_{1,3}}{h}\frac{w_{3,3}}{h}$	8.54	-6.35	0	0	0	0	-.931	.0818	0	0	0	0
$\frac{w_{1,1}}{h}\frac{w_{1,3}}{h}\frac{w_{1,5}}{h}$	0	-2.015	.0815	0	0	0	0	0	0	0	0	0
$\frac{w_{1,1}}{h}\frac{w_{1,3}}{h}\frac{w_{5,1}}{h}$	4.29	0	0	0	0	36.71	-17.47	0	0	0	0	0
$\frac{w_{1,1}}{h}\left(\frac{w_{3,1}}{h}\right)^2$	0	0	0	0	0	-.480	0	0	0	0	0	0

TABLE 2B .- RELATION BETWEEN THE MOMENT COEFFICIENTS, THE DEFLECTION COEFFICIENTS, AND THE AVERAGE COMPRESSIVE STRESS IN THE x-DIRECTION

$$\left[\frac{b}{a} = 1.5;\ \mu = 0.316\right]$$

	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =
$\frac{w_{1,1}}{h}\ \frac{w_{3,1}}{h}\ \frac{w_{3,3}}{h}$	-5.757	0	16.19	0	-13.86	-4.42	0	0	0	0	0	0	0	0	0	0	5.103	-1.943	0	0	0	0	0
$\frac{w_{1,1}}{h}\ \frac{w_{3,1}}{h}\ \frac{w_{1,5}}{h}$	0	0	4.148	-13.87	-11.56	0	7.88	0	0	0	0	22.59	-9.84	0	0	0	5.871	-7.144	.945	0	0	0	0
$\frac{w_{1,1}}{h}\ \frac{w_{3,1}}{h}\ \frac{w_{5,1}}{h}$	-.432	4.215	2.788	-4.42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\frac{w_{1,1}}{h}\left(\frac{w_{3,3}}{h}\right)^2$	.915	0	0	0	5.16	6.48	-8.76	0	0	0	0	0	0	0	0	0	0	0	-.657	0	0	0	0
$\frac{w_{1,1}}{h}\ \frac{w_{3,3}}{h}\ \frac{w_{1,5}}{h}$	1.316	-13.857	-4.69	10.32	0	8.23	-6.60	6.932	0	0	0	0	11.14	-7.70	0	0	-5.15	0	-3.62	.433	0	0	0
$\frac{w_{1,1}}{h}\ \frac{w_{3,3}}{h}\ \frac{w_{5,1}}{h}$	.513	-4.42	-7.511	12.97	8.24	0	0	0	0	0	0	-9.90	0	0	0	0	0	0	0	0	0	0	0
$\frac{w_{1,1}}{h}\left(\frac{w_{1,5}}{h}\right)^2$	3.526	-5.78	0	0	0	0	0	4.52	-1.589	0	0	0	0	-.656	.0400	0	0	0	0	0	0	0	0
$\frac{w_{1,1}}{h}\ \frac{w_{1,5}}{h}\ \frac{w_{5,1}}{h}$	0	0	0	8.23	0	0	0	0	0	0	0	-18.41	10.13	0	0	0	-24.22	44.28	-20.94	0	0	0	0
$\frac{w_{1,1}}{h}\left(\frac{w_{5,1}}{h}\right)^2$	8.428	0	-9.50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{1,3}}{h}\right)^3$	0	0	5.38	-5.06	0	0	0	-.316	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{1,3}}{h}\right)^2\ \frac{w_{3,1}}{h}$	-2.686	15.22	0	0	-7.68	-6.36	5.06	0	0	0	0	8.64	-4.97	0	0	0	0	-1.977	.480	0	0	0	0
$\left(\frac{w_{1,3}}{h}\right)^2\ \frac{w_{3,3}}{h}$	0	0	-13.18	26.41	0	0	0	3.24	0	0	0	0	0	-3.56	0	0	-15.89	0	0	.0791	0	0	0
$\left(\frac{w_{1,3}}{h}\right)^2\ \frac{w_{1,5}}{h}$	2.871	-7.68	0	0	23.03	0	0	0	-.994	0	0	-16.05	0	0	.00095	0	0	0	0	0	0	0	0
$\left(\frac{w_{1,3}}{h}\right)^2\ \frac{w_{5,1}}{h}$	0	-6.363	0	0	0	19.45	0	0	0	0	0	-7.59	7.68	0	0	0	0	16.27	-13.02	0	0	0	0
$\frac{w_{1,3}}{h}\left(\frac{w_{3,1}}{h}\right)^2$	-4.849	0	15.22	0	-14.72	-7.68	0	0	0	0	0	0	0	0	0	0	12.40	-5.06	0	0	0	0	0

TABLE 2C .- RELATION BETWEEN THE MOMENT COEFFICIENTS, THE DEFLECTION COEFFICIENTS, AND THE AVERAGE COMPRESSIVE STRESS IN THE x-DIRECTION

$\left[\frac{b}{a} = 1.5;\ \mu = 0.316\right]$

	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0=
$\frac{w_{1,3}}{h}\frac{w_{3,1}}{h}\frac{w_{3,3}}{h}$	16.20	0	0	0	35.58	36.70	-36.49	0	0	0	0	0	0	0	0	0	0	11.52	-8.17	0	0	0	0
$\frac{w_{1,3}}{h}\frac{w_{3,1}}{h}\frac{w_{1,5}}{h}$	4.15	-29.46	-15.36	35.59	0	19.67	-27.33	15.84	0	0	0	0	31.02	-15.71	0	0	-15.38	0	-8.49	2.466	0	0	0
$\frac{w_{1,3}}{h}\frac{w_{3,1}}{h}\frac{w_{5,1}}{h}$	2.788	-15.36	-12.72	36.72	19.66	0	0	0	0	0	0	-28.57	0	0	0	0	0	0	0	0	0	0	0
$\frac{w_{1,3}}{h}\left(\frac{w_{3,3}}{h}\right)^2$	0	0	26.41	0	0	0	0	-17.79	0	0	0	0	0	0	0	0	28.12	0	0	-3.24	0	0	0
$\frac{w_{1,3}}{h}\frac{w_{3,3}}{h}\frac{w_{1,5}}{h}$	-4.68	35.58	0	0	-62.87	-31.34	0	0	9.74	0	0	88.4	0	0	-9.45	0	0	-36.59	0	0	581	0	0
$\frac{w_{1,3}}{h}\frac{w_{3,3}}{h}\frac{w_{5,1}}{h}$	-7.50	36.74	0	0	-31.35	0	36.24	0	0	0	0	28.42	-29.64	0	0	0	0	0	0	0	0	0	0
$\frac{w_{1,3}}{h}\left(\frac{w_{1,5}}{h}\right)^2$	0	0	23.03	-31.44	0	0	18.23	0	0	-1.026	0	0	-7.67	0	0	.00095	0	0	0	0	0	0	0
$\frac{w_{1,3}}{h}\frac{w_{1,5}}{h}\frac{w_{5,1}}{h}$	0	19.66	0	-31.34	0	-59.27	0	0	0	0	0	0	-36.40	31.09	0	0	58.68	0	55.18	-40.25	0	0	0
$\frac{w_{1,3}}{h}\left(\frac{w_{5,1}}{h}\right)^2$	-9.50	0	19.44	0	-29.60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{3,1}}{h}\right)^3$	0	25.71	0	-25.65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{3,1}}{h}\right)^2\frac{w_{3,3}}{h}$	0	-76.95	0	104.1	0	0	0	0	0	0	0	-77.3	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{3,1}}{h}\right)^2\frac{w_{1,5}}{h}$	0	0	-14.72	0	39.77	0	-24.65	0	0	0	0	0	0	0	0	0	-19.43	27.49	-10.25	0	0	0	0
$\left(\frac{w_{3,1}}{h}\right)^2\frac{w_{5,1}}{h}$	2.107	0	-7.68	0	0	79.2	0	0	0	0	0	0	0	0	0	0	-73.1	0	0	0	0	0	0
$\frac{w_{3,1}}{h}\left(\frac{w_{3,3}}{h}\right)^2$	0	104.1	0	0	0	0	0	0	0	0	0	80.5	-78.02	0	0	0	0	0	0	0	0	0	0
$\frac{w_{3,1}}{h}\frac{w_{3,3}}{h}\frac{w_{1,5}}{h}$	-13.86	0	35.58	0	0	-50.35	66.27	-56.11	0	0	0	0	0	0	0	0	32.31	0	23.06	-16.48	0	0	0

TABLE 2D .- RELATION BETWEEN THE MOMENT COEFFICIENTS, THE DEFLECTION COEFFICIENTS, AND THE AVERAGE COMPRESSIVE STRESS IN THE x-DIRECTION

$\left[\frac{b}{a} = 1.5;\ \mu = 0.316\right]$

	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =	0 =
$\frac{w_{3,1}}{h}\frac{w_{3,3}}{h}\frac{w_{5,1}}{h}$	-4.42	0	36.72	0	-50.35	-177.3	0	0	0	0	0	0	0	0	0	0	324.4	-159.6	0	0	0	0	0
$\frac{w_{3,1}}{h}\left(\frac{w_{1,5}}{h}\right)^2$	-5.784	39.76	0	0	0	-26.90	0	-19.44	10.24	0	0	0	0	23.70	-11.58	0	0	0	0	-7.67	2.345	0	0
$\frac{w_{3,1}}{h}\frac{w_{1,5}}{h}\frac{w_{5,1}}{h}$	0	0	19.67	-50.35	-53.78	0	51.10	0	0	0	0	91.9	-56.33	0	0	0	0	0	0	0	0	0	0
$\frac{w_{3,1}}{h}\left(\frac{w_{5,1}}{h}\right)^2$	0	72.23	0	-88.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{3,3}}{h}\right)^3$	0	0	0	30.71	0	0	0	0	0	0	0	0	0	-25.6	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{3,3}}{h}\right)^2\frac{w_{1,5}}{h}$	5.14	0	0	0	66.6	38.9	0	0	-26.07	0	0	0	0	0	0	0	0	31.41	0	0	-6.23	0	0
$\left(\frac{w_{3,3}}{h}\right)^2\frac{w_{5,1}}{h}$	6.482	0	0	0	38.9	94.0	-53.2	0	0	0	0	0	0	0	0	0	0	75.4	-79.4	0	0	0	0
$\frac{w_{3,3}}{h}\left(\frac{w_{1,5}}{h}\right)^2$	0	0	-31.42	66.59	0	0	-38.9	0	0	6.24	0	0	51.17	0	0	-6.50	-48.70	0	-20.80	0	0	.656	0
$\frac{w_{3,3}}{h}\frac{w_{1,5}}{h}\frac{w_{5,1}}{h}$	8.24	-50.36	-31.34	77.8	0	0	-83.6	78.10	0	0	0	0	65.52	-57.08	0	0	0	0	0	0	0	0	0
$\frac{w_{3,3}}{h}\left(\frac{w_{5,1}}{h}\right)^2$	0	-88.6	0	94.0	0	0	0	0	0	0	0	-152.2	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{1,5}}{h}\right)^3$	0	0	0	0	39.4	0	0	0	0	0	-.3162	-39.1	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{1,5}}{h}\right)^2\frac{w_{5,1}}{h}$	0	-26.86	0	0	0	89.63	0	0	0	0	0	0	0	-38.8	27.9	0	0	0	0	51.42	-33.21	0	0
$\frac{w_{1,5}}{h}\left(\frac{w_{5,1}}{h}\right)^2$	0	0	-29.64	0	89.6	0	-72.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\left(\frac{w_{5,1}}{h}\right)^3$	0	0	0	0	0	198.1	0	0	0	0	0	0	0	0	0	0	-197.8	0	0	0	0	0	0

Table 3 - Relations derived by substituting from Table 2 in equations (15).

	0 =	0 =	0 =
$\frac{b^4}{\pi^4 Eh^4}$	$-3.010k_1$	$-.9428k_3$	$-.566k_5$
$\frac{\bar{p}_x b^2}{Eh^2}$	$-.233\frac{w_{1,1}}{h}$	$-.0491\frac{w_{3,1}}{h}$	$-.01875\frac{w_{5,1}}{h}$
$\frac{\bar{p}_x b^2}{Eh^2}$	$-.058\frac{w_{1,3}}{h}$	$-.0778\frac{w_{3,3}}{h}$	$-.0433\frac{w_{5,3}}{h}$
$\frac{\bar{p}_x b^2}{Eh^2}$	$-.017\frac{w_{1,5}}{h}$	$-.0540\frac{w_{3,5}}{h}$	$-.0466\frac{w_{5,5}}{h}$
$\frac{\bar{p}_x b^2}{Eh^2}$	$-.007\frac{w_{1,7}}{h}$	$-.0323\frac{w_{3,7}}{h}$	$-.0388\frac{w_{5,7}}{h}$
$\frac{\bar{p}_x b^2}{Eh^2}$	$-.003\frac{w_{1,9}}{h}$	$-.0195\frac{w_{3,9}}{h}$	$-.0294\frac{w_{5,9}}{h}$
$(\frac{w_{1,1}}{h})^3$	+ .307	- .001497	0
$(\frac{w_{1,1}}{h})^2(\frac{w_{1,3}}{h})$	- .453	- .01243	0
$(\frac{w_{1,1}}{h})^2(\frac{w_{3,1}}{h})$	- .0689	- .0235	-.000687
$(\frac{w_{1,1}}{h})^2(\frac{w_{3,3}}{h})$	- .1147	- .1112	-.00365
$(\frac{w_{1,1}}{h})^2(\frac{w_{1,5}}{h})$	- .0374	- .0217	0
$(\frac{w_{1,1}}{h})^2(\frac{w_{5,1}}{h})$	0	+ .00458	-.0329
$(\frac{w_{1,1}}{h})(\frac{w_{1,3}}{h})^2$	+ 2.546	- .0768	0
$(\frac{w_{1,1}}{h})(\frac{w_{1,3}}{h})(\frac{w_{3,1}}{h})$	- .090	+ .196	-.00802
$(\frac{w_{1,1}}{h})(\frac{w_{1,3}}{h})(\frac{w_{3,3}}{h})$	- 1.528	+ .5125	-.0317

TABLE 3.- continued.

	0 =	0 =	0 =
$(\frac{w_{1,1}}{h})(\frac{w_{1,3}}{h})(\frac{w_{1,5}}{h})$	- .428	- .1096	0
$(\frac{w_{1,1}}{h})(\frac{w_{1,3}}{h})(\frac{w_{5,1}}{h})$	0	- .1047	+.0740
$(\frac{w_{1,1}}{h})(\frac{w_{3,1}}{h})^2$	+ 2.424	0	+.00330
$(\frac{w_{1,1}}{h})(\frac{w_{3,1}}{h})(\frac{w_{3,3}}{h})$	- 5.89	0	+.0083
$(\frac{w_{1,1}}{h})(\frac{w_{3,1}}{h})(\frac{w_{1,5}}{h})$	+ .448	- .086	-.0073
$(\frac{w_{1,1}}{h})(\frac{w_{3,1}}{h})(\frac{w_{5,1}}{h})$	+ .272	- .0668	0
$(\frac{w_{1,1}}{h})(\frac{w_{3,3}}{h})^2$	+ 1.059	0	+.0168
$(\frac{w_{1,1}}{h})(\frac{w_{3,3}}{h})(\frac{w_{1,5}}{h})$	+ .051	+ .1630	-.0346
$(\frac{w_{1,1}}{h})(\frac{w_{3,3}}{h})(\frac{w_{5,1}}{h})$	- .798	+ .125	0
$(\frac{w_{1,1}}{h})(\frac{w_{1,5}}{h})^2$	+ 3.657	- .1442	0
$(\frac{w_{1,1}}{h})(\frac{w_{1,5}}{h})(\frac{w_{5,1}}{h})$	0	- .013	+.0351
$(\frac{w_{1,1}}{h})(\frac{w_{5,1}}{h})^2$	+ 6.19	0	0
$(\frac{w_{1,3}}{h})^3$	+ 1.374	- .1918	0
$(\frac{w_{1,3}}{h})^2(\frac{w_{3,1}}{h})$	- 3.158	+ .5140	-.0338
$(\frac{w_{1,3}}{h})^2(\frac{w_{3,3}}{h})$	- 3.84	+ .968	-.1206
$(\frac{w_{1,3}}{h})^2(\frac{w_{1,5}}{h})$	+ 4.602	- .607	0

TABLE 3.- continued.

	0 =	0 =	0 =
$(\frac{w_{1,3}}{h})^2(\frac{w_{5,1}}{h})$	0	- .2313	+.1082
$(\frac{w_{1,3}}{h})(\frac{w_{3,1}}{h})^2$	- 2.13	0	+.0277
$(\frac{w_{1,3}}{h})(\frac{w_{3,1}}{h})(\frac{w_{3,3}}{h})$	+18.11	0	+.1594
$(\frac{w_{1,3}}{h})(\frac{w_{3,1}}{h})(\frac{w_{1,5}}{h})$	- .26	+ .984	-.0975
$(\frac{w_{1,3}}{h})(\frac{w_{3,1}}{h})(\frac{w_{5,1}}{h})$	+ 1.03	+ .272	0
$(\frac{w_{1,3}}{h})(\frac{w_{3,3}}{h})^2$	+ 6.51	0	+.1973
$(\frac{w_{1,3}}{h})(\frac{w_{3,3}}{h})(\frac{w_{1,5}}{h})$	- 9.28	+3.126	-.4005
$(\frac{w_{1,3}}{h})(\frac{w_{3,3}}{h})(\frac{w_{5,1}}{h})$	- 8.91	+1.161	0
$(\frac{w_{1,3}}{h})(\frac{w_{1,5}}{h})^2$	+ 6.43	-1.314	0
$(\frac{w_{1,3}}{h})(\frac{w_{1,5}}{h})(\frac{w_{5,1}}{h})$	0	- .996	+.420
$(\frac{w_{1,3}}{h})(\frac{w_{5,1}}{h})^2$	- 6.88	0	0
$(\frac{w_{3,1}}{h})^3$	0	- .356	0
$(\frac{w_{3,1}}{h})^2(\frac{w_{3,3}}{h})$	0	+ .073	0
$(\frac{w_{3,1}}{h})^2(\frac{w_{1,5}}{h})$	- 1.59	0	+.0070
$(\frac{w_{3,1}}{h})^2(\frac{w_{5,1}}{h})$	+ .188	0	-.295
$(\frac{w_{3,1}}{h})(\frac{w_{3,3}}{h})^2$	0	+3.384	0

TABLE 3.- concluded.

	0 =	0 =	0 =
$(\frac{w_{3,1}}{h})(\frac{w_{3,3}}{h})(\frac{w_{1,5}}{h})$	- 3.96	0	+.152
$(\frac{w_{3,1}}{h})(\frac{w_{3,3}}{h})(\frac{w_{5,1}}{h})$	+ 1.22	0	+.58
$(\frac{w_{3,1}}{h})(\frac{w_{1,5}}{h})^2$	- 6.10	+1.108	-.1192
$(\frac{w_{3,1}}{h})(\frac{w_{1,5}}{h})(\frac{w_{5,1}}{h})$	+ 2.58	- .375	0
$(\frac{w_{3,1}}{h})(\frac{w_{5,1}}{h})^2$	0	-1.459	0
$(\frac{w_{3,3}}{h})^3$	0	+ .922	0
$(\frac{w_{3,3}}{h})^2(\frac{w_{1,5}}{h})$	+ 9.90	0	+.3618
$(\frac{w_{3,3}}{h})^2(\frac{w_{5,1}}{h})$	+ 7.92	0	+.386
$(\frac{w_{3,3}}{h})(\frac{w_{1,5}}{h})^2$	- 9.14	+3.305	-.511
$(\frac{w_{3,3}}{h})(\frac{w_{1,5}}{h})(\frac{w_{5,1}}{h})$	- .90	+2.234	0
$(\frac{w_{3,3}}{h})(\frac{w_{5,1}}{h})^2$	0	-2.570	0
$(\frac{w_{1,5}}{h})^3$	+ 2.864	-1.030	0
$(\frac{w_{1,5}}{h})^2(\frac{w_{5,1}}{h})$	0	- .846	+.434
$(\frac{w_{1,5}}{h})(\frac{w_{5,1}}{h})^2$	- 3.18	0	0
$(\frac{w_{5,1}}{h})^3$	0	0	-.855

TABLE 4.- VALUES OF COEFFICIENTS IN DEFLECTION FUNCTION, EQUATION (10), FOR PLATE UNDER EDGE COMPRESSION WITH UNLOADED EDGES RIGIDLY CLAMPED

$$\left[\frac{b}{a} = 1.5;\ \mu = 0.316\right]$$

$\frac{\bar{p}_x b^2}{Eh^2}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{1,7}}{h}$	$\frac{w_{1,9}}{h}$	$\frac{w_{1,11}}{h}$	$\frac{w_{1,13}}{h}$	$\frac{w_{1,15}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{3,5}}{h}$
6.37	0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6.38	.100	-.0138	-.0035	-.0014	-.0007	-.0004	-.0002	-.0001	.0000	.0000	.0000
6.80	.400	-.0534	-.0145	-.0056	-.0027	-.0015	-.0009	-.0006	.0004	-.0001	.0000
7.33	.600	-.0774	-.0225	-.0088	-.0042	-.0023	-.0014	-.0009	.0011	-.0002	-.0001
8.00	.800	-.0966	-.0308	-.0121	-.0058	-.0032	-.0020	-.0013	.0023	-.0003	-.0002
8.84	1.000	-.1130	-.0401	-.0158	-.0075	-.0042	-.0026	-.0017	.0047	-.0003	-.0004
10.16	1.250	-.1251	-.0526	-.0212	-.0100	-.0056	-.0034	-.0022	.0086	-.0003	-.0010
11.64	1.500	-.1308	-.0656	-.0272	-.0127	-.0071	-.0043	-.0028	.0139	.0005	-.0018
13.21	1.750	-.1272	-.0788	-.0341	-.0158	-.0088	-.0054	-.0035	.0206	.0020	-.0030
15.02	2.000	-.1180	-.0925	-.0418	-.0192	-.0106	-.0065	-.0043	.0308	.0050	-.0048
16.91	2.250	-.1002	-.1052	-.0501	-.0229	-.0127	-.0077	-.0051	.0410	.0106	-.0064
18.93	2.500	-.0794	-.1181	-.0594	-.0271	-.0149	-.0091	-.0060	.0569	.0191	-.0088
21.05	2.750	-.0525	-.1295	-.0693	-.0315	-.0174	-.0106	-.0070	.0766	.0327	-.0104
23.25	3.000	-.0211	-.1400	-.0801	-.0362	-.0200	-.0123	-.0080	.1041	.0535	-.0110

$\frac{\bar{p}_x b^2}{Eh^2}$	$\frac{w_{3,7}}{h}$	$\frac{w_{3,9}}{h}$	$\frac{w_{3,11}}{h}$	$\frac{w_{3,13}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{5,3}}{h}$	$\frac{w_{5,5}}{h}$	$\frac{w_{5,7}}{h}$	$\frac{w_{5,9}}{h}$	$\frac{w_{5,11}}{h}$
6.37	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6.38	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6.80	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
7.33	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8.84	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
10.16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
11.64	-.0001	-.0001	.0000	.0000	.0001	.0000	.0000	.0000	.0000	.0000
13.21	-.0003	-.0002	-.0001	-.0001	.0001	.0000	.0000	.0000	.0000	.0000
15.02	-.0008	-.0004	-.0003	-.0002	.0002	.0001	-.0001	.0000	.0000	.0000
16.91	-.0012	-.0007	-.0004	-.0003	.0002	.0002	-.0001	.0000	.0000	.0000
18.93	-.0022	-.0012	-.0007	-.0005	.0004	.0005	-.0002	.0000	.0000	.0000
21.05	-.0036	-.0020	-.0012	-.0008	.0006	.0009	-.0003	.0000	.0000	.0000
23.25	-.0059	-.0033	-.0019	-.0012	.0009	.0016	-.0005	-.0001	.0000	.0000

TABLE 5.- VALUES OF COEFFICIENTS IN MOMENT FUNCTION, EQUATION (6), FOR PLATE UNDER EDGE COMPRESSION WITH UNLOADED EDGES RIGIDLY CLAMPED

$$\left[\frac{b}{a} = 1.5; \qquad \mu = 0.316\right]$$

$\frac{\overline{p}_x b^2}{Eh^2}$	$\frac{b^4 k_1}{\pi^4 Eh^4}$	$\frac{b^4 k_3}{\pi^4 Eh^4}$	$\frac{b^4 k_5}{\pi^4 Eh^4}$
6.37	0	0	0
6.38	-.0474	.0000	.0000
6.80	-.1937	.0000	.0000
7.33	-.300	.000	.000
8.00	-.411	-.001	.000
8.84	-.536	-.002	.000
10.16	-.709	-.005	.000
11.64	-.903	-.008	.000
13.21	-1.118	-.020	.000
15.02	-1.358	-.044	.000
16.91	-1.620	-.066	.000
18.93	-1.908	-.118	-.001
21.05	-2.218	-.193	-.003
23.25	-2.546	-.311	-.005

TABLE 6.- VALUES OF COEFFICIENTS IN STRESS FUNCTION, EQUATION (11), FOR PLATE UNDER EDGE COMPRESSION WITH UNLOADED EDGES RIGIDLY CLAMPED

$$\left[\frac{b}{a} = 1.5;\ \mu = 0.316\right]$$

$\frac{\bar{p}_x b^2}{Eh^2}$	$\frac{4\pi^2 b_{0,2}}{Eh^2}$	$\frac{16\pi^2 b_{0,4}}{Eh^2}$	$\frac{36\pi^2 b_{0,6}}{Eh^2}$	$\frac{64\pi^2 b_{0,8}}{Eh^2}$	$\frac{100\pi^2 b_{0,10}}{Eh^2}$	$\frac{144\pi^2 b_{0,12}}{Eh^2}$	$\frac{4\pi^2 b_{2,0}}{Eh^2}$	$\frac{4\pi^2 b_{2,2}}{Eh^2}$	$\frac{16\pi^2 b_{2,4}}{Eh^2}$	$\frac{36\pi^2 b_{2,6}}{Eh^2}$	$\frac{64\pi^2 b_{2,8}}{Eh^2}$
6.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6.38	.04	-.01	.00	.00	.00	.00	.01	.00	.00	.00	.00
6.80	.56	-.09	-.01	.00	.00	.00	.10	-.03	-.02	.00	.00
7.33	1.25	-.19	-.02	.00	.00	.00	.24	-.09	-.03	-.01	.00
8.00	2.19	-.30	-.06	.00	.00	.00	.41	-.12	-.07	-.01	.00
8.84	3.38	-.42	-.09	-.02	.00	.00	.63	-.18	-.11	-.03	.00
10.16	5.16	-.52	-.18	-.04	-.01	.00	.98	-.21	-.20	-.05	-.01
11.64	7.27	-.56	-.28	-.07	-.02	.00	1.38	-.24	-.31	-.10	-.01
13.21	9.67	-.49	-.41	-.12	-.02	-.02	1.85	-.21	-.43	-.14	-.03
15.02	12.34	-.30	-.56	-.19	-.04	-.02	2.39	-.13	-.58	-.26	-.04
16.91	15.22	.05	-.68	-.28	-.07	-.02	2.96	.02	-.69	-.39	-.06
18.93	18.39	.57	-.83	-.39	-.10	-.03	3.60	.23	-.76	-.54	-.13
21.05	21.73	1.31	-.93	-.53	-.17	-.05	4.33	.56	-.71	-.69	-.23
23.25	25.26	2.29	-.98	-.70	-.25	-.08	5.07	1.03	-.44	-.87	-.36

$\frac{\bar{p}_x b^2}{Eh^2}$	$\frac{100\pi^2 b_{2,10}}{Eh^2}$	$\frac{144\pi^2 b_{2,12}}{Eh^2}$	$\frac{16\pi^2 b_{4,0}}{Eh^2}$	$\frac{16\pi^2 b_{4,2}}{Eh^2}$	$\frac{16\pi^2 b_{4,4}}{Eh^2}$	$\frac{36\pi^2 b_{4,6}}{Eh^2}$	$\frac{64\pi^2 b_{4,8}}{Eh^2}$	$\frac{36\pi^2 b_{6,0}}{Eh^2}$	$\frac{36\pi^2 b_{6,2}}{Eh^2}$	$\frac{36\pi^2 b_{6,4}}{Eh^2}$	$\frac{36\pi^2 b_{6,6}}{Eh^2}$
6.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6.38	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
6.80	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
7.33	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
8.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
8.84	.00	.00	.01	.00	.00	.00	.00	.00	.00	.00	.00
10.16	.00	.00	.01	-.01	.00	.00	.00	.00	.00	.00	.00
11.64	.00	.00	.02	-.01	-.01	.00	.00	.00	.00	.00	.00
13.21	.00	-.01	.05	-.01	-.01	.00	.00	.00	.00	.00	.00
15.02	-.02	-.01	.07	.00	-.03	-.01	.00	.00	.00	.00	.00
16.91	-.04	-.02	.11	.01	-.07	-.03	-.01	.00	.00	.00	.00
18.93	-.06	-.02	.19	.04	-.11	-.04	-.01	.00	.00	.00	.00
21.05	-.12	-.05	.26	.09	-.15	-.09	-.02	.01	.00	-.01	.00
23.25	-.21	-.10	.40	.19	-.24	-.17	-.04	.02	.02	-.01	-.01

TABLE 7.- STRESSES AT A CORNER OF PLATE $(x = 0,\ y = 0)$

[Shear stress τ_{xy}' is 0 because of symmetry and shear stress τ_{xy}'' is 0 because of clamping at edge]

	Membrane stresses		Extreme fiber bending stresses		Extreme fiber stresses outside of buckle		Extreme fiber stresses inside of buckle	
$\frac{\bar{p}_x b^2}{Eh^2}$	$\frac{\sigma_x' b^2}{Eh^2}$	$\frac{\sigma_y' b^2}{Eh^2}$	$\frac{\sigma_x'' b^2}{Eh^2}$	$\frac{\sigma_y'' b^2}{Eh^2}$	$\frac{\sigma_x b^2}{Eh^2}$	$\frac{\sigma_y b^2}{Eh^2}$	$\frac{\sigma_x b^2}{Eh^2}$	$\frac{\sigma_y b^2}{Eh^2}$
6.37	- 6.37	0.00	0.00	0.00	- 6.37	0.00	- 6.37	0.00
6.38	- 6.41	- .02	.00	.00	- 6.41	- .02	- 6.41	- .02
6.80	- 7.21	- .16	.00	.00	- 7.21	- .16	- 7.21	- .16
7.33	- 8.24	- .32	.00	.00	- 8.24	- .32	- 8.24	- .32
8.00	- 9.63	- .61	.00	.00	- 9.63	- .61	- 9.63	- .61
8.84	-11.37	- .97	.00	.00	-11.37	- .97	-11.37	- .97
10.16	-14.10	- 1.60	.00	.00	-14.10	- 1.60	-14.10	- 1.60
11.64	-17.31	- 2.36	.00	.00	-17.31	- 2.36	-17.31	- 2.36
13.21	-20.99	- 3.46	.00	.00	-20.99	- 3.46	-20.99	- 3.46
15.02	-25.17	- 4.79	.00	.00	-25.17	- 4.79	-25.17	- 4.79
16.91	-29.84	- 6.32	.00	.00	-29.84	- 6.32	-29.84	- 6.32
18.93	-35.11	- 8.26	.00	.00	-35.11	- 8.26	-35.11	- 8.26
21.05	-40.94	-10.76	.00	.00	-40.94	-10.76	-40.94	-10.76
23.25	-47.44	-13.82	.00	.00	-47.44	-13.82	-47.44	-13.82

TABLE 8.- STRESSES AT MIDPOINT OF LOADED EDGE $(x=0,\ y = \frac{b}{2})$

[Shear stresses τ_{xy}' and τ_{xy}'' are 0 because of symmetry]

	Membrane stresses		Extreme fiber bending stresses		Extreme fiber stresses outside of buckle		Extreme fiber stresses inside of buckle	
$\frac{\bar{p}_x b^2}{Eh^2}$	$\frac{\sigma_x' b^2}{Eh^2}$	$\frac{\sigma_y' b^2}{Eh^2}$	$\frac{\sigma_x'' b^2}{Eh^2}$	$\frac{\sigma_y'' b^2}{Eh^2}$	$\frac{\sigma_x b^2}{Eh^2}$	$\frac{\sigma_y b^2}{Eh^2}$	$\frac{\sigma_x b^2}{Eh^2}$	$\frac{\sigma_y b^2}{Eh^2}$
6.37	-6.37	0.00	0.00	0.00	-6.37	0.00	-6.37	0.00
6.38	-6.33	- .02	.00	.00	-6.33	- .02	-6.33	- .02
6.80	-6.17	- .29	.00	.00	-6.17	- .29	-6.17	- .29
7.33	-5.98	- .72	.00	.00	-5.98	- .72	-5.98	- .72
8.00	-5.63	-1.15	.00	.00	-5.63	-1.15	-5.63	-1.15
8.84	-5.21	-1.78	.00	.00	-5.21	-1.78	-5.21	-1.78
10.16	-4.68	-2.63	.00	.00	-4.68	-2.63	-4.68	-2.63
11.64	-4.05	-3.53	.00	.00	-4.05	-3.53	-4.05	-3.53
13.21	-3.21	-4.54	.00	.00	-3.21	-4.54	-3.21	-4.54
15.02	-2.53	-5.51	.00	.00	-2.53	-5.51	-2.53	-5.51
16.91	-1.78	-6.41	.00	.00	-1.78	-6.41	-1.78	-6.41
18.93	- .99	-7.40	.00	.00	- .99	-7.40	- .99	-7.40
21.05	- .31	-8.35	.00	.00	- .31	-8.35	- .31	-8.35
23.25	.28	-9.14	.00	.00	.28	-9.14	.28	-9.14

TABLE 9.- STRESSES AT MIDPOINT OF RIGIDLY CLAMPED EDGES ($x = \frac{a}{2}$, $y = 0$)

[Shear stresses τ_{xy}' and τ_{xy}'' are 0 because of symmetry]

	Membrane stresses		Extreme fiber bending stresses		Extreme fiber stresses outside of buckle		Extreme fiber stresses inside of buckle	
$\frac{\bar{p}_x b^2}{Eh^2}$	$\frac{\sigma_x' b^2}{Eh^2}$	$\frac{\sigma_y' b^2}{Eh^2}$	$\frac{\sigma_x'' b^2}{Eh^2}$	$\frac{\sigma_y'' b^2}{Eh^2}$	$\frac{\sigma_x b^2}{Eh^2}$	$\frac{\sigma_y b^2}{Eh^2}$	$\frac{\sigma_x b^2}{Eh^2}$	$\frac{\sigma_y b^2}{Eh^2}$
6.37	- 6.37	0.00	0.00	0.00	- 6.37	0.00	- 6.37	0.00
6.38	- 6.41	.02	- .70	- 2.20	- 7.11	- 2.18	- 5.71	2.22
6.80	- 7.31	.16	- 2.85	- 9.01	-10.16	- 8.85	- 4.46	9.17
7.33	- 8.50	.32	- 4.41	- 13.95	-12.91	-13.63	- 4.09	14.27
8.00	-10.03	.61	- 6.03	- 19.07	-16.06	-18.46	- 4.00	19.68
8.84	-12.01	.92	- 7.85	- 24.84	-19.86	-23.92	- 4.16	25.76
10.16	-15.04	1.60	-10.35	- 32.74	-25.39	-31.14	- 4.69	34.34
11.64	-18.62	2.36	-13.16	- 41.63	-31.78	-39.27	- 5.46	43.99
13.21	-22.63	3.33	-16.14	- 51.07	-38.77	-47.74	- 6.49	54.40
15.02	-27.25	4.61	-19.31	- 61.11	-46.56	-56.50	- 7.94	65.72
16.91	-32.20	6.14	-22.84	- 72.27	-55.04	-66.13	- 9.36	78.41
18.93	-37.70	7.81	-26.32	- 83.30	-64.02	-75.49	-11.38	91.11
21.05	-43.41	10.05	-29.81	- 94.32	-73.22	-84.27	-13.60	104.37
23.25	-49.34	12.64	-32.92	-104.18	-82.26	-91.54	-16.42	116.82

TABLE 10.- STRESSES AND DEFLECTION AT CENTER OF PLATE ($x = \frac{a}{2}$, $y = \frac{b}{2}$)

[Shear stresses τ_{xy}' and τ_{xy}'' = 0 from symmetry]

	Membrane stresses		Extreme fiber bending stresses		Extreme fiber stresses outside of buckle		Extreme fiber stresses inside of buckle		
$\frac{\bar{p}_x b^2}{Eh^2}$	$\frac{\sigma_x' b^2}{Eh^2}$	$\frac{\sigma_y' b^2}{Eh^2}$	$\frac{\sigma_x'' b^2}{Eh^2}$	$\frac{\sigma_y'' b^2}{Eh^2}$	$\frac{\sigma_x b^2}{Eh^2}$	$\frac{\sigma_y b^2}{Eh^2}$	$\frac{\sigma_x b^2}{Eh^2}$	$\frac{\sigma_y b^2}{Eh^2}$	$\frac{w_{center}}{h}$
6.37	-6.37	0.00	0.00	0.00	- 6.37	0.00	- 6.37	0.00	0.0000
6.38	-6.33	.02	1.66	1.36	- 4.67	1.38	- 7.99	- 1.34	.1113
6.80	-6.15	.29	6.58	5.40	.43	5.69	-12.73	- 5.11	.4425
7.33	-5.84	.72	9.70	7.86	3.86	8.58	-15.54	- 7.14	.6601
8.00	-5.51	1.15	12.64	10.06	7.13	11.21	-18.15	- 8.91	.8722
8.84	-5.01	1.73	15.38	12.04	10.37	13.77	-20.39	-10.31	1.080
10.16	-4.58	2.54	18.51	14.06	13.93	16.60	-23.09	-11.52	1.320
11.64	-4.01	3.44	21.47	15.88	17.46	19.32	-25.48	-12.44	1.574
13.21	-3.45	4.32	24.16	17.31	20.71	21.63	-27.61	-12.99	1.808
15.02	-2.97	5.33	26.51	18.52	23.54	23.85	-29.43	-13.19	2.035
16.91	-2.50	6.23	29.11	19.85	26.61	26.08	-31.61	-13.62	2.258
18.93	-2.07	7.13	31.29	20.94	29.22	28.07	-33.36	-13.81	2.475
21.05	-1.79	8.10	33.51	22.25	31.72	30.35	-35.30	-14.15	2.688
23.25	-1.42	8.93	35.35	23.34	33.93	32.27	-36.77	-14.41	2.897

TABLE 11.- AVERAGE COMPRESSIVE DISPLACEMENT PER UNIT LENGTH IN THE x-DIRECTION AND CORRESPONDING EFFECTIVE WIDTH FOR PLATE UNDER EDGE COMPRESSION WITH UNLOADED EDGES RIGIDLY CLAMPED

[$b/a = 1.5$; $\mu = 0.316$]

$\frac{\overline{p}_x b^2}{Eh^2}$	$\overline{\epsilon}_x \frac{b^2}{h^2}$	$\frac{\text{Effective width}}{\text{Initial width}}$
6.37	6.37	1.000
6.38	6.41	.996
6.80	7.25	.938
7.33	8.35	.878
8.00	9.81	.816
8.84	11.66	.758
10.16	14.55	.698
11.64	17.95	.648
13.21	21.79	.606
15.02	26.22	.573
16.91	31.08	.544
18.93	36.44	.520
21.05	42.29	.498
23.25	48.66	.478

TABLE 12.- CONVERGENCE OF SOLUTION FOR EFFECTIVE WIDTH OF PLATE UNDER EDGE COMPRESSION WITH UNLOADED EDGES RIGIDLY CLAMPED ($b/a = 1.5$; $\mu = 0.316$) AS THE NUMBER OF CUBIC TERMS USED IN THE EQUATIONS OF TABLES 2 AND 3 ARE INCREASED FROM 1 TO 56

Edge strain ratio $\frac{\overline{\epsilon}_x b^2}{h^2}$	$\frac{\text{Effective width}}{\text{Initial width}}$ (1 cu term)	$\frac{\text{Effective width}}{\text{Initial width}}$ (10 cu terms)	$\frac{\text{Effective width}}{\text{Initial width}}$ (56 cu terms)
6.37	1.000	1.000	1.000
11.66	.730	.761	.758
26.22	.544	.568	.573
48.66	.477	.470	.478

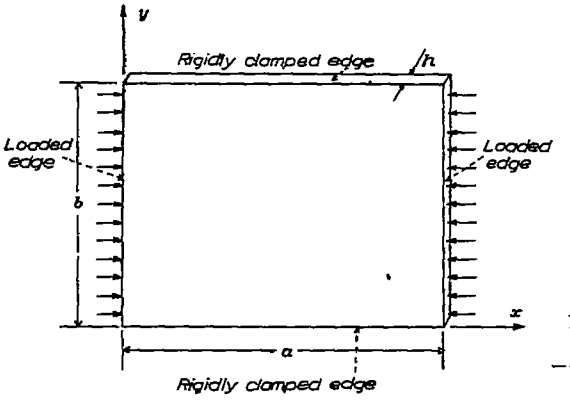


Figure 1.- Rectangular plate subjected to compressive load in plane of plate. Loaded edges simply supported; other two edges rigidly clamped against rotation.

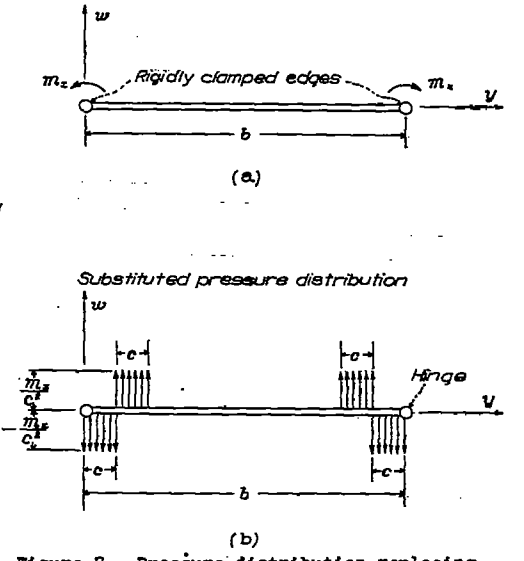
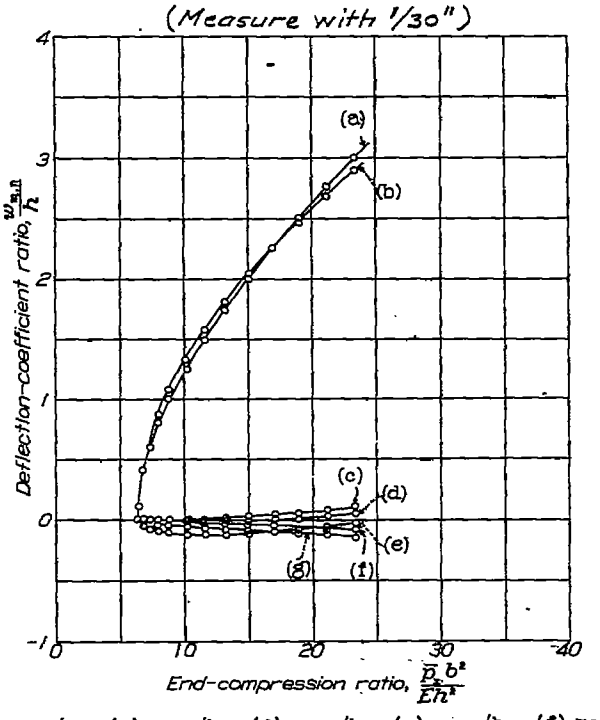
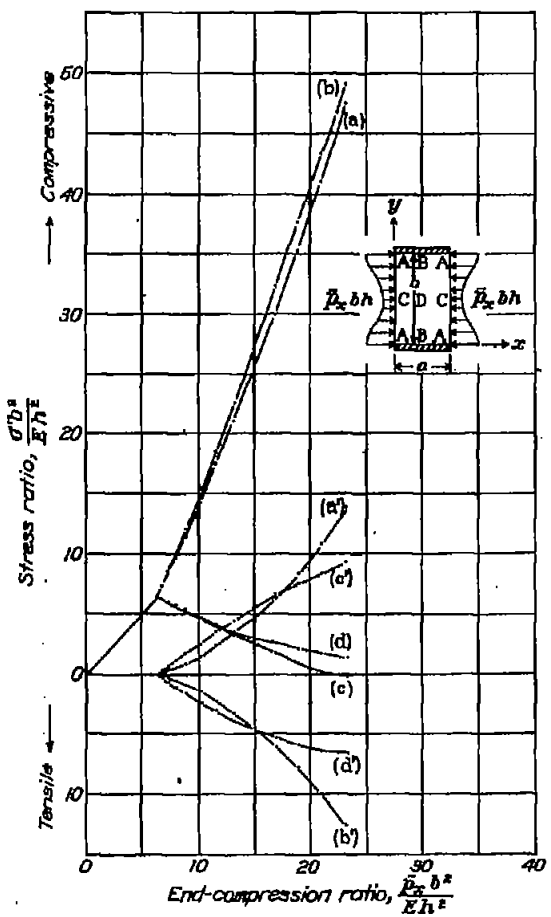


Figure 2.- Pressure distribution replacing rigid clamping at edges.



(a) $w_{1,1}/h$ (b) w_{center}/h (c) $w_{3,1}/h$ (d) $w_{3,3}/h$ (e) $w_{1,3}/h$ (f) $w_{1,7}/h$ (g) $w_{1,5}/h$

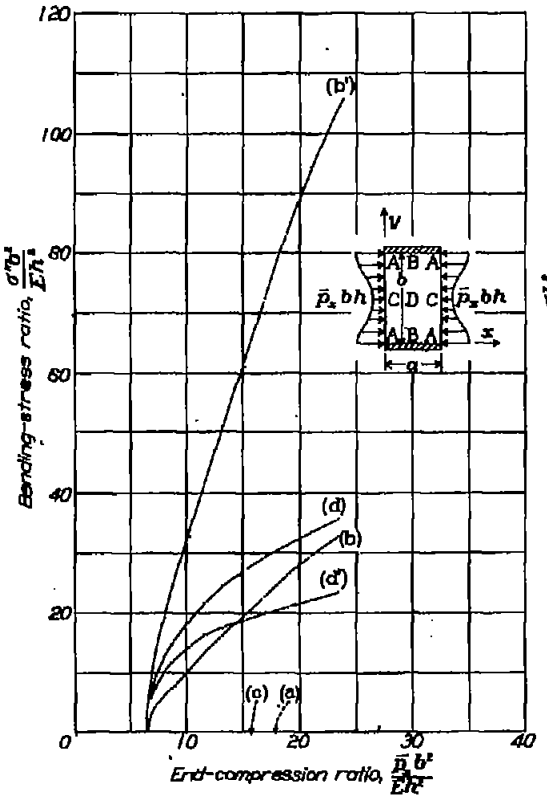
Figure 3.- Values of deflection coefficients in equation (10) as a function of the average compressive stress $\bar{p}_x$ on the loaded edges for a plate under edge compression having the unloaded edges rigidly clamped. ($b/a = 1.5$, $\mu = 0.316$)



(a) $\sigma'_x b^2/Eh^2$(A) (b) $\sigma'_x b^2/Eh^2$(B) (c) $\sigma'_x b^2/Eh^2$(C) (d) $\sigma'_x b^2/Eh^2$(D)

(a') $\sigma'_y b^2/Eh^2$(A) (b') $\sigma'_y b^2/Eh^2$(B) (c') $\sigma'_y b^2/Eh^2$(C) (d') $\sigma'_y b^2/Eh^2$(D)

Figure 4.- Membrane stress ratios for the corner of the plate (A at x = 0, y = 0), for the midpoints of the edges (B at x = a/2, y = 0 and C at x = 0, y = b/2), and for the center of the plate (D at x = a/2, y = b/2) against the end-compression ratio, $\bar{p}_x b^2/Eh^2$.

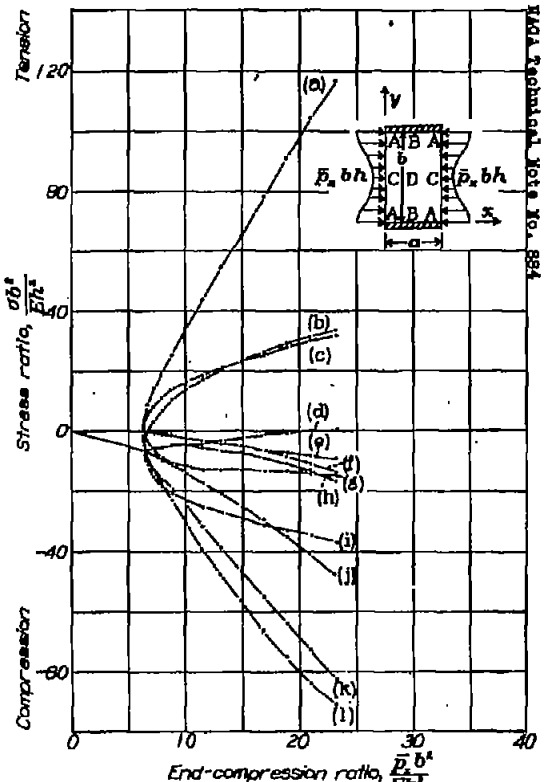


(a) $\sigma''_x b^2/Eh^2$(A) (b) $\sigma''_x b^2/Eh^2$(B) (c) $\sigma''_x b^2/Eh^2$(C) (d) $\sigma''_x b^2/Eh^2$(D)

(b') $\sigma''_y b^2/Eh^2$(B) (d') $\sigma''_y b^2/Eh^2$(D)

(Measure with 5/16")

Figure 5.- Extreme-fiber bending stresses for the corner of the plate (A at x = 0, y = 0), for the midpoints of the edges (C at x = 0, y = b/2 and B at x = a/2, y = 0), and for the center of the plate (D at x = a/2, y = b/2) against the end-compression ratio, $\bar{p}_x b^2/Eh^2$.



Inside of buckle	Outside of buckle
(j) $\sigma_x b^2/Eh^2$ at A	(j) $\sigma_x b^2/Eh^2$ at A
(f) $\sigma_y b^2/Eh^2$ at A	(f) $\sigma_y b^2/Eh^2$ at A
(k) $\sigma_x b^2/Eh^2$ at B	(h) $\sigma_x b^2/Eh^2$ at B
(l) $\sigma_y b^2/Eh^2$ at B	(a) $\sigma_y b^2/Eh^2$ at B
(d) $\sigma_x b^2/Eh^2$ at C	(d) $\sigma_x b^2/Eh^2$ at C
(e) $\sigma_y b^2/Eh^2$ at C	(e) $\sigma_y b^2/Eh^2$ at C
(b) $\sigma_x b^2/Eh^2$ at D	(i) $\sigma_x b^2/Eh^2$ at D
(c) $\sigma_y b^2/Eh^2$ at D	(g) $\sigma_y b^2/Eh^2$ at D

Figure 6.- Extreme-fiber stresses.

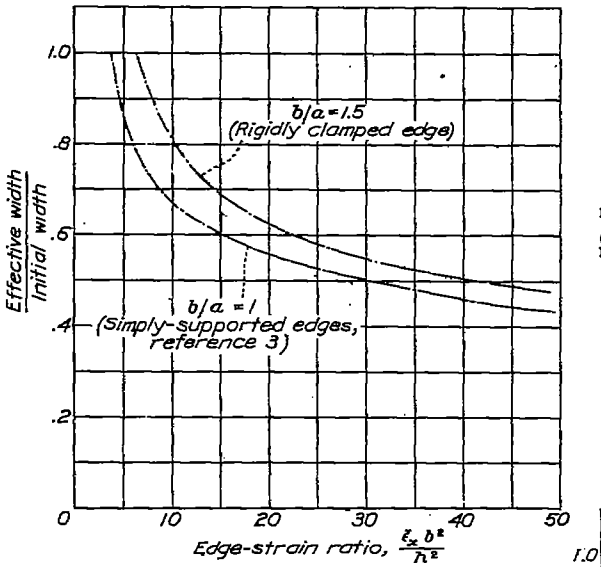


Figure 7.- Ratio of effective width to initial width for plates having the unloaded edges rigidly clamped (b/a = 1.5) and for plates simply-supported (b/a = 1). Poisson's ratio = 0.316.

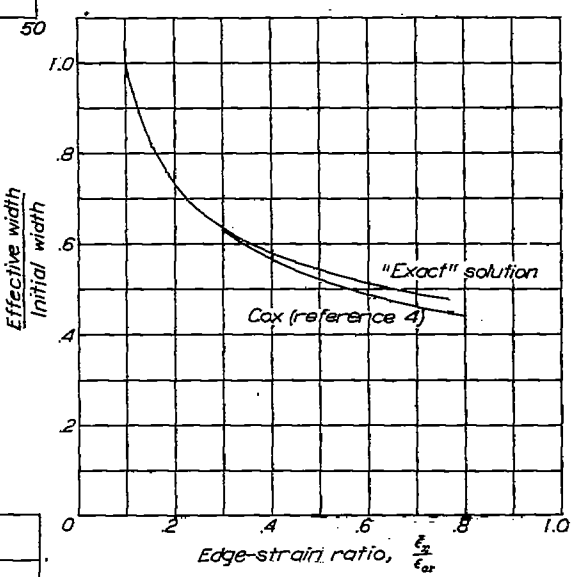


Figure 8.- Effective-width comparison of present solution with Cox's approximate solution for unloaded edges rigidly clamped. $\epsilon_{cr} = 6.37h^2/b^2$

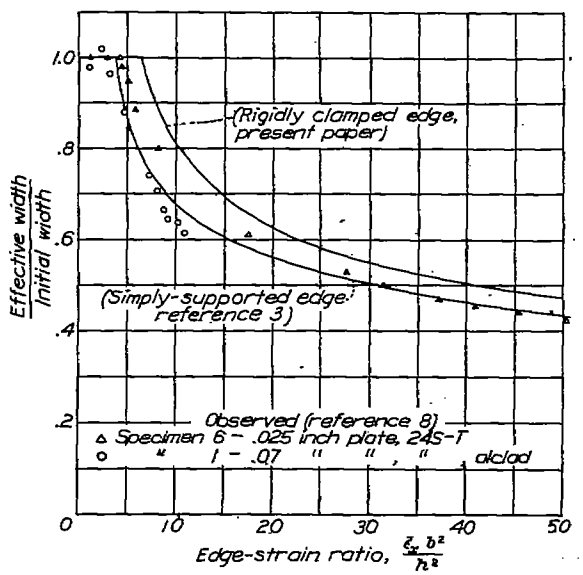


Figure 9.- Comparison with experimental results given in reference 8. (ϵ_x = average stringer strain.)

(Measure with 1/40")

~~RESTRICTED~~

ATI-

REVISION none

ORIG. AGENCY NO. TN-884

TITLE: Large-deflection Theory for End Compression of Long Rectangular Plates Rigidly Clamped along Two Edges

AUTHOR(S): Levy, Samuel; Krupen, Philip

ORIGINATING AGENCY: National Advisory Committee for Aeronautics, Washington, D. C.

DATE	DOC. CLASS	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Jan '43	Restr.	U.S.	Eng.	33	tables, diagrs, graphs

ABSTRACT:

For the extreme case of rigid clamping of flat plates along the edges parallel to the load, equations are solved for edge strains equal to 8 times the buckling strain. Theoretical values of effective width are compared with values for simple support along edges parallel to the load. Experimental values are between theoretical curves for the extremes of simple support and rigid clamping.

Plates, Deflection

P20/11

DISTRIBUTION: Requests copies of this report only from Originating Agency

DIVISION: Stress Analysis and Structures (7)

SECTION: Structural Theory and Analysis Methods

ATI SHEET NO.: R-7-2-6 (2)

SUBJECT HEADINGS: Plates - Stresses (71975)

240 400

INDEX

Wright-Patterson Air Force Base
Dayton, Ohio

AD-B805 391

Classification cancelled
or changed to _____

AUTH:

By _____
Signature and Grade

Date APR 29 1949

Classification cancelled per authority
of List NACA dd. 28 Sept 1945
George R. Jordan, USCO. 29 Apr 1949

RESTRICTED

TITLE: Large-deflection Theory for End Compression of Long Rectangular Plates Rigidly Clamped along Two Edges

AUTHOR(S): Levy, Samuel; Krupen, Philip

ORIGINATING AGENCY: National Advisory Committee for Aeronautics, Washington, D. C.

ATI- 8644

REVISION none

ORIG. AGENCY NO. TN-884

DATE	DOC. CLASS	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
Jan '43	Restr.	U.S.	Eng.	33	tables, diagrs, graphs

ABSTRACT:

For the extreme case of rigid clamping of flat plates along the edges parallel to the load, equations are solved for edge strains equal to 8 times the buckling strain. Theoretical values of effective width are compared with values for simple support along edges parallel to the load. Experimental values are between theoretical curves for the extremes of simple support and rigid clamping.

DISTRIBUTION: Requests copies of this report only from Originating Agency

DIVISION: Stress Analysis and Structures (7)

SECTION: Structural Theory and Analysis Methods

ATI SHEET NO.: R-7-2-6 (2)

SUBJECT HEADINGS: Plates - Stresses (71975)

Air Materiel Command
U.S. Air Force

AIR TECHNICAL INDEX

Wright-Patterson Air Force Base
Dayton, Ohio

RESTRICTED