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NAVAL RESEARCH LABORATORY,
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(NRL REPORT NO. S-2113)

THE PROPAGATION OF UNDERWATER SOUND AT LOW FREQUENCIES, AS A FUNCTION OF THE ACOUSTIC PROPERTIES OF THE BOTTOM - AND APPENDIXES A-E - AND ADDENDA

JOHN M. IDE; RICHARD F. POST; WILLIAM J. FRY 15 AUG '43
188 PP. PHOTO, GRAPHS, DRWGS, MAP

SCIENCES, GENERAL (33)
PHYSICS (2)

WAVES, SOUND - PROPAGATION IN FLUIDS
ACOUSTICS, UNDER WATER

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ABSTRACT

This report presents an experimental and theoretical analysis of the transmission of underwater sound in shallow water, taking into account the influence of the bottom. The analysis is particularly concerned with frequencies at which the wavelengths are comparable to the physical dimensions of the acoustic system.

The examination of 96 range run records from two locations in the Potomac River area shows, within the frequency range 70-400 cps, different rates of attenuation of sound pressure level with distance, depending upon the acoustic properties of the bottom.

The records indicate a low rate of attenuation over hard bottom, in pronounced disagreement with the rate predicted from the transmission theory based upon normal acoustic impedance. It is concluded that the impedance is insufficient to determine the general propagation constants required for solving transmission problems.

A new analysis of underwater sound propagation is given in terms of the normal modes of vibration of the acoustic system of the sea between surface and bottom. The bottom is characterized only by a density and a velocity of sound. The experimental results are interpreted successfully and in detail in terms of the initial stimulation, the relative attenuation, and the phase velocities of the modes.

It is shown that the observed phenomena cannot be explained as the result of interference between direct and surface-reflected sound beams, except under special limiting conditions such as great water depth or complete absorption of sound at the bottom.

Methods are described for specifying the character of the transmission and computing the propagation constants, when the density and the velocity of sound of the bottom material are known.

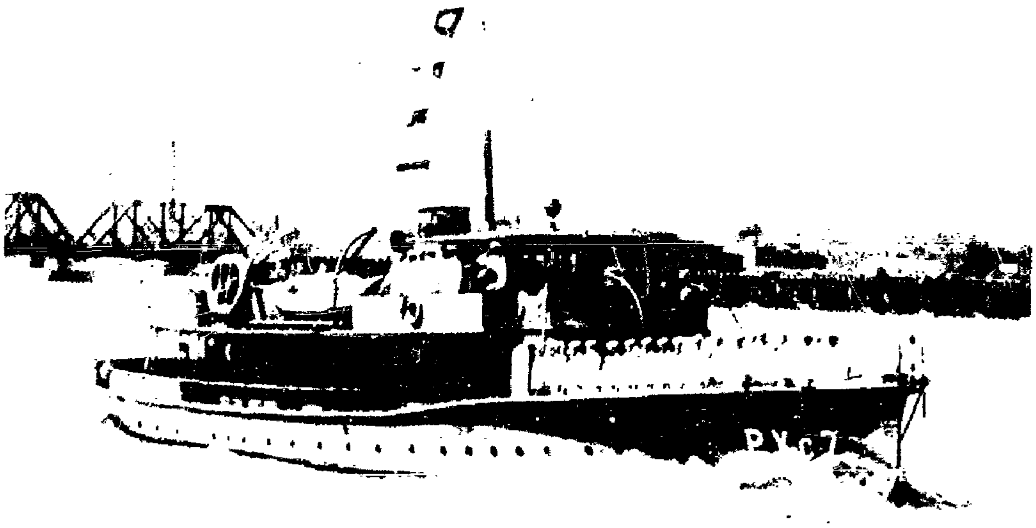
The use of acoustic measurements as transmission criteria is discussed, together with the estimation of acoustic constants from hydrophone soundings, bottom samples, and hydrographic data.

Computation of transmission characteristics from the data is facilitated by the use of six charts which are reproduced in the report.

Applications of the analysis to problems of specific interest to the Navy are discussed. The primary application is to the interpretation of measurements of underwater sound fields in the acoustic system of the sea. Such measurements must be made in the testing of acoustic minesweeping devices, and in the study of ship noises. The analysis may be applied to the prediction of minesweeping ranges from hydrographic and acoustic data, and to the estimation of submarine listening ranges.

PERSONNEL

The experimental work on which this report is based was performed by Dr. John M. Ide and Messrs. Donald E. Albert and Richard F. Post, with the able cooperation of the officers and men of the USS Aquamarine (PYc-7), commanded by Lt.(sg) Britton B. Wood (USNR). The theoretical derivations were obtained by Mr. William J. Fry, and the computations were made by Messrs. Morton S. Raff, Post, and Albert. The report was written by Dr. Ide and Mr. Post with assistance from Mr. Albert in the preparation of the plates.



USS AQUAMARINE

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I. INTRODUCTION.

I. INTRODUCTION.

1. Understanding of the relationships between the horizontal propagation of underwater sound from a ship-mounted source and the acoustic characteristics of the underlying sea bottom is vital for the correct interpretation of the transmission measurements which must be made in the testing of acoustic minesweeping devices, the study of ship noises, and the estimation of submarine listening ranges.

2. For example, the effectiveness of acoustic minesweeping operations may depend as much on the geometric configurations and on the acoustic behavior of the bottom as on the sound output of the gear itself. Although the characteristics of the sweeping gear may be determined by acoustic measurements at short distance (e.g. 6 ft), the influence of the boundaries (surface and bottom) on the sound pressures at greater distances gives rise to extremely complicated phenomena. These phenomena, particularly at frequencies where the wavelengths are comparable to the physical dimensions involved, have not heretofore been adequately explained.

3. The underwater sound pressure distributions directly beneath a ship-mounted source have been investigated by this Laboratory, and a formal report has been submitted (Bibliog. 4). The results of experiment were shown to be in close agreement with the free field acoustic impedance theory (Bibliog. 11).

4. A study has now been made of the related problem of the propagation of underwater sound as a function of the horizontal distance from the ship-mounted source. A preliminary report has been submitted (Bibliog. 5), in which the methods and results of the NRL analysis are briefly outlined. In the formal report presented herewith, much of the experimental evidence is reproduced and discussed, the consequences of the new theory are deduced and fully explored, and the analysis is applied to the interpretation of the observed phenomena.

5. The report is divided into a number of sections, many of which are provided with separate summaries for convenience in following the argument. The reader who does not have time for detailed study of the report is advised to read all of Sections I, II, and III, the summaries only of Sections IV and V, all of Section VI, the summary of Section VII, and all of Sections VIII, IX, X, and XI. By following this procedure the gist of the new analysis may be obtained.

6. Applications of the results to problems of Naval interest are discussed in Section X. The mathematical derivations relevant to various aspects of the subject will be found in the five appendices.

7. It is planned to continue the study of low frequency sound propagation by making additional range tests and hydrophone soundings in the Chesapeake Bay area and along the nearby sea coasts.

II. EXPERIMENTAL PROCEDURE: CONDITIONS OF TESTS.

II. EXPERIMENTAL PROCEDURE; CONDITIONS OF TESTS.

8. For the investigation of underwater sound propagation described in this report the experimental procedure consisted of setting up a sound field of high intensity, and recording the pressure level registered by a hydrophone placed at different depths and distances from the source.

9. During the tests the sound source was mounted at an effective depth of 10-12 ft beneath the USS AQUAMARINE (PYc 7), a converted yacht assigned to the Sound Division of the Naval Research Laboratory, or from the bow of the USS ACCENTOR (AMc 36), a coastal minesweeper. The XUR-2 mechanically driven sound generator, mounted in the 36" well of the AQUAMARINE or swung from the bow of the ACCENTOR by a 24 ft boom, was employed as sound source in most of the tests. For a few tests, the NRL Model X-3 magnetic type underwater loud-speaker, mounted in the AQUAMARINE, was employed. These devices have been previously described (Bibliog. 1, 2, and 3).

10. Both the loud-speaker and the mechanical generator were in effect point sources of sound, since their dimensions were small in comparison with the wavelength. The output from both may be characterized as polyphonic, a word here used to designate a fundamental frequency accompanied by harmonics the intensity of which decreases with the order.

11. The experimental records obtained were of two types, range run recordings and hydrophone soundings. The range run recordings show the sound pressure level received by a hydrophone planted 1½ ft above the bottom, as the test vessel traversed a straight course several thousand feet long and passed directly over the hydrophone. During each run the source was operated at a constant level and frequency. The hydrophone soundings are records of the vertical distribution of sound pressure level. These were made by raising the hydrophone from the sea bottom to the surface, at constant rate, with the output of the sound source maintained constant and the test vessel at anchor. Typical hydrophone soundings made directly beneath the source have been reproduced and discussed in detail in a recent report (Bibliog. 4). The hydrophone soundings described in Section IX of the present report were made at considerable distance from the source.

12. The measurement equipment, including NRL tourmaline hydrophones, ERPI Sound Frequency Analyzer (RA 277 F), and ERPI Graphic Level Recorder (RA 246), was the same as that described in previous reports (Bibliog. 2, 4, and 5). The calibrations of the hydrophones were checked by comparison with the Bell Laboratories Type 3A standard crystal hydrophone, which had been compared with the MIT standards. Extraneous noise was eliminated by tuning the ERPI analyzer to the frequency of the source, with the filter set to pass a frequency band only 5 cycles wide. The combination of a constant sound source of high intensity and a calibrated sharply-tuned receiving channel was found to be highly satisfactory for the quantitative study of transmission phenomena.

13. The receiving and recording apparatus was arranged to be battery powered, and was deployed in a 12 ft surf boat for many of the range runs and for hydrophone soundings made at a distance from the test vessel. For the latter, the hydrophone was lowered and raised by hand from a bathythermograph cable reel. For some of the range runs the receiving apparatus was located in the NRL range house on the main east pier of the Potomac River Bridge, and a cable run to the hydrophone planted near the bottom in a position 200 ft west of the pier.

14. The tests described in this report were made in two locations, 1) at the Potomac River Bridge near Morgantown, Maryland, and 2) near the Potomac River Mouth, centered about a position 3000 yards south of St. George Island. Portions of the hydrographic charts for the areas in which the tests were made are reproduced in Plate 1. The range courses, the hydrophone locations, and the bottom contours are shown. The first course was along the channel between the deep water piers of the Potomac River Bridge, with the hydrophone planted 200 ft west of the pier on the Maryland side of the main span.

15. The Bridge was a particularly convenient location because the geometry of the course could be determined accurately by taking bearings and stadimeter sights on the piers and girders. It was found by experience that a reliable distance scale for the range run records was obtained by marking the record as the bow and as the stern of the test vessel passed the recording station on the pier. The speeds of the vessel and of the recording paper were substantially constant during each run. On the River Mouth range course the passage of the ship

gave the only data for establishing the distance scales. The accuracy of the latter is believed to be of the order of 10%.

16. At the center of the River Bridge course the water depth was 57 ft, and at the center of the River Mouth course, 55 ft. At the Bridge, the bottom sloped up stream about 3 ft per mile; at the River Mouth, the bottom sloped down stream about 10 ft per mile. The area of substantially flat bottom was larger at the River Mouth than at the Bridge. Since in both locations the bottom irregularities along the course were small in comparison with the depth or with the wavelength of the propagated sound, the transmission measurements are considered to be characteristic of those which might be obtainable over an ideally flat bottom.

17. The locations chosen for this investigation represent different acoustic conditions. It was shown by the measurements of bottom impedance described in the previous report (Bibliog. 4) that the soft mud bottom at the River Bridge is acoustically "soft" in summer and highly absorbing in winter, and that the hard sandy mud bottom at the River Mouth location is acoustically "hard" both in summer and in winter. At the Bridge, the loss per normal bottom reflection is 6-9 db in summer, and 16 db in winter; at the River Mouth, relatively independent of season, it is 8-14 db. At the Bridge the normal bottom inpedance at low frequencies is smaller than the impedance of water. This impedance increases with frequency, and a phase transition takes place above 500 cps in summer and at about 150 cps in winter. At the River Mouth location the measured normal impedance is real, and equal to two or three times the water impedance. The terms "soft", "hard", "transitional", and "impedance" are defined in Bibliog 4, q.v.

18. The experimental material on which the conclusions of this report are based consists of 96 range run recordings listed in Table I, and about 80 hydrophone soundings made along the range courses at the River Mouth and at the Potomac River Bridge. About 50 additional range run recordings and several sets of hydrophone soundings have recently been made in the Chesapeake Bay and Rappahannock River areas. A full discussion of the recent records will be reserved for a subsequent report.

TABLE I. RANGE RUN RECORDINGS.

Number of Recordings	Location	Date	Sound Source	Frequency Range
10	P.R. Bridge	Aug 6, 1942	X-3 Speaker	200-400 cps
10	P.R. Bridge	Sept 30, 1942	XUR-2	70-186 cps
13	River Mouth	Oct 1, 1942	XUR-2	70-200 cps
*51	P.R. Bridge	Oct 13-15, 1942	XUR-2	70-200 cps
2	River Mouth	April 7, 1943	XUR-2	70-100 cps
10	P.R. Bridge	April 8, 1943	XUR-2	70-200 cps

*USS ACCENTOR acting as test vessel.

19. Bottom impedance data from soundings directly beneath the source were obtained at the River Mouth in February 1943, and at the Bridge in August 1942, and February 1943. On April 7, 1943, hydrophone soundings for frequencies in the range 70-300 cps were made at the River Mouth at horizontal distances from the source of 100 ft, 350 ft, 1000 ft, and 2000 ft. On April 8, 1943, hydrophone soundings were made at the Bridge at horizontal distances from the source of 200 ft, 500 ft, and 1000 ft, for frequencies in the range 70-300 cps.

20. Eleven range run recordings selected from the groups made in August and September at the Potomac River Bridge are reproduced in Plates 2, 3, and 4. Nine recordings made at the River Mouth in October are reproduced in Plates 5, 6, and 7. Typical hydrophone soundings at considerable distance from the source made in April 1943 are reproduced in Plate 11.

21. The db reference level for these and for all reproduced records is 0.0002 dynes/sq. cm., unless specifically stated to be otherwise.

III. DISCUSSION OF EXPERIMENTAL RESULTS.

SUMMARY

Experimental Results

22. The study of the range run records shows that the observed attenuation over soft bottom at the Potomac River Bridge is in substantial agreement with that computed from measured values of bottom impedance. The observed attenuation over hard bottom at the River Mouth is extraordinarily low compared with values computed from the theory involving impedance.

23. This discrepancy between observed and computed results is traced to the inherent limitations of the normal acoustic impedance. A new analysis of underwater sound propagation is proposed in which it is assumed that the bottom is a homogeneous medium completely characterized for acoustic behavior by a density and a velocity of sound. Although details of the new analysis are not presented in this section (See Section IV), it is shown from elementary considerations that total internal reflection of sound waves at a hard bottom should be expected under certain conditions, and that the occurrence of this type of reflection would explain the low rates of attenuation observed at the River Mouth.

24. Additional experimental results, such as the "interaction loops" which are prominent on the records, the "initial drop" of sound pressure level in the first 250 ft of source-receiver separation, and the consistency and reproducibility of the data, are briefly discussed.

III. DISCUSSION OF EXPERIMENTAL RESULTS:

A. Characteristics and Consistency of Range Run Data.

25. The salient features which emerge from a study of the range run records for the two locations are: (a) that all records except those at the lowest frequencies (70 and 80 cps) exhibit a succession of strongly marked loops, with maxima and minima of sound pressure alternating as the distance from the source increases; (b) that all records exhibit an "initial drop", or rapid decrease in sound level in the first 250 ft of source-receiver separation, followed at greater distances by a more gradual rate of decrease; (c) that the records made at the River Mouth, showing propagation over hard bottom, exhibit a much lower rate of attenuation of sound pressure level with distance than those made at the Bridge.

26. Interaction Loops. The alternating loops which characterize most of the records are graphically illustrated in Plates 2, 3, 4, 5, 6, 7, and 8. The phenomenon was first reported by this Laboratory about a year ago (Bibliog. 2). Although indications of the effect have been observed (e.g. Bibliog. 13), the cause does not appear to have been generally recognized. A detailed discussion of the "interaction loops", as they may be termed, is given in a later section of this report, where they are explained as due to the interaction of the normal modes of vibration which characterize the sound waves confined between the surface and the sea bottom.

27. Initial Drop. The relative sound pressure levels obtained from range runs at various frequencies over various types of bottom are superposed, and plotted in Plates 9 and 10 as a function of the distance between source and receiver. In Plate 10 the plotted levels represent the envelopes of the respective range records expressed in db referred to the peak level directly over the hydrophone. In Plate 9 the envelope curves are expressed in db referred to the sound output level at a distance of 6 ft from the source mounted on the ship. A comparison of the envelope curves, as well as of the original records, shows that the "initial drop", or the decrease in sound level in the first 250 ft of source-receiver separation, is relatively independent of the frequency or of the character of the bottom. The amount of the "initial drop", from the average of 60 determinations is 17 ± 2 db. The water depth was 55 ft in all cases.

28. Attenuation at Considerable Distance. In order to show the influence of different bottom conditions on the propagation for a constant frequency, the envelopes of three characteristic range records are shown in Plate 9 for 100 cps and for 70 cps, respectively. These curves show pronounced differences in the attenuation at considerable distances from the source, corresponding to differences in the acoustic properties of the bottom. Curves (a) representing transmission at the River Mouth, over hard bottom, indicate much smaller rates of attenuation than the curves representing transmission at the Potomac River Bridge, over soft mud. For example the average slope of curve (a) in Plate 9 for transmission over hard bottom at 100 cps, is less than 4 db per 1000 ft at considerable distances (1000 - 2000 ft), and the rate of attenuation decreases with increasing distance. It may be seen from Plate 10 (b) that the curve for 100 cps is representative of those obtained over hard bottom, since the curves for other frequencies also correspond to low rates of attenuation. The original records of the two exceptionally long runs over hard bottom, for 70 cps and 100 cps, respectively, are reproduced in Plate 7 together with the fathometer record of the depth. Curves (a) in Plate 9 are the envelopes of these records.

29. The rates of attenuation indicated by the records made over soft bottom at the Potomac River Bridge are larger by an order of magnitude than those observed at the River Mouth. Curves (b) in Plate 9, for the Bridge in September, have average slopes of about 18 db/1000 ft after the "initial drop"; curves (c) for the same location in April have average slopes of about 40 db/1000 ft. It may be seen from Plate 10 (a) and (d) that these average slopes are representative for all frequencies in the range 70-200 cps. The bottom impedance data indicate "soft" bottom reflections at all seasons in the River Bridge location at these frequencies, although the absorption per bottom reflection is about twice as large in April as in September. The difference in average slope between the two sets of curves for soft bottom, (b) and (c) of Plate 9, is explicable in terms of the higher absorption at the River Bridge in April compared with that in September. On the other hand, the extraordinarily low values of attenuation shown by the curves for transmission over hard bottom at the River Mouth (e.g. curves (a) Plate 9) suggest that the factors which govern transmission over hard bottom may differ in some essential way from those which control transmission over soft bottom.

30. Consistency of Range Run Data. The consistency of the data, and consequently the significance which may be attached to the trends indicated by the records, may be gauged by the superposed envelope curves shown in Plate 10, (a), (b), (c), and (d). Records at various frequencies are shown grouped as follows: (a) records from the Potomac River Bridge in September, in the frequency range 70-186 cps; (b) records from the River Mouth in October, in the frequency range 70-200 cps; (c) records from the Potomac River Bridge in April, in the range 70-100 cps; and (d) records from the Bridge in August, in the frequency range 200-400 cps. The study of Plate 10 shows that each group of records is self-consistent, and that in general there appears to be a small systematic reduction in attenuation as frequency increases. The trend with frequency is best illustrated by record groups (a) and (d) in Plate 10, made at the Bridge in summer.

31. The envelopes of the range run records at various frequencies (Plate 10), for a given location and acoustic condition of the bottom, show the same general course except for the slight downward trend of attenuation with frequency noted above. A few curves, such as that for 200 cps in Plate 10 (b), show unaccoutably broad central peaks. This is probably explained by the failure of the test vessel to pass directly over the hydrophone during the run.

32. When the frequencies as well as the bottom conditions were maintained constant, repeated range run records closely duplicated each other in all except the most insignificant details. Records made at the Bridge with the X-3 loud-speaker, reproduced in a previous report (Bibliog. 2), showed close duplication of interaction spacings, levels at various distances, and other details when runs were made at the same frequency with the test vessel running alternately north and south along the course. Many duplicate runs were made in the large group obtained with the source mounted on the USS ACCENTOR, particularly at 70 cps and at 90 cps. Those for the same frequency were almost identical except for the region close to the central peak. The minor differences which did occur may be attributed to the occasional departure of ACCENTOR from the course, especially when passing over the hydrophone. The AQUAMARINE, having had more experience in this type of maneuver, was relatively more successful in repeating runs. The effects of surface roughness, interference from the ship's hull, wakes, direction of tidal flow, and other possible cause of variation in range run records, appeared to be relatively unimportant.

B. Correlation of Transmission Attenuation With Bottom Impedance.

33. A method of computing the horizontal attenuation for transmission of sound over a bottom characterized by a given normal acoustic impedance is outlined in Bibliog . If the attenuation to be expected at the River Mouth and at the Bridge in summer and in winter be computed by this method from the bottom impedance measurements for these locations (Bibliog 4), the following results are obtained:

TABLE II.

Attenuation Between 500 feet and 1500 feet from Sound Source, Computed From Theory Based on Normal Impedance, With Addition of Cylindrical Spreading. 100 cps.

	Computed	Observed
Potomac River Bridge Soft Bottom Summer Conditions	14 db/1000 ft	14 db/1000 ft
Potomac River Bridge Soft Bottom Winter Conditions	33 db/1000 ft	38 db/1000 ft
Potomac River Mouth Hard Bottom Winter Conditions	59 db/1000 ft	4 db/1000 ft

34. The MIT method, based upon normal acoustic impedance, gives computed results in substantial agreement with experiment for the rate of attenuation over soft bottom at the Potomac River Bridge, in summer and in winter. The method gives completely erroneous results, however, for the attenuation to be expected over hard bottom at the Potomac River Mouth. The computed rate of attenuation for this location, 59 db/1000 ft, differs by an order of magnitude from the observed attenuation of 2-4 db/1000 ft at considerable distances from the source. Indeed, the observed decrease in sound pressure level with distance, for source-receiver separations greater than about 500 feet, may be entirely accounted for by the cylindrical spreading of the sound waves in the medium enclosed by the surface and the bottom. This is shown by the dotted curve on Plate 9, which was computed for a decrease in sound

pressure proportional to the square root of the distance from the source, and superimposed at arbitrary level. There is no evidence on curve (a) of Plate 9 of appreciable loss of energy by absorption at either surface or bottom, after the first few hundred feet. Since in April the water temperature at the surface was 35°F, temperature gradients could not have played a significant role. The prevailing acoustic conditions at the Potomac River Mouth were such as to result in the propagation of underwater sound with total internal reflection at the lower boundary, the sea bottom. All of the records made in this location, in October and in April, at frequencies from 70 cps to 200 cps, showed this type of transmission.

C. Transmission Over Hard Bottom; Proposed Explanation.

35. The observed phenomenon of propagation of underwater sound with negligible attenuation over hard bottom cannot be explained by the transmission theory based upon normal bottom impedance. Observed and computed slopes for curve (a), Plate 9, differ by 55 db, corresponding to a pressure factor of almost 600. When theory and experiment give results differing by such a wide margin, the basis of the theory requires re-examination.

36. Careful study shows that the difficulties may be traced to the inherent limitations of the normal acoustic impedance concept. Indeed, the usefulness of this concept for the evaluation of boundary conditions in transmission problems is found to be limited to certain special cases. For air-borne sound waves incident on acoustic absorbing material, and for water-borne sound waves incident on a "soft" bottom, the wave velocity relations at the boundary are such that the latter may be usefully charcterized by a normal acoustic impedance, independent of the angle of incidence. For the special case of the "soft" bottom at the River Bridge (curves (b) and (c) of Plate 9,) it has been shown above that values of horizontal attenuation in substantial agreement with range run data may be computed from measured values of normal impedance by employing the method outlined in Bibliog 10 . When used to compute the transmission over hard bottom, this method gives erroneous results because the normal impedance does not adequately express the boundary conditions for the propagation of sound waves impinging at large angles of incidence on the boundary between a medium of low velocity and a medium of higher velocity.

37. Although the sound pressure distribution directly beneath a ship-mounted source is best interpreted in terms of normal impedance, the adequacy of the concept for expressing the acoustic properties of the sea bottom for other angles of incidence has been shown to be open to question.

38. In a recent NOL Report (Bibliog 12) the normal impedance is computed for various types of sea bottom, including viscous fluid and elastic medium. The impedances are derived as complicated expressions involving the angle of incidence and the densities and velocities in the media. It is shown that if the velocity in the bottom is comparable to or greater than the velocity in the water, the bottom impedance will depend appreciably on the angle of incidence. A normal impedance can still be measured, and a reflection coefficient (ratio of reflected to incident wave pressure) may be formulated in terms of it, but under these conditions the impedance alone is not sufficient to determine the propagation constants which are required for solving transmission problems.

39. Fortunately the situation is not so unfavorable as is implied in the NOL Report (Bibliog 12). The bottom impedance measurements made in the Potomac River area (Bibliog 4) indicate the common occurrence, in land-locked waters, of "soft" bottoms in which the wave velocity is much smaller than that in water. In this case the impedance is relatively independent of the angle of incidence, and the transmission theory based upon impedance should be valid. The experimental results depicted in Plate 9 curves (b) and (c) verify this expectation. In contrast, if the bottom impedance measurements indicate acoustically "hard" bottom, it is at least probable that the wave velocity in the bottom is greater than that in water. In this case it is to be expected that the transmission theory based upon normal impedance will not give valid results. The failure of the theory to account for the experimental results at the River Mouth is thus explained.

40. The problem can be solved, however, by a theoretical analysis in which the appropriate boundary conditions are formulated without characterizing the bottom by a normal acoustic impedance. Such an analysis of low frequency sound propagation, valid for both "soft" and "hard" bottoms, in which the solutions of the wave equations are obtained in the form of normal modes of free vibration between the surface and a homogeneous fluid bottom, is presented in Section IV and in Appendices A and D. The term "fluid bottom" is used to denote a bottom in which compressional waves alone need be considered and for which the energy in shear waves is negligible.

41. This analysis is valid for a wide range of acoustic conditions at the bottom, since the formulation of boundary conditions is quite general. For convenience, the results are obtained in terms of the densities and velocities of sound in the water and in the material of the bottom. It is shown (a) that if the velocity in the bottom is appreciably less than that in water, the transmission should be essentially the same as that predicted by the theory based on normal impedance; and (b) if the velocity in the bottom is greater than in water, the transmission at all frequencies above a certain critical frequency should be characterized by negligible horizontal attenuation at considerable distances from the source. An explanation is thus provided for the low-loss propagation observed experimentally over the hard bottom.

42. Although the detailed analysis in terms of normal modes is necessarily complicated, the basic phenomena may be visualized from the classical theory of reflection and refraction of sound at the boundary between two homogeneous media. It is pointed out in the standard texts on acoustics, beginning with Rayleigh, that plane waves of sound incident on the boundary between two homogeneous media will, under certain conditions, suffer total internal reflection. These conditions are: (a) that the velocity of sound in the refracting medium (the bottom) is larger than that in the incident medium (the water) and; (b) that the angle of incidence of the waves at the boundary is larger than a critical angle defined by the relation $\sin \theta = c_1/c_2$, where θ is the angle of incidence and c_1 and c_2 are the velocities in the incident and refracting media, respectively. It may be shown that the effective angles of incidence and the expected velocity ratios are favorable for the occurrence of total internal reflection.

43. The velocity of sound in the bottom material is seldom directly known, although the probable range of velocities, from geophysical data, is 5000-6500 ft/sec for compact sandy mud, and may be as high as 20,000 ft/sec for rock. It is entirely reasonable that bottoms classified by normal impedance measurements as "hard" ($Z/\rho c > 1$) may have sound velocities greater than that of water. The transmission of low frequency sound over them at considerable distances may therefore be characterized by total internal reflection. Under these conditions the only causes of attenuation would be the cylindrical spreading of the waves, and the losses (usually small) from reflection or scattering at the surface. The experimental record envelopes reproduced in curves (a) of Plate 1 demonstrate this type of propagation.

44. Phenomena involving total internal reflection at the sea bottom have received practically no notice in the literature of underwater sound transmission. From the elementary considerations discussed above this type of transmission should be extremely common. For example, the entire continental shelf along the Atlantic coast of the United States, outside the harbors and river mouths, is characterized by bottoms of sand, shell, and hard sandy mud. Similar conditions obtain along the Pacific Coast, at the approaches to San Francisco Bay, on both sides of the English Channel, and along much of the southern shore of the Mediterranean Sea. The acoustic conditions for "hard" bottom transmission of low frequency sound should be realized over most of these areas.

45. Although long ranges have been observed, particularly for explosion waves employed in sound ranging, the possibility that total internal reflection at the sea bottom may be the dominant factor in the transmission of underwater sound at low frequencies seems to have been entirely overlooked by prior investigators. The scarcity of relevant observations on this subject may perhaps be accounted for by the complexity of the experimental equipment and of the arrangements required for the quantitative study of underwater sound propagation.

IV. INTRODUCTION TO NORMAL MODE THEORY OF PROPAGATION OF UNDERWATER SOUND OVER HOMOGENEOUS BOTTOM

SUMMARY

Introduction to Normal Mode Theory

46. The problem of the propagation of low frequency sound in water, taking into account the influence of the boundaries on the transmission, is analyzed in terms of the normal modes of vibration of the acoustic system of the sea between surface and bottom. This analysis is particularly adapted to frequencies at which the wavelengths are comparable to the physical dimensions of the system. The results should be valid for a wide range of acoustic conditions at the sea bottom, and free from the limitations inherent in the use of normal impedance.

47. The acoustic field surrounding a ship-mounted source in shallow water is treated theoretically by studying the sound pressure field set up by a point source placed at any point in a non-dissipative homogeneous medium, the water, confined between two infinite plane boundaries. Reference is made to Appendix D for details of the general derivations. In order to surmount the difficulties which arise in the integration of the general equations, the actual acoustic system is replaced by a rectangular tube with rigid side walls. The derivations for sound propagation in such a tube are outlined in this section and given in detail in Appendix A.

48. The principal result of both the general analysis and that in which the pipe artifice is employed is a transcendental equation relating the distribution constants of the acoustic system to the elastic constants (density and velocity of sound) of the bottom. Five charts (Plates 12-16 inc.) have been computed and plotted from this equation, by means of which the distribution constants for the first and second modes may be evaluated once the acoustic properties of the bottom are known. The propagation constants for each mode (damping and phase velocity) may be determined from the distribution constants by means of a conformal chart, Plate 17. The analysis thus permits the propagation to be computed by adding the contributions from the separate modes, each determined as above.

49. Brief discussions are given of propagation over an elastic bottom, and of propagation over a stratified bottom. Reference is made to Appendices B and C, respectively, for additional details.

IV. INTRODUCTION TO NORMAL MODE THEORY OF PROPAGATION OF UNDERWATER SOUND OVER HOMOGENEOUS BOTTOM.

A. Statement of the Problem.

50. The problem of the propagation of low frequency sound in water, taking into account the influence of the boundaries on the transmission, may be satisfactorily analyzed in terms of the normal modes of vibration of the acoustic system of the sea between surface and bottom. This is a method of great power, because the individual modes of vibration can be separately considered and their effects combined to give explanations of complex phenomena in terms of relatively simple component factors. The analysis is particularly effective for frequencies at which the wavelengths are comparable to the physical dimensions of the system. At these frequencies other theoretical treatments, such as the method of images, are virtually inapplicable.

51. The use of normal modes - "eigenvalues" or characteristic functions - for the solution of vibration problems has long been a standard method in many fields of physics. A classical illustration from mechanics is the consideration of the normal modes of vibration of a taut string excited by plucking. Although these methods were first applied to acoustics during the last century[1], the application of normal modes to room acoustics has undergone extensive development in the last decade[2]. The attempt to adapt these methods to the treatment of underwater sound transmission has heretofore achieved only partial success, owing to the inadequacy of general formulations in terms of normal impedance. The need for a theory which is valid for a wide range of acoustic conditions at the sea bottom, and free from the limitations inherent in the use of normal impedance, has been demonstrated in the foregoing discussion.

52. It is the purpose of this section to describe such a theory of low frequency sound propagation in shallow water. The method and the results of the analysis are discussed in the text and the mathematical derivations are given in Appendices A, B, C, and D.

53. The acoustic field surrounding a ship-mounted source in shallow water may be treated theoretically by studying the sound pressure field set up by a point source placed at any point in a

1. For example, Rayleigh, Bibliog. 8.

2. Notably by P.M. Morse, R.H. Bolt, and their co-workers. See for example Bibliog. 7, 10, 17 and 18.

non-dissipative homogeneous medium, the water, confined between two infinite plane boundaries. At the upper boundary, the sea surface, the sound pressure is taken to be everywhere zero. The lower boundary, the sea bottom, is visualized for this analysis as the upper surface of a second homogeneous medium extending indefinitely downward. The pressure field in the neighborhood of the source and directly beneath it has been previously discussed (Bibliog. 4).

54. The field equations for the acoustic system described above may be set up and the general solution developed as a summation of terms[3]. Since these terms involve integrals which have thus far not been evaluated, this form of the analysis does not at present lead to solutions useful for the computation of amplitudes or of attenuation. The relations between the constants which govern the pressure field distribution and the acoustic constants of the bottom are, however, derived for the general case.

55. The behavior of the acoustic system may be alternatively derived by extension from the solutions for the propagation of sound in a rectangular tube[4]. These solutions may be obtained more easily than those for the general case described above. The tube is assumed infinite in length, bounded at its two sides by rigid walls, at its top by a free surface, and at its bottom by a homogeneous fluid medium. The replacement of the actual system by the tube with rigid side walls is an artifice which enables the difficulties of the general treatment to be surmounted.

56. The analysis is based upon the assumption that the bottom is a homogeneous medium. The medium may be described as a fluid, since its acoustic behavior is considered to be completely determined by the density and the velocity of sound (compressional wave velocity). This amounts to neglecting the effects of shear waves in the material of the bottom.

57. The analysis may also be applied, at least in principle, to a stratified bottom consisting of a single layer of homogeneous fluid, underlaid by an extended medium of different acoustic properties[5]. Although numerical computations from the solutions would be cumbersome, they could be made in special instances. In practice, however, the effects arising from stratification of the bottom are seldom pronounced.

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3. For details see Appendix D and Addenda.
4. For details see Appendix A.
5. For details see . ppendix C.

58. A complete analysis of propagation over an isotropic solid bottom has not been made, but the modifications required to adapt the propagation theory derived for a homogeneous fluid bottom to the special case of an isotropic solid bottom in which shear waves occur are discussed in Section VII (Reflection Laws For Different Types of Bottom). The discussion is based upon an analysis, given in Appendix B, of the boundary conditions for this case. The principal result is that the shear waves may usually be neglected, in the types of bottom to be expected. The basic acoustic phenomena may be explained, to a close approximation, in terms of the analysis for a fluid medium.

59. Consideration of the propagation of sound in a rectangular tube of the type chosen to represent the acoustic system bounded by the surface and the sea bottom, shows that the simplest class of "transverse" acoustic waves which may be set up will have a uniform distribution of pressure across the tube normal to the rigid walls. In other words the pressure variations will be two-dimensional, with a pattern of nodes and loops between the top and the bottom but not along lines perpendicular to the rigid walls. The waves are "transverse" in the sense that in general the particle displacements have components perpendicular to the axis of the pipe. The waves propagate by alternate reflection at the top and bottom. This is also the class of waves which will be propagated through a medium enclosed between parallel planes such as the sea bounded by surface and bottom.

60. The transmission in the sea will, however, be modified by cylindrical spreading which is not present in the idealized acoustic system, the pipe. Physical considerations indicate that the attenuation due to cylindrical spreading will be independent of that due to absorption at the boundaries, and that the total attenuation may therefore be obtained by simple addition. With this modification, the solutions for the pipe may be employed to give the transmission in the actual acoustic system[6].

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6. A comparable problem of sound transmission in pipes, with the boundary conditions expressed in terms of impedance, has been treated by P. M. Morse (Bibliog. 18). "Transverse" acoustic waves in rigid tubes, analogous to the electro-magnetic oscillations in hollow metal tubes and dielectric columns or "wave guides", have been treated by H. E. Hartig and C. E. Swanson (Bibliog. 19). Hartig's derivations, following Rayleigh (Bibliog. 8), were restricted to propagation in tubes with rigid walls.

B. Outline of Derivations.

61. The actual derivations for the propagation of sound in a rectangular tube bounded by two rigid walls, a free top surface, and a homogeneous fluid bottom, are presented in Appendix A. The solution of the wave equation is obtained for the pressure distribution in a single mode of the simplest class of "transverse" waves. The solution represents the product of (a) a progressive wave, damped exponentially as it travels down the pipe parallel to the long axis, and (b) a standing wave system between the top and the bottom of the pipe. The progressive wave has a complex propagation constant $\sigma + j\tau$. The real part σ determines the damping along the pipe, and the imaginary part determines the phase velocity of the wave c/τ. The latter is always larger than c, the velocity of sound in the "free" medium. The phase velocity may be defined as the frequency times the distance between wave crests. The standing wave system between surface and bottom consists of a pattern of nodes and loops given by the complex distribution constant $\kappa - j\mu$.

62. It is found that the propagation constant is related to the distribution constant by the equation[7]

$$(\sigma + j\tau)^2 + \frac{1}{\eta_1^2}(\kappa - j\mu)^2 + 1 = 0 \tag{5}$$

where $\eta_1 = 2h/\lambda$ is the water depth measured in half wavelengths. This equation has the same form regardless of how the boundary conditions are evaluated. The same equation was obtained from the impedance theory (Bibliog. 10). The phase velocity and the damping can be found for any value of η_1, once κ and μ are known, by computation from equation (5) or by consulting a conformal chart such as that reproduced in Plate 17.

63. Assuming a pressure distribution in the bottom represented by an exponential decay both along the axis of the pipe and in the direction normal to the bottom, a relation similar to that of equation (5) is found between the propagation constant and distribution constant for the second medium.

64. A relation (Equation (7)) between the distribution and propagation constants of the second medium and those of the first

7. The numbers of all equations are those used in the Appendices.

medium is then obtained by the application of the conditions that the pressure and the normal particle velocity must be continuous at the lower boundary. One result of this is an equation (16) which relates the propagation constants of the two media. A further and most important result, after applying this relation and making appropriate transformations, is

$$\frac{\tanh^2 \pi(\kappa - j\mu)}{(\kappa - j\mu)^2} = \frac{A^2}{(\kappa - j\mu)^2 - B^2} \tag{20}$$

in which

$$A = \rho_2/\rho_1 \quad \text{and} \quad B = (\eta_2^2 - \eta_1^2)^{1/2} = \frac{2fh}{c_1}\left[(c_1/c_2)^2 - 1\right]^{1/2} \tag{21}$$

65. Comparison of equation (20) with the corresponding relation (Equation (135)) derived in Appendix D shows that the two expressions are identical. The validity of the pipe artifice employed for the analysis in Appendix A is confirmed by this agreement, since the general theory for the pressure field surrounding a point source between two infinite planes is shown to result in the same relation between the distribution constants as the special theory restricted to propagation in a pipe.

66. By means of equations (5), (7), (16), and (20) in Appendix A, the distribution and propagation constants for any mode belonging to the simplest type of "transverse" waves in the first medium (density ρ_1 and velocity c_1) may be determined in terms of the acoustic properties of the bottom (density ρ_2 and velocity c_2), and the geometrical conditions (η_1 , the depth of the water, measured in half wavelengths). The effective density and velocity of sound of the material of the bottom may be estimated with sufficient accuracy from hydrographic data, sampling, and acoustic measurements of $\rho_2 c_2$, as discussed in Section IX (The Use of Acoustic Measurements as Transmission Criteria).

67. For convenience in the interpretation of the solution for the distribution constants in terms of the acoustic properties of the bottom, the charts shown in Plates 12, 13, 14, 15, and 16 have been prepared. These charts were derived from equation (20) by computing and plotting the values of A and B which correspond to various assumed values of κ and μ . Plates 13 and 14, for values of μ between 0 and 1, and Plates 15 and 16, for values of μ between 1 and 2, correspond to the solutions for the first and second modes, respectively. An assembly of Plates 13-16, on a reduced scale is shown in Plate 12.

68. Once the components of the distribution constant are determined from these charts, the damping and the phase velocity for the particular mode may be obtained from another chart, Plate 17, which gives σ and τ in terms of κ and μ. This chart is identical with that reproduced in Bibliog. 10, p 315, and corresponds to the relations given by equation (5). Plate 17 gives solutions for a wide range of values of μ, and consequently for several lower modes.

69. The final pressure distribution may be built up by adding the contributions of the various modes, each determined by the method outlined above.

NOTE: Since the foregoing was written it has been found possible to evaluate the general pressure integrals in Appendix D in terms of Hankel functions. The general treatment is thus completed, and it is now possible to compute, in terms of the normal modes, the pressure at any point in the acoustic field surrounding a point source between two infinite flat plates. The mathematical key to the integrations was obtained from Report No. 65, recently received from the Minesweeping Section of the Bureau of Ships. This is a theoretical study by G.M. Roe, entitled "The Propagation of Sound in Shallow Water".

After evaluation of the integrals, the normal mode solutions in Appendix D, may be shown to be identical with the solutions for pressure obtained by a different method in Report No. 65. Details of the integrations will be found in the Addenda to the present report.

The treatment in Report No. 65 makes use of a generalized bottom impedance, which is allowed to vary with the angle of incidence. Although the method is different from that employed in Section IV above, the final result is similar in that the behavior of the bottom is not characterized by normal impedance alone, but by a more complex function which includes additional acoustic information.

The generalized impedance would appear to be a mathematical abstraction which is not useful for computation. If, however, the plane wave reflection law involving densities and velocities as given in equation (48) be inserted into equation (34) of Roe's report, and appropriate transformations be carried out, the same transcendental equation as that from which the charts were computed is obtained. The charts in Plates 12-16, representing plots of this equation, therefore provide a simple means of obtaining numerical results, whether the pressure equations are derived directly in terms of normal modes or by combining elementary plane waves.

V. THE CHARACTER OF THE SOLUTION

SUMMARY

Character of the Solution

70. The character of the sound propagation predicted by the theoretical analysis is discussed. Interpretation of the charts (Plates 12-16 inc) shows that for any one mode the transmission may be classified as either "guided", "damped", or "hybrid", depending upon the acoustic constants of the bottom. "Guided" transmission, with very low attenuation and no bottom absorption, should occur over hard bottom at frequencies higher than a critical frequency. This type was observed experimentally at the Potomac River Mouth. "Damped" transmission, with considerable damping due to bottom absorption, should occur over soft bottom at all frequencies. This type was observed experimentally at the Potomac River Bridge. "Hybrid" transmission, also with considerable damping, should occur over hard bottom at frequencies below the critical frequency. This type was recently observed experimentally in the Rappahannock River area. The critical frequency is slightly higher than that for which the water depth is a quarter wavelength.

71. A physical picture of the sound pressure fields which correspond to "guided" and "damped" transmission in the first and second modes is given in the form of isobar plots which show the distribution of the lines of constant pressure between surface and bottom and along the direction of propagation.

72. In a discussion of the relative attenuation of the various modes it is shown that the rate of damping with increasing distance from the source will be greater the higher the order of the mode. As a result, the higher modes tend to disappear more rapidly than the lower ones. At distances greater than a few times the water depth, most of the energy is carried in the first one or two modes. The vertical standing wave pattern between surface and bottom becomes progressively simpler as higher modes disappear. Although the relative degree of initial stimulation of the modes depends upon the nature and location of the source, the lowest modes are usually the most strongly stimulated as well as the least damped. A descriptive explanation of the observed transmission phenomena is given in terms of the relative stimulation and damping of the normal modes.

73. An interesting analogy is presented between sound transmission in the acoustic system of the sea between surface and bottom, and electromagnetic wave transmission in "wave guides".

V. THE CHARACTER OF THE SOLUTION.

A. Three Types of Transmission, Illustrated by the First and Second Modes.

74. The transmission of underwater sound in the first and second modes will depend upon the acoustic properties of the bottom in accordance with the distribution and propagation constants plotted in Plates 12, 13, 14, 15, 16, and 17. Physically, the distribution constant $\kappa - j\mu$ determines the character of the standing wave pattern between the surface and the sea bottom. The real part κ is a measure of the damping of the pattern, and the imaginary part μ determines the distribution of nodes and loops between the boundaries. Small values of μ (between 0 and 1) are associated with the first mode, and correspond to the simplest distributions. Larger values correspond to higher modes and to more complicated distributions. For all modes of vibration of the acoustic system the surface is a pressure node; and the bottom a node if "soft", and an approximate anti-node if "hard". The vertical pattern of sound pressure for the first mode is the simplest which will fit the boundary conditions, i.e. a "half-wave" loop above a "soft" bottom, with nodes at surface and bottom; and a "quarter-wave" pattern above a "hard" bottom, with a node at the surface and an anti-node at the bottom. The distribution patterns for higher modes will be discussed later.

75. The chart for each mode, in the assembly Plate 12, shows three types of solutions of equation (20). These solutions correspond to different acoustic conditions:

(a) Values of B imaginary and greater than a critical value ($\frac{1}{2}$ for the first mode and $1\frac{1}{2}$ for the second mode). This corresponds to sound velocities in the bottom greater than in the water ($c_1 < c_2$), and to exciting frequencies greater than a critical or "cut-off" frequency.

(b) Positive values of B, corresponding to lower sound velocity in the bottom than in the water ($c_1 > c_2$).

(c) Values of B imaginary and less than the critical values stated in (a). This corresponds to sound velocities in the bottom greater than in the water ($c_1 < c_2$), and to exciting frequencies lower than the critical frequency.

76. The three portions of each chart correspond to three types of sound propagation in each mode. These may be classified by their physical characteristics as follows:

(a) "Guided" transmission, associated with "hard" bottom.

(b) "Damped" transmission, associated with "soft" bottom.

(c) "Hybrid" transmission, associated with "hard" bottom, at very low frequencies.

The reasons for this nomenclature will be clear after consideration of the physical character of the waves whose distribution constants are represented by the various portions of the charts.

77. In "guided" transmission, κ, the real part of the distribution constant, is zero for the allowed values of (between $\frac{1}{2}$ and 1 for the first mode). The standing wave patterns between surface and bottom therefore have zero damping, and no absorption at the boundaries. If κ is zero, σ, the real part of the propagation constant, must also be zero. This is shown in Appendix A, Part 2. Under these conditions the progressive wave is propagated with zero absorption at the boundaries. This means that the cylindrical spreading from the source, amounting to 3 db per distance double, is the sole cause of attenuation. Transmission takes place by means of "guided" waves, confined between the surface and the bottom, with total internal reflection at the lower boundary. This type of transmission is illustrated by curves (a) Plate 9, for the experimental results over hard bottom at the Potomac River Mouth.

78. "Guided" transmission can occur only if the real part of the distribution constant is equal to zero. The sections of the charts which correspond to this type of transmission are derived from the solutions of equation (15) for which κ is equal to zero. Such solutions exist only for imaginary values of B, greater than certain critical values. From the definition of B (equation (21)) it may be seen that this corresponds to exciting frequencies higher than the critical frequency for the first mode given by

$$f_c = \frac{c_1}{4h}\sqrt{\frac{1}{1-(c_1/c_2)^2}} \qquad (31)$$

79. "Guided" transmission will occur if the velocity in the bottom is greater than that in the water and if the source frequency is higher than the critical frequency, f_c determined primarily by the water depth as compared to the wavelength. This type of transmission cannot occur at frequencies below the first mode critical frequency given by equation (31). It is clear from this equation that, if $c_2 \gg c_1$, the critical frequency for the first mode will be that for which the water depth is a quarter wavelength. For example the critical frequency may be 8 cps if the depth is 150 ft, or 24 cps if the depth is 50 ft. If c_2 is only slightly larger than c_1 the critical frequencies will be substantially higher than those given by the quarter wavelength rule. The relation between the critical frequency for any mode and the critical angle for total internal reflection is discussed in Section VII-B. The equation for the critical frequency of any mode is $(2n - 1)f_c$, where n is the order of the mode and f_c is given above.

80. "Damped" transmission is represented by those portions of the charts (Plate 12) for which the real part of the distribution constant, κ, is not zero, and for which the velocity of sound in the bottom is less than that in water. The standing wave patterns between surface and bottom are damped, owing to absorption at the lower boundary. Under these conditions transmission takes place by means of a progressive wave train proceeding with phase velocity c/τ, and with damping determined by the value of σ associated with each value of κ. The attenuation with distance is made up of cylindrical spreading (pressure varying as $1/\sqrt{r}$, decreasing 3 db per distance double) plus absorption at the bottom (pressure amplitude decreasing in accordance with $\epsilon^{-2\pi\sigma r}$, or 54.6 σ db per wavelength.

81. "Damped" transmission occurs over "soft" bottom, since the acoustic condition required is that the velocity of sound in the bottom be less than that in the water (B positive, $c_1 > c_2$). This type of transmission is illustrated by curves (b) and (c) of Plate 9, for the experimental results over soft bottom at the Potomac River Bridge.

82. "Hybrid" transmission is represented by those portions of the charts which lie between the sections identified with "guided" and "damped" transmission. The characteristics of "hybrid" transmission are mixed. In general the wave patterns are highly damped ($K>0$), and they occur only over hard bottom ($c_1 < c_2$, B imaginary and less than $\frac{1}{2}$ for the first mode and less than $1\frac{1}{2}$ for the second mode) at frequencies lower than the critical frequency of that particular mode for "guided" transmission. The experimental study of "hybrid" transmission is difficult since

very low excitation frequencies are required. A series of range runs which demonstrate the transition of "guided" transmission into "hybrid" transmission as the frequency is decreased below the "cut-off" for the first mode, was obtained recently over hard bottom in the Rappahannock River area. Some of these records will be reproduced in a later report.

83. The previous discussion has shown that one of three types of transmission, "guided", "damped", or "hybrid", may occur in the propagation of underwater sound, corresponding to the simple distribution patterns which characterize the first and second modes. Additional consideration of the transcendental equation (20), the roots of which are shown on the charts (Plate 12), indicates that the roots are cyclic. If curves are plotted for successively larger values of the complex distribution constant $\kappa - j\mu$, they will cross over the curves previously plotted and again cover the plane of the chart. The equation thus actually defines a multiplicity of charts, resembling a family of Riemann surfaces.

84. The physical significance of the charts is that each "sheet" corresponds to one mode. In order completely to describe the characteristics of the system, an infinite number of such charts must be plotted. Fortunately, for reasons which will be discussed later, in most instances only the lowest modes need be considered.

85. For any one mode, the propagation can be classified as corresponding to one or the other of the three types of transmission. For any given depth, the type of transmission will be determined by the exciting frequency and the velocity of sound in the bottom, which together fix the value of B on the chart. The ordinate A is the specific gravity of the material of the bottom.

86. The chart for the second mode is similar in general form to that for the first mode. Plate 12 gives the assembly, and Plates 13-16 the detailed charts. The following differences between transmission in the first and second modes are indicated by the charts:

(a) the critical frequency for the second mode, corresponding to $\mu = 1\frac{1}{2}$, $B = j1\frac{1}{2}$, is three times the critical frequency for the first mode (equation (31));

(b) the damping constant for "damped" and "hybrid" transmission, primarily a function of κ , is about twice as great for the second mode as for the first, and

(c) the portion which represents "hybrid" transmission is relatively more extensive in the chart for the second mode than in that for the first.

The range of frequencies, density ratios and velocity ratios for which "hybrid" transmission can occur is therefore greater for the second mode than for the first. Indeed, this range increases with the order of the mode. In the transmission over hard bottom of frequencies below the critical frequency for the first mode, the energy may be carried by several lower modes in "hybrid" form, and these will in general be highly damped.

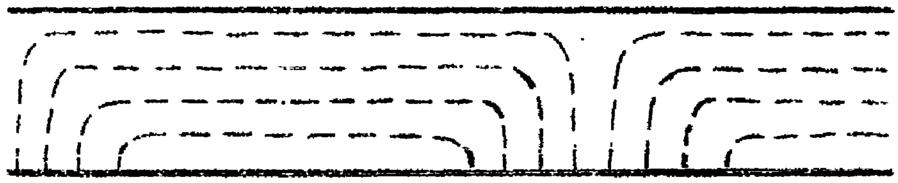
87. The distribution constants which correspond to the higher modes may be worked out in detail from equation (20). Values of μ between 2 and 3 correspond to the third mode, values of μ between 3 and 4 to the fourth mode, etc. The chart for each mode will have the same general form as those reproduced in Plate 12, with sections corresponding to the three types of transmission. "Guided" transmission, without damping by bottom absorption, will occur over hard bottom, for each mode, at frequencies above the critical frequency for that mode. The critical frequency increases with the order of the mode in accordance with the ratios 1:5:7: etc. "Damped" and "hybrid" transmission by means of each of the higher modes will be characterized by a damping constant σ governed by the value of κ from the appropriate chart. The damping constant will increase with the order of the mode.

B. A Physical Picture of the Sound Pressure Fields.

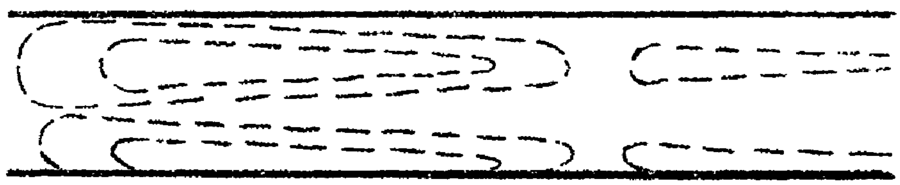
88. A physical picture of the sound pressure fields which correspond to "damped" and "guided" transmission in the first and second modes may be obtained from Fig. 1, (a), (b), (c), and (d).

89. The figure shows the distribution of the lines of constant sound pressure, or isobars, between the surface and bottom and along the direction of propagation of the waves. The representation in each sketch is for a single instant of time. Actually the entire distribution progresses at the group velocity, in the direction of propagation. The group velocity, at which the energy is transferred, is always smaller than the velocity of sound in the free medium (the water). In fact, the latter is equal to the geometric mean of the group velocity and the phase velocity of the waves.

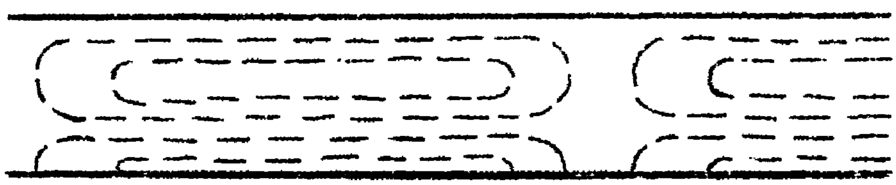
(a) First mode - soft bottom



(b) First mode - hard bottom



(c) Second mode - soft bottom



(d) Second mode - hard bottom

Figure 1

90. As shown by the sketches, the patterns repeat at intervals of a half wavelength, computed on the basis of the phase velocity, c/τ . Since the phase velocity increases as frequency decreases, all the patterns lengthen as the frequency is lowered. At the critical frequency of the first mode, over hard bottom, the phase velocity approaches the velocity in the bottom, and the group velocity becomes c_1/c_2. For the case of rigid bottom (c_2 infinite), the isobars become straight lines parallel to surface and bottom. For this limiting condition and at lower frequencies no wave of this

type is propagated. The sketches indicate the greater complexity of the pressure patterns for the second mode than for the first, as well as the relative damping rates for the two modes over soft bottom.

91. The isobar plots for the pressure field distribution corresponding to higher modes will be similar to those shown in Fig. 1, except that the number of loops between surface and bottom will increase with the order of the mode.

C. Relative Damping of the Modes.

92. It is necessary to consider the distribution of the sound energy among the normal modes of the system, since the actual sound pressure at any point is the sum of the pressures associated with each of the modes present. The partitioning of energy in the acoustic system under consideration is closely analogous to the distribution of vibrational energy among the normal modes of a plucked string. The overall decay of the sound energy with distance is determined by a summation of the decay rates of the individual modes. From basic considerations it may be shown that the higher modes have more rapid decay rates than the lower modes.

93. The lowest mode corresponds to a wave traveling nearly tangentially to the boundaries, with comparatively few reflections. The higher modes correspond to waves traveling with relatively frequent reflections between surface and bottom. The higher the order of the mode, the more nearly the angle of incidence of the corresponding waves approaches normal incidence. When sound is propagated a considerable distance in shallow water, that part of the total energy which has been reflected the largest number of times in transit will be carried by the highest mode present, and the energy which travels at nearest grazing incidence will be carried by the lowest mode.

94. It is evident that the rate of attenuation with increasing distance will be greater the higher the order of the mode. For example, the assembled charts on Plate 12 show that the damping associated with the second mode is very much greater than that expected for the first mode, for transmission over soft bottom. Because of their higher decay rates, the higher modes tend to disappear more rapidly than the lower ones as the distance from the source increases. The propagation of low frequency sound in shallow water usually takes place under conditions which ensure that, at distances greater than a few times the water depth, most of the energy is carried in the first one or two modes. When total internal reflection takes place in the propagation of such frequencies over hard bottom, either the lowest or the first two modes may travel with zero attenuation due to the boundaries, while the higher modes may be very highly damped.

95. As the higher modes disappear with increasing distance from the source, the vertical standing wave pattern of sound pressure between surface and bottom should become progressively simpler. The vertical pattern, determined by hydrophone soundings, shows the maximum number of nodes and loops directly beneath the source, where the maximum number of modes are present. These patterns, for various acoustic conditions, are discussed in detail in a recent report (Bibliog. 4). There is again a close analogy between this pattern and the normal modes of a vibrating string. The first allowed mode of the string consists of a single loop between the suspension points, and higher modes appear as more complex distributions of loops and nodes between the supports. Similarly the sound pressure distribution beneath a ship-mounted source may be a complicated sequence of nodes and loops, in which the complexity of the pattern serves as an index of the number and strength of the modes present. It is evident, then, that if hydrophone soundings are made at progressively greater distances from the source, the vertical pattern should become simpler as the higher modes are damped out.

96. It is shown in later sections (Sections VI, VIII, and IX) that the experimental records may be interpreted in terms of the expected relationships between the modes. The relatively higher damping rates of higher modes, and the progressive simplification of the vertical pressure distributions with increasing distance from the source, are confirmed by experiment.

D. Initial Stimulation of the Modes.

97. The initial stimulation of each mode, that is, the relative amount of the total energy which is carried by it at the origin, is determined by the nature and location of the source. Consider, for example, the analogous system of the vibrating string. The lowest mode of a string stretched between rigid supports is of course a single loop with nodes at the ends. Maximum stimulation of the first mode will occur if the string is bowed or otherwise excited at a point midway between the supports. If the string is bowed at any other point, except precisely at the points of support, the first mode will be stimulated to some extent, although not so strongly as when bowed at the center. The second allowed mode of the string is two loops with a nodal point midway between the supports. This second mode will receive maximum stimulation if the exciting force is applied at a point one quarter of the string length from either support. Although bowing the string at the center (or at the end points) will not stimulate the second mode, bowing at any other point will stimulate it to some extent.

98. Similar considerations apply to the stimulation of the modes of vibration of the actual acoustic system, although the boundary conditions are different from those for the string. The analogy of the vibrating string indicates that in the acoustic system of the sea between surface and bottom: (a) the first mode will always be stimulated, and the degree of stimulation will not be a critical function of the source depth (in wavelengths) or of the character of the bottom; and (b) any given higher mode may or may not be effectively stimulated, depending upon the exciting frequency and upon whether or not the sound source is located at a nodal point for that mode. The location of the nodal points, determined by the order of the mode and by the boundary conditions, may be computed in desired cases and may be demonstrated experimentally by hydrophone soundings.

99. Since the nodal points for the various higher modes will seldom coincide with the location of the source, most of the modes will be stimulated to some extent by a given sound source at any depth. Although there exist an infinite number of modes of the acoustic system, the amount of energy carried by each mode in general decreases rapidly with its order, and most of the energy is carried by the lowest modes. That this is true of the acoustic system of the sea, in common with most vibrating systems, may be seen from the general mathematical treatment in Appendix D.

100. The relative degree of stimulation of the various modes is also a function of the frequency of the sound source. Referring again to the mechanical analogue, if the vibrating string is excited at various frequencies by a periodic force, that mode whose "natural frequency" corresponds to the frequency of excitation will be strongly stimulated, and modes whose "natural frequencies" differ from this frequency will be less strongly stimulated. The "natural frequency" of a mode in the acoustic system of the sea is that frequency for which the half wavelength (in free water) is approximately equal to the distance between pressure maxima (or minima) in its vertical standing wave system. It is therefore the variation of sound pressure with time which most nearly corresponds to the desired pressure distribution in space.

101. In the actual acoustic system the sound pressure at a given point will not necessarily be greatest under conditions of "resonance" with a given mode, but the proportion of the total energy carried by that mode will tend to be a maximum. The concepts of "resonance" and "natural frequency" are not strictly applicable to the normal modes of an acoustic system excited inter-

nally, and it should be kept in mind that the modes correspond primarily to spacial rather than to temporal distributions of sound pressure.

102. A descriptive explanation may now be given for the underwater sound pressure phenomena to be expected in the vicinity of the source. It has been shown that: (a) close to the source all the normal modes will be stimulated in greater or lesser degree, and the amount of total radiated energy carried by a given mode will, in general, decrease with the order of the mode; and (b) the rate of attenuation of each mode with increasing distance from the source will, in general, be greater the higher the order of the mode. The effects of spreading of the sound waves from the source must be added to the distributions governed by the normal modes.

103. If the resultant pressure field be probed by moving a hydrophone horizontally outward from the source, the following effects should occur:

(a) In the immediate vicinity of the source the primary factor in the pressure variation should be geometric spreading, with the influence of the boundaries playing a minor role.

(b) At distances greater than that between the source and the surface or bottom, a further sharp decrease in resultant pressure with distance from the source should occur, caused by the combination of wave spreading and the rapid decay of the higher normal modes. The influence of the modes, representing the effects of reflection from the bounding surfaces, becomes increasingly important in this region.

(c) At yet greater distances from the source, of the order of several times the water depth, the rate of attenuation with distance should become much smaller, corresponding to cylindrical wave spreading combined with the relatively gradual decay of the remaining low order modes. At moderately low frequencies the first and second modes usually persist to considerable distances.

104. The general character of the observed transmission, shown on the experimental curves of Plates 9 and 10, is in accord with these predictions. A more detailed discussion of the attenuation to be expected under various acoustic conditions will be found in a later section (Section VIII, Attenuation).

E. Acoustic Analog of Electromagnetic Wave Guides.

105. In the foregoing theoretical and experimental analysis, it has been shown that under certain physically realizable conditions the propagation of low frequency sound in shallow water over a hard bottom may be of the type encountered in the electromagnetic "wave guide", i.e. the waves may be guided or channelled between two bounding surfaces without loss from absorption at the boundaries. The analogy between the acoustic and the electromagnetic "wave guide" systems is very close, if the latter be visualized as a dielectric medium bounded at the top by a perfectly conducting surface having infinite dielectric constant, and at the bottom by a second dielectric medium having higher velocity than the first. Density and compressibility of the media in the acoustic system are analogous to permeability and dielectric constant in the electromagnetic system. Viscous losses in the media of the acoustic system are analogous to dielectric losses in the media of the electromagnetic system. Similar relations exist between pressure, particle velocities, phase velocities, attenuation, critical frequencies, and other constants of the two systems.[8]

106. In brief, the analysis of propagation of electromagnetic waves in hollow guides, tubes, or pipes shows that these waves may be divided into several classes or "modes", the distinctions being made on the basis of the configuration of the electric and magnetic fields within the guide. The lowest modes correspond to the simplest field configurations, and higher modes to increasingly more complex field distributions. The "cut-off" phenomenon is encountered in "wave guides", and under ideal conditions the guide may be completely "transparent" to a given mode at frequencies higher than a critical frequency, and completely "opaque" to the same mode at lower frequencies.

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8. Although special cases of "guided" transmission in both acoustic and electromagnetic systems were treated by Rayleigh many years ago, attention was first directed to the specific analogies between them by L. Brillouin (Bibliog. 21). For discussions of the electromagnetic case, reference is made to the basic studies of G. C. Southworth (Bibliog 22), J. R. Carson, S. P. Mead, and S. A. Schelkunoff (Bibliog. 23), and L. Page and N. I. Adams (Bibliog. 24).

107. The physical process involved in the transmission of waves through a guide is difficult to visualize from a description of the field configurations within the guide. A physical picture of this type of transmission has been suggested by Page and Adams (Bibliog. 24) in which the "guided" waves are shown to be analyzable into two sets of "elementary plane waves", traveling at an angle with the axis of the tube, and propagated down the tube by alternate reflections at the walls of the guide as shown in the accompanying figure.

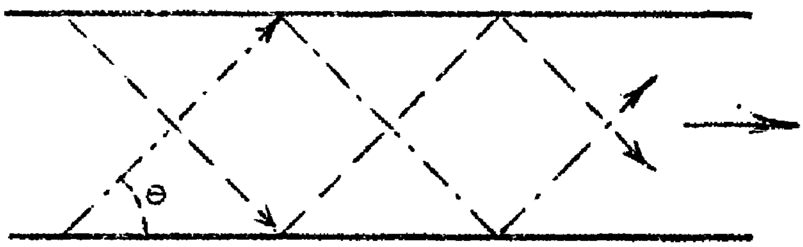

Figure 2.

108. Although this picture of "guided" transmission is open to certain objections discussed by Schelkunoff (Bibliog 25) the mathematical and physical simplicity of the conception recommends it. For example the relation between the critical angle and the critical frequency for propagation in the analogous acoustic system can be clarified from this standpoint.[9]

109. Some differences exist between sound propagation in the acoustic system of the sea between surface and bottom, and the propagation of electromagnetic waves in "guides". The most important difference is that, in the development of the acoustic theory, provision for various values of density of the bottom material must be made. Density in the acoustic system corresponds to permeability in the analogous dielectric system. Since dielectric media with permeability different from unity do not exist, the exact electromagnetic analog of the acoustic system is not physically realizable.

110. Additional differences between the acoustic system under discussion and the usual wave guides are: (a) that the acoustic system is non-symmetrical in the sense that the two bounding surfaces have in general different acoustic properties, (b) that types of waves may be propagated in the electromagnetic system which have no acoustic analog, and (c) that the acoustic system is not closely analogous to the types of wave guides of most practical importance, those with conducting metal walls.

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9. (See Section VI, and Appendix A, Part 4).

111. The phenomena which occur in hollow metal guides are in many ways simpler than those to be expected in the dielectric guide which is the actual analog of the acoustic system.

112. Study of the literature on electromagnetic wave propagation also reveals an interesting optical analogy with the "hybrid" sound transmission discussed in Section V-A above. T. C. Fry (Bibliog. 20) has used the term "hybrid" to denote the type of plane waves which occur in nature on the dark side of a prism within which "elementary plane waves" are being subjected to total internal reflection. S. A. Schelkunoff (Bibliog. 25) has suggested that the term may logically be extended to include the electric field distribution which exists in a dissipative wave guide, or in a dissipationless wave guide at frequencies below the critical frequency. The parallel is almost exact between the latter use of the term "hybrid" and its use in this report to describe transmission of sound over a hard bottom at frequencies below the critical frequency.

VI. INTERPRETATION OF RANGE RECORDS IN TERMS OF THE MODES.

SUMMARY

Interpretation of Range Records

113. Since the modes travel along different effective "paths", and hence with different phase velocities, it is to be expected that interference phenomena will occur when low frequency sound is propagated in water. This type of interference between the various modes is graphically illustrated by the "interaction loops" which appear on most of the experimental records.

114. Excellent quantitative agreement is obtained between observed and computed spacings for interaction between the lower modes, particularly between the first and the second. The observed spacings cannot be explained by interference between sound "beams" or "rays" from a succession of virtual sources.

115. Considerable information about the number and relative importance of the modes present in any given case may be obtained from the interaction spacings on the experimental records. Various characteristics of the modes are illustrated by these spacings, for example (a) the relatively higher damping rates for the higher than for the lower modes, (b) the critical frequencies of the first three modes over hard bottom, and (c) the occurrence of "hybrid" transmission.

VI. INTERPRETATION OF RANGE RECORDS IN TERMS OF THE MODES.

A. Interaction Loops. - Computation of Spacings.

116. The propagation of low frequency sound in shallow water has been resolved by this analysis into the transmission of a series of distinct modes, which may be considered separately or may be added together to give the actual sound pressure at any point in the acoustic system. The primary advantage of this analysis is that it makes possible the explanation of extremely complex phenomena in terms of the interactions and combinations of relatively simple components. This is illustrated by the interpretation of the "interaction loops" graphically shown on most of the range records on Plates 2-8.

117. The simultaneous existence of two or more distinct modes traveling through the same system but along different effective individual "paths" and hence with different phase velocities, suggests that interference phenomena may be expected. The pressure level for each mode, considered by itself, is in general smoothly attenuated with distance from the source. The resultant sound level from two co-existent modes, traveling with different phase velocities, should show maxima and minima as the pressure due to the two modes combine alternately in phase and out of phase with each other.

118. Consider for example the spacing of the minima to be expected from the interaction of the first and second modes. At low audio frequencies this should be the most common and the most persistent spacing, since the lowest modes in general carry most of the sound energy. Minima can be recorded only at points at which the waves add in opposite phase independent of the time, because the hydrophone used to probe the field measures only rms values of pressure. Obviously the spacing between successive minima will be the distance required for the "faster" of the two modes to gain a phase wavelength on the "slower" mode. By "phase wavelength" is meant the wavelength, λ/τ, which is associated with the phase velocity, c/τ. These constants for the acoustic system between boundaries are always larger than λ and c, the wavelength and the velocity of sound in an infinitely extended body of water.

119. If the phase wavelength of the faster mode is c/τ_1, and that of the slower mode is c/τ_2, the distance S between minima will be given by the relation

$$S = n \frac{\lambda}{\tau_1} = (n+1) \frac{\lambda}{\tau_2}$$

From one minimum to the next there will be n phase wavelengths of the "faster" wave, and n + 1 phase wavelengths of the "slower" wave. The above two forms of the expression for S may easily be combined, with elimination of n, to give

$$S = \frac{\lambda}{\tau_2 - \tau_1} \qquad \tau_2 > \tau_1$$

The "interaction spacing" between successive minima characteristic of the interference between any two modes may be computed from this formula, once the appropriate values of phase constant τ are known from measurements of the acoustic properties of the bottom (e.g. from the results of hydrophone soundings combined with the use of the charts, Plates 13-17 inc).

120. The interaction spacings corresponding to the first two modes appear prominently on the records reproduced in Plates 2 and 3. Observed and computed values of these spacings for the frequencies of the records are shown in Table III below, together with the corresponding values of τ_1 and τ_2. The latter were computed from the charts, Plates 13, 14, and 17, using an assumed density ρ_2 equal to 1.2, and a velocity ratio c_1/c_2 equal to 0.291, corresponding to $\rho_2 c_2/\rho_1 c_1$ equal to 0.35. This was obtained from hydrophone soundings (Bibliog. 4).

TABLE III.

Observed and Computed Interaction Spacings For Interference Between the First and Second Modes.

Frequency	τ_2	τ_1	$S = \frac{\lambda}{\tau_2 - \tau_1}$	Observed Spacing
93 cps	0.882	0.355	98 ft	110 ft
100	0.897	0.500	120	120
110	0.918	0.510	142	150
135	0.945	0.755	188	200
186	0.972	0.885	297	300

The agreement between observed and computed spacings is obviously within the accuracy of the distance scale on the records.

121. A pronounced variation in spacing with frequency is indicated by the table. The reason for this is that the phase velocity for each mode decreases as frequency increases and tends to approach asymptotically to the velocity of sound in the extended medium. The closer the approach of the two phase velocities to

each other, the larger the number of phase wavelengths which will be required between successive cancellations, and the larger the value of the spacing S.

122. In addition to the spacings given by the interaction of the first and second modes, other spacings may be observed close to the central peak on the records for 135 and 186 cps on Plate 3, and on the records for 200-400 cps on Plate 4. These correspond to interactions between higher modes, notably between the first and third, and between the second and third. A detailed check between computed and observed spacings is hardly to be expected at frequencies higher than 200 cps, owing to the complicated interactions which occur when several modes are simultaneously present. The component spacings for the higher frequencies could doubtless be extracted from the records by an elaborate statistical analysis, but this does not seem worthwhile for present purposes.

B. Interpretation of Records.

123. The relatively high rate of attenuation with increasing distance from the source, which has been shown to be characteristic of the higher modes, is clearly evident from the distribution of the interaction spacings on all the higher frequency records. This is shown by smaller spacings close to the source which fade out with increasing distance. These are visible on all the records for frequencies higher than 135 cps. The relatively greater persistence of the lower modes is indicated by the simplification of the loop pattern with increasing distance from the source.

124. The records reproduced in Plates 2 and 3 graphically illustrate the decrease in the damping of the second mode as the source frequency increases from 70 cps to 186 cps. The bottom in this location, the Potomac River Bridge, is acoustically "soft" and the "natural frequencies" of the first and second modes are 45 cps, and 90 cps, respectively. For an exciting frequency of 70 cps all the energy is carried by the first mode, and the pressure level decreases uniformly after the initial drop. At 80 cps the second mode is stimulated, but is very rapidly damped out. At distances greater than about 100 ft the energy is carried by the first mode. At 93 cps, however, the stimulation of the second mode is sufficient, and its damping low enough, to cause well developed interaction spacings out to 500 ft from the source. At 100 cps, and progressively at the higher frequencies, these

spacings become better developed and persist to greater and greater distances. The spacings shown by range run records may thus be made to yield considerable information about the number and relative importance of the modes present in any given case.

125. The occurrence of pressure maxima and minima from the interaction of the modes may be demonstrated even when a complex underwater sound source is employed. For example, the records reproduced in Plate 8 were made at the Potomac River Bridge, employing both the parallel pipe device towed by USS ACCENTOR, and the ship noise produced by USS AQUAMARINE. The loops characteristic of interference between the first and second modes are clearly observable on the records of noise from the parallel pipes, when recorded through a 5 cycle band filter. The spacings, 260 ft at 150 cps and 120 ft at 100 cps, are in exact agreement with those determined from the records for the single frequency source presented in Table III above. The loops are observable although not pronounced in the record of ship noise analyzed in a 50 cycle band centered at 140 cps. The loops do not appear in the broad record (200 cycles wide) of ship noise, on account of the averaging effect of the many superposed frequencies.

126. These records indicate that the interaction loops may be expected to appear whenever either the noise source or the receiving equipment has an effective band width of less than 50 cps. It is probable, for example, that a resonant receiving unit such as the German acoustic mine would be responsive to the interaction loops produced by ship noise or by the complex sources used for minesweeping. If the band width of either source or receiver is only a few cycles, the interaction loops will be a prominent feature of the records of sound transmission.

127. The loops which occur on the records taken over hard bottom at the Potomac River Mouth illustrate the relations of the modes for "guided" and "hybrid" transmission. In this location the critical frequencies of the first three modes are about 30 cps, 90 cps, and 150 cps, respectively. The records on Plates 5, 6, and 7 show that the interaction between the first and second modes is characterized by loops with a spacing of 230-300 ft. These are prominent on the records for 90, 100, 110, and 135 cps, and persist to distances of at least 4000 ft (Plate 7). This spacing does not occur at frequencies lower than 90 cps, presumably because these frequencies are below the critical frequency for the second mode. Similarly the record for 200 cps shows a spacing of 150 ft, which does not occur at lower frequen-

cies. This spacing, probably representing interaction between the first and third modes, is shown only by the 200 cps record, since the critical frequency for the third mode occurs at about 150 cps.

128. The critical frequency for the first mode in this location (about 30 cps) could not be reached with the available sound sources. Hence all the records taken at the River Mouth primarily illustrate "guided" transmission. It is probable, however, that some of the rudimentary interaction loops which occur on the records for 70 and 80 cps represent partial interference between the "hybrid" form of the second mode and the "guided" form of the first mode. These loops fade out rapidly with distance, owing to the high damping rate associated with "hybrid" transmission. Interaction spacings computed from the charts (Plates 15, 16, and 17) - using a velocity ratio of 1.42 computed from the critical frequency of the second mode, and a density ratio 2.0, - are 240 ft for the first vs second modes of "guided" waves, and 129 ft for the first mode of "guided" waves interfering with the "hybrid" form of the second mode. The observed spacings agree with these values within the accuracy of the distance scale of the records. Computation shows that over hard bottom the spacings should increase only slightly with frequency, and this also was found to be the case.

129. "Hybrid" transmission is also illustrated by recordings (not reproduced in this report) made recently over hard bottom in the Rappahannock River area. In this location the critical frequency for the first mode was 72 cps. At frequencies below this "cut-off", interaction spacings were obtained which correspond to interference between the first and second modes, both in "hybrid" form. At yet lower frequencies (38 cps), these spacings were not present, presumably because the second mode was not stimulated and all the energy was carried in the first mode ("hybrid" form). High damping rates were observed at all frequencies below the first mode critical frequency, and negligible damping at higher frequencies.

130. The records made over hard bottom (Plates 5, 6, and 7) do not show quite such smooth loops as those over soft bottom (Plates 2, 3, and 4). The reason is that the phenomena which occur over hard bottom are critical functions of the exciting frequency and of the geometry of the system. Small variations in depth along the range course or minor changes in velocity and density of the bottom material would be sufficient to cause observable anomalies in the records. An acoustic system which includes a soft bottom, being in general more highly damped, should be less sensitive to small irregularities.

131. The interaction spacing S, characteristic of any pair of interfering modes, rema... the same for any given exciting frequency regardless of the distance from the source. This is true because the phase velocity is a constant for the mode, and does not depend upon source distance. The fact that the experimental records show constant spacings is acceptable proof that the observed maxima and minima are <u>not</u> caused by interference between sound "beams" or "rays" from a succession of virtual sources in a vertical line with the actual source.

132. The spacings which may be derived from a group of image sources must become larger as the distance from the line of sources increases, and these spacings should be smaller the higher the frequency of excitation. The observed spacings follow neither of these rules, and have dimensions which cannot be checked by computation from any possible disposition of fixed image sources. The image theory is inadequate to account for the observed interaction phenomena because the sound pressure distribution in the actual acoustic system presents a three dimensional diffraction problem, which cannot in general be successfully treated in terms of the optical analogy of simple rays from point sources.

133. The image theory may, however, be expected to give an acceptable acount of underwater sound propagation phenomena under a few special conditions, such as: (a) when the exciting frequencies are so high that the wavelengths are short in comparison with the image distances; (b) when the bottom absorption is very high and the proportion of reflected energy so small that the normal modes are weakly stimulated and strongly damped; and (c) when the water depth is so great that the effects of reflections from the bottom may be neglected. The image theory also gives good agreement with the results of experiment for the sound pressure distributions directly beneath a ship-mounted source, a case which is discussed in detail in a recent NRL report (Bibliog. 4).

134. The general theory of underwater sound propagation by means of normal modes, as developed in the present report, should be valid for all frequencies, for all distances from the sound source, and for transmission over the most commonly encountered types of bottom. Detailed computation from this theory may be difficult or unwieldy if many modes are simultaneously stimulated at high frequencies, or if the bottom has a complicated layered structure. The derivations of Appendix A are not strictly valid if a significant proportion of the refracted energy is converted into shear waves in the material of the bottom, or if there is scattering or absorption in the medium itself (the water) as distinct from the boundaries. Other factors than the normal modes

may be important under special conditions such as those mentioned in the preceding paragraph. In general, however, the normal mode theory presented in this report gives an account of underwater sound transmission which is in satisfactory agreement with observation, and which permits quantitative prediction of the principal phenomena to be expected over different types of sea bottom.

VII. REFLECTION LAWS FOR DIFFERENT TYPES OF BOTTOM.

SUMMARY

Reflection Laws

135. Different laws for the reflection of plane waves at the sea bottom may be derived depending upon whether (a) the bottom is assumed to be a homogeneous medium of specified density and velocity of sound, or (b) the bottom is assumed to be a plane boundary having impedance Z independent of the angle of incidence. The analysis presented in Section IV is based upon the first assumption; the transmission theory involving impedance is based upon the second assumption.

136. If the bottom is soft, the two laws give substantially the same variation of reflection coefficient vs angle of incidence. The higher the absorption coefficient the greater the difference between reflection coefficients computed from the two laws. If the bottom is hard, the two laws give completely different reflection coefficients, and may be reconciled only by adopting a most improbable variation of Z with angle of incidence. Even if such a variation is accepted, the resultant impedance is not useful for computing transmission.

137. It is concluded that the acoustic behavior of the sea bottom is not completely determined by its impedance, although it is determined to a close first approximation by the density and velocity of sound of the material of the bottom.

138. The problem of sound propagation over an elastic bottom, in which both compressional and shear waves may be set up by the incident sound, is discussed in terms of the reflection law computed for this case. Reference is made to Appendix B for details. It is shown that the loss of energy to shear waves will be negligible unless the velocity of these waves in the bottom is greater than about 2400 ft/sec. or 1/2 the velocity of sound in water. Such velocities are improbable in the mud and sand commonly encountered at the sea bottom. It is concluded that in general the effect of shear waves may be neglected. The transmission in such cases may then be correctly computed in the manner described in Section IV.

139. A discussion is given of the relation between the critical angle for total internal reflection of plane waves, and the critical frequency below which "guided" transmission over hard bottom is replaced by "hybrid" transmission. It is shown that the critical angle is the smallest angle of incidence at which propagation of the "guided" type can take place.

140. The effects of viscous losses in the material of the bottom on sound transmission over it are discussed and the conclusion reached that these effects will in general be of the second order, although they may be observable over hard bottom as extra damping.

A. Reflection Laws - Soft, Hard, and Elastic Bottoms.

141. The plane wave reflection laws which result from the application of boundary conditions in various ways provide means for visualizing the difference between the analysis of underwater sound propagation presented in this report and that based upon the use of normal impedance (Bibliog. 10). The computation of the reflection laws for different types of bottom gives means for estimating the effects of special acoustic conditions on sound transmission. These include the case of relatively high bottom absorption and the case in which shear waves are excited in addition to the usual compressional waves in the bottom.

142. The reason why the propagation theory based upon acoustic impedance gives correct results for transmission over "soft" bottom, and erroneous results for transmission over "hard" bottom, may be seen from examination of the pressure reflection coefficients computed as a function of the angle of incidence for (a) a sea bottom characterized by density and velocity ratios for two homogeneous fluids; and (b) the same sea bottom characterized by a normal acoustic impedance Z. The plot of these coefficients in Fig. 3 shows the equivalence of the two characterizations for reflection problems involving "soft" bottoms, as well as the complete failure of the impedance theory to give the correct reflection law for "hard" bottoms.

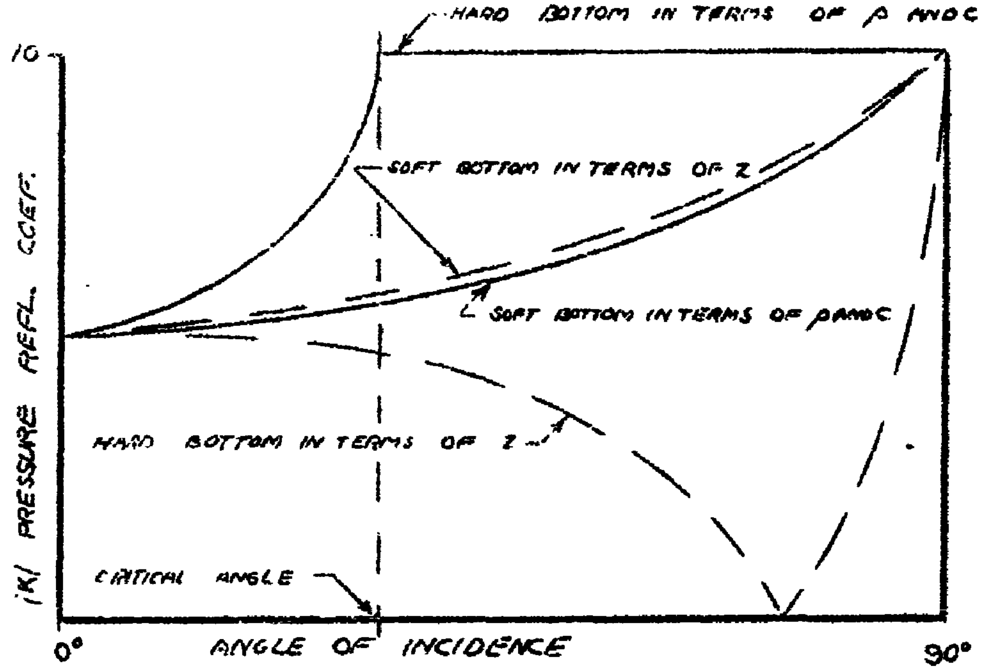

FIG. 3

143. Figure 3 was computed by assigning reasonable values to the constants in the formulas for the ratio K of reflected to incident wave pressure. The formulas which correspond to cases (a) and (b) above are:

Case (a)

$$K = \frac{1 - \frac{\rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1}}{1 + \frac{\rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1}} \tag{48}$$

Case (b)

$$K = \frac{1 - \frac{\rho_1 c_1}{Z \cos\theta_1}}{1 + \frac{\rho_1 c_1}{Z \cos\theta_1}} \tag{49}$$

where the densities and velocities in the water and in the bottom have their usual designations, θ_1 and θ_2 are the angles of incidence and refraction, and Z is the normal impedance of the bottom. The derivations of these formulas may be found elsewhere (e.g. Bibliog. 6, 7, and 12.)

144. From Snell's law for refraction we have

$$\cos\theta_2 = \left\{1 - \left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_1\right\}^{1/2} \tag{50}$$

The comparison of the reflection formulas after the introduction of Snell's law into the expression for case (a), shows clearly that the impedance Z can be independent of the angle of incidence only under the condition that

$$\left(\frac{c_1}{c_2}\right)^2 \gg \sin^2\theta_1 \tag{51}$$

This will be true only over soft bottoms, in which the velocities of sound are considerably smaller than in water. The reflection law given by equation (48) should be valid for any bottom which is homogeneous and in which the energy converted into shear waves is negligible.

145. The analysis presented in Section IV, as well as the experimental results discussed in Section III, show that low frequency sound may be propagated over hard bottom with virtually no loss due to absorption at the bottom.

This type of propagation is to be expected if the reflection law for such a bottom is given by equation (48). If it is given by equation (49) there is no single value of impedance which is consistent with the observed transmission.

146. A comparison of the two equations shows that the general relation (Equation (48)) may be reconciled with the impedance law (Equation (49)) either (a) if the velocity of sound in the bottom is sufficiently small for relation (51) to be valid, or (b) if the impedance Z varies inversely with the cosine of the angle of refraction. If c_2 is greater than c_1 (hard bottom) the angle of refraction becomes and remains imaginary as the angle of incidence reaches and exceeds the critical angle. In order to yield the general reflection law, the impedance in a typical instance would have to vary as follows: as the angle of incidence increases from 0° to 90° Z must increase from its normal value (assumed real) to infinity at the critical angle, must become imaginary at that angle, and then, continuing to be imaginary, must decrease smoothly to somewhat less than the original absolute value. It is obvious that an impedance having such properties is a mathematical artifice rather than an expression of physical relationships.

147. Another problem arises, however, if the attempt is made to compute transmission by using an impedance which varies with angle of incidence. The resulting transmission equations are indeterminate, and their solution by the usual methods impracticable. The most satisfactory course is to abandon the use of impedance in transmission problems except for the special case of soft bottom for which the relation (51) is valid. It has been shown in Section IV that transmission may be correctly computed in terms of two quantities, the velocity of sound and the density of the bottom. The impedance alone is insufficient.

148. Although the relevance of impedance for transmission over "hard" and "soft" bottoms has been discussed, little has been said about intermediate conditions. The intermediate conditions correspond to the transmission of considerable energy into the bottom. This energy may either be reflected back from underlying layers or be absorbed in the material of the bottom. The reflection law will be determined in the first case by the acoustic properties of the dominant layer or layers, and in the second case by the effective absorption coefficient of the bottom.

149. Reflection coefficients corresponding to three values of bottom absorption are plotted in Fig. 4, the full line curves being computed from equation (48) and the dashed line curves from equation (49). The absorption coefficients (α_o) for the three cases are 0.1, 0.2, and ∞ , or 6 db, 12 db, and complete loss per bottom reflection, respectively. The corresponding impedance ratios ($Z/\rho c$) are 0.3, 0.55, and 1.0. These absorption coefficients and impedance ratios apply to reflections at normal incidence.

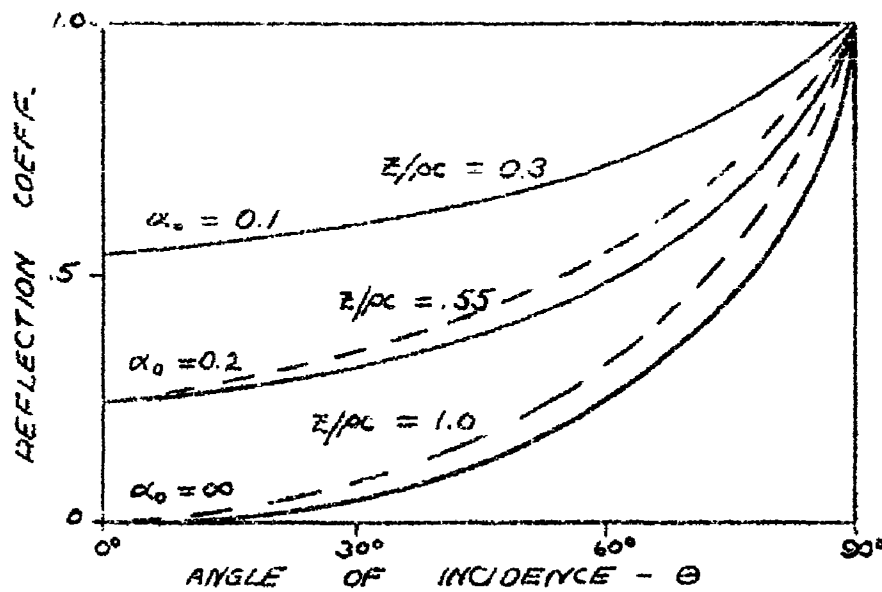

FIG. 4

150. The figure shows that the closer the impedance of the bottom approaches that of water the greater is the departure of the two formulas from each other, and the greater must therefore be the error in computing transmission attenuation from impedance. The figure shows also that considerable reflection may be obtained at large angles of incidence, even from a bottom whose impedance perfectly "matches" the impedance of water. There will be no reflections if both density and velocity of sound are the same in the bottom as in the water, but an impedance "match" is not sufficient to insure negligible reflections at all angles at the boundary.

151. <u>Elastic Bottom</u>. Computation of the reflection law for a bottom in which both compressional and shear waves may be set up by the incident sound gives the plot shown in Figure 5 . The method by which the reflection coefficient for such a bottom was computed is outlined in Appendix B, <u>Propagation Over An Elastic Bottom</u>.

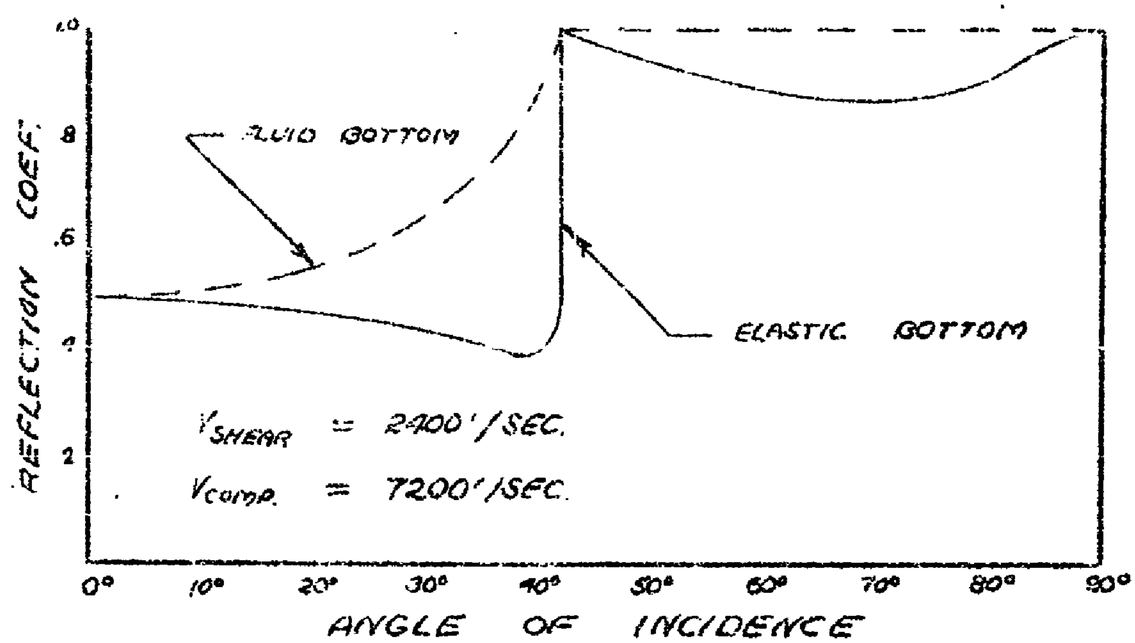

FIG. 5

152. This figure shows that for the computed cases: (a) the critical angle at which reflection is total and beyond which the refracted compressional wave in the bottom is imaginary, is not altered by the presence of shear waves ; (b) at angles of incidence less than the critical angle, the reflection coefficient is smaller than at the same angles over a fluid bottom; (c) at angles of incidence midway between the critical angle and grazing incidence, the reflection coefficient is reduced by the loss of energy to shear waves.

153. The loss of energy to shear waves is negligible unless the velocity of these waves in the bottom material is greater than about 2400 ft/sec, or one-half the velocity of sound in water. Shear wave velocities larger than this are highly improbable in the mud and sand commonly encountered at the sea bottom. It is probable that observable indications of loss of energy to shear waves could be obtained by making transmission measurements over a hard rock bottom. Such measurements will be attempted if a suitable location is found.

B. <u>Relation Between Critical Angle and Critical Frequency.</u>

154. The general law for the reflection of plane waves, illustrated in Fig. 3, shows that sound waves are totally reflected at a hard bottom ($c_2 > c_1$) if their angles of incidence are greater than a critical angle, given by $\sin\theta = c_1/c_2$. The normal mode solutions for the sound pressure distribution in the acoustic system of the sea between the surface and a hard bottom showed that for each mode there exists a critical frequency below which the transmission is highly damped (hybrid) and above which the pressure waves are propagated without absorption at the bottom (guided). Since critical phenomena in transmission over hard bottoms are predicted from two completely different methods of analysis, it is reasonable to inquire if there may be a relation between the critical angle and the critical frequency. That such a relation exists may be shown by making use of a result from the analysis of electromagnetic "wave guides".

155. In the discussion in Section V, <u>Acoustic Analog of Electromagnetic Wave Guides</u>, and also in Appendix A, Part 4, the suggestion is made that the "guided" wave associated with any one mode in either the acoustic or electromagnetic system may be interpreted as the synthesis of two sets of plane waves. The two sets travel at the same velocity and at the same angle with the normal to the boundaries, making a criss-cross pattern as they propagate horizontally away from the source by alternately reflecting at surface and at bottom. The angle of reflection is determined by the phase velocity, which is in turn determined by the ratio of the depth to the wavelength, and by the order of the mode.

156. At the critical frequency of the first mode, the phase velocity is equal to the velocity of sound in the bottom. This is shown in Appendix A, Part 3. In Part 4 it is shown that, at the critical frequency, the angle which the direction of the waves makes with the normal to the boundaries is given by the relation

$$\sin\theta = c/c_2 \qquad (60)$$

But this expression is exactly the same as that given above for the critical angle for total internal reflection.

157. The relation between the two critical phenomena is now clear. The critical angle is the angle which the direction of the waves makes with the normal to the boundaries, at the critical frequency, if the "guided" wave be visualized as broken up into its constituent sets of plane waves. Since "guided"

transmission can occur only if there is no absorption at the bottom, the critical angle is the smallest angle of incidence at which propagation of the "guided" type can take place. At frequencies below the critical frequency of the first mode, the pressure field distribution will change, absorption at the bottom will occur, and the "guided" waves will disappear.

C. Effects of Bottom Viscosity on Transmission.

158. Viscous losses in the material of the bottom, which have been neglected thus far in the discussion, undoubtedly play some role in the propagation of underwater sound, since, although the water itself may be considered to be non-dissipative at audio frequencies, viscous losses must occur in the material of the bottom. The experimental results indicate that the influence of such losses on sound transmission in the acoustic system of the sea between surface and bottom will in general be small. These, together with other second order effects, will be investigated further as opportunity arises.

159. These may be compared with dielectric losses in the boundary material of the analogous electromagnetic wave guide system. The dielectric material of the guide itself is visualized as dissipationless, like the water, but the lower boundary consists of an imperfect dielectric medium, corresponding to the sea bottom in the acoustic system.

160. The effect of viscous losses in the material of a "soft" bottom should, so far as transmission measurements in the water are concerned, be indistinguishable from transmission into the bottom. Thus, regardless of the degree to which viscosity may contribute to the total loss of energy, "damped" transmission over a soft bottom may be computed successfully from measured acoustic constants.

161. The effect of viscous losses in the material of a "hard" bottom should be to introduce a small amount of damping per unit distance. The experimental records made over hard bottom at the Potomac River Mouth do not show any effects which may be definitely traced to viscous damping. It is possible, however, that additional transmission studies may reveal such effects.

VIII. ATTENUATION, A SYNTHESIS OF THE COMPONENT FACTORS.

SUMMARY

Attenuation

162. The attenuation of underwater sound propagated in shallow water is shown to result from the operation of two factors, damping and spreading. Damping is differentiated into first order damping, due to absorption at the bottom; and second order damping, due to a number of causes such as scattering, viscous losses and shear waves in the bottom. Second order damping is ordinarily important only at the higher frequencies. Cylindrical spreading, amounting to 3 db per distance double, is present in all cases of propagation between two plane bounding surfaces.

163. If the bottom is acoustically hard the attenuation of low frequencies is dominated by cylindrical spreading. If the frequency is sufficiently high for many modes to be stimulated, the second order damping terms may combine to give a pressure-distance relation which approximates an inverse power curve. Such a curve may be characterized by a "transmission exponent".

164. If the bottom is acoustically soft the attenuation is dominated by first order damping, although cylindrical spreading and second order damping are also present. If many modes are present the data may be represented by a "transmission exponent" for a few hundred feet near the source.

165. At extremely low frequencies, over both hard and soft bottom, extraordinarily high attenuations may be expected, and the damping is governed primarily by the ratio of the water depth to the wavelength.

166. If the bottom is strongly sound absorbent or the water depth is great, the attenuation is characterized by "dipole spreading", which amounts to 12 db per distance double

VIII. ATTENUATION, A SYNTHESIS OF THE COMPONENT FACTORS.

A. The Primary Factors: Spreading and Damping.

167. Understanding of the physical factors which determine the attenuation of sound with distance in the acoustic system of the sea should make possible the explanation of the observed transmission and the prediction of the expected sound levels at various distances and over various types of bottom. In this report the term "attenuation" is used to signify any overall decrease in the rms sound pressure level which occurs in moving away from the source from one position in the system to another. The system is assumed to be in the steady state. In this section only envelope or average levels are considered, since the interference phenomena which may take place have been separately discussed in Section VI.

168. In general, the observed attenuation of sound level with increasing distance from the source results from the influence of two primary factors, spreading and damping.

169. Spreading occurs in all cases, and is that part of the attenuation which arises from the geometrical divergence of the waves with increasing distance. Spherical spreading, in which the sound pressure varies inversely with the distance from the source, is the type which occurs if the medium is infinitely extended. Cylindrical spreading, in which the pressure varies inversely with the square root of the distance from the source, is the type which occurs in a medium enclosed by two infinite flat planes, such as the sea between surface and bottom. It is obvious that cylindrical spreading is to be expected when sound is propagated through shallow water over a substantially flat bottom.

170. Damping occurs under certain conditions, for example over soft bottoms, and represents the "tapping off" or leakage of wave energy from the acoustic system. It is assumed throughout this discussion that there are no losses due to absorption or scattering in the water itself as distinct from the bounding surfaces. The total attenuation between two points in the sea will in general be the result of cylindrical spreading between them plus the damping which may exist.

171. Physically the two components of the attenuation manifest themselves in quite different ways.

172. The component due to spreading, being a geometrical divergence phenomenon, depends merely upon the ratio of the distances - measured from the source - of the two points under consideration. Therefore the sound pressure decreases inversely with some power of this ratio. When sound pressure levels are measured in decibels, it is often convenient to describe attenuation due to spreading in terms of "db per distance double". This constant, provided that the attenuation is caused primarily by spreading, should be independent of the distance from the source. For cylindrical spreading, the geometric attenuation constant is 3 db per distance double; for spherical spreading, it is 6 db per distance double.

173. Damping is evidenced by a uniform percentage reduction of the sound energy as distance increases. The amount of damping is independent of position in the system, and may therefore be conveniently expressed in decibels per unit distance (e.g. db per 1000 ft). This is equivalent mathematically to a pressure-distance law having the form $P \sim e^{-kx}$, where x is the distance and k is the damping constant.

174. The actual attenuation in the acoustic system of the sea results from a syntheses of the two factors, damping and spreading. The attenuation of sound between any two points lying on a line through the source may be derived from a pressure formula having the mathematical form $P \sim e^{kx} \cdot (x)^{-1/2}$

175. This expression, involving a single damping factor, expresses the physical relationships for transmission in a single mode. If many modes are present the resultant pressure will be the sum of a series of pressure terms, each having the above form and a distinctive damping constant, one for each mode.

176. The form of the transmission curve obtained will be determined primarily by the exponential damping factors $e^{-K_n x}$ if the bottom is soft, and by the spreading factor $x^{-1/2}$ if the bottom is hard. Assume for simplicity that only one mode is present. Then, if the bottom is soft, a decibel (logarithmic) plot of pressure level versus distance (linear) will, except for the "initial drop" near the source, be substantially a straight line. If the bottom is hard, the slope of the transmission curve plotted as db vs distance will decrease gradually with increasing distance. "Damped" transmission over soft bottom, with a linear decline of pressure level with distance, is illustrated by the lower curves on Plate 2. "Guided" transmission over hard bottom, with a rate of attenuation which decreases as distance increases, is illustrated by curves (a) on Plate 9.

177. Various combinations of damping and spreading are illustrated by the synthetic transmission curves in Figs. 6 and 7.

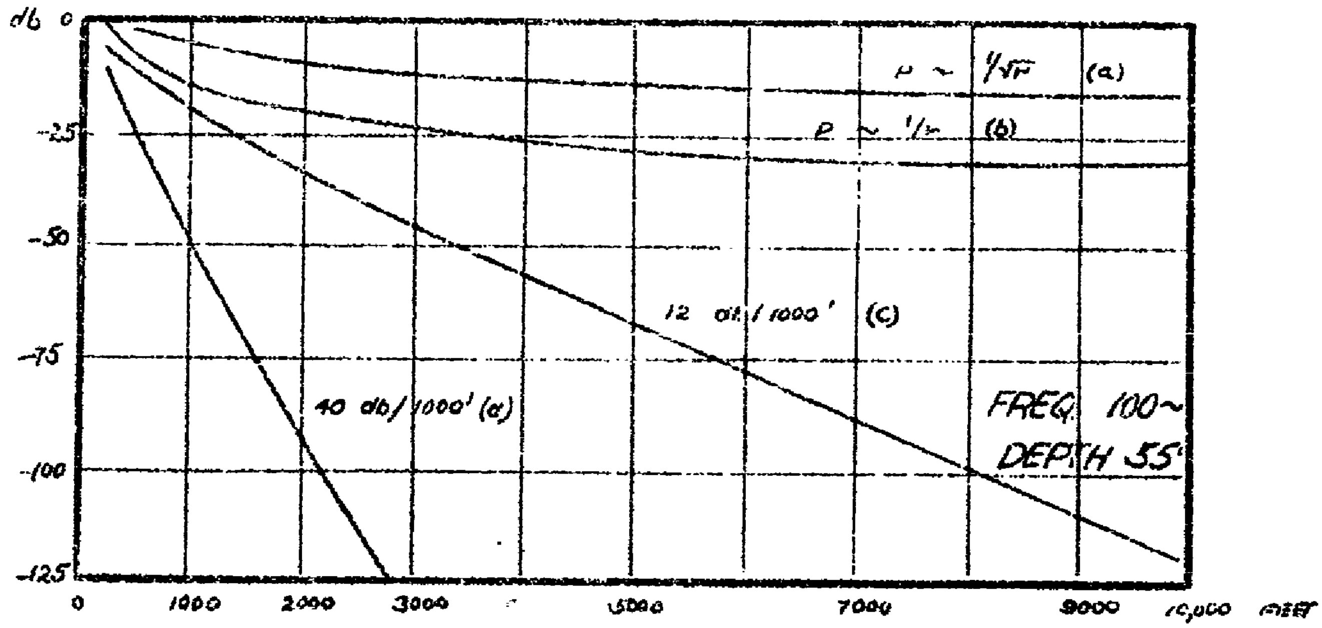

FIG. 6

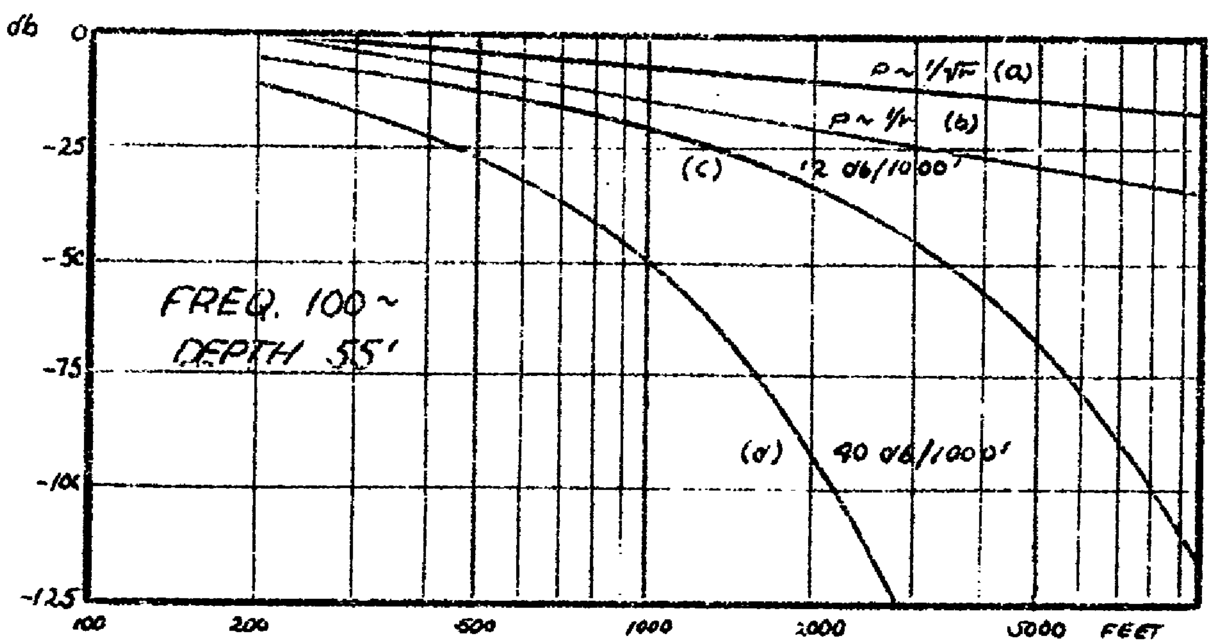

FIG. 7

178. The ordinates in both figures are pressure levels in db. The abscissas are distances, plotted to a linear scale in Feg. 6 and to a logarithmic scale in Fig. 7. Curves (a) and (b), beginning at a point 200 ft distant horizontally from the source, illustrate attenuation due to spreading alone. Curve (a), corresponding to cylindrical spreading, defines the ideal lower limit of attenuation in "guided" propagation over a flat bottom. Transmission over a hard bottom may approach this curve if losses from scattering and other second order effects are sufficiently small. Curve (b), corresponding to spherical spreading, is assumed to start from the same initial level as curve (a).

179. Curves (c) and (d) are typical examples of the combination of spreading and damping to give "damped" transmission, with small and large damping, respectively. These curves indicate the upper and lower limits within which the observed transmission over soft bottoms will probably lie, at frequencies for which the wavelength is of the order of the water depth or less. Although the effects of spreading are included in curves (c) and (d), the transmission is dominated by exponential damping which causes the curves to approximate straight lines. The slopes of these lines are the damping constants, 12 db/1000 ft and 40 db/1000 ft, which correspond at these frequencies to minimum and maximum damping respectively. Reasonable initial levels are assumed for the starting points of the synthetic curves.

180. The differences between "damped" and "guided" transmission, over soft and hard bottom respectively, are graphically illustrated by the synthetic curves. The superiority of the propagation over hard bottom to that obtainable over soft bottom is well shown. It is evident that the nature of the physical processes involved should be kept clearly in mind when discussing the decrease of sound pressure level with distance. Although a "db per distance double" relation may closely approximate the attenuation to be expected over hard bottom, or in other special cases described below, such a relation cannot adequately represent "damped" transmission (e.g. the lower curves of Plate 2). Similarly, an attenuation expressed in "db per 1000 ft" may accurately represent low frequency propagation over soft bottom at considerable distance from the source, although this form of expression has little physical significance if applied to "guided" transmission over hard bottom. When spreading is the dominant factor, the rate of attenuation per unit distance changes with distance from the source.

181. The difference between the two basic types of transmission is illustrated equally well by Fig. 6 and by Fig. 7. It would seem most convenient to use a linear distance scale in plotting curves showing "damped" transmission, and a logarithmic distance scale in plotting curves showing "guided" transmission.

182. It should be remembered that the synthetic curves do not give the attenuations expected at extremely low frequencies, for which the wavelengths are considerably longer than the water depth.

B. Influence of Higher Modes - Transmission Exponents.

183. The synthetic curves, Figs 6 and 7 are idealized. They are strictly applicable only if all the energy is carried in the lowest mode, since the observed attenuation has no component causes other than damping and spreading. In spite of the restrictions on their generality, the synthetic curves provide a clear picture, in terms of physical processes, of attenuation in shallow water at moderately low frequencies.

184. The primary damping factor is the loss of energy from the system to the bottom. The first order damping is computable from the elastic properties of the bottom, in terms of the constant σ. Additional damping may result from a number of other factors which cannot be ignored, although their effects are usually of secondary importance. The second order damping factors include: (a) the influence, increasing with frequency, of viscous losses in the bottom; (b) the loss of sound energy to shear waves in the material of the bottom; (c) the scattering of sound waves at the surface of the sea; (d) the possible scattering and irregular reflection at the bottom; and (e) the absorption and scattering in the medium itself. Factor (a) is discussed in detail in Section VII-C; factor (b) is discussed in Section IV-A and in Appendix B; and factors (c), (d), and (e) are considered to be negligible at the frequencies of primary concern for this analysis, although they may be very important at higher frequencies.

185. The combined influence of the second order loss factors may reasonably be simulated by the superposition of a single damping term on the overall cylindrical spreading of the energy. The shapes of the curves for transmission over soft bottom will

not be altered by the effect of additional damping. The shapes of the curves for hard bottom will be altered if the damping term is comparable to the inverse power term in the pressure equation. The experimental curves (Plate 9 (a)) indicate that spreading was the dominant factor in transmission over hard bottom, as far as the records were carried. It is concluded that, under the conditions of the measurements, the second order damping was very small.

186. The actual transmission curves may differ from the idealized synthetic curves because of the different decay rates of the constituent modes. It has been shown that if only one mode is present the rate of attenuation is determined by a single damping constant. At moderate frequencies, however, many modes are stimulated, each of which contributes to the overall transmission. It was shown in Section V-C that the damping rate increases with the order of the mode. The modes are propagated by alternate reflections at top and bottom; moreover the number of reflections per unit horizontal distance increases with the order of the mode. Except for absorption in the water itself, the second order loss terms all depend upon reflection. The effect of these terms must therefore increase with the order of the mode, and also with the frequency. The increase of scattering with frequency is particularly pronounced.

187. The actual transmission curve may be considered to be the envelope of a group of curves, one for each mode. Each curve starts from a different initial level, and each has a characteristic downward slope. The slopes of these curves increase with the order of the mode because both first and second order damping terms increase in this manner. The combination of a number of straight lines, each with a different slope, may have an overall envelope which closely approximates an inverse power curve. It is therefore possible for the envelope transmission curve, if many modes are present, to slope downward in accordance with any preassigned inverse power of the distance. A curve similar to (b) in Figs 6 and 7 may thus result from the combination of several curves similar to (c) and (d). In this case the observed transmission phenomena may appear to be represented by a pressure-distance relation of the "spreading" type, although actually the attenuation may be dominated by the effects of damping.

188. The fact that an inverse power curve may represent, over certain ranges, the transmission of sound in many modes, is the sole physical justification for presenting the results

of tests in the form of "transmission exponents". It has been reported that the results of transmission measurements in the approaches to San Francisco Harbor may be characterized by inverse power curves over ranges from 500 to 2000 yards. The results (Bibliog. 13) were plotted as pressure level in db versus the logarithm of the range. In general, straight lines were obtained, the slopes of which determined the "transmission exponents". The latter were estimated by reading the ordinates of the curves at two ranges differing by a factor of 10. The difference between the two ordinates, 10 n (db), gave the transmission exponent, n. This exponent is associated with the attenuation of sound intensity. The pressure exponent is of course one half of the intensity exponent. The intensity exponent for cylindrical spreading should be 1, and for spherical spreading, 2. In the San Francisco study, transmission exponents were obtained ranging from 1.4 to 3.4, with a median value of 2.3. The median value corresponds to a rate of attenuation slightly higher than that associated with spherical spreading.

189. The results of the San Francisco survey may be readily interpreted in terms of the normal modes of the system. The frequency at which the measurements were made, although not stated, is presumed to have been in the range 300-1000 cps, where many modes would be stimulated. The bottom in the area studied is marked "hard Sand" on the hydrographic chart.

190. Inverse power attenuation with an exponent of 2.3 could not have been obtained if all the sound energy had been carried in a single mode, because the superposition of a single damping term (expressing second order losses) on cylindrical spreading of the sound energy cannot yield an inverse power relation with such a high exponent as 2.3. Inverse power attenuation with this exponent can, however, be explained as the resultant of many modes, each characterized by cylindrical spreading and a distinctive damping term.

191. If measurements had been made in this area at lower frequencies, for example 100 cps, the median transmission exponent would doubtless have been considerably smaller. The NRL data suggest that the ideal transmission exponent, unity, may be closely approximated when sound is propagated over hard bottom at sufficiently low frequencies. Additional measurements at the Potomac River Mouth at higher frequencies (200-1000 cps) would obviously be desirable in order to clarify these relationships.

192. The record at the bottom of Plate 4 for transmission over a soft bottom at 400 cps is another example of propagation in which the sound pressure varies inversely with a power of the source-distance. It also satisfies the conditions that a number of modes shall be stimulated, and that the damping of each mode shall increase with the order of the mode. The envelope of this record can be accurately fitted by an inverse power curve. A plot of pressure level in db versus log distance, using data from this record, is a straight line with a slope corresponding to a transmission exponent of 2.5. This exponent fits the data, although it has no direct interpretation in terms of spreading alone.

193. Even at high frequencies, the propagation over soft bottom is not so good as that over hard bottom. This is indicated by the fact that the transmission exponent noted above is higher than the exponents found by the San Francisco study for corresponding source distances (less than 1000 ft).

194. The probable mechanism of attenuation at moderately high frequencies is now clear:

a) The physical relationships involved in sound transmission require that the spreading which can occur between two infinite parallel planes be cylindrical, corresponding to a transmission exponent of unity.

b) They also require that the effect of all loss terms on the propagation of any one mode may be represented by a single damping term.

c) Each mode has a characteristic damping constant, and the rate of damping increases with the order of the mode, regardless of whether the damping is caused by bottom absorption or by second order effects such as scattering.

d) If the measured transmission curve declines in accordance with a higher inverse power than 1/2 (i.e. with a higher transmission exponent than unity), the presence of several modes is indicated.

e) The combined decay of several modes at different rates is able to produce an overall transmission record which is indistinguishable from an inverse power curve, the transmission exponent of which may be considerably higher than unity - the exact value depending upon the relative degree of stimulation and the relative damping rates of the various modes.

C. Computation of Attenuation For Various Types of Bottom.

195. The influence of the boundaries, particularly of the bottom, on the attenuation of underwater sound may be estimated in terms of damping and spreading, and actual levels at different distances from the source may be computed for special cases. This is done as follows. The bottom in a given area is classified on the basis of hydrographic data and hydrophone soundings as acoustically "soft" or "hard". The soundings provide values of the effective $\rho_2 c_2$, the normal impedance of the bottom. Although density and velocity (ρ_2 and c_2) are interdependent, these quantities may be separately estimated.

196. Damping constants (σ) and phase velocities (c/τ) are determined for each mode, once the density (ρ_2), the velocity (c_2), and the depth in half wavelengths (η_1) are known. Numerical computations are facilitated by the use of charts (Plates 12-17 inc.), the origin and significance of which are discussed in Sections IV and V.

197. Computed values of attenuation are shown in Table IV for an illustrative special case. It is assumed that all the energy is carried by the first mode. The water depth is taken to be 60 ft, the exciting frequency 100 cps, and computations are made for bottoms having a wide range of acoustic properties. Cylindrical spreading is assumed, and the total attenuation is the sum of this term plus the damping due to absorption at the bottom. Secondary damping factors such as scattering are neglected.

198. The pressure level differences between 6 ft from the source and a distance equal to about three times the water depth were estimated for Table IV from representative experimental data. The procedure described on the preceding page sufficed to evaluate the attenuations between this point and greater distances. If a technique now being developed proves successful, the short range pressure differences may eventually be computable entirely from theoretical considerations. The would make possible the exact computation of attenuation over wide ranges, starting from the known levels at 5 ft from the source.

TABLE IV.

COMPUTED ATTENUATION OVER VARIOUS TYPES OF BOTTOM.

Depth 60 ft, Frequency 100 cps, First Mode, Effective Source Depth 12 ft.

(Attenuation is expressed as db below the pressure level at 6 ft distance from the sound source.

MATERIAL	CLASSIFICATION	IMPEDANCE RATIO (Assumed)	DENSITY RATIO (Est)	VELOCITY RATIO (Est)	DAMPING 1st. MODE	ATTENUATION 500 feet from source	1000	2000
Soft Mud	Soft	0.3	1.3	0.231	11 db/1000'	36 db	45 db	56 db
	"	0.5	1.3	0.384	19	42	52	71
	"	0.7	1.3	0.538	35	55	72	--
Blue Clay	Absorbent	1.0	1.6	0.625	--	50	62	74
Hard Sand	Hard	2.5	2.0	1.250	0	33	36	39
Rock	"	4.0	3.0	1.330	0	33	36	39

199. The tabulated attenuations are quantitatively reliable for propagation at the assumed frequency and water depth, and illustrate what may be expected under related conditions. For example, if the depth 60 ft be doubled to 120 ft, the results in the table will be approximately correct for a frequency of 50 cps, and all values of total attenuation from 6 ft will increase about 6 db owing to the spreading of the available energy through a volume twice as great as before. If the depth be increased the central peak on the range runs should become less prominent, but the slopes of the curves at any considerable distance (500 to 1000 ft) should be altered only slightly. If the frequency be increased without alteration of the depth the attenuations will be slightly decreased, as shown by the superposed range runs on Plate 9. This effect is due to the influence of the higher modes. If both frequency and depth be somewhat increased, the changes in attenuation will be small, since the results of the various effects tend to counteract each other.

200. The differences in attenuation caused by variations in the acoustic character of the bottom are very much more important than those which may be expected from variations in depth or frequency, provided the frequency is appreciably higher than the "natural frequency" for the first mode. The range of attenuations shown in Table IV for different types of bottom should be generally valid, and with some reservations the values given in the table should be approximately correct for other depths and frequencies than the 60 ft and 100 cps for which they were computed. The close agreement between the values in Table IV and the experimental results shown in Plates 9 and 10 is worth noting.

D. Attenuation at Extremely Low Frequencies.

201. The theory predicts that, if the exciting frequency is substantially lower than that for which the depth is a wavelength, the damping constant for the first mode over soft bottom will be very large and strongly dependent upon frequency. This is not true at the higher frequencies discussed in the preceding paragraphs. It is to be expected that pronounced changes will occur in the attenuation over soft bottom as the frequency is reduced, and that these effects will predominate if the free wavelength approaches or exceeds twice the water depth.

203. The expected variation of damping for transmission over soft bottom as a function of frequency may be computed with the aid of the charts (Plates 12-17 inc.). Computed results, for the acoustic conditions which exist at the Potomac River Bridge in summer, are plotted in Figure 8. The damping rates, in db per 1000 ft, are shown for the frequency range 20 to 100 cps.

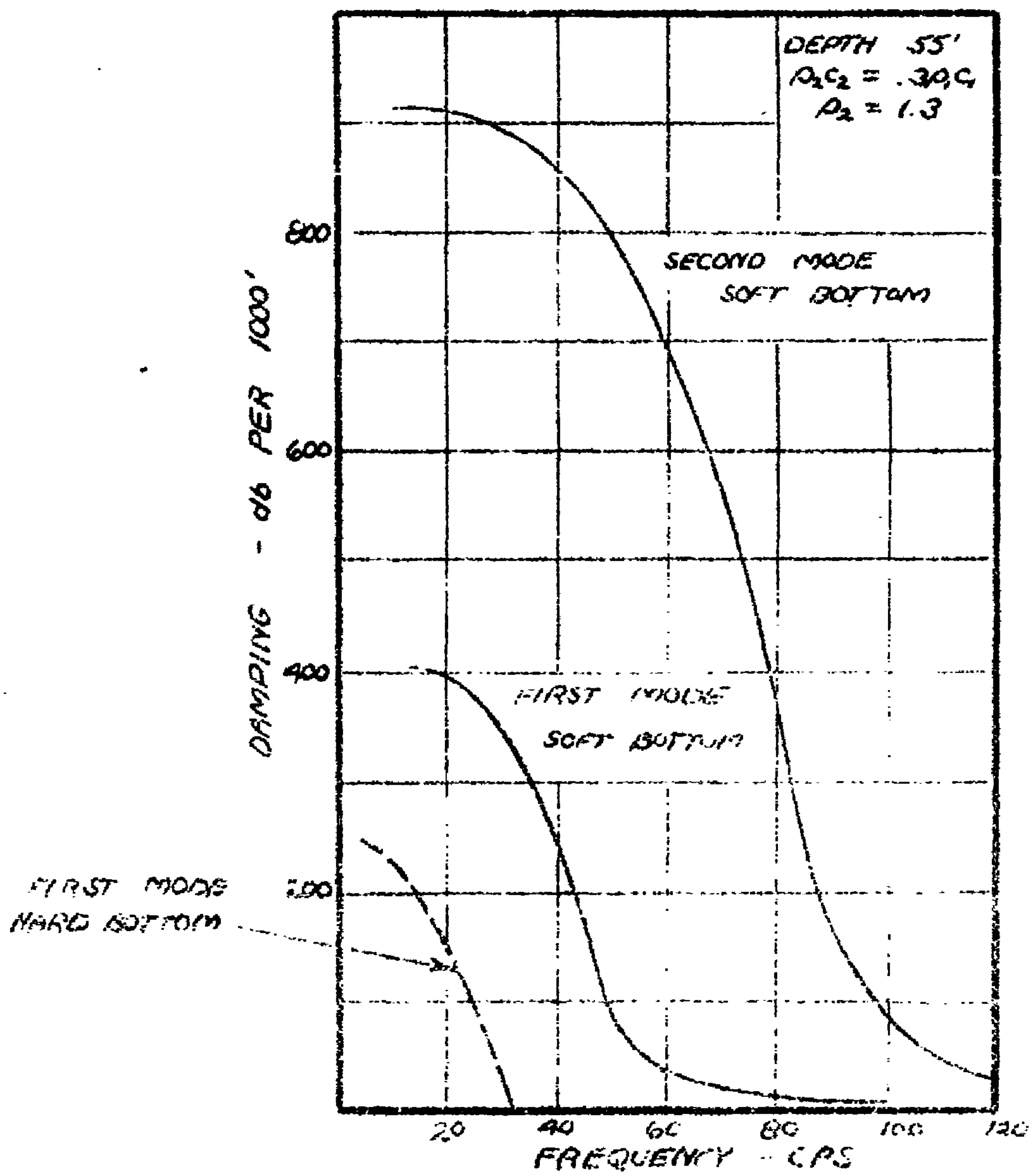

FIG. 8

203. The striking feature of the plot is the extremely rapid increase in the damping rate of each mode as the frequency decreases below a certain value. At any given frequency, the damping rate of the second mode is much greater than that of the first mode. The damping of each mode becomes pronounced as the frequency approaches the "natural frequency" of the particular mode. In this case the "natural frequencies" which correspond to the first and second modes are 45 cps and 90 cps, respectively.

204. The significance of the curves in Fig. 8 will be apparent if they are compared with the experimental records in Plate 2. At these frequencies the overall transmission is governed primarily by the damping rate of the first mode, although the second mode may also be present, especially near the source. At frequencies above 100 cps, the first two modes are strong enough to give interaction loops to considerable distances. As the frequency is lowered from 100 cps to 80 cps, the damping constant of the second mode increases so rapidly that the second mode is present only in the immediate vicinity of the source at frequencies lower than about 90 cps. The transmission is carried by the first mode alone, without excessive damping, as the frequency is lowered toward 60 cps. At frequencies lower than 60 cps (wavelength equal to about 1 1/2 times the depth) the damping constants of the first mode increase at extraordinary rates, reaching 80 db/1000 ft at 50 cps, 250 db/1000 ft at 40 cps, and 350 db/1000 ft at 30 cps.

205. If the water depth were halved, these damping rates would apply to double the indicated frequencies. Similarly if the water depth were doubled, the damping would be that for half the indicated frequencies. The basic variable is of course neither depth nor frequency, but the ratio of water depth to wavelength.

206. The interaction loops on the records in Plate 2 illustrate the disappearance of the second mode as the frequency decreases to 70 cps. Recent range runs at the River Bridge, not reported in detail at this time, confirm the expected high damping rates at frequencies lower than 70 cps. For example, the following attenuations were observed: 44 db/1000 ft at 56 cps, 83 db/1000 ft at 48 cps, 210 db/1000 ft at 45 cps, and 280 db/1000 ft at 32 1/2 cps. These values are in good agreement with the computed curves.

207. It is thus predicted from theory, and confirmed by observation, that: (a) for a soft bottom of given acoustic properties the factor which determines the propagation at low

frequencies is the ratio of the water depth to the free wavelength of the sound; (b) for frequencies higher than that for which the depth is a wavelength, the damping rate is small relative to that at lower frequencies, and is not a pronounced function of frequency or depth; and (c) for extremely low frequencies, below the "natural frequency" of the lowest mode, the damping rate is extraordinarily high and is a pronounced function of frequency and depth.

208. Over hard bottom, the transmission of extremely low frequencies is characterized by a sharp transition at a definite critical frequency. This frequency, as previously noted, is slightly higher than that for which the depth is one quarter wavelength. Above the critical frequency the damping is zero and below it the damping increases with extraordinary rapidity as the frequency decreases. This variation, for a typical instance of hard bottom transmission, is illustrated by the dashed curve in Figure 8.

E. Attenuation Over Sound Absorbent Bottom.

209. If the bottom is almost completely sound absorbent for normally incident waves, there will be no appreciable standing wave pattern between surface and bottom, and the normal modes will be weakly stimulated and highly damped. This corresponds to a normal reflection coefficient which is almost zero and a bottom impedance nearly equal to that of water. In this special case the attenuation may be closely approximated by assuming a dipole source, the actual source and its image above the surface, radiating sound into a semi-infinite medium. The computation results in a inverse power curve characterized by a transmission exponent (intensity) of 4. If the bottom is sufficiently sound absorbent an attenuation of 12 db per distance double should therefore be expected.

210. At high frequencies or at great distances from the source the interference patterns may become imperfect. In this case the "dipole spreading" described above degenerates into spherical spreading with an attenuation of 6 db per distance double. Also, at large distances from the source, reflections from the bottom at high angles of incidence may become appreciable.

It is shown by equation (48) in Appendix A, that there may be considerable reflection at large angles of incidence, even if the ρc of the bottom "matches" that of the water.

211. The values of attenuation in Table IV, for an assumed impedance ratio of unity, were computed in accordance with the dipole spreading law, equation (52) in Appendix A. Locations have been found in the Chesapeake Bay area and at the NMWTS acoustic range, near Solomons Island, where the measured bottom impedance is very nearly equal to that of water, and where the standing wave system between surface and bottom is very poorly developed. In these locations the material of the bottom, Chesapeake "blue clay", is highly dissipative.

212. Range runs in these areas at frequencies between 38 cps and 300 cps resulted in good experimental confirmation of the attenuation predicted by the dipole spreading law. One of the records is illustrated in Fig. 9 in Section X (Applications of Results). Additional discussion of the tests in the Chesapeake Bay and at the NMWTS range will be reserved for a subsequent report.

213. The attenuation which occurs in transmission over a strongly sound absorbent bottom approaches as a limit the attenuation to be expected if the bottom were replaced by additional water extending indefinitely downward. The propagation of low frequency sound in deep water should therefore approximate that over a completely absorbing bottom. In this special case the normal modes are not important, since they depend upon reflections from the bottom.

NOTE: The integration of the general pressure equations of Appendix D, accomplished after the report was written (See Addenda), makes possible the computation of the resultant sound pressure at __any__ point in the field of the acoustic system of the sea between surface and bottom.

It is shown in Section VIII-C that the methods already described are entirely adequate for the computation of transmission curves for source-distances substantially greater than the depth. The complete pressure equations are required, however, in computations for shorter source-distances. Transmission data, including actual pressure levels, may now be computed for all types of bottom and for all source-depths and source-distances.

IX. THE USE OF ACOUSTIC MEASUREMENTS AS TRANSMISSION CRITERIA.

SUMMARY

Transmission Criteria

214. The acoustic information required for the computation of propagation constants may be obtained by making hydrophone soundings beneath a sound source mounted on an experimental vessel. The effective density of the bottom and the velocity at which sound is transmitted through it may be obtained from such soundings, combined with available knowledge of the general character of the bottom (mud, sand, rock, etc.).

215. By making use of these values of density and velocity, and the water depth in half wavelengths, the distribution constants for the first and second modes may be obtained from charts (Plates 12-16 inc), and converted into damping and phase constants by means of another chart (Plate 17). If the bottom is definitely "soft" or "hard", the transmission may be computed with considerable accuracy; if it is "transitional", the acoustic behavior may be estimated by considering the influence of lower layers or strata.

216. Although soundings beneath the source are sufficient to characterize the acoustic behavior of the bottom, it is desirable if facilities permit, to supplement these with similar soundings made at a considerable horizontal distance from the source. The interpretation of these soundings in terms of the physics of sound propagation is simple and direct, provided that they are made under conditions which ensure that only the first mode is present, and that the effective angles of incidence are large.

IX. THE USE OF ACOUSTIC MEASUREMENTS AS TRANSMISSION CRITERIA.

A. Hydrophone Soundings Beneath the Source.

217. Consider an area in which the water is comparatively shallow and the acoustic character of the bottom is unknown. What acoustic measurements are required to enable a fair estimate to be made of the low frequency sound transmission which may be expected? An attempt to answer this question is made in this section.

218. It has been shown by the foregoing analysis that the character of the transmission can be specified, and the propagation constants computed, provided that the density of the bottom material and the velocity of sound in it are known. It is possible that the effective viscosity will also have to be known in order to give a complete account of transmission over hard bottom, but the experimental results indicate that a close approximation may be obtained in terms of density and velocity alone.

219. The most practical acoustic measurements which give the desired information are hydrophone soundings beneath a sound source mounted on the experimental vessel. These measurements give records at various frequencies of the vertical distribution of sound pressure level between the source and the sea bottom.

220. The method of making such soundings and their interpretation in terms of the normal acoustic impedance of the bottom has been discussed in a previous report (Bibliog. 4). The frequency range in which the vertical soundings give the most interpretable records is that for which the water depth is between one and four wavelengths. The source depth should be a quarter wavelength, although this is not critical, and the output of the source should preferably be a simple rather than a complex sound "spectrum". If a complex sound source is used, the receiving system must include a sharp filter. The receiver should include a hydrophone, an amplifier, a band pass filter, and a level recorder. The procedure consists of recording the sound level as a function of time (or depth) while slowly raising the hydrophone from the bottom to the surface.

221. The analysis of the hydrophone sounding records permits the evaluation of the normal acoustic impedance of the sea bottom in the desired location. The impedance concept is of course entirely valid if restricted to systems in which the sound waves impinge at normal incidence. On the basis of hydrophone soundings the bottom may be classified as "soft", "hard", or "transitional", corresponding to three types of bottom reflection, "free-boundary", "rigid-boundary" and "transitional". Values of impedance which are predominantly real and smaller than the radiation resistance (ρc) of water correspond to "soft" bottom; values which are predominantly real and greater than the radiation resistance of water correspond to "hard" bottom; and values which are predominantly imaginary (reactive) and of the same order of magnitude as the radiation resistance of water correspond to "transitional" acoustic conditions at the bottom. Yet another condition is occasionally encountered, in which the bottom is strongly sound-absorbent, with an impedance nearly equal to that of water. Fortunately the intermediate conditions occur infrequently, and in most instances the bottom can be definitely classified from hydrophone soundings as "soft" or "hard". If the measured impedance varies with frequency, the impedance at the lowest frequency may usually be taken as most representative of the influence of the bottom upon sound transmission.

222. The impedance determined from hydrophone soundings is an effective value of ρc (density times velocity of sound) for the material of the bottom. The evaluation of transmission constants requires that ρ and c be separately known. The density may easily be determined from the weight and volume of a sample of the bottom material. A bottom sample also gives a desirable cross check on the impedance measurements and on the data given by the hydrographic charts. Once the density is known; the velocity in the bottom may be computed from the measured value of ρc.

223. If it is undesirable or difficult to take samples, the density of the bottom may be guessed within close limits from its general character and from the impedance data. Thus the density of mud and clay may safely be assumed to lie between 1.2 and 1.6; that of sand and gravel between 1.8 and 2.2; and that of rock between 2.4 and 3.0 depending upon the type. Mixtures such as sandy mud may be assumed to have intermediate densities. A rough rule of this sort should be adequate for most practical cases, although an impedance measurement would give an additional indication. The lowest density range should correspond to impedance ratios less than unity; the intermediate density range to impedance ratios between one and four; and the high density range to impedance ratios greater than four.

224. Two definitions of "soft" and "hard" bottom have been given, one in terms of impedance (Bibliog. 4) and one in terms of velocity ratios (Section IV). The definition in terms of velocity ratios would appear to have greater physical significance than that in terms of impedance. There is no conflict between the definitions if the bottom is definitely either soft or hard. Some confusion may arise, however, if the impedance is greater than that of water, and the velocity slightly less than that in water. In practice this should cause no difficulty because such bottoms will be classified as either strongly absorbent or "transitional". The acoustic behavior of these types of bottom is discussed in a later paragraph.

225. The velocity of sound in, and the density of, the material of the bottom, and the depth of the water in half wavelengths are sufficient to determine A and B through equation (21). The quantity A is the specific gravity of the bottom. The excitation frequency enters into the computation of the depth in half wavelengths. The latter, multiplied by the factor $\left[(c/c_2)^2 - 1\right]^{1/2}$ gives B.

226. Upon consultation of the charts for distribution constants of the first and second modes (Plates 12-16 inc), a set of values of κ and μ may be obtained, corresponding to each set of values of A and B. These values applied in turn to the chart which gives the relations between the distribution and propagation constants (Plate 17) determine the corresponding values of σ and τ for the first and second modes.

227. The damping constant σ and the phase constant τ for each mode may thus be determined from the basic physical constants, density ρ_2, velocity c_2, and depth in half wavelengths η. Sample computations are given in Appendix E. The phase constant τ is chiefly useful for the computation of interaction spacings as described in Section VI. The phase velocity of the mode is c/τ, and the phase wavelength is λ/τ.

228. The damping of the pressure wave in a distance of one wavelength λ is given by the expression $\varepsilon^{-2\pi\sigma}$, or 54.6 σ db per wavelength. This is easily converted into db per 1000 ft or into other desired units. The attenuation caused by geometrical spreading about the source (3 db per distance double) should be added to the damping term in order to obtain the expected rate of attenuation

for the particular mode. The final composite transmission curve may also include the effects of other factors discussed in Section VIII.

229. If the bottom is hard ($c_2 > c_1$), and the frequency is higher than the critical frequency, it will be found that $\kappa = 0$, $\sigma = 0$, and that cylindrical spreading is the only attenuation factor. At frequencies below the critical frequency for a particular mode, that mode can exist only in the "hybrid" form discussed in Section V-A (Three Types of Transmission).

230. Computation of the propagation constants as described above should enable the most important transmission phenomena to be quantitatively predicted with considerable accuracy in case the bottom is "soft", and with fair accuracy in case the bottom is "hard". Fortunately, rough estimates of density are usually sufficient to give reliable values of damping constant σ, since under most conditions this quantity does not very in a critical way with density for a given ρc.

231. If the hydrophone soundings indicate intermediate acoustic conditions at the bottom, the detailed computation of transmission constants may be difficult. One intermediate case, that of the strongly sound-absorbent bottom, was discussed at the end of Section VIII (Attenuation). Another intermediate case is that for which the measured normal acoustic impedance of the bottom is predominantly reactive. The acoustic behavior of such a bottom will be dominated, at least at certain frequencies, by reflections from an underlying layer or layers. In this case the transmission will vary with frequency, and "resonance" effects will take place at frequencies for which the "path" lengths in the bottom to the reflecting boundary, are half wavelengths or multiples thereof.

232. Reflections from lower layers will govern transmission only at certain frequencies, distances, and angles of incidence. For example, if two modes are present in transmission over hard bottom, one mode may seem to disappear, and to reappear at a greater distance from the source. The overall attenuation may not be greatly affected if considerable energy is transmitted in the other mode.

233. The records made at the River Mouth show evidence of this type of "selective fading", some of which may arise from nonhomogeneities of the system and some from the influence of underlying layers. Although such effects may exist, they seldom dominate

the transmission. Detailed computation of propagation constants for the layered bottom could doubtless be made from the final equations of Appendix C (Propagation Over Stratified Bottom).

B. Hydrophone Soundings at a Distance From Source.

234. Sufficient information about the bottom to enable its influence on sound transmission to be predicted may be obtained from hydrophone soundings made directly beneath a ship-mounted source. It is desirable however, when practicable, to supplement these by hydrophone soundings made at a considerable horizontal distance from the source. This may be done by deploying the receiving equipment (battery powered) in a small boat, and lowering and raising the hydrophone with a small cable reel.

235. The hydrophone soundings at a distance from the source permit the determination of (a) the actual pressure level at a distance, which may be compared with the expected level at that distance over hard or soft bottom; and (b) the vertical distribution of pressure level between surface and bottom. The acoustic properties of the bottom may be derived from interpretation of this pattern. Samples of hydrophone soundings at a distance are reproduced in Plate 11.

236. The interpretation of vertical pressure distributions made at considerable distance from the source is difficult if more than one mode is represented on the records. Since the higher modes are damped out more rapidly with increasing distance than the first mode, the soundings over soft bottom may easily be made at sufficient distance to ensure that the sound pressure distribution is due almost entirely to the first mode. Over a hard bottom any mode which is excited above the critical frequency will be only slightly attenuated with distance. In this case the hydrophone soundings at a distance should be made at frequencies low enough to ensure that only the first mode is stimulated above the critical frequency. In all cases, it is advisable to make soundings at several frequencies to obtain representative averages.

237. An analysis of the vertical distribution is given in Appendix A, Part 5. It is shown, as might be expected, that the pressure pattern in the first mode should have an approximate

antinode at a hard bottom, and a node at a soft bottom. The form of the pattern should correspond analytically to a section of a sine curve

$$\sin\left[j\pi\left(\frac{x}{h_1}\right)(k - j\mu)\right]$$

Since $k = 0$, the undamped distribution over hard bottom is represented by the sine of a real angle, and the damped distribution over soft bottom is represented by the sine of a complex angle. Observed and computed patterns for the first mode are illustrated in Plate 11, for the soft bottom conditions at the Potomac River Bridge. The computations were made from the distribution constants derived from hydrophone soundings beneath the source, and the agreement between theory and experiment is shown to be very close. Although similar computations may be made from the constants of the hard bottom, experimental data have shown that in this instance the agreement will probably be less satisfactory. The general patterns of records obtained over the two types of bottom (Plate 11) are, however, entirely distinct.

238. Hydrophone soundings made at considerable distance from the source possess certain advantages as experimental criteria for the study of sound transmission. The conditions under which they are made insure that the effective angles of incidence of the sound waves striking the bottom between the source and the hydrophone will be large, and that only the first mode will be present at the point of measurement. Hydrophone soundings beneath the source are made under conditions which insure normal incidence of the sound waves at the bottom, and a stimulation of the maximum number of modes. Although hydrophone soundings beneath the source are sufficient to determine the acoustic constants of the bottom, soundings made at a distance from the source provide an experimental measure of the pressure field distribution at a selected location on an actual transmission path. The interpretation of such soundings in terms of the physics of sound propagation is therefore direct and immediate.

239. If practical, actual range run recordings should be made, since they offer the most effective means of investigating the conditions which govern underwater sound transmission in any given area. A complete experimental study should include range run recordings, hydrophone soundings at a distance from the source, and hydrophone soundings directly beneath the source. If test facilities are limited, soundings beneath the source should be sufficient to characterize the acoustic behavior of the bottom.

Experimental recordings should be made at several frequencies, and particularly at frequencies lower than 200 cps. The interpretation of the physical phenomena will in general be most clear and satisfactory from records made at those frequencies which correspond to the simplest distribution patterns and to the modes of lowest order, especially the first and second.

X. APPLICATIONS OF THE RESULTS.

X. APPLICATIONS OF THE RESULTS.

240. The primary application of the foregoing analysis is to the interpretation of measurements of underwater sound fields in the acoustic system of the sea between surface and bottom. The transmission of sound in the sea is in general strongly influenced by the acoustic properties of the bottom although the effect of the bottom is least important if the water is deep or if the bottom is strongly sound-absorbent. The analysis applies to all situations in which there exist vertical standing-wave patterns of sound pressure, corresponding to the normal modes of vibration of the system.

241. The analysis is applicable to the sound pressure fields produced by effective or approximate point sources such as ship's propellers and auxiliaries, ship's hulls in vibration, torpedoes, Fessenden Oscillators, non-directional underwater projectors, and acoustic minesweeping devices (hammerboxes, parallel pipes, and kindred devices). The relatively narrow beams from standard echo ranging projectors are of course much less influenced by bottom reflections.

242. Although applicable to all frequencies, the analysis in terms of normal modes is most effective applied to sound fields of low audio frequency, where the wavelengths are comparable to the physical dimensions of the acoustic system. This is precisely the frequency range in which other methods of analysis are virtually inapplicable.

243. The measurement and analysis of underwater sound transmission, making use of the principles described in this report, may contribute to the solution of many specific problems of interest to the Navy. Such problems arise for example in tests of acoustic minesweeping devices and other low frequency sound sources; in the design of acoustic mines; the study of ship noises; the prediction of minesweeping ranges from hydrographic and acoustic data; the estimation of listening ranges for submarines over various types of sea and harbor bottom, and of the effectiveness of echo ranging in shallow water with small wide beam projectors (e.g. QBG and WEA-1). It may also be possible for our submarines to make use of the acoustic properties of the bottom in evasion tactics.

244. In the study of ship noises and in the testing of acoustic minesweeping devices, the acoustic properties of the range course (water depth, velocity of sound, and density of the bottom) will determine the rate of attenuation of the sound level with distance and will strongly influence the actual sound levels measured at various points in the system.

245. The measured sound level, for example, at a distance of 1000 ft from a given minesweeping source operating at 100 cps in water 55 ft deep may be as much as 30 db higher (factor of 32) if the bottom is hard than if it is strongly sound-absorbent. The difference between the level if the bottom is hard and the level if the bottom is soft but a good reflector, may be about 15 db at 1000 ft. Transmission curves are shown in Fig 9 for three cases for which experimental data are available: (a) hard bottom at the Potomac River Mouth, (b) soft reflecting bottom at the Potomac River Bridge, and (c) strongly sound-absorbent bottom in the Chesapeake Bay south of Smith Point.

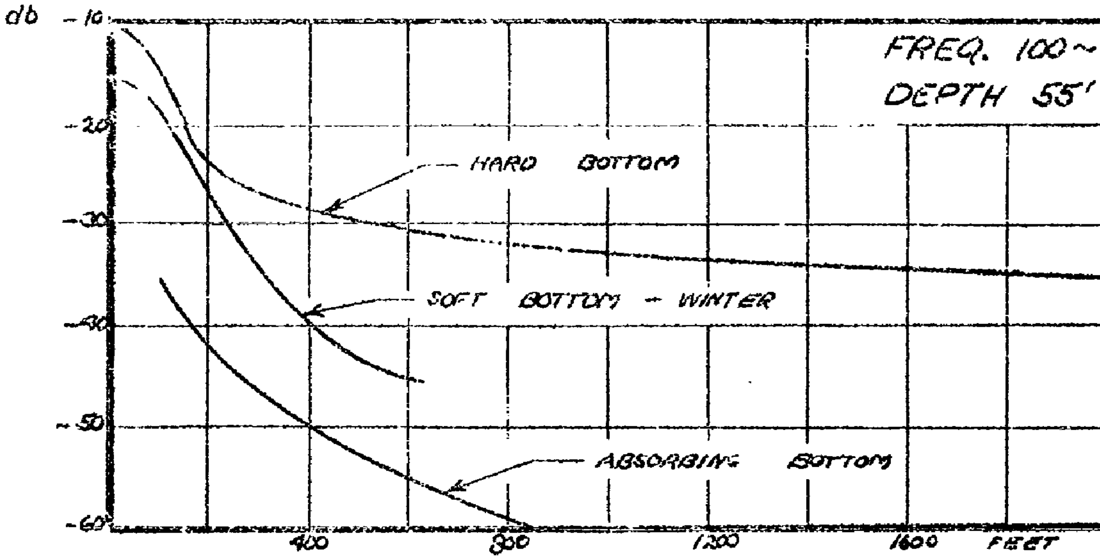

FIG. 9

246. It is clear from the figure that a sound source of sufficient intensity to sweep a 100-cycle mine at a distance of 1800 ft over the hard bottom, would have to approach within about 400 ft to fire the same mine over the soft bottom, and within 200 ft to fire it over the absorbing bottom. Another conclusion which may be drawn is that a given increase, say 6 db, in the pressure level obtainable from a sound source for minesweeping, may increase the firing range from 500 ft to 1800 ft if over a hard bottom, but only from 130 ft to 200 ft if over an absorbing bottom. The increased range obtained from a small improvement in the equipment is therefore much greater over hard bottom than over soft bottom.

247. At very low frequencies (i.e. when the depth is substantially less than a wavelength) the difference in sound level at a given distance from the source may be very much greater than that indicated in Fig. 9, depending upon whether the bottom is

hard or soft. At these frequencies conditions may be imagined for which the difference between the sound level over a hard and over a soft bottom may amount theoretically to more than 200 db at a distance of 1000 ft. Under such conditions the sound level from a powerful source would, at distances of the order of 200 to 300 ft from the source, drop below the water noise level. Recent experimental results are consistent with this expectation.

248. The effectiveness of a sweeping device may depend more upon the acoustic properties of the bottom over which it is used than upon its inherent capacity for generating sound. Measurements of the latter, including the sound output at 6 ft distance from the device, are entirely insufficient to enable the sweeping range to be estimated. Acoustic measurements of the performance of a device on a given range course will not in general represent its performance on other courses or in other areas. For some sound sources, such as the parallel pipes used in acoustic minesweeping, the performance tests must be made with the devices in motion on a range. Measurements made on different courses may, however, be reconciled and correlated, in accordance with the principles discussed in this report, by interpretation in terms of the acoustic properties of the bottom.

249. If measurements are made in an area where the bottom is acoustically soft, the acoustic properties of the bottom, and hence the sound transmission characteristics of the range, will change with the season (Bibliog. 4). The best propagation will occur in summer, and the poorest in winter. The seasonal difference may be very large at moderately low frequencies. This fact, heretofore unrecognized, has resulted in the reporting of apparent anomalies in the performance of acoustic noise-makers.

250. It should be possible to predict with reasonable accuracy the effectiveness of a sweeping device in any area for which adequate acoustic data can be obtained. In areas for which no acoustic data are available, a rough estimate of the sound transmission can frequently be made from hydrographic information alone. This should include the water depth and the material of the bottom, classified for example as clay, sand, hard, or sticky. There appears to be a close correlation between physical and acoustic "softness", and between physical and acoustic "hardness". "Soft" bottoms are in general associated with fine mud or clay deposits in semi-stagnant landlocked basins such as estuaries and river channels, and "hard" bottoms are associated with sands, gravels, and other coarse deposits common along the sea coasts and the continental shelves. The information on the hydrographic charts is, however, frequently incomplete, and in many cases out of date.

251. If the information on the charts could be supplemented with acoustic data, it should be possible to make reliable estimates of the underwater sound transmission to be expected in any desired area. Such data, obtained from a program of hydrophone soundings in key harbors and operational waters, would have great value for the problems of acoustic minesweeping and of underwater listening. Methods of making and interpreting the soundings are described in Section IX and in Bibliog. 4.

252. If studies of ship noise, particularly at low frequencies, are to be significant or valuable, the exact conditions under which they are made must be stated and all possible information about the acoustic properties of the range course and the test equipment should be recorded. Correct interpretation of experimental results may sometimes depend upon factors the relevance of which is not apparent at the time of measurement.

253. The character of measured transmission curves, for example, may be drastically altered by the effects of harmonic frequencies which have slipped through the filters without being recognized. This effect should be taken into account in acoustic-mine design by making sure that the mine cannot be swept or fired by higher frequency components than those for which it was designed. This is particularly important if the transmission varies markedly with frequency, i.e. if harmonics may be received at much higher level than the fundamental.

254. A few of the factors which may influence the results of ship-noise studies or transmission tests are: the frequency or frequency range involved in the tests, the placement of the hydrophone in relation to the bottom, the season of the year and temperature of the water (the acoustic properties may change with season and with gas content of the mud), and the roughness of both water surface and bottom. The effects of roughness, although unimportant at low frequency, determine the influence of scattering at higher frequencies.

255. An example of the difficulties which may arise in the interpretation of transmission measurements is given by tests made near New London. The measurement of very high "transmission losses" at frequencies of about 200 cps in this area has been reported (Bibliog. 15). It seem probable that the high attenuations resulted from the fact that the wavelengths were of the order of twice the water depth, and that the first mode was therefore strongly damped (See Fig. 8, Section VIII). The results of the measurements, made in very shallow water (less than 16 ft) over soft bottom,

are not at all representative of the transmission which would have been obtained over the same bottom in water 50 ft or more in depth.

256. Other difficulties may arise if "transmission losses" are estimated from point by point observations made close to the source. The dangers of the point by point method will be obvious after studying the records reproduced in Plates 2 and 3 of this report.

257. For most of the ranges on which ship-noise studies have been made, neither low frequency range runs of adequate length nor bottom impedance data are at present available. It seems clear from published ship-noise contours that the MIT range at Nahant has a relatively "hard" bottom, the acoustic properties of which are not uniform along the course. The Wolf Trap range seems to have a relatively "soft" bottom, with considerable absorption. It would be a simple matter to characterize these ranges by making appropriate acoustic measurements at several points along the courses. The properties which determine the influence of the bottom on underwater sound transmission may be estimated either from hydrophone soundings, or from range runs made under controlled conditions. Long runs at low frequencies are required to separate the effects of the modes. Such measurements should be made and interpreted for all the ranges employed for ship noise studies.

258. In the location of new range sites, the choice of an acoustically suitable location for ship noise or transmission studies would be facilitated by a preliminary survey of the acoustic properties of the bottom, using the methods described in this report (Section IX, The Use of Acoustic Measurements as Transmission Criteria).

259. Underwater sound measurements may be influenced by two other factors which have not always received adequate consideration. These are the position of the measuring hydrophone with reference to the bottom, and the effective depth of the sound source beneath the water surface.

260. In order to measure representative average values of sound pressures, hydrophones for the study of underwater sounds should be located a substantial fraction of a wavelength away from the bounding surfaces (surface and bottom). If the hydrophone is placed close to such a boundary it is necessary to correct for, or otherwise take into account, the position of the hydrophone in the standing wave pattern. This factor has been discussed in detail in a previous report (Bibliog.4).

261. The effective depth of the sound source should also be considered, since it determines the relative degree of stimulation of the possible modes. If the source depth is subject to control, as in acoustic minesweeping tests, it should be made at least a quarter wavelength for the lowest emitted frequency, in order to assure adequate stimulation of the lowest mode.

262. If the source is ship-mounted, it should be located at least a quarter wavelength from the nearest "pressure release" surface. At frequencies lower than about 1000 cps the usual ship's hull behaves acoustically in a manner similar to the surface of the sea. The effective normal impedance of the hull is very low, and the surface of the hull is an approximate pressure node at these frequencies. These considerations have been shown to be of the utmost importance to the design of mounting gear for acoustic minesweeping, especially at the lower frequencies.

263. The ranges at which propeller sounds from enemy vessels may be heard will depend upon the properties of the bottom in much the same manner as did the minesweeping ranges discussed previously. Extraordinarily long listening ranges have been reported from our submarine patrols in the southwest Pacific. These were undoubtedly made possible by "guided" transmission over the hard bottom which is prevalent in the area. Comparatively short listening ranges are to be expected over "soft" or strongly absorbing bottoms, the transmission over which may be strongly damped.

264. Estimates and predictions of submarine listening ranges, based upon tests in a given area, will in general be valid only for that area, unless changes in the character of the bottom are properly allowed for. Obviously much work remains to be done before the effect of the bottom on long range transmission can be reliably computed. The importance of this factor, and the magnitude of the errors which may arise from neglecting it, are illustrated by the results of the present analysis.

265. The possibility that low audio frequencies may suffer considerable distortion in transmission over hard bottom may be deduced from the analysis of the critical phenomenon (Section V and Section VII-B). It has been shown that under certain conditions a critical frequency may exist below which the transmission is highly damped ("hybrid" transmission) and above which propagation takes place with very low attenuation ("guided" transmission). The system of the sea between surface and bottom may therefore act as if it were a high-pass acoustic filter with a definite "cut-off" frequency. Sound waves of lower frequency than that for which the

water depth is a quarter wavelength will be poorly transmitted over hard bottom because of the critical phenomenon. The damping of such frequencies over soft bottom is extraordinarily high (See Fig. 8). Advantage could be taken of the poor transmission of the extremely low frequencies, in the design of an acoustic mine which would be effective but difficult to sweep.

266. Another type of distortion which may occur over both hard and soft bottom arises from the fact that the modes travel with different phase velocities. This effect may be visualized as a type of dispersion, in which case the resulting distortion will be somewhat analogous to that exhibited by a long non-loaded telephone cable. The distortion, which should manifest itself as a loss of intelligibility of speech or signals, should be most pronounced under conditions which involve transmission in a small number of modes.

267. The application of the normal mode analysis to very high frequencies is complicated by the strongly directional character of supersonic beams, and by the existence of additional factors which are unimportant at low frequencies. These are refraction due to gradients of temperature, hydrostatic pressure, and salinity, and attenuation caused by absorption and scattering in the water itself as distinct from the bounding surfaces. It is not profitable to distinguish between the individual modes at supersonic frequencies, because many are stimulated and adjacent modes are close together. Since the wavelengths are short in comparison with the dimensions of the acoustic system, the optical analogy of beams and rays from point sources is valid, and computations based upon the image theory (Bibliog. 14) give reasonable agreement with experiment.

268. Although the analysis into individual modes is not immediately helpful in problems of high frequency sound transmission, the acoustic characterization of the sea bottom on the bases of hydrophone soundings may have important applications. For example it is possible that familiarity with the types of bottom associated with poor sound propagation might be of assistance to a submarine employing evasion tactics. This could take the form of rendering the enemy listening less effective by "hiding" above a soft or strongly absorbent bottom, or of rendering the submarine difficult to pick out from the background by "matching" the impedance of the submarine to that of the bottom. Although little is known at present about the impedance of submarines, the possibility exists that this quantity may be subject to alteration by acoustic treatment of the hull surfaces. The impedance of the bottom in the

areas of operation could of course be measured and known beforehand. Tests are under way to determine the extent to which the acoustic classification of bottoms, made from low frequency hydrophone soundings, are valid at echo ranging frequencies.

269. As a result of the analysis given in this report the interference patterns observed on the experimental records have been explained quantitatively and in detail in terms of the interactions of the normal modes of vibration between surface and bottom. Recent attempts, both in this country and abroad, to interpret low frequency transmission phenomena as the result of interference between direct and surface reflected "beams", would appear from our analysis to be doomed to failure. It was stated in Section VI (Interpretation of Range Records) that the observed patterns could not result from any possible combination of direct and reflected "rays" from fixed point sources. The image theory is valid only under special limiting conditions, such as very great water depth or complete absorption of sound at the bottom. Under these limiting conditions the effects of bottom reflections on transmission are negligible, and the normal modes are weakly stimulated and highly damped. The image theory cannot be expected to give valid explanations of transmission phenomena which occur when bottom reflections are important.

270. In general the influence of bottom reflections is pronounced, and the normal modes play a dominant role in transmission. Although this was pointed out by the MIT group in 1941 (Bibliog. 10), the importance of the modes as aids to interpretation does not appear to have been generally recognized. This may be attributed, at least in part, to the inadequacy of the transmission theory formulated in terms of normal impedance. The shortcomings of the theory involving impedance have been discussed (Sections III and VII) and remedied (Section IV), and a new analysis in terms of the modes has been presented in detail (Sections IV, V, VI, and IX). The new analysis gives reasonable interpretations of the phenomena which have been observed to date.

XI. CONCLUSIONS

XI. CONCLUSIONS.

271. The correlation and theoretical analysis of a large number of transmission measurements made in the Potomac River and Chesapeake Bay areas warrants the conclusions which follow.

272. Given a source of audio frequency sound in water, the sound pressure level which may be measured at appreciable distances will be influenced in a complicated way by the boundaries, especially by the bottom. The sea bottom may be classified by its behavior for sound as "soft", "hard", or intermediate in character. Specifically, the velocity of sound and the density of the bottom may be estimated from acoustic measurements (hydrophone soundings and range runs), by sampling, and from hydrographic data.

273. The effect of the boundaries on sound propagation from a ship-mounted source may be determined from an analysis of the normal modes of vibration of the acoustic system of the sea between surface and bottom. The analysis is particularly effective for frequencies at which the wavelengths are comparable to the physical dimensions of the system. For low frequencies and given bottom conditions, the most important factor governing the propagation is the ratio of the depth of the water to the free wavelength of the sound. A detailed interpretation of the observed transmission phenomena may be given in terms of the initial stimulation, the relative attenuation, and the phase velocities of the modes. Damping and phase constants may be determined for each mode by means of special charts which give the propagation and distribution constants in terms of the acoustic properties of the system. The overall transmission is given as the sum of the effects produced by the individual modes.

274. "Damped" transmission, which occurs over "soft" bottom, is always accompanied by considerable attenuation, the amount ranging in practice from about 12 db/1000 ft as a lower limit to about 40 db/1000 ft under conditions of strong bottom absorption. The attenuation over soft bottoms is relatively independent of frequency in the range where the wavelength is substantially less than twice the depth. The attenuation becomes much greater than the above limits and the system becomes more difficult to excite as the frequency is lowered below the "natural frequency" of the first mode, at which the water depth is approximately a half wavelength. "Soft" bottoms are frequently encountered in landlocked basins such as fiords, estuaries, and river channels.

275. "Guided" transmission, which occurs over "hard" bottom at all frequencies higher than a minimum critical frequency, is characterized by negligible damping due to bottom absorption and by an attenuation with distance caused primarily by cylindrical spreading from the source. The latter amounts to 3 db per distance double. "Hybrid" transmission, which occurs over hard bottom at frequencies below the lowest critical frequency, is associated with large and erratic damping from bottom absorption. The critical frequency depends upon the hardness of the bottom, and is slightly higher than the frequency for which the water depth is equal to a quarter wavelength. The acoustic system of the sea between the surface and a hard bottom thus acts as if it were a high-pass filter, analogous to an electromagnetic "wave guide". This type of sound transmission is very commonly encountered, since hard bottom predominates along the sea coasts of all the continents.

276. The measurements and analysis of underwater sound transmission, making use of the principles described in this report, may contribute to the solution of specific problems of Naval interest arising, for example, from tests of acoustic minesweeping devices, the design and location of acoustic mines, the study of ship noises, the prediction of minesweeping ranges from hydrographic and acoustic data, the estimation of listening ranges for submarines, and the effectiveness of echo ranging in shallow water.

277. Previous analyses of underwater sound transmission in shallow water have been inadequate to explain the results of observations. The analysis described in this report based upon accepted physical principles gives consistent, coherent, and reasonably complete explanations of the observed transmission phenomena. It not only explains certain apparent anomalies of underwater sound transmission but resolves many discrepancies between the results previously obtained by various laboratories.

APPENDIX A.

NORMAL MODE THEORY OF PROPAGATION OF UNDERWATER SOUND OVER HOMOGENEOUS BOTTOM.

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NORMAL MODE THEORY OF PROPAGATION OF UNDERWATER SOUND OVER HOMOGENEOUS BOTTOM.

1. Derivation of the Equation Which Expresses the Influence of the Boundaries on the Field Distribution.

278. The mathematical derivations applicable to the propagation of low-frequency sound in shallow water are presented in this section. The theory for the actual acoustic system, consisting of an infinite expanse of water confined between the surface and the sea bottom, may be derived by extension from the general theory of the propagation of sound in rectangular tubes. The transmission in the sea will be modified by cylindrical spreading which is not present in the idealized system, the pipe. The conditions under which the analysis is valid and the significance of the theoretical results are discussed in the text.

279. The wave equation for sound pressure (See for example Bibliog. 7) may be written

$$\nabla^2 P = \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} \qquad (1)$$

280. For the detailed analysis we shall consider the rectangular pipe in the coordinate system shown in Fig. 10. The origin is taken at an upper corner. Let the height be h, measured along the y axis; the width w, measured along the z axis; and the length infinite in the direction parallel to the x axis. Let it be assumed that the two sides, parallel to the x-y plane, are rigid; that the top, parallel to the x-z plane, is a free surface (perfectly reflecting); and that the bottom of the tube is bounded by a homogeneous fluid infinitely extended in a direction normal to the x-z plane. The positive direction for y is taken downwards.

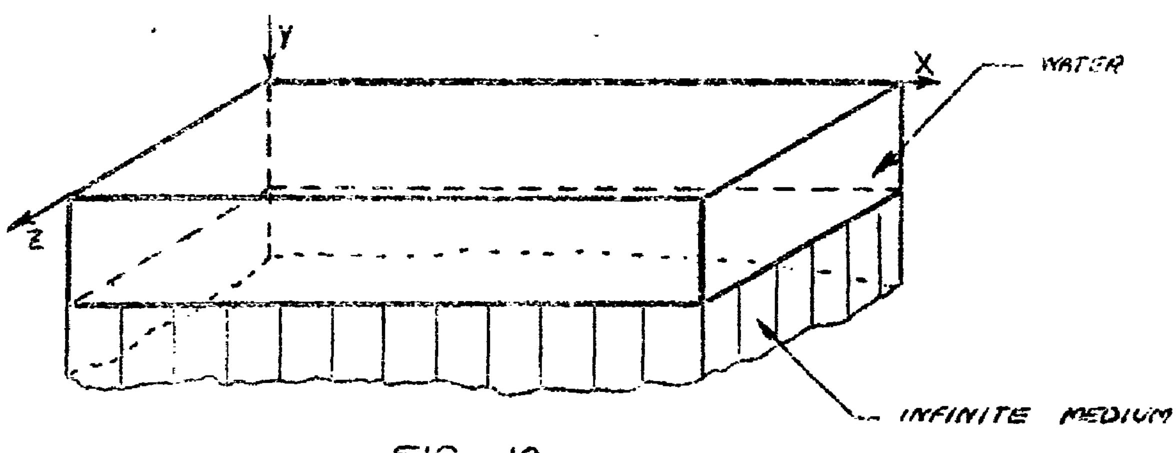


FIG. 10

281. The simplest class of "transverse" waves which may be propagated down the tube will have uniform pressure distribution in the z direction, and the pressure variations in the tube may therefore be expressed as functions of x and y only. The solution of the wave equation (1) for the pressure distribution in a single mode of the "transverse" class of waves may be written, for the nth mode,

$$
\begin{aligned}
P_{(n)} &= X(x)\;Y(y) \\
X(x) &= X_0\,\epsilon^{[j\omega t - \omega/c(\sigma_n + j\tau_n)x]} \\
Y(y) &= Y_0 \sinh(\pi y/h)(\kappa_n - j\mu_n)
\end{aligned}
\qquad (2)
$$

282. The function X(x) represents a progressive wave, attenuated exponentially as it travels down the pipe parallel to the x axis. A sinusoidal variation of pressure with time, which does not limit the generality of the expression, is assumed. The angular velocity ω is equal to 2π times the frequency of the sound source. The complex propagation constant $\sigma + j\tau$ for the wave consists of a real part σ which determines the attenuation in the x direction, and an imaginary part τ which determines the phase velocity. The phase velocity of the wave c/τ is always greater than c, the velocity in free space.

283. The function Y(y) represents a standing wave system between the top and the bottom of the pipe, having a pattern of nodes and loops given by the complex distribution constant $\kappa - j\mu$. The sinh rather than the cosh form of expression is used in order to make the pressure reduce to zero at the free surface, y = 0.

284. Each mode is represented by an expression of the type shown in equations (2), where the values of the distribution and propagation constants determine the transmission properties of the system for the particular mode. The components of the distribution constant $\kappa - j\mu$ are determined for each mode by the boundary conditions in a manner discussed later. The components of the propagation constant $\sigma + j\tau$ are related to κ and μ, and through them to the boundary conditions. The actual sound pressure at any given point inside the pipe will be given by the summation of the contributions from all the modes, taking both magnitude and phase into account.

285. The discussion will continue in terms of the solution for a single mode, with the understanding that the behavior of the system for different modes will differ only through the values of

the distribution and propagation constants. Subscripts indicating the order of the mode are dropped, and hereafter the subscript 1 is used for quantities in the first medium, inside the pipe, and the subscript 2 for quantities in the second medium which fills the region y>h.

286. The pressure distribution in the medium enclosed by the pipe, for a typical mode belonging to the simple "transverse" class of waves, is then

$$P_1 = P_{01} \sinh[\pi y/h\,(\kappa - j\mu)]\, \varepsilon^{[j\omega t - \omega/c_1(\sigma + j\tau)x]} \quad (3)$$

287. To determine the manner in which κ and μ depend upon σ and τ, equation (3) is substituted into the wave equation, (1). This results in a relation between the constants.

$$(\pi/h)^2(\kappa - j\mu)^2 + (\omega/c_1)^2(\sigma + j\tau)^2 = -(\omega/c_1)^2 \quad (4)$$

The further substitution $\eta_1 = 2h/\lambda$, where $\lambda = \frac{2\pi c_1}{\omega}$ is the wave-length in free space having velocity c_1, yields the form

$$(\sigma + j\tau)^2 + 1/\eta_1^2(\kappa - j\mu)^2 + 1 = 0 \quad (5)$$

288. The relation between $\sigma + j\tau$ and $\kappa - j\mu$ given by this equation has exactly the same form, whether or not the boundary conditions are evaluated in terms of an impedance. Consequently the chart reproduced in Bibliog. (10) p. 315, giving the conformal transformation between the components of the distribution constant $\kappa - j\mu$ and those of the propagation constant $\sigma + j\tau$ is equally applicable to the problem under discussion. By means of such a chart, or by computation from equation (5), the phase velocity and the attenuation can be found for any value of η_1 once κ and μ are known.

289. The components of the distribution constant $\kappa - j\mu$ must now be evaluated in terms of the boundary conditions. Since these will not be represented by a normal impedance, it will be necessary to assume a pressure distribution in the second medium, the homogeneous fluid. Since the second medium extends to infinity in the y direction there can be no reflection terms, and the distribution

for any one mode may be represented by a general exponential decay term in the y direction as well as in the x direction. The general expression for the pressure distribution in the second medium may therefore be written:

$$P_2 = P_{o2}\,\varepsilon^{-\pi y/h\,[\kappa'-j\mu']}\,\varepsilon^{[j\omega t - \omega/c_1(\sigma'+j\tau')x]} \tag{6}$$

where the primed quantities represent the components of the distribution constant $\kappa'-j\mu'$ and propagation constant $\sigma'+j\tau'$ in the second medium. The significance of these quantities is similar to that of the analogous constants in the first medium.

290. Substitution of equation (6) into the wave equation for the second medium results in a relation between the constants of the second medium which is of the same form as equation (5). Thus

$$(\sigma'+j\tau')^2 + 1/\eta_2^2(\kappa'-j\mu')^2 + 1 = 0 \tag{7}$$

$$\eta_2 = 2h/\lambda_2$$

291. A relation between the distribution and propagation constants of the second medium and those of the first medium may now be obtained. In order to do this, the boundary conditions of continuity of pressure and of normal particle velocity must be applied. These are

$$\left.\begin{matrix} P_1 = P_2 \\ \dot{\xi}_1 = \dot{\xi}_2 \end{matrix}\right\} \text{AT} \quad y = h \tag{8}$$

292. The normal component of the particle velocity is given in terms of the pressure and density in each medium by the relations

$$\dot{\xi}_1 = \frac{-j}{\rho_1\omega}\cdot\frac{\partial P_1}{\partial y} \quad \text{and} \quad \dot{\xi}_2 = \frac{-j}{\rho_2\omega}\cdot\frac{\partial P_2}{\partial y} \tag{9}$$

Applying the second boundary condition

$$\frac{1}{\rho_1}\frac{\partial P_1}{\partial y} = \frac{1}{\rho_2}\frac{\partial P_2}{\partial y} \tag{10}$$

[ρ_1 and ρ_2 are densities]

Applying the first boundary condition, $P_1 = P_2$ at $y = h$

$$P_{01} \sinh \pi(\kappa - j\mu) \cdot \varepsilon^{[j\omega t - \omega/c_1(\sigma + j\tau)x]} = P_{02} \varepsilon^{-\pi(\kappa' - j\mu')} \cdot \varepsilon^{[j\omega t - \omega/c_2(\sigma' + j\tau')x]} \qquad (11)$$

293. This equation, considered as a relation between products of the form $[X(x) \cdot Y(y)]_1 = [X(x) \cdot Y(y)]_2$, can be separated into two $[X(x)]_1 = [X(x)]_2$ and $[Y(y)]_1 = [Y(y)]_2$; which must be true independently since the functions of x are not functions of y and vice versa. This means, from the physical standpoint, that the character of the pressure distribution in the y direction is not a function of position along the tube, or of time.

294. Separating equation (11) into two parts we have

$$1/c_1(\sigma + j\tau) = 1/c_2(\sigma' + j\tau') \qquad (12)$$

$$P_{01} \sinh \pi(\kappa - j\mu) = P_{02}\, \varepsilon^{-\pi(\kappa' - j\mu')} \qquad (13)$$

295. Application of equation (10), for continuity of normal particle velocity at the boundary, to equation (11) gives

$$1/\rho_1 P_{01}(\kappa - j\mu) \cosh \pi(\kappa - j\mu) = -1/\rho_2 P_{02}(\kappa' - j\mu') \varepsilon^{-\pi(\kappa' - j\mu')} \qquad (14)$$

296. If equation (13) be divided by equation (14), a transcendental equation is obtained relating the distribution constants in the two media

$$\frac{\rho_1 \tanh \pi(\kappa - j\mu)}{\kappa - j\mu} = \frac{-\rho_2}{\kappa' - j\mu'} \qquad (15)$$

297. This equation can be expressed in a form more convenient for computation by eliminating κ' and μ' by means of the previously determined relations between the constants, in equations (5), (7), and (12).

298. Since $\eta/\eta_2 = c_2/c_1$ equation (12) may be rewritten

$$(\sigma + j\tau) = \eta_2/\eta_1 (\sigma' + j\tau'), \qquad (16)$$

by combination of (16) with (7) and (5) we obtain

$$(\kappa - j\mu)^2 + \eta_1^2 = (\kappa' - j\mu') + \eta_2^2 \qquad (17)$$

299. The separation of the real and imaginary parts of equation (17) provides the relations

$$\kappa^2 - \mu^2 + \eta_1^2 = \kappa'^2 - \mu'^2 + \eta_2^2$$
$$\mu\kappa = \mu'\kappa' \qquad (18)$$

The substitution of these equations into the square of equation (16) gives

$$\left[\frac{\rho_1}{\rho_2}\right] \cdot \frac{\tanh^2 \pi(\kappa - j\mu)}{(\kappa - j\mu)^2} = \frac{1}{\kappa^2 - \mu^2 - 2j\kappa\mu - (\eta_2^2 - \eta_1^2)} \qquad (19)$$

whence

$$\frac{\tanh^2 \pi(\kappa - j\mu)}{(\kappa - j\mu)^2} = \frac{A^2}{(\kappa - j\mu)^2 - B^2} \qquad (20)$$

in which

$$A = \rho_2/\rho_1 \quad \text{and}$$
$$B = [\eta_2^2 - \eta_1^2]^{1/2} = \eta_1 [(c_1/c_2)^2 - 1] = \frac{2fh}{c_1} [(c_1/c_2)^2 - 1]^{1/2} \qquad (21)$$

300. With the derivation of equation (20) the object of the analysis is achieved, since a relationship has been obtained between the acoustic properties of the media (density and velocity ratios), the geometrical conditions (the depth of the water measured in half wavelengths), and the distribution constant $\kappa - j\mu$ for any mode belonging to the simplest type of "transverse" waves in the first medium.

301. Once the components κ and μ of the distribution constant are known from equation (20), the attenuation and the phase velocity of the wave, for the particular mode, may be obtained by computation from equation (5) or more conveniently from a chart (Plate 17) giving σ and τ in terms of κ and μ. The final pressure distribution may be built up by adding the contributions of the various modes, each determined by the method outlined above.

2. Forms of the Equation Convenient For Computation.

302. Although there are no simple analytic means of solving equation (20) for κ and μ in terms of A and B, it may be solved

for A and B in terms of κ and μ. If this is done the following expressions are obtained:

$$A = \frac{\rho_2}{\rho_1} \frac{\cosh 2\pi\kappa - \cos 2\pi\mu}{\left\{\left[\frac{\mu^2-\kappa^2}{\mu\kappa}\right]\left[\sinh 2\pi\kappa \sin 2\pi\mu\right] + \left[(\sinh 2\pi\kappa + \sin 2\pi\mu)(\sinh 2\pi\kappa - \sin 2\pi\mu)\right]\right\}^{1/2}}$$

$$B = \frac{2fh}{c_1}\left[\left(\frac{c}{c_1}\right)^2 - 1\right] = \left\{\frac{\left[(\mu+\kappa)\sinh 2\pi\kappa + (\mu-\kappa)\sin 2\pi\mu\right]\left[(\mu-\kappa)\sinh 2\pi\kappa - (\mu+\kappa)\sin 2\pi\mu\right]}{\left[\frac{\mu^2-\kappa^2}{\mu\kappa}\right]\left[\sinh 2\pi\kappa \sin 2\pi\mu\right] + \left[(\sinh 2\pi\kappa + \sin 2\pi\mu)(\sinh 2\pi\kappa - \sin 2\pi\mu)\right]} - \mu^2 + \kappa^2\right\}^{1/2}$$

These expressions were used in computing those portions of the distribution charts which correspond to "damped" and "hybrid" transmission. The charts were plotted for convenience on a logarithmic scale for B and a linear scale for A. If, however, the computed values are plotted with linear scales for both A and B, the curves approach straight lines for values of B greater than those appearing on the charts. This fact might be useful in obtaining approximate extrapolation curves for values of B beyond the limits of the published charts (Plates 12-16 inc).

303. The portions of the charts which correspond to "guided" transmission cannot be obtained directly from equation (20). They may be obtained, however, by setting equal to zero in equation (15), which is then written in the form

$$\tan(\pi\mu) = \frac{-A\mu}{\kappa' - j\mu'} \qquad (24)$$

304. The distribution constants for the second medium, κ' and μ' must be eliminated from this equation. If reals and imaginaries are separated from equations (5), (7), and (16) the following relations are obtained:

$$\sigma = \frac{\eta}{\eta_1}\sigma' \qquad (25)$$

$$\tau = \frac{\eta}{\eta_1}\tau' \qquad (26)$$

$$\mu\kappa = \mu'\kappa' = \eta_1^2\sigma\tau \qquad (27)$$

On substitution of $K = 0$ into (27) it may be seen that σ must equal zero, since η and τ are always greater than zero. If σ is zero, the traveling wave will be propagated without damping, and the transmission will be of the "guided" type.

305. In all but the limiting case, K' must be positive and finite, since it determines the rate of decay of energy with increasing distance into the bottom. Negative values of K' would correspond to an increase in energy with increasing distance downward. Since this is physically impossible K' must be always positive. Equation (27) requires that, if K equals zero, the product $\mu' K'$ must also equal zero. Since K' is known to be finite, μ' must equal zero and equation (24) therefore reduces to

$$\tan \pi\mu = -A \frac{\mu}{K'}$$

This equation can be satisfied only by values of μ between certain limits

$$\mu = \left\{ n - \left[\tfrac{1}{2} > a > 0 \right] \right\} \qquad n = 1,\ 2,\ 3,\ \ldots \qquad (28)$$

Thus $\mu = 1/2$ to 1, 1 1/2 to 2, etc. Only values of μ within the allowed limits may be used for computing points for those portions of the charts which represent "guided" transmission.

306. The actual expressions used for computing the curves on Plates 13 and 14 are obtained by setting $K = 0$ in the transcendental equation, (20). Thus

$$B = \frac{-j\mu}{\tan \pi\mu} \sqrt{A^2 + \tan^2 \pi\mu} \qquad (29)$$

or

$$jB = \frac{\mu}{\tan \pi\mu} \sqrt{A^2 + \tan^2 \pi\mu} \qquad (30)$$

where the allowed values of μ are those given by (28) above.

307. Each range of values of μ is limited by a critical value, 1/2, 1 1/2, 2 1/2, etc. at which the constant μ curve degenerates into a straight line parallel to the A axis and having the abscissa $jB = -\mu$. Each of these lines defines the critical frequency for a mode. Since the lines are parallel to the A axis, the critical frequency must be independent of the density ratio, and dependent only upon the water depth and the velocity ratio. The critical frequency for the n th mode may be obtained from the definition of B, equation (21).

308. The critical values of μ are $\mu_n = 1/2\ (2n - 1)$ and the corresponding critical frequencies are

$$f_n = \frac{c_1 (2n-1)}{4h} \quad \frac{1}{\left[1 - (c_1/c_2)^2\right]^{1/2}} \tag{31}$$

If c_2 is appreciably greater than c_1 the expression $\left[1 - (c_1/c_2)^2\right]^{1/2}$ is nearly equal to unity, and the critical frequency for the first mode will have a value slightly higher than that for which the depth is a quarter wavelength.

309. Forms of equations (22) and (23) which are valid for computations in the limiting case when $K \to 0$ are

$$\underset{K \to 0}{A} \longrightarrow \frac{(1 - \cos 2\pi\mu)}{\left\{\sin 2\pi\mu (2\pi\mu - \sin 2\pi\mu)\right\}^{1/2}} \tag{32}$$

$$\underset{K \to 0}{B} \longrightarrow \frac{\mu\ (2\pi\mu)^{1/2}}{(\sin 2\pi\mu - 2\pi\mu)^{1/2}} \tag{33}$$

In the limiting case when $\mu \to 0$, these become

$$\underset{\mu \to 0}{A} \longrightarrow \frac{(\cosh 2\pi K - 1)}{\left\{\sinh 2\pi K (\sinh 2\pi K - 2\pi K)\right\}^{1/2}} \tag{34}$$

$$\underset{\mu \to 0}{B} \longrightarrow \frac{K\ (2\pi K)^{1/2}}{(2\pi K - \sinh 2\pi\mu)^{1/2}} \tag{35}$$

3. <u>Relations Between the Constants.</u>

310. <u>The relation between the propagation and the distribution constants</u> is given by equation (5) in Appendix A, part one. This relation is shown graphically in Plate 17 for a considerable range of values. Extrapolation to values not given on the chart

should be facilitated by using the following forms of equation (5).

311. Separating reals and imaginaries in equation (5) we have

$$\left.\begin{aligned}\tau^2-\sigma^2-1 &= \left(\frac{K}{\eta_1}\right)^2-\left(\frac{\mu}{\eta_1}\right)^2\\ \tau &= \mu K/\eta_1^2\sigma\end{aligned}\right\} \quad (36)$$

Let $\quad R = \left[\frac{\mu K}{\eta_1^2}\right]^2 \quad (37)$

and $\quad S = \left(\frac{K}{\eta_1}\right)^2-\left(\frac{\mu}{\eta_1}\right)^2 \quad (38)$

312. Employing these abbreviations it may be shown that equations (36) yield the relations

$$\sigma^4+\sigma^2(S+1)-R = 0 \quad (39)$$

$$\tau^4-\tau^2(S+1)-R = 0 \quad (40)$$

from which

$$\sigma = \frac{1}{\sqrt{2}}\left\{-(S+1)+\sqrt{(S+1)^2+4R}\right\}^{1/2} \quad (41)$$

$$\tau = \frac{1}{\sqrt{2}}\left\{(S+1)+\sqrt{(S+1)^2+4R}\right\}^{1/2} \quad (42)$$

In the special case for which $\sigma \ll \tau$, equations (36) reduce to

$$\sigma = \left(\frac{R}{1+S}\right)^{1/2} = \frac{R}{\tau} \quad (43)$$

$$\tau = \sqrt{1+S} \quad (44)$$

313. <u>The computation of the phase velocity of any mode at its critical frequency</u> exhibits an interesting relation.

314. The phase velocity of the n th mode has been defined, in paragraph 61, Section IV, as

$$V_N \equiv c_1/\tau \quad (45)$$

The quantity τ_N may be expressed, from equation (36), as

$$\tau_N^2 = 1 - \mu_N^2/\eta_1^2 \qquad (46)$$

which is valid if K and σ are zero. By definition $\eta_1^2 = 4\xi^2 h^2/c_1^2$. At the critical frequency equation (45) may be rewritten, with τ_N expressed in terms of μ_N and f_c through equations (28) and (31). The result of these substitutions is

$$V_N = \frac{c_1}{\sqrt{1 - \frac{(1/2)^2(2N-1)^2}{4f_c^2\, h^2/c_1^2}}} = \frac{c_1}{\sqrt{1 - \left[1 - (c_1/c_2)^2\right]}} = C_2 \qquad (47)$$

315. This formula shows that for any mode at its critical frequency, the phase velocity of the "guided" waves becomes equal to the free space wave velocity in the bottom.

316. The general reflection law for a sea bottom characterized by density and velocity ratios is:

$$K = \frac{1 - \frac{\rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1}}{1 + \frac{\rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1}} \qquad (48)$$

317. The reflection law for the same sea bottom, characterized by a normal acoustic impedance, Z, is

$$K = \frac{1 - \rho_1 c_1/Z \cos\theta_1}{1 + \rho_1 c_1/Z \cos\theta_1} \qquad (49)$$

Snell's law for sound refraction is

$$\cos\theta_2 = \left\{1 - (c_2/c_1)^2 \sin^2\theta_1\right\}^{1/2} \qquad (50)$$

The condition under which the impedance Z is independent of the angle of incidence is

$$(c_1/c_2)^2 \gg \sin^2\theta_1 \qquad (51)$$

318. <u>The pressure at any point, x, y, in the pressure field of an acoustic dipole</u> (source and its virtual image above the surface) may be written

$$P_{x,y} \sim \left\{\frac{1}{r_1^2}+\frac{1}{r_2^2}-\frac{2}{r_1 r_2}\cos\left[\frac{\omega}{c}(r_1-r_2)\right]\right\}^{1/2} \qquad (52)$$

where $r_1 = \sqrt{x^2+(y-d)^2}$

$r_2 = \sqrt{x^2+(y+d)^2}$

x is the horizontal distance from the source,

d is the source depth,

y is the receiver depth.

4. <u>Analysis of Mode Propagation in Terms of Elementary Plane Waves.</u>

319. In the text, Section VII-B, the "guided" wave associated with any one mode of the acoustic system was interpreted as the synthesis of two sets of elementary plane waves. The derivations which justify this interpretation are presented in this portion of Appendix A. It will be shown that the critical frequency of a mode in "guided" transmission corresponds to that frequency for which the elementary plane waves are incident at the bottom at the critical angle for total internal reflection. The latter is given by Snell's law, equation (50).

320. For "guided" transmission, k and σ are both zero, and the expression for the sound pressure in the n th mode (equation 3) takes the form

$$P_N = P_{ON} \sinh\left[-j\left(\frac{\pi\mu_N}{h}\right)y\right] \varepsilon^{-j\left[\left(\frac{\omega\tau_N}{c_1}\right)x-\omega t\right]} \qquad (53)$$

This may be written

$$P_N = C_N \sin(my)\cos(\ell x-\omega t) \qquad (54)$$

where

$$m = \pi\mu_N/h \quad \text{and} \quad \ell = \omega\tau_N/c_1$$

321. Equation (54) may be rearranged as follows

$$P_N = \frac{C_N}{2}\left[\sin(\ell x + my - \omega t) - \sin(\ell x - my - \omega t)\right] \qquad (55)$$

The form of this expression shows that the pressure field for each mode may be considered to result from the combination of two sets of plane waves reflected alternately at the surface and the bottom, and making an angle θ with the normal to the boundaries. For the angle θ

$$\tan\theta = \pm \ell/m \qquad \sin\theta = \pm \ell/\sqrt{m^2+\ell^2} \qquad (56)$$

The positive sign corresponds to one set of waves and the negative sign to the other set.

322. Equation (56) above may be expressed in terms of the constants of the system, c_1/c_2 and h, as follows: Equation (46) becomes, at the critical frequency of any mode

$$\tau_N^2 = 1 - \frac{c_1^2\pi^2}{\omega_c^2 h^2}\mu_N^2 \qquad (57)$$

From the definitions of m and l:

$$\ell^2 = \left(\frac{\omega_c}{c_1}\right)^2\tau_N^2 = \left(\frac{\omega_c^2}{c_1^2} - \frac{\pi^2\mu_N^2}{h^2}\right) = \left(\frac{\omega_c^2}{c_1^2} - m^2\right)$$
$$\ell^2 + m^2 = \omega_c^2/c_1^2 \qquad (58)$$

Thus at the critical angle θ_c

$$\sin\theta_c = \frac{\sqrt{\omega_c^2/c_1^2 - m^2}}{\omega_c/c_1} \qquad (59)$$

Inserting values of ω_c and μ_N from equations (28) and (31) the above expression may be reduced to

$$\sin\theta_c = c/c_2 \qquad (60)$$

This is just the critical angle for total internal reflection of plane waves, in accordance with Snell's law of refraction.

5. Vertical Distributions of Sound Pressure.

323. The form of the pressure distribution between surface and bottom, which corresponds to a given mode in the acoustic system, has been given in equation (3). Writing only the term which gives the variation of sound pressure with depth, y, we have

$$P \sim \sinh\left[\pi y/h\,(\kappa - j\mu)\right] \tag{61}$$

324. Equation (61) may be expanded into the form

$$P \sim \left[\sinh\left(\tfrac{\pi y}{h}\kappa\right)\cos\left(\tfrac{\pi y}{h}\mu\right) - j\cosh\left(\tfrac{\pi y}{h}\kappa\right)\sin\left(\tfrac{\pi y}{h}\mu\right)\right] \tag{62}$$

The rms sound pressure measured by the hydrophone is proportional to the square root of the sum of the squares of the real and imaginary parts of equation (62). Thus

$$|P| \sim \left[\sinh^2\left(\tfrac{\pi y}{h}\kappa\right)\cos^2\left(\tfrac{\pi y}{h}\mu\right) + \cosh^2\left(\tfrac{\pi y}{h}\kappa\right)\sin^2\left(\tfrac{\pi y}{h}\mu\right)\right]^{1/2} \tag{63}$$

This may be simplified to the form

$$|P| \sim \left[\cosh^2\left(\tfrac{\pi y}{h}\kappa\right) - \cos^2\left(\tfrac{\pi y}{h}\mu\right)\right]^{1/2} \tag{64}$$

Applying the half-angle formulae, this reduces to

$$|P| \sim \left[\cosh\left(\tfrac{2\pi y}{h}\kappa\right) - \cos\left(\tfrac{2\pi y}{h}\mu\right)\right]^{1/2} \tag{65}$$

325. This expression gives the vertical distribution of sound pressure corresponding to any given mode, in terms of the distribution constants for that mode. In general the measured pressure distribution will be a summation of the pressures due to each mode present. This will be difficult to interpret unless the conditions of measurement are such that only the first mode contributes appreciably to the pattern. Experimental curves made under these conditions are illustrated by Plate 11, and are discussed in Section IX of the text.

326. Measured and computed distributions for a specific case are shown in the lower half of Plate 11. The distribution constants for this case were obtained from hydrophone soundings beneath the ship. The agreement between measured and computed curves is very close.

327. For the case of transmission over hard bottom at frequencies higher than the critical frequency, is zero, and equation (64) reduces to

$$|P| \sim \sin\left(\frac{\pi \gamma u}{h}\right) \tag{66}$$

This shows that the vertical sound pressure distributions over hard bottom, for frequencies above the critical frequency, should be sections of undamped sine waves. The experimental results confirm this.

APPENDIX B

PROPAGATION OVER AN ELASTIC BOTTOM.

APPENDIX B.

PROPAGATION OVER AN ELASTIC BOTTOM.

328. The simplest approach to the problem of underwater sound propagation over an elastic bottom, taking into account the influence of shear waves in the bottom, is through the computation of the plane wave reflection law which applies to this case. The bearing of the reflection laws on sound transmission is discussed in Section VII of the text.

329. The pressure coefficient of reflection - the ratio of reflected to incident pressure amplitudes - may be computed as a function of the angle of incidence for plane waves incident from water on the boundary of an elastic medium. This may be accomplished conveniently by using an analysis of the general boundary conditions for the reflection and refraction of elastic waves published by C.G. Knott many years ago (Bibliog. 26). A summary of this analysis is given in "Exploration Geophysics", by J.J. Jakosky.

330. The special case which is relevant to the underwater sound problem is that in which a compressional wave, originating in the fluid, is incident at the interface of a fluid and an elastic solid. In general, the energy of the incident wave will be distributed among three new waves, a reflected compressional wave in the water, a refracted compressional wave in the solid, and a refracted shear wave in the solid. There is no reflected shear wave.

331. The formulas given by Knott utilize the following abbreviations:

C - Cotangent of angle of incidence of compressional wave in the first medium (the water).

C' - Cotangent of angle of refraction of compressional wave in the second medium (the bottom).

γ' - Cotangent of angle of refraction of shear wave in the second medium.

n' - Modulus of rigidity of the second medium.

ρ, ρ' - Density of the first and second medium, respectively.

A - Energy factor of the incident compressional wave.

A_1 - Energy factor of the reflected compressional wave.

A' - Energy factor of the refracted compressional wave.

B' - Energy factor of the refracted shear wave.

332. The geometric relations at the interface are indicated on Fig. 11, in which θ_1 is the angle of incidence, θ_2 the angle of refraction for the compressional wave, and ϕ the angle of refraction for the shear wave.

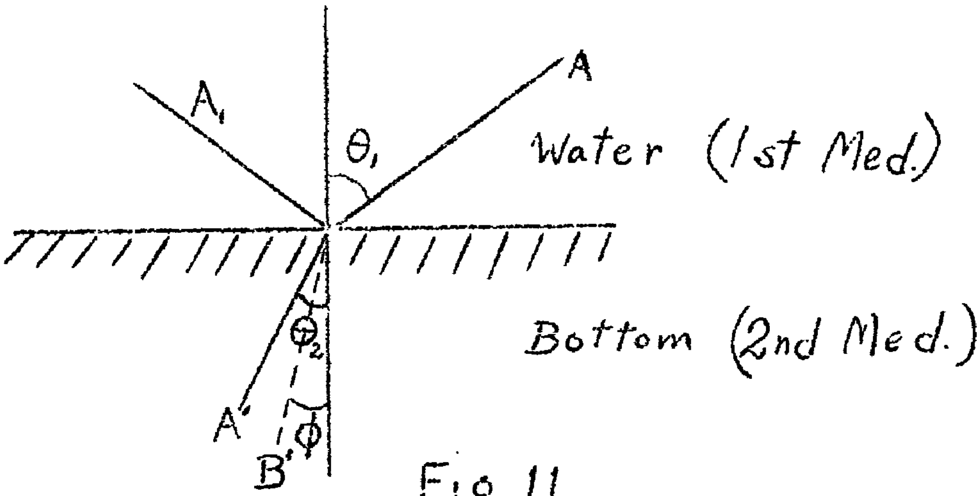


Fig. 11

333. The angles made by the rays with the normal to the boundary are related by the equation which expresses Snell's law:

$$\sin\theta_1 : \sin\theta_2 : \sin\phi = V_1 : V_2 : v_2$$

where the angles are shown in Fig. 11, V_1 and V_2 are the velocities in the first and in the second medium, respectively, and v_2 is the velocity of shear waves in the second medium.

334. The energy equation states that:

$$c\rho A^2 - c\rho A_1^2 = c'\rho' A'^2 + \gamma'\rho' B'^2 \quad (67)$$

335. The boundary conditions require that the sum of the components of the displacement on both sides of the boundary must be the same and that the sum of the normal and the tangential components of stress must be the same. The satisfaction of the boundary conditions leads to three equations:

$$B' = \frac{2c'}{\gamma'^2 - 1} A' \tag{68}$$

$$A - A_1 = \frac{c'}{c} \frac{\gamma'^2 + 1}{\gamma'^2 - 1} A' \tag{69}$$

$$A + A_1 = \frac{\rho'}{\rho} \left\{ \frac{4c'\gamma'}{\gamma'^4 - 1} + \frac{\gamma'^2 - 1}{\gamma'^2 + 1} \right\} A' \tag{70}$$

336. By making use of these three equations, together with Snell's law and the energy equation, it is possible to compute the ratios of the energy factors A/A', A_1/A', A_1/A and B'/A', which correspond to assigned values of density and velocity ratios for the two media. The pressure reflection coefficient discussed in the text (Section VII) is A_1/A. At certain angles of incidence some of the energy factors are imaginary or complex. In these cases the absolute magnitudes alone are considered.

337. Computations have been made for various angles of incidence, for the following assumed ratios of density and velocity:

(a) $V_1 : V_2 : v_2 = 2 : 3 : 2$, and $\rho'/\rho = 2$

(b) $V_1 : V_2 : v_2 = 2 : 3 : 2$, and $\rho'/\rho = 1$

(c) $V_1 : V_2 : v_2 = 2 : 3 : 1$, and $\rho'/\rho = 2$

The results of the computations are shown in Table V:

(See Table V on page 119.)

TABLE V.

PERCENTAGE ENERGY DISTRIBUTION BETWEEN REFLECTED, REFRACTED, AND SHEAR WAVES. ELASTIC BOTTOM.

Θ_1	Θ_2	Reflected	Refr. Comp.	Refr. Shear	Reflected	Refr. Comp.	Refr. Shear	Reflected	Refracted Compressional	Refracted Shear
		%	%	%	%	%	%	%	%	%
0°	0°	25.0	75.0	0.0	4.0	96.0	0.0	25.0	75.0	0.0
10°	15°8'	24.2	69.9	6.1	3.6	88.8	7.6	24.1	73.1	3.0
20°	30°52'	21.9	54.8	23.4	2.5	68.4	29.0	22.5	69.6	8.0
30°	48°35'	18.2	32.3	49.6	1.2	39.2	60.0	21.0	61.6	17.0
40°	74°35'	14.1	10.0	77.2	0.2	11.6	88.0	19.8	34.6	46.0
41°	79°45'	15.1	9.7	77.4	-	-	-	-	-	-
* 41.75	90°	100	0	0	100	0	0	100	0	0
50°	Imaginary Angle	61.8	Refracted Wave Imaginary	38.4	42.8	Refracted Wave Imaginary	57.2	91.6	Refracted Wave Imaginary	8.4
60°		37.0		63.0	33.1		66.9	81.5		18.5
70°		24.8		75.2	27.0		73.0	73.4		26.6
80°		46.8		53.2	64.3		35.7	87.6		12.4
90°		100		0	100		0	100		0
		case (a)			case (b)			case (c)		

* Critical angle

APPENDIX C.

PROPAGATION OVER A STRATIFIED BOTTOM.

APPENDIX C.

PROPAGATION OVER A STRATIFIED BOTTOM.

338. Although most cases of low frequency sound propagation in shallow water are adequately described by the theory derived in Appendix A for a simple bottom, the derivations may readily be extended to include the more general case of a stratified bottom. In these derivations the effects of reflection from a stratum below the actual bottom are taken into account by treating an acoustic system which comprises three media. The three media are the water, a single layer of homogeneous fluid of specified density and velocity of sound, and an underlying fluid of different density and velocity of sound. The underlying fluid extends indefinitely downward.

339. The form of the solution in terms of the normal modes of the two upper layers is similar to that derived for the simple bottom, although it is more complicated algebraically because of the additional parameters. The derivation is carried out for a typical mode of the "transverse" class of waves transmitted through a rectangular tube.

340. Consider, for the analysis, the pipe system shown in Fig. 12. This includes the water; the intermediate bottom layer, and the underlying bottom material extending to infinity. Let the water depth be h_1, and the thickness of the bottom layer be h_2. The thicknesses are measured along the y axis with positive y being taken downward. The length of the tube system is considered infinite in the x direction.

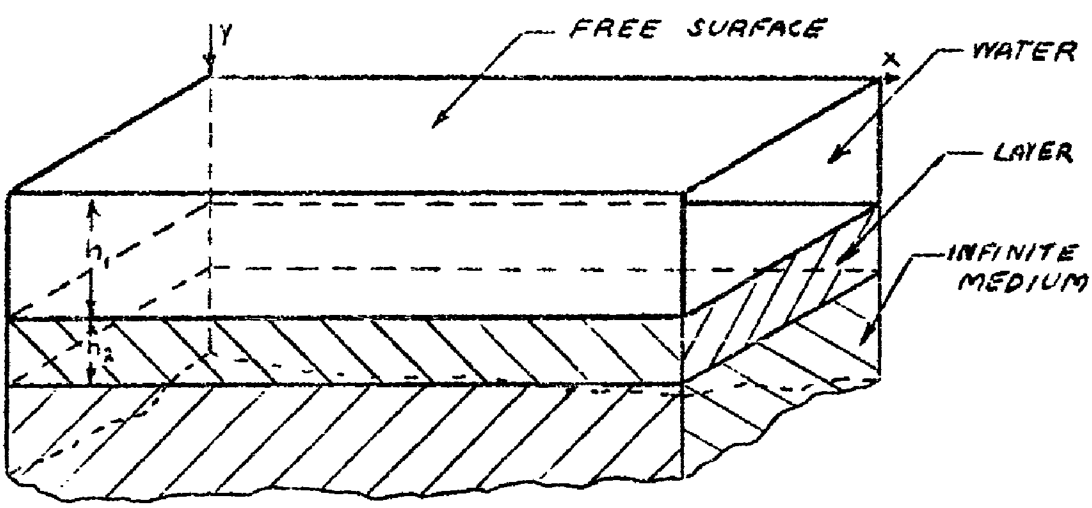


FIG. 12

341. For the simplest class of "transverse" waves, the pressure distributions in the water, the bottom layer, and the underlying structure, will have the following forms:

$$P_I = P_{o1} \sinh\left[\frac{\pi y}{h_1}(\kappa_1 - j\mu_1)\right] \varepsilon^{\left[j\omega t - \frac{\omega}{c_1}(\sigma_1 + j\tau_1)x\right]} \tag{71}$$

$$P_{II} = P_{o2} \sinh\left[\frac{\pi y}{h_2}(\kappa_2 - j\mu_2)\right] \varepsilon^{\left[j\omega t - \frac{\omega}{c_2}(\sigma_2 + j\tau_2)x\right]} \tag{72}$$

$$P_{III} = P_{o3}\, \varepsilon^{-\left[\frac{\pi y}{h_2}(\kappa_3 - j\mu_3)\right]}\, \varepsilon^{\left[j\omega t - \frac{\omega}{c_3}(\sigma_3 + j\tau_3)x\right]} \tag{73}$$

The distribution constants κ and μ, and the propagation constants σ and τ have the meaning given in Appendix A.

342. Substituting these relations into the wave equation yields the following relations between the distribution and propagation constants.

$$(\sigma_1 + j\tau_1) + \frac{1}{\eta_1^2}(\kappa_1 - j\mu_1) + 1 = 0 \tag{74}$$

$$(\sigma_2 + j\tau_2) + \frac{1}{\eta_2^2}(\kappa_2 - j\mu_2) + 1 = 0 \tag{75}$$

$$(\sigma_3 + j\tau_3) + \frac{1}{\eta_3^2}(\kappa_3 - j\mu_3) + 1 = 0 \tag{76}$$

where

$$\eta_1 = \frac{2h_1}{\lambda_1} \qquad \eta_2 = \frac{2h_2}{\lambda_2} \qquad \eta_3 = \frac{2h_3}{\lambda_3}$$

343. The general boundary conditions to be applied are those of continuity of pressure and of particle velocity at the boundaries.

$$\begin{array}{ll} \text{at } y = h_1 & \text{at } y = h_2 \\ P_I = P_{II} & P_{II} = P_{III} \\ \dfrac{1}{\rho_1}\dfrac{\partial P_I}{\partial y} = \dfrac{1}{\rho_2}\dfrac{\partial P_{II}}{\partial y} & \dfrac{1}{\rho_2}\dfrac{\partial P_{II}}{\partial y} = \dfrac{1}{\rho_3}\dfrac{\partial P_{III}}{\partial y} \end{array} \tag{77}$$

344. Applying the first boundary condition, continuity of pressure, to the water-bottom boundary there results

$$P_{01}\sinh[\pi(K_1-j\mu_1)]\,\varepsilon^{[j\omega t-\frac{\omega}{c_1}(\sigma_1+j\tau_1)x]} = P_{02}\sinh[\pi\left(\frac{h_1}{h_2}\right)(K_2-j\mu_2)+\varphi_2]\,\varepsilon^{[j\omega t-\frac{\omega}{c_2}(\sigma_2+j\tau_2)x]} \qquad (78)$$

This expression must be an identity independent of time and distance along the tube. For this to be true the following relations must hold

$$\frac{\sigma_1+j\tau_1}{c_1} = \frac{\sigma_2+j\tau_2}{c_2} \qquad (79)$$

$$P_{01}\sinh[\pi(K_1-j\mu_1)] = P_{02}\sinh\left[\pi\left(\frac{h_1}{h_2}\right)(K_2-j\mu_2)\right] \qquad (80)$$

Applying the second boundary condition, continuity of particle velocity, to the water-bottom boundary, there results

$$\frac{P_{01}(K_1-j\mu_1)}{\rho_1 h_1}\cosh[\pi(K_1-j\mu_1)] = \frac{P_{02}(K_2-j\mu_2)}{\rho_2 h_2}\cosh\left[\pi\left(\frac{h_1}{h_2}\right)(K_2-j\mu_2)+\varphi_2\right] \qquad (81)$$

345. Continuity of pressure at the lower surface of the bottom layer, yields the relation

$$P_{02}\sinh[\pi(K_2-j\mu_2)+\varphi_2]\,\varepsilon^{[j\omega t-\frac{\omega}{c_2}(\sigma_2+j\tau_2)x]} = P_{03}\,\varepsilon^{-\pi(K_3-j\mu_3)}\,\varepsilon^{[j\omega t-\frac{\omega}{c_3}(\sigma_3+j\tau_3)]} \qquad (82)$$

Again, this expression must be an identity independent of time and distance along the tube system. From this the following relations are obtained.

$$\frac{\sigma_2+j\tau_2}{c_2} = \frac{\sigma_3+j\tau_3}{c_3} \qquad (83)$$

$$P_{02}\sinh[\pi(K_2-j\mu_2)+\varphi_2] = P_{03}\,\varepsilon^{-\pi(K_3-j\mu_3)} \qquad (84)$$

Continuity of particle velocity at the lower surface of the bottom layer gives the expression

$$\frac{1}{\rho_2}(K_2-j\mu_2)P_{o2}\cosh[\pi(K_2-j\mu_2)+\varphi_2] = \frac{-1}{\rho_3}(K_3-j\mu_3)P_{o3}\varepsilon^{-\pi(\sigma_3-j\zeta_3)} \tag{85}$$

346. Dividing (80) by (81) there results the transcendental equation

$$\frac{\rho_1 h_1 \tanh \pi(K_1-j\mu_1)}{K_1-j\mu_1} = \frac{\rho_2 h_2 \tanh[\pi(\frac{h_1}{h_2})(K_2-j\mu_2)+\varphi_2]}{K_2-j\mu_2} \tag{86}$$

Dividing (84) by (85) results in

$$\frac{\rho_2 h_2 \tanh[\pi(K_2-j\mu_2)+\varphi_2]}{K_2-j\mu_2} = \frac{-\rho_3 h_3}{K_3-j\mu_3} \tag{87}$$

Combining (79) and (83)

$$\frac{\sigma_1+j\tau_1}{c_1} = \frac{\sigma_2+j\tau_2}{c_2} = \frac{\sigma_3+j\tau_3}{c_3} \tag{88}$$

347. From (74), (75), (76), and (88)

$$\frac{1}{c_1^2\eta_1^2}(K_1-j\mu_1)^2 = \frac{1}{c_2^2\eta_2^2}(K_2-j\mu_2)^2 = \frac{1}{c_3^2\eta_3^2}(K_3-j\mu_3)^2 \tag{89}$$

From (86), let

$$V = \tanh\left[\pi\left(\frac{h_1}{h_2}\right)(K_2 - j\mu_2) + \varphi_2\right] = \frac{\rho_1 h_1}{\rho_2 h_2}\left(\frac{K_2 - j\mu_2}{K_1 - j\mu_1}\right)\tanh\left[\pi(K_1 - j\mu_1)\right] \tag{90}$$

$$\tanh \varphi_2 = \frac{V - \tanh\left[\pi\left(\frac{h_1}{h_2}\right)(K_2 - j\mu_2)\right]}{1 - V\tanh\left[\pi\left(\frac{h_1}{h_2}\right)(K_2 - j\mu_2)\right]} \tag{91}$$

From (87), let

$$U = \tanh\left[\pi(K_2 - j\mu_2) + \varphi_2\right] = \frac{\rho_3}{\rho_2}\left(\frac{K_2 - j\mu_2}{K_3 - j\mu_3}\right) \tag{92}$$

$$\tanh \varphi_2 = \frac{U - \tanh\left[\pi(K_2 - j\mu_2)\right]}{1 - U\tanh\left[\pi(K_2 - j\mu_2)\right]} \tag{93}$$

Combining (91) and (93), there results

$$\frac{V - \tanh\left[\pi\left(\frac{h_1}{h_2}\right)(K_2 - j\mu_2)\right]}{1 - V\tanh\left[\pi\left(\frac{h_1}{h_2}\right)(K_2 - j\mu_2)\right]} = \frac{U - \tanh\left[\pi(K_2 - j\mu_2)\right]}{1 - U\tanh\left[\pi(K_2 - j\mu_2)\right]} \tag{94}$$

348. Substituting for U and V yields the final transcendental equation,

$$\frac{\frac{\rho_1 h_1}{\rho_2 h_2}\left(\frac{K_2-j\mu_2}{K_1-j\mu_1}\right)\tanh \pi(K_1-j\mu_1) - \tanh\left[\pi\left(\frac{h_1}{h_2}\right)(K_2-j\mu_2)\right]}{1-\frac{\rho_1 h_1}{\rho_2 h_2}\left(\frac{K_2-j\mu_2}{K_1-j\mu_1}\right)\tanh \pi(K_1-j\mu_1)\tanh\left[\pi\left(\frac{h_1}{h_2}\right)(K_2-j\mu_2)\right]} =$$

$$\frac{\frac{-\rho_3}{\rho_2}\left(\frac{K_2-j\mu_2}{K_3-j\mu_3}\right) - \tanh \pi(K_2-j\mu_2)}{1+\frac{\rho_3}{\rho_2}\left(\frac{K_2-j\mu_2}{K_3-j\mu_3}\right)\tanh \pi(K_2-j\mu_2)} \qquad (95)$$

349. Because of the additional parameters introduced, this expression is much more complicated than that obtained for the simpler conditions of Appendix A. For this reason it is not possible to plot the general results on a single series of charts such as those plotted in Plates 12-16. Charts could be computed, although with difficulty, for specific special conditions, for example a definite assumed thickness of intermediate layer. Some simplification of the expression results if attention is confined to transmission over hard bottom above the critical frequency. Detailed computations for these cases have not been made.

APPENDIX D

GENERAL NORMAL MODE THEORY FOR A POINT SOURCE BETWEEN INFINITE PLATES

APPENDIX D.

GENERAL NORMAL MODE THEORY FOR A POINT SOURCE BETWEEN INFINITE PLATES.

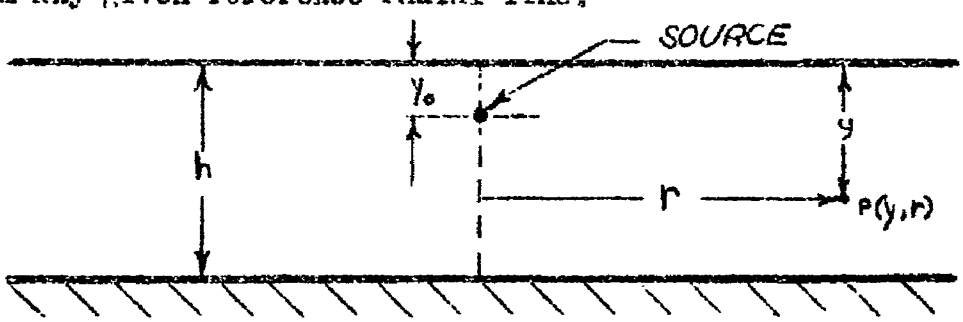
350. The purpose of this appendix is to derive directly the propagation relations for the general case of a point source of sound located between two parallel planes, the surface and the bottom. In Appendix A the propagation of sound in shallow water was derived by extension from the theory for transmission in a rectangular pipe. The artifice of the pipe is convenient because it permits complete solutions to be obtained. It is not entirely general, however. The treatment in this appendix is general, although the results are expressed in the form of integrals which have not yet been evaluated.[10] The most relevant result of the derivations, the equation which expresses the influence of the boundaries on the field distribution, will be found to be identical with that derived under the simpler assumptions of Appendix A. The generality of this equation is thus greatly enhanced.

351. The wave equation for a region in which there is a sound source is

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\rho_1 \frac{\partial q}{\partial t} \tag{96}$$

where q is the source function and p is the pressure.

352. The region considered is defined by two infinite parallel planes distant h from each other, as in Fig. 13. The space between the two planes is assumed to be filled with water. The upper boundary is taken to be a free surface. The lower plane is taken to be the boundary of a homogeneous fluid infinitely extended in a direction normal to the boundary. Cylindrical coordinates are employed, and the origin is taken at the surface on a vertical line passing through the source. The location of any point in the system is described in terms of its distance from the surface (y), its distance radially from the origin (r), and its azimuth angle (θ) from any given reference radial line.



10. The integrals were evaluated after the text was written. See Addenda.

353. The theory of partial differential equations shows that the entire solution of equation (96) for the steady state may be obtained by solving the equation with the right hand side equal to zero, and by expanding the expression on the right hand side in terms of the characteristic functions(normal modes) thus obtained.

354. Since the treatment is concerned only with the steady state solution of (96) for a sinusoidal source (a complex source may be analyzed into a series of sinusoidal sources if the system is linear), p and q may be defined as follows

$$
\begin{aligned}
p &= P(\gamma, r, \theta)\, \varepsilon^{-j\omega t} \\
q &= Q(\gamma, r, \theta)\, \varepsilon^{-j\omega t}
\end{aligned} \qquad (97)
$$

The equation (96) now becomes

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right) P(\gamma, r, \theta) = j\omega\rho_1 Q(\gamma, r, \theta) \qquad (98)$$

355. For any system the solution in cylindrical coordinates of (98) with the right-hand side equal to zero is

$$P(\gamma, r, \theta) = \sum_{\gamma_\theta} \sum_{\gamma_\gamma} \sum_{\gamma_r} A(\gamma_\theta) \cos(\gamma_\theta \theta) B(\gamma_\gamma) \sin(\gamma_\gamma \gamma + \phi) C(\gamma_r) J_{(\gamma_\theta)}\left(\frac{\gamma_r r}{c}\right) \qquad (99)$$

This equation describes the pressure at any point (γ,r,θ) in the region, in terms of summations, in which the values of the terms are determined by the separation parameters γ_γ , γ_r , and γ_θ . These parameters correspond physically to complex distribution constants, whose values depend upon the boundary conditions. If the pressure at any point in a given acoustic system is defined by the summation of the pressures at that point due to a number of normal modes, then each term of the summation (99) may be considered to represent one of the normal modes of the system. It follows that the pressure at any point due to the n^{th} mode will be given by an expression of the form

$$P_N(\theta, \gamma, r) = \Theta(\theta) \cdot \Upsilon(\gamma) \cdot R(r) \qquad (100)$$

356. The admissible values of the complex distribution constants γ_y, γ_r, γ_θ are determined by the boundary conditions imposed by the physical constants and by the geometrical configuration of the system, namely:

(a) The system is symmetrical about the origin. P will therefore by independent of Θ.

(b) The upper boundary is considered to be an infinite, plane, free surface. P is therefore equal to zero at all points on the upper boundary.

(c) The lower boundary is taken to be the plane upper surface of an infinitely extended homogeneous fluid. The bottom is considered parallel to, and at a distance h from the upper surface. The conditions which must be satisfied at the lower boundary are those of continuity of pressure and of normal particle velocity.

$$\begin{aligned} P_1 &= P_2 \\ \dot{\xi}_1 &= \dot{\xi}_2 \end{aligned} \qquad (101)$$

357. These boundary conditions impose four restrictions on the distribution constants:

(a) Condition (a) requires that $\gamma_\theta = 0$, since the system is symmetrical about the origin.

(b) Condition (b) requires that $\phi = 0$, since the pressure must reduce to zero at the surface.

(c) The third condition gives rise to a transcendentel equation relating the distribution constant γ_y and the physical constants of the bottom. The roots of this equation will give the allowed values of γ_y. The derivation of the expression will be given in a later section.

(d) Since the dimension in the y direction is finite, discrete values of the distribution constant γ_y are obtained, corresponding to the allowed configurations of the pressure field in the y direction. Since the medium is infinite in the r direction, there are no discrete values of γ_r. The values of γ_r therefore form a "continuous" spectrum.

358. Consider now the form of equation (99) when the above restrictions have been placed upon it. We have

$$P(y,r) = \sum_{\gamma_y} \sum_{\gamma_r} A(\gamma_y, \gamma_r) \sin\left(\frac{\gamma_y y}{c}\right) J_0\left(\frac{\gamma_r r}{c}\right) \tag{102}$$

Since equation (99) is the most general solution for the pressure at any point in the field, obtained without specifying boundary conditions, equation (102) must be the most general solution for the pressure under the assumed boundary conditions. Since the values of the distribution constants γ_y form a discrete spectrum, and the values of γ_r form a "continuous" spectrum, the most general solution must contain an infinite summation of terms involving γ_y, and an integral over the range of possible values of γ_r. Equation (102) may therefore be written

$$P(y,r) = \sum_{n=1}^{\infty} \sin\left(\frac{\gamma_{yn} y}{c}\right) \int_0^{\infty} F_n\left(\frac{\gamma_r}{c}\right) J_0\left(\frac{\gamma_r r}{c}\right) d\left(\frac{\gamma_r}{c}\right) \tag{103}$$

where the function $F_n\left(\frac{\gamma_r}{c}\right)$ is to be determined. Since this expression satisfies the above requirements, it must be the most general form of the solution.

359. The characteristic functions of the above expression are now used to obtain an expansion for the source function.

$$Q(y,r) = \sum_{n=1}^{\infty} \sin\left(\frac{\gamma_{yn} y}{c}\right) \int_0^{\infty} G_n\left(\frac{\gamma_r}{c}\right) J_0\left(\frac{\gamma_r r}{c}\right) d\left(\frac{\gamma_r}{c}\right) \tag{104}$$

where $G_n\left(\frac{\gamma_r}{r}\right)$ is to be determined. Multiply both sides of this equation by $\sin\left(\frac{\gamma_{yn} y}{c}\right)$

$$Q(y,r) \sin\left(\frac{\gamma_{yn} y}{c}\right) = \sin\left(\frac{\gamma_{yn} y}{c}\right) \sum_{n=1}^{\infty} \sin\left(\frac{\gamma_{yn} y}{c}\right) \int_0^{\infty} G_n\left(\frac{\gamma_r}{c}\right) J_0\left(\frac{\gamma_r r}{c}\right) d\left(\frac{\gamma_r}{c}\right) \tag{105}$$

360. A mathematical device, similar to that used to obtain Fourier series expansions, will now be employed. The device consists of the assumption of orthogonality for terms of the type

$$\sin(mx)\cdot\sin(nx)$$

The condition for orthogonality which must be satisfied is

$$\int_0^{x_1}\sin(mx)\sin(nx)\,dx \begin{array}{ll} = 0 & \text{for all } m \neq n \\ \neq 0 & \text{for all } m = n \end{array}$$

Assuming orthogonality of the terms $\sin\left(\frac{\gamma_{ym}y}{c}\right)\sin\left(\frac{\gamma_{yn}y}{c}\right)$ and integrating over x from 0 to h, and noting that all terms of the summation vanish except those for m = n, the following expression is obtained:

$$\int_0^h Q(y,r)\sin\left(\frac{\gamma_{yn}y}{c}\right)dx = \int_0^h \sin^2\left(\frac{\gamma_y y}{c}\right)dy\int_0^\infty G_n\left(\frac{\gamma_r}{c}\right)J_0\left(\frac{\gamma_r r}{c}\right)d\left(\frac{\gamma_r}{c}\right) \quad (106)$$

Let

$$H_n(r) = \frac{\int_c^h Q(y,r)\sin\left(\frac{\gamma_{yn}y}{c}\right)dy}{\int_c^{h}\sin^2\left(\frac{\gamma_{yn}y}{c}\right)dy} \quad (107)$$

Equation (106) then becomes

$$H_n(r) = \int_0^\infty G_n\left(\frac{\gamma_r}{c}\right)J_0\left(\frac{\gamma_r r}{c}\right)d\left(\frac{\gamma_r}{c}\right) \quad (108)$$

361. The function $G_n\left(\frac{\gamma_r}{c}\right)$ may now be obtained by applying the Fourier-Bessel theorem (Bibliog. 9, p 385, ex 44). This theorem gives a transformation which allows solution for an unknown function occurring inside an integral. Thus if $f(x)$ and $g(t)$ are two functions, the theorem states that

$$f(x) = \int_0^\infty t\, g(t)\, J_0(tx)\, dt$$

$$g(t) = \int_0^\infty x\, f(x)\, J_0(tx)\, dt$$

Letting $f(x) = H_n(r)$, $t\,g(t) = G_n\left(\frac{\gamma_r}{c}\right)$, $x = r$, $t = \left(\frac{\gamma_r}{c}\right)$, $G_n\left(\frac{\gamma_r}{c}\right)$ may now be obtained in the form

$$G_n\left(\frac{\gamma_r}{c}\right) = \frac{\gamma_r}{c}\int_0^\infty H_n(r)\, J_0\left(\frac{\gamma_r r}{c}\right) r\, dr \tag{109}$$

362. For any given source $Q(y,r)$, $H_n(r)$ may be evaluated by means of (107) and then substituted into (109) to calculate $G_n\left(\frac{\gamma_r}{c}\right)$. For the special case of a point source, there results

$$Q(y,r) = \frac{Q_0}{\Delta y\, \pi (\Delta r)^2} \quad \text{over the range} \left\{ \begin{matrix} (y_0 - \frac{1}{2}\Delta y) < y < (y_0 + \frac{1}{2}\Delta y) \\ 0 < r < \Delta r \end{matrix} \right\}$$

and $\quad Q(y,r) = 0 \quad$ at all other points $\qquad$ (110)

y_0 = source depth.

From equation (107)

$$H_n(r) = \frac{Q_0 \int_{y_0 - \frac{1}{2}\Delta y}^{y_0 + \frac{1}{2}\Delta y} \sin\left(\frac{\gamma_n y}{c}\right) dy}{\pi\, \Delta y\, (\Delta r)^2\, K_n} \tag{111}$$

where

$$K_n = \int_0^h \sin^2\left(\frac{\gamma_{yn} y}{c}\right) dy = \frac{1}{2}\left[h - \frac{c}{2\gamma_{yn}} \sin\left(\frac{2\gamma_{yn} h}{c}\right)\right] \quad (112)$$

Placing this value of $H_n(r)$ into equation (109) results in the following expression for $G_n(\frac{\gamma_r}{c})$.

$$\frac{\gamma_r Q_o \sin\left(\frac{\gamma_{yn} y_o}{c}\right)}{\pi c\ K_n} \int_0^{\Delta r} \frac{J_o\left(\frac{\gamma_r r}{c}\right) r\,dr}{(\Delta r)^2} \approx \frac{\gamma_r Q_o \sin\left(\frac{\gamma_{yn}}{c} y_o\right)}{2\pi c\ K_n} \quad (113)$$

363. This expression may be substituted into equation (104) to obtain an explicit expansion for the source function.

$$Q(y,r) = \sum_{n=1}^{\infty} \sin\left(\frac{\gamma_{yn} y}{c}\right) \int_0^{\infty} G_n\left(\frac{\gamma_r}{c}\right) J_o\left(\frac{\gamma_r r}{c}\right) \quad (114)$$

364. Continuing with the development of the solution, return to equation (103), where $F_n\ (\frac{\gamma_r}{c})$ is to be determined. Substitute equations (104) and (103) into the wave equation (96).

$$\left\{\frac{1}{c^2} \sum_{n=1}^{\infty} \sin\left(\frac{\gamma_{yn} y}{c}\right) \int_0^{\infty} F_n\left(\frac{\gamma_r}{c}\right) J_o\left(\frac{\gamma_r r}{c}\right) d\left(\frac{\gamma_r}{c}\right)\right\} \left\{\omega^2 - \gamma_{yn}^2 - \gamma_r^2\right\} = \quad (115)$$

$$j\omega\rho_1 \sum_{n=1}^{\infty} \sin\left(\frac{\gamma_{yn} y}{c}\right) \int_0^{\infty} G_n\left(\frac{\gamma_r}{c}\right) J_o\left(\frac{\gamma_r r}{c}\right) d\left(\frac{\gamma_r}{c}\right)$$

To make this expression an identity, $F_n\left(\frac{\gamma_r}{c}\right)$ must have the following form

$$F_n\left(\frac{\gamma_r}{c}\right) = \frac{j\omega\rho_1 G_n\left(\frac{\gamma_r}{c}\right)c^2}{\omega^2 - \gamma_{yn}^2 - \gamma_r^2} \tag{116}$$

365. The final solution for the pressure then becomes

$$P(y,r) = \sum_{n=1}^{\infty}\left[\frac{j\omega c^2\rho_1 Q_o}{2\pi K_n}\sin\left(\frac{\gamma_{yn}y}{c}\right)\sin\left(\frac{\gamma_{yn}y_o}{c}\right)\int_0^{\infty}\frac{\gamma_r J_o\left(\frac{\gamma_r r}{c}\right)d\left(\frac{\gamma_r}{c}\right)}{\omega^2 - \gamma_{yn}^2 - \gamma_r^2}\right] \tag{117}$$

This expression gives the pressure at any point in the system as the summation of the pressures due to all the modes of the system. The integration over the distribution constant (γ_n) may be taken for real values only, with no loss of generality. For the first mode the expression has the form

$$P_1(y,r) = \left[\frac{j\omega c^2\rho_1 Q_o}{2\pi K_1}\right]\sin\left(\frac{\gamma_{y1}y}{c}\right)\sin\left(\frac{\gamma_{y1}y_o}{c}\right)\int_0^{\infty}\frac{\gamma_r J_o\left(\frac{\gamma_r r}{c}\right)d\left(\frac{\gamma_r}{c}\right)}{\omega^2 - \gamma_{y1}^2 - \gamma_r^2} \tag{118}$$

366. The definite integral in the expression has not been evaluated[11] but it is possible to deduce some important properties of the solution by inspection. These will be discussed, following the derivation of an equation which gives the admissible values of the distribution constant, (γ_1).

367. The solution given by equation (117) for the pressure field at any point in the system depends upon the boundary conditions at the bottom, which, in turn, determine the values of the

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11. See Addenda.

distribution constant (γ_y). The formulation of the boundary conditions therefore determines the nature of the complete solution.

368. Consider the n^{th} term of the series represented by the equation for the pressure at any point in the system (117). This term represents the pressure due to the n^{th} mode of the system. It may be rewritten more simply in the form

$$P_I = C_n \sin\left(\frac{\gamma_{yn} y}{c}\right) \int_0^{\infty} \frac{\gamma_r J_0\left(\frac{\gamma_r r}{c}\right) d\left(\frac{\gamma_r}{c}\right)}{\omega^2 - \gamma_{yn}^2 - \gamma_r^2} \tag{119}$$

The boundary conditions of continuity of pressure and normal particle velocity give rise to the following expressions

$$\text{at} \quad y = h$$

$$\begin{aligned} P_I &= P_{II} \\ \frac{1}{\rho_1}\frac{\partial P_I}{\partial y} &= \frac{1}{\rho_2}\frac{\partial P_{II}}{\partial y} \end{aligned} \tag{120}$$

As in Appendix A, the pressure distribution in the second medium, the bottom, must be known. The requirements that the pressure must decrease with increasing downward distance from the boundary, and that the pressure must be continuous at the boundary at all points in the system, necessitate the following form for the sound pressure distribution in the bottom:

$$P_{II} = C_n' \varepsilon^{-\left(\frac{\gamma_y' y}{c}\right)} \int_0^{\infty} \frac{\gamma_r' J_0\left(\frac{\gamma_r' r}{c}\right) d\left(\frac{\gamma_r'}{c}\right)}{\omega^2 - \gamma_{yn}' - \gamma_r'} \tag{121}$$

369. Applying the <u>first boundary condition</u>, continuity of pressure, yields

$$C_n \sin\left(\frac{\gamma_{yn} h}{c_1}\right) \int_0^{\infty} \frac{\gamma_r J_0\left(\frac{\gamma_r r}{c_1}\right) d\left(\frac{\gamma_r}{c_1}\right)}{\omega^2 - \gamma_{yn}^2 - \gamma_r^2} = C_n' \varepsilon^{-\left(\frac{\gamma_{yn}' h}{c_2}\right)} \int_0^{\infty} \frac{\gamma_r' J_0\left(\frac{\gamma_r' r}{c_2}\right) d\left(\frac{\gamma_r'}{c_2}\right)}{\omega^2 - \gamma_{yn}'^2 - \gamma_r'^2} \tag{122}$$

Since this must be true independent of the distance from the origin, the following relations are obtained

$$\frac{\gamma_r}{c_1} = \frac{\gamma_r'}{c_2} \tag{123}$$

$$\left(\frac{\omega}{c_1}\right)^2 - \left(\frac{\gamma_{yn}}{c_1}\right)^2 = \left(\frac{\omega}{c_2}\right)^2 - \left(\frac{\gamma_{yn}'}{c_2}\right)^2 \tag{124}$$

$$C_n \sin\left(\frac{\gamma_{yn} h}{c_1}\right) = C_n' \varepsilon^{-\left(\frac{\gamma_{yn}' h}{c_2}\right)} \tag{125}$$

370. The <u>second boundary condition</u> requires that

$$\frac{1}{\rho_1} \frac{\gamma_{yn}}{c_1} C_n \cos\left(\frac{\gamma_{yn} h}{c_1}\right) = -\frac{1}{\rho_2} \frac{\gamma_{yn}'}{c_2} \varepsilon^{-\left(\frac{\gamma_{yn}' h}{c_2}\right)} \tag{126}$$

dividing (125) by (126) gives the equation

$$\frac{\rho_1 c_1}{\gamma_{yn}} \tan\left(\frac{\gamma_{yn} h}{c_1}\right) = \frac{-\rho_2 c_2}{\gamma_{yn}'} \tag{127}$$

371. Additional relations are obtained by setting the expressions for the distributions in the two media into the wave equation (96), with the right-hand side equal to zero. In cylindrical coordinates the wave equation has the form

$$\frac{\partial^2 P}{\partial y^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) + \frac{\omega^2}{c^2}P = 0 \qquad (128)$$

Substituting the expression for the distribution in the first medium (equation (118)) into the wave equation results in

$$\left\{C_n \frac{1}{c_1^2}\sin\left(\frac{\gamma_{yn} y}{c_1}\right)\int_0^\infty \frac{\gamma_r J_0\left(\frac{\gamma_r r}{c_1}\right) d\left(\frac{\gamma_r}{c_1}\right)}{\omega^2 - \gamma_{yn}^2 - \gamma_r^2}\right\}\left\{\omega^2 - \gamma_{yn}^2 - \gamma_r^2\right\} = 0 \qquad (129)$$

From this arises the relation

$$\omega^2 - \gamma_{yn}^2 - \gamma_r^2 = 0 \qquad (130)$$

Substituting the expression (equation (121)) for the pressure in the second medium into the wave equation a similar relation is obtained

$$\omega^2 - \gamma_{yn}'^2 - \gamma_r'^2 = 0 \qquad (131)$$

Combining equations (123), (130), and (131) results in

$$(\gamma_{yn}')^2 = \left(\frac{c_2}{c_1}\right)^2 \gamma_{yn}^2 + \omega^2\left[1 - \left({}^{c_2}\!/\!_{c_1}\right)^2\right] \qquad (132)$$

(N.B. This relation also arises from the combination of (123) and (124), because of the conditions which were imposed in obtaining the final pressure expression.)

372. Now let $\gamma_{yn} = j\pi(K - j\mu)\frac{c_2}{h}$

Equation (127) reduces to

$$\frac{\tanh^2 \pi(K - j\mu)}{(K - j\mu)^2} = \frac{(\rho_2/\rho_1)^2}{\left(\frac{h}{c_1\pi}\right)^2\left\{(K - j\mu)^2 + \omega^2\left[1 - (c/c_2)^2\right]\right\}} \tag{133}$$

Let $A = \left(\rho_2/\rho_1\right)$ $B = \eta_1\left[(c/c_2)^2 - 1\right]^{1/2}$ where $\eta_1 = \frac{2h}{\lambda_1}$ (134)

Equation (133) reduces to

$$\frac{\tanh^2 \pi(K - j\mu)}{(K - j\mu)^2} = \frac{A^2}{(K - j\mu)^2 - B^2} \tag{135}$$

373. This equation is identical with equation (20) in Appendix A. Since equation (135) was derived on much more general assumptions than equation (20), the validity of the relatively simple treatment in Appendix A is considerably enhanced. An additional result is that the solutions of equation (20), computed and plotted in charts (Plates 12-16), are applicable to the general analysis of Appendix D as well as to the special analysis of Appendix A. It may be shown that the same equation is valid also for the conditions assumed in Appendix C (Propagation Over A Stratified Bottom).

374. An equation in integral form (equation (117)) has been obtained for the pressure at any point in the acoustic system, and also a relation (equation (135)) which expresses the influence of the boundaries on the field distribution. Consider equation (117), the final expression for the complete pressure field. Examination of the admissible values of the distribution constant γ_{yn} shows that this constant increases consistently with the order of the mode. Examination of the denominator of the integral expression shows that for moderately low exciting frequencies, for which $\omega^2 \ll |\gamma_{1n}^2|$ and n is large, the stimulation decreases with the order of the mode. The stimulation of any mode may be defined as the proportionate part of the total energy which resides in that mode.

375. One corollary of the stimulation law is that the modes of high order are weakly stimulated at low frequencies, and their excitation requires only a small fraction of the total radiated energy. Another corollary is that the modes of low order are less strongly stimulated and the modes of high order are more strongly stimulated, as the exciting frequency increases. At high frequencies many modes are strongly stimulated. The stimulation of modes of low order is not negligible, however, even at high frequencies. The small damping rates of the lower modes insure that most of the energy at large distances over soft bottom will be carried by these modes.

376. The term sin $\left(\frac{\gamma_m y_0}{c}\right)$ involving the source depth y_0, gives the variation in stimulation of the modes as a function of the source location. It may be seen from this term that the stimulation of the system is zero for all modes when the source is at the surface. It is also clear that the source should be located as nearly as possible midway between surface and bottom, in order to provide the maximum stimulation, under varying bottom conditions, of the modes of lowest order and smallest decay rates. It also follows that if the source is located at a point for which $\left(\frac{\gamma_m y_0}{c}\right)$ is small for a given mode, the stimulation of that mode will be weak.

377. The transcendental term $\sin\left(\frac{\xi y}{c}\right)$ which involves the distance from the surface, y, gives the vertical distribution of the sound pressure for any given mode. This expression may be reduced to the vertical pressure distribution relation given in Appendix A, Part 5. The interaction spacings caused by interference between the modes may be determined from examination of the coincidence points of the maxima of the Bessel function terms in the integral expression. Good agreement is obtained between observed interaction spacings and spacings computed by the above method.

378. <u>Summary</u>. An integral expression has been derived which gives the sound pressure at any point in the acoustic system defined by two infinite parallel planes, surface and bottom, with a point source located between them. In formulating the boundary conditions it was assumed that the bottom is homogeneous to an infinite depth (i.e. reflections from lower strata were assumed negligible), and that shear waves in the bottom may be neglected. Both assumptions appear justified by the results. It seems probable that the transmission of underwater sound in the actual physical system closely approximates the predictions of the theory.

APPENDIX E

SAMPLE CALCULATIONS

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SAMPLE CALCULATIONS.

379. The purpose of this appendix is to present a typical calculation of the attenuation of sound pressure level between two given points in line with a sound source, using the propagation theory developed in the text. The example chosen for computation is represented by the third record from the top of Plate 2, made at the Potomac River Bridge Range under summer conditions, at a frequency of 80 cps. The transmission is of the "damped" type and is dominated by the first mode at distances greater than two or three times the water depth.

380. The data are:

Frequency	f	80 cps
Water depth	h	55 ft
Velocity of Sound in water	c_1	4800 ft/sec
Normal Impedance Ratio (from hydrophone soundings)	$\rho_2 c_2 / \rho_1 c_1$	0.3
Specific Gravity of Bottom (estimated from samples)	$\rho_2/\rho_1 =$ A	1.3

381. The computations follow:

Free Field Wavelength	$\lambda = c_1/f$	60 ft
Depth in Half Wavelengths	$\eta_1 = 2h/\lambda$	1.83
Velocity Ratio	$c_1/c_2 = A/0.3$	4.33
Abscissa on Chart (Plate 15)	$B = \frac{2fh}{c_1}\left[\left(\frac{c_1}{c_2}\right)^2 - 1\right]^{1/2}$	
	$= \frac{2 \times 80 \times 55}{4800}\left(4.33^2 - 1\right)^{1/2} =$	7.73

382. The above values of A and B are entered on the appropriate chart (Plate 15), and values of κ and μ are determined. From the chart $\kappa = 0.054$ and $\mu = 0.997$.

383. Next the propagation constants σ and τ are determined, using the chart on Plate 17, or the approximate relations in equations (43) and (44). The values obtained are $\tau = 0.839$ and $\sigma = 0.0188$.

384. The damping per wavelength caused by absorption at the bottom is $54.6\,\sigma = 1.03$ db per wavelength. In terms of db/1000 ft, this is $54.6\,\sigma/\lambda$, or 17.1 db/1000 ft.

385. Consider a point 200 ft from the source and another point 1200 ft from the source. The total attenuation between these two points is the sum of that caused by damping, 17.1 db in this case, and that caused by cylindrical spreading, amounting to 7.7 db in this case. The total attenuation should be the sum of these figures, or 24.8 db. The measured attenuation between 200 ft and 1200 ft from the source, on the corresponding experimental record, is 23 db.

386. The deviation between observed and computed values, 1.8 db is well within the experimental error of the acoustic measurements, the distortions caused by irregularities of the system, and errors in the estimation of distances and water depths.

387. The final expression, equation (117) in Appendix D, for the sound pressure at any point in the field of the acoustic system of the sea, contains an integral of the form

$$\int_0^\infty \frac{\gamma_r J_o\left(\frac{\gamma_r r}{c}\right) d\left(\frac{\gamma_r}{c}\right)}{\omega^2 - \gamma_{yn}^2 - \gamma_r^2}$$

where the meaning of the symbols is defined in Appendix D.

388. This integral may be evaluated by following the method employed for a similar case in a report (No. 65) recently received from the Minesweeping Section of the Bureau of Ships. The method is to write the Bessel function in the integrand as the sum of two Hankel functions, and then to integrate the Hankel functions around appropriate contours. Thus, if $w = u + jv$ is a complex variable,

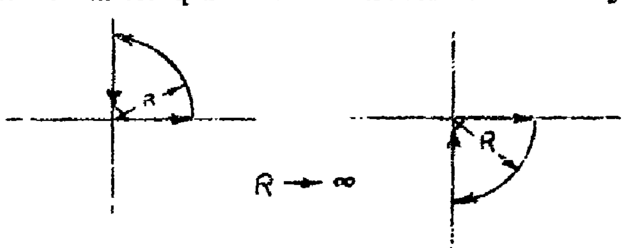
$$J_o(wr) = \tfrac{1}{2} H_o^{(1)}(wr) + \tfrac{1}{2} H_o^{(2)}(wr) \tag{136}$$

389. By making this type of transformation the evaluation of the integral in equation (117) may be reduced to the evaluation of the pair of integrals

$$\int_0^\infty \frac{w H_o^{(1)}(rw)\,dw}{a^2 - w^2} + \int_0^\infty \frac{w H_o^{(2)}(rw)\,dw}{a^2 - w^2}$$

where the abbreviations $w = \gamma_r/c$ and $a^2 = \frac{\omega^2 - \gamma_{yn}^2}{c}$ have been made for convenience.

390. The problem takes the form of finding a contour around which each integrand may be integrated, to give the above integrals as a resultant. The proper contours are indicated in the accompanying sketches. The first quadrant contour is employed for the first integrand and the fourth quadrant contour is employed for the second integrand.



391. Provision must be made for the fact that the Hankel functions have branch points at the origin, which the contour must avoid. The contribution of the path around the branch points may be shown to approach zero in the limit. The integrals over the segments of the circles vanish if the radius is taken sufficiently large, because the Hankel functions approach zero at infinity. Since $H_0(jv) = -H_0(-jv)$ the integrals along the imaginary axes cancel, leaving the sums of the integrals along the real axes. This is equal to $2\pi j$ times the sum of the residues. The residue at the pole in the first quadrant is all that remains. Thus

$$\int_0^{\infty} \frac{w H_0^{(1)}(rw)\,dw}{a^2 - w^2} + \int_0^{\infty} \frac{w H_0^{(1)}(rw)\,dw}{a^2 - w^2} = j\pi H_0^{(1)}(ra) \qquad (137)$$

392. The final solution for the pressure at any point in the field becomes, if the integrals are evaluated in the above manner,

$$P_{(y,r)} = \sum_{n=1}^{\infty} \frac{-\rho_1 Q_0 \omega c}{4K_N} \sin\left(\frac{\delta_{yn} y}{c}\right) \sin\left(\frac{\delta_{yn} y}{c}\right) H_0^{(1)}\left(r\frac{\delta_n}{c}\right) \qquad (138)$$

where the substitution $a = \left(\frac{\omega^2 - \delta_{yn}^2}{c}\right)^{1/2} = \frac{\delta_n}{c}$ has been made. It may be shown from the relations between the constants (Appendix D) that $\delta_n = 2\pi j(\sigma + j\tau)\frac{c}{h}$, and δ_{yn}, Q_0 and K_N may be evaluated in terms of κ_n and μ_n. Making the necessary substitutions

$$P_{(y,r)} = \sum_{n=1}^{\infty} \frac{\rho_0 Q_0 \omega c}{2h\left\{1 - \frac{\sinh[2\pi(\kappa - j\mu)]}{2\pi(\kappa - j\mu)}\right\}} \sinh\left[\frac{\pi y_0}{h}(\kappa - j\mu)\right] \sinh\left[\frac{\pi y}{h}(\kappa - j\mu)\right] H_0\left[j\frac{2\pi r}{h}(\sigma + j\tau)\right] \qquad (139)$$

393. At source-distances greater than a wavelength the Hankel function may be replaced, with little error, by its asymptotic expansion

$$H_0(z) = \frac{\varepsilon^{-jz - j\pi/4}}{\sqrt{\pi/2\, z}} \qquad (140)$$

When $r > \lambda$, the pressure at any point in the field may be written

$$P_{(y,r)} = \sum_{n=1}^{\infty} \frac{\rho_1 Q_0 \pi^{-1/2} \omega^{1/2} c^{1/2}}{\sqrt{2}\, h\left\{1 - \frac{\sinh[2\pi(\kappa - j\mu)]}{2\pi(\kappa - j\mu)}\right\}} \sinh\left[\frac{\pi y_0}{h}(\kappa - j\mu)\right] \sinh\left[\frac{\pi y}{h}(\kappa - j\mu)\right] \frac{\varepsilon^{-\frac{2\pi}{h}(\sigma + i\tau) r}}{\sqrt{r}} \qquad (141)$$

Inspection of this equation shows that the pressure formula expresses the influence of three factors,

(a) stimulation, involving source-strengths and source depths,

(b) propagation, involving damping, phase velocities and vertical distributions, and

(c) spreading, involving attenuation which varies inversely with the square root of the source-distance.

394. In the theoretical treatment employing the pipe artifice (Appendix A) only the propagation factors (b) were rigorously derived, although the effects of cylindrical spreading were included as a result of physical reasoning. For source-distences greater than one wavelength the simpler treatment of underwater transmission is adequate except for the computations of stimulation terms. It has now been demonstrated by a general derivation that factors (b) and (c) are correctly analyzed in the simpler treatment. In addition, equation (139) is an exact expression, valid for all source-distances. This permits complete computations, including the exact propagation function and the stimulation terms.

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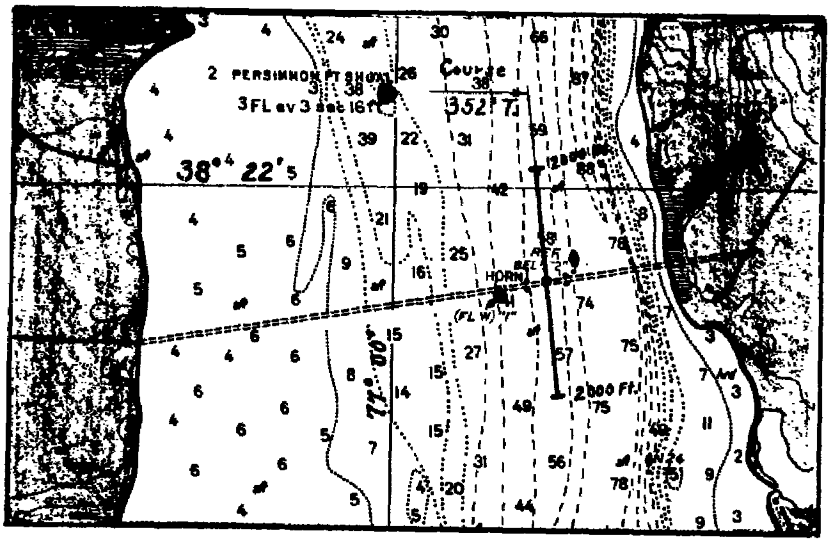
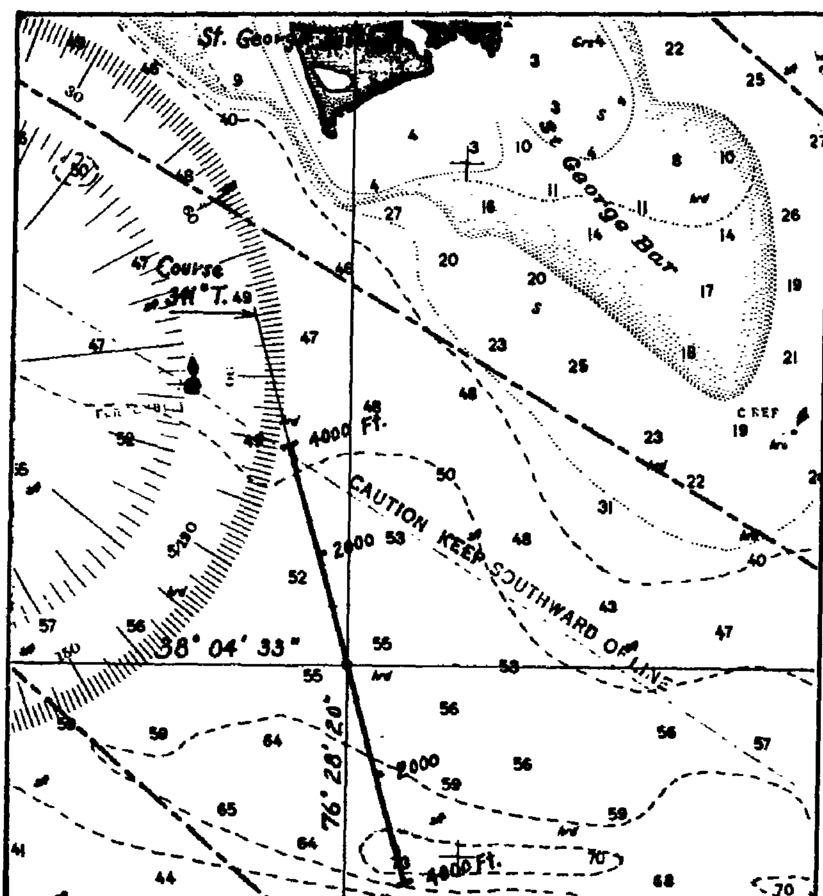
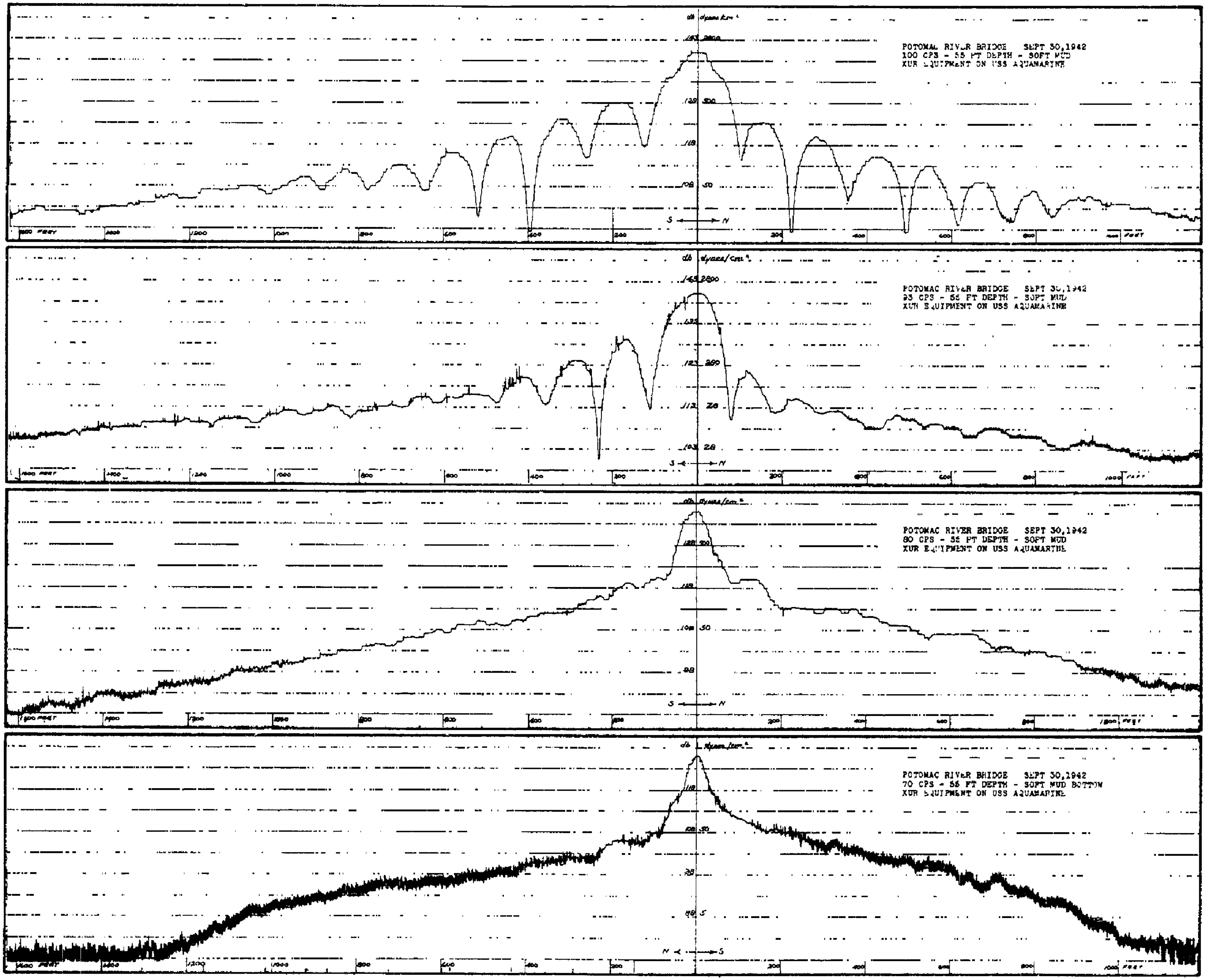


POTOMAC RIVER BRIDGE COURSE

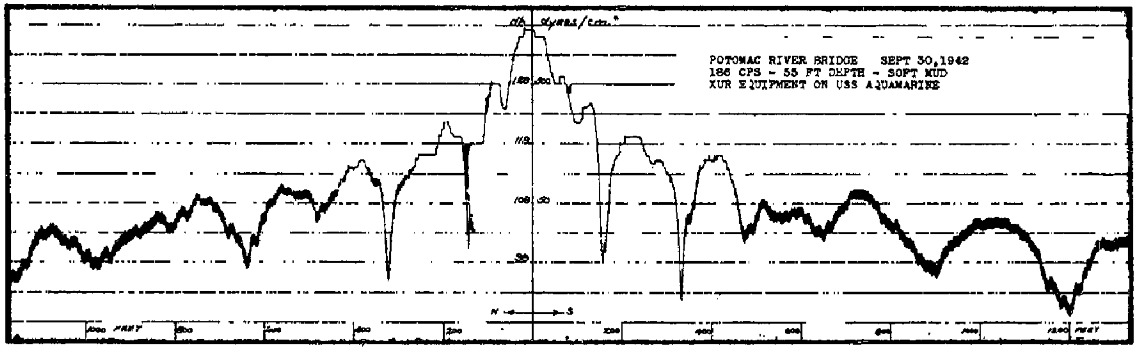


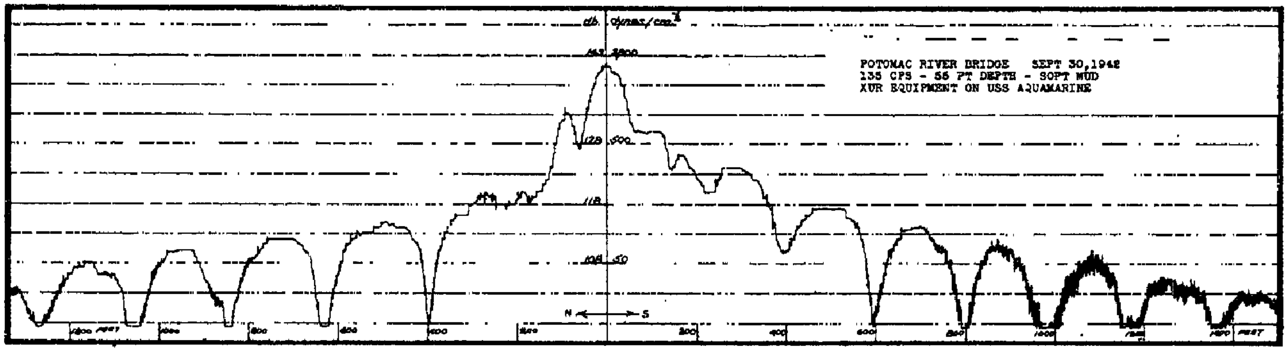
POTOMAC RIVER MOUTH COURSE

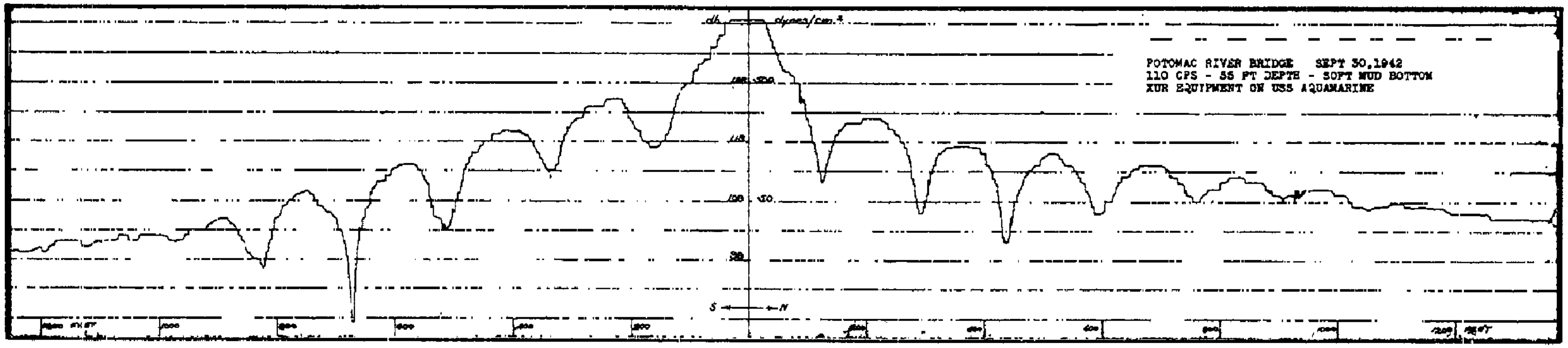
PLATE 1



RANGE RECORDS OVER SOFT BOTTOM ~ POTOMAC RIVER BRIDGE

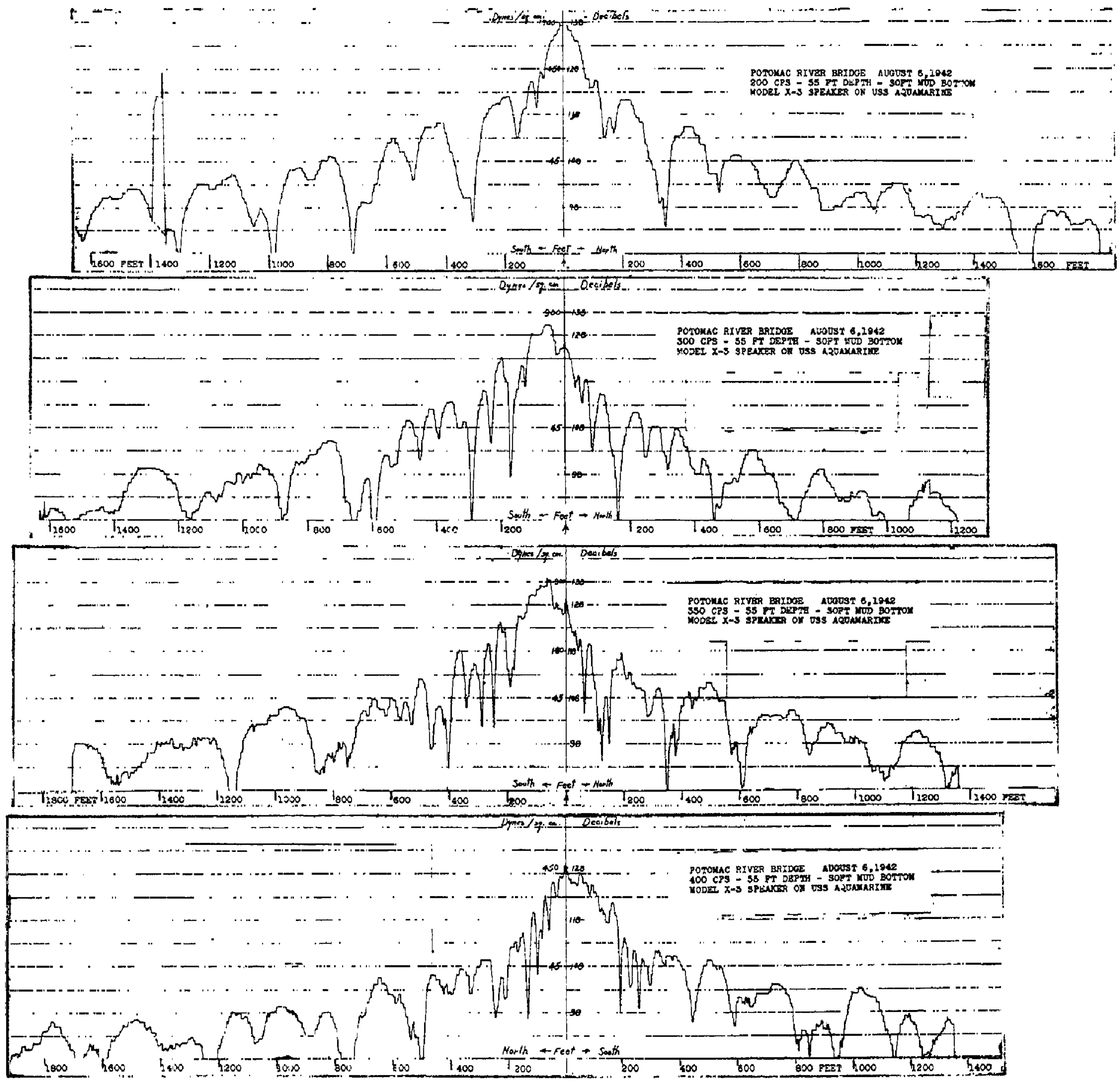






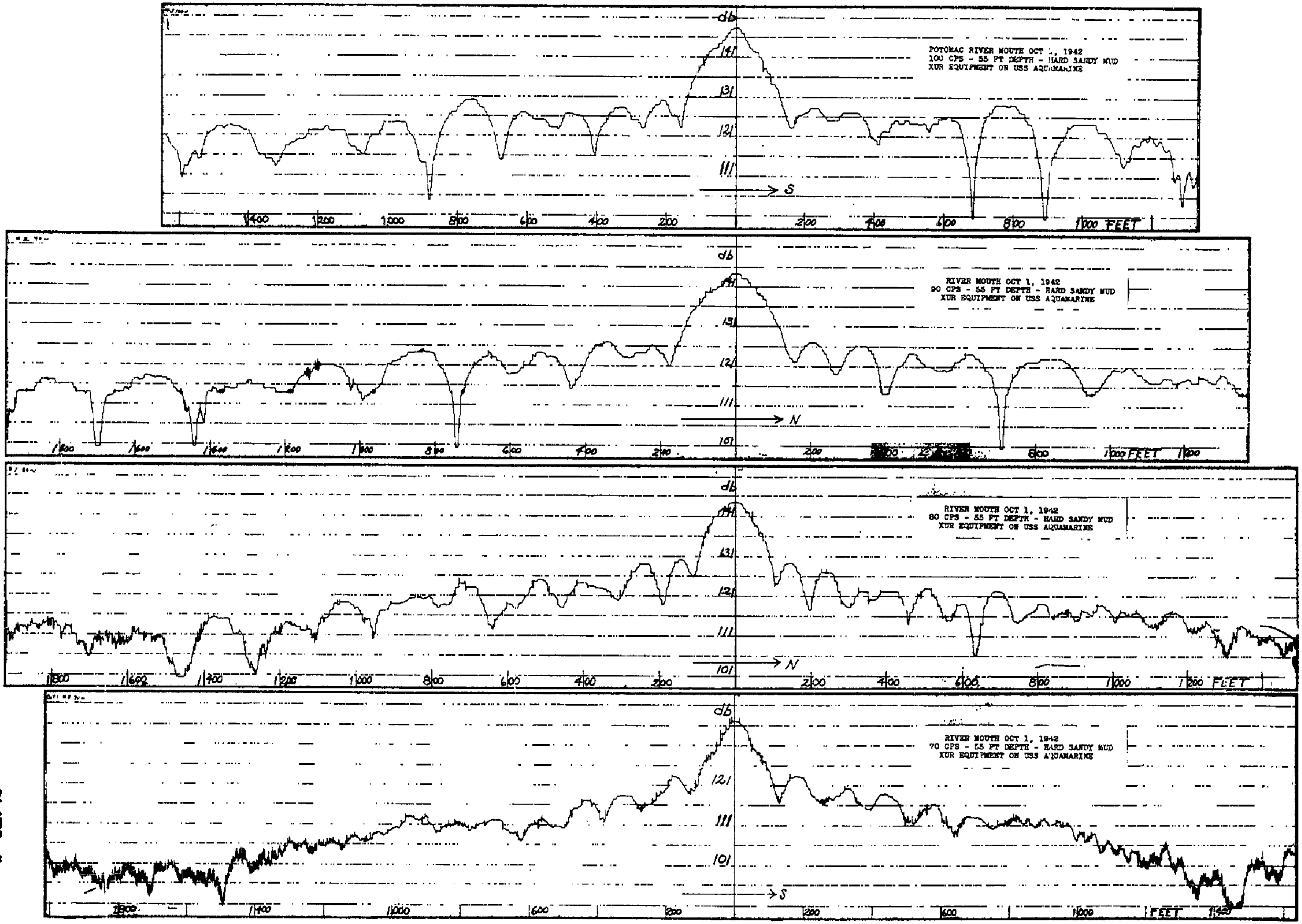
RANGE RECORDS OVER SOFT BOTTOM ~ POTOMAC RIVER BRIDGE

PLATE 3



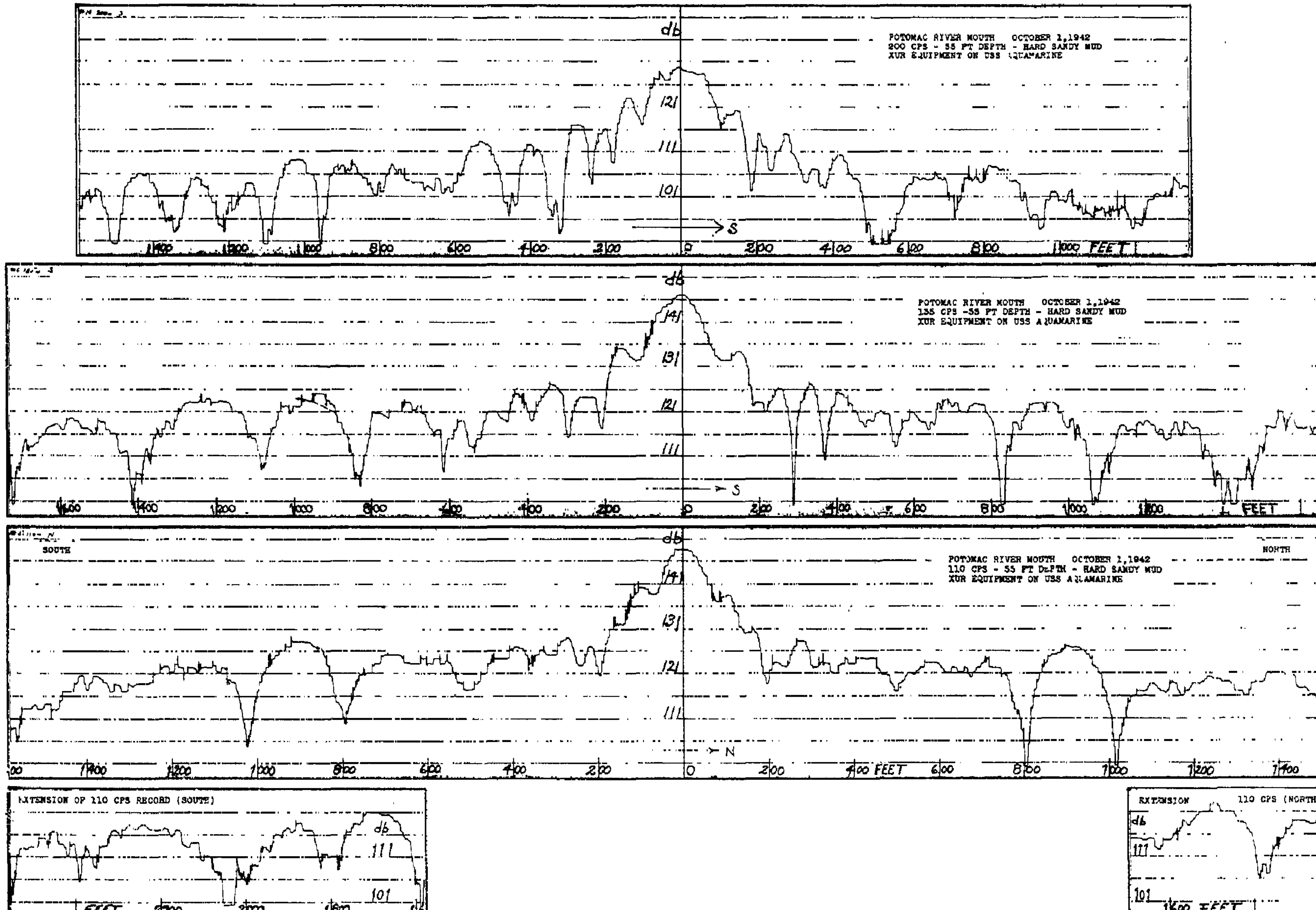
RANGE RECORDS OVER SOFT BOTTOM~POTOMAC RIVER BRIDGE

PLATE 4



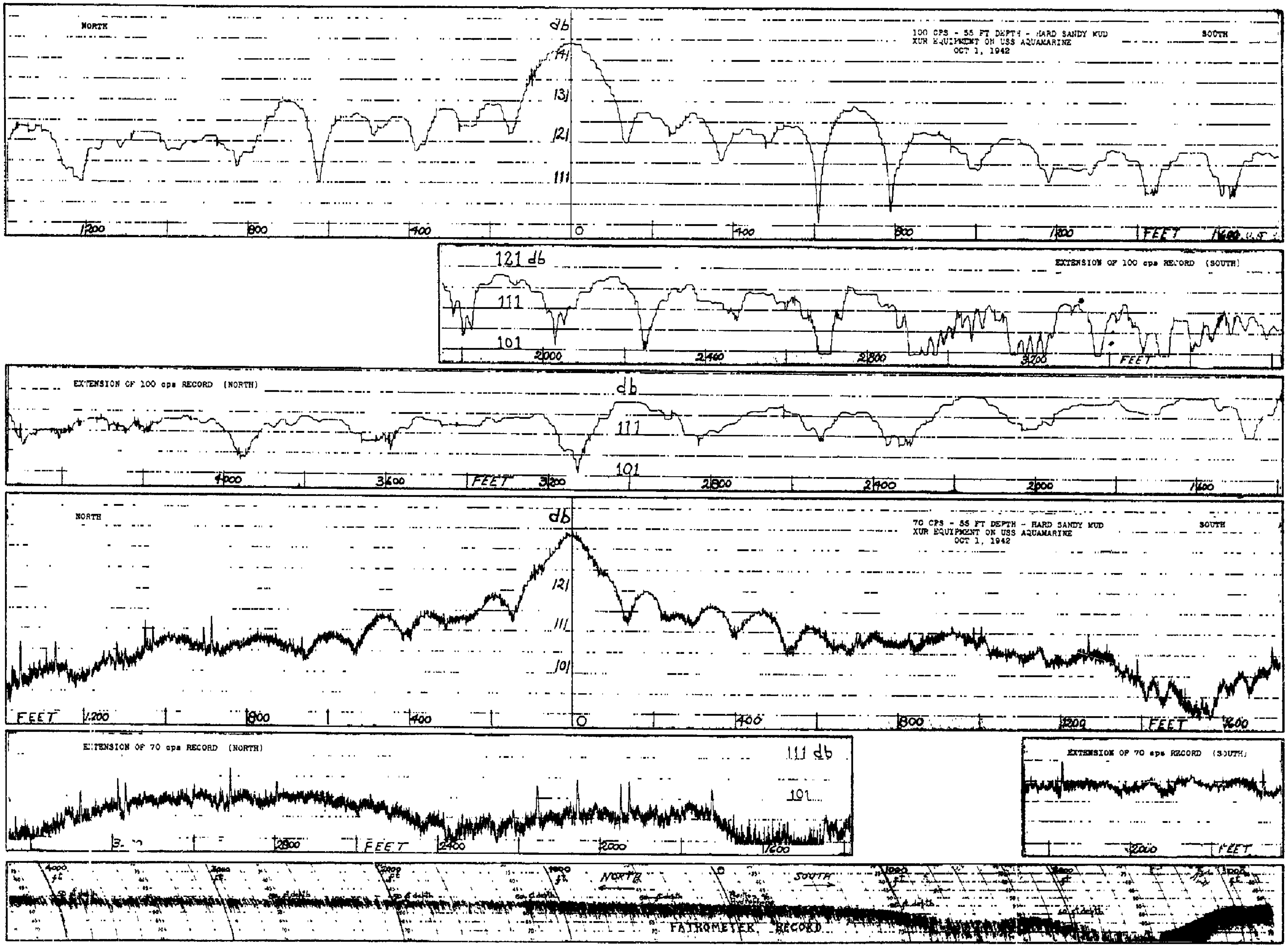
RANGE RECORDS OVER HARD BOTTOM ~ POTOMAC RIVER MOUTH

PLATE 5



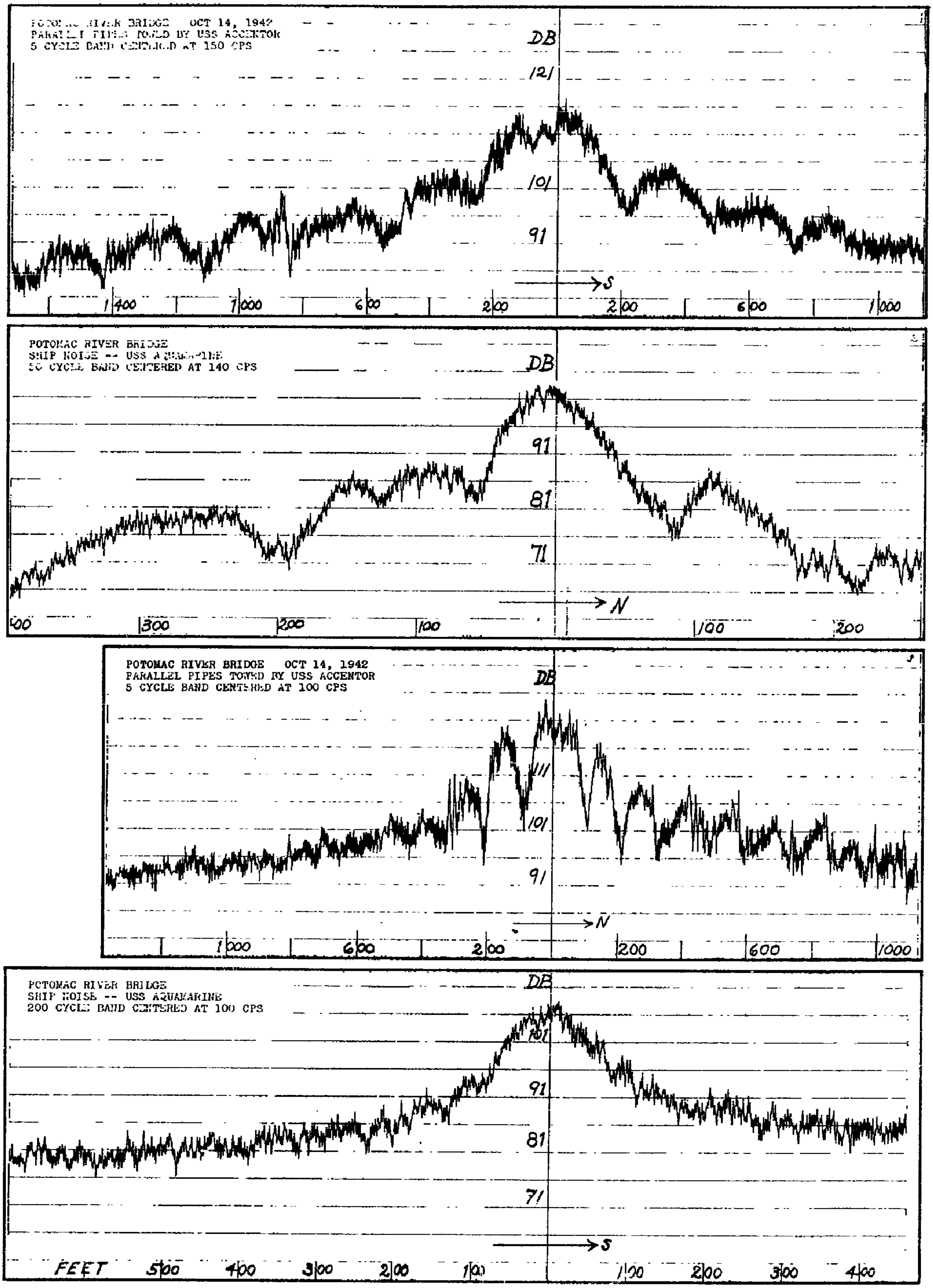
RANGE RECORDS OVER HARD BOTTOM ~ POTOMAC RIVER MOUTH

PLATE 6



RANGE RECORDS OVER HARD BOTTOM ~ POTOMAC RIVER MOUTH

PLATE 7



RANGE RECORDS FOR COMPLEX SOUND SOURCES

PLATE 8

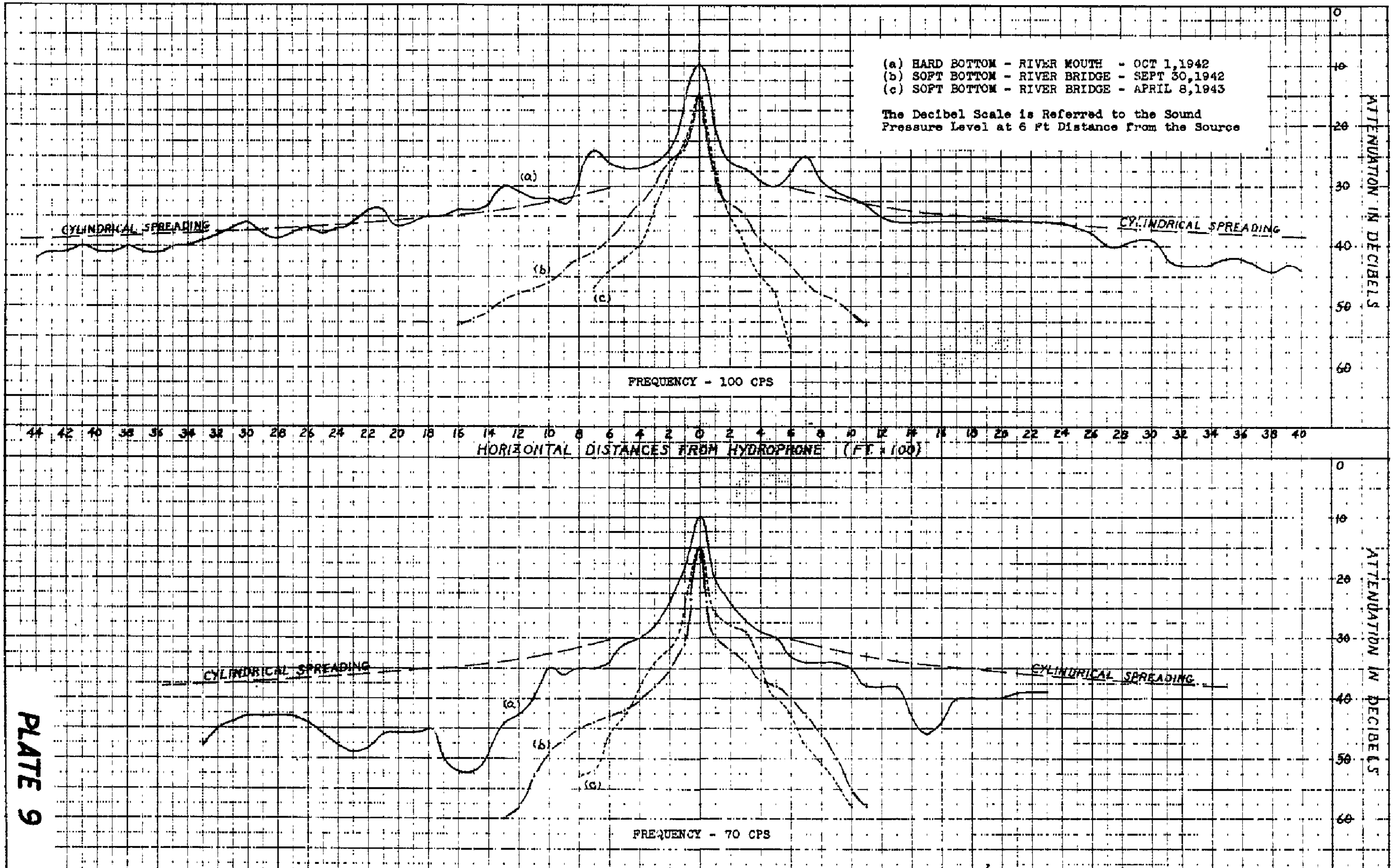
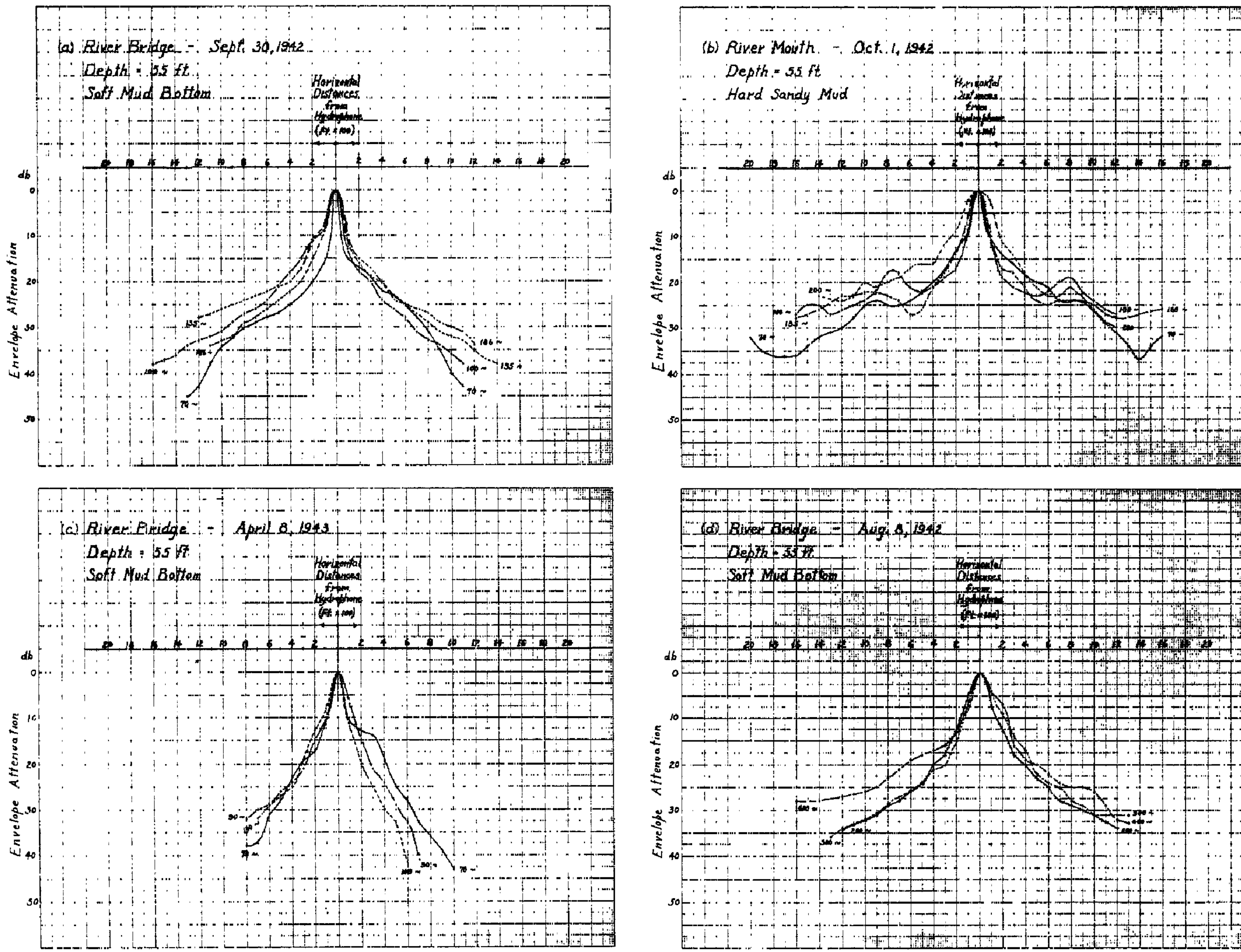


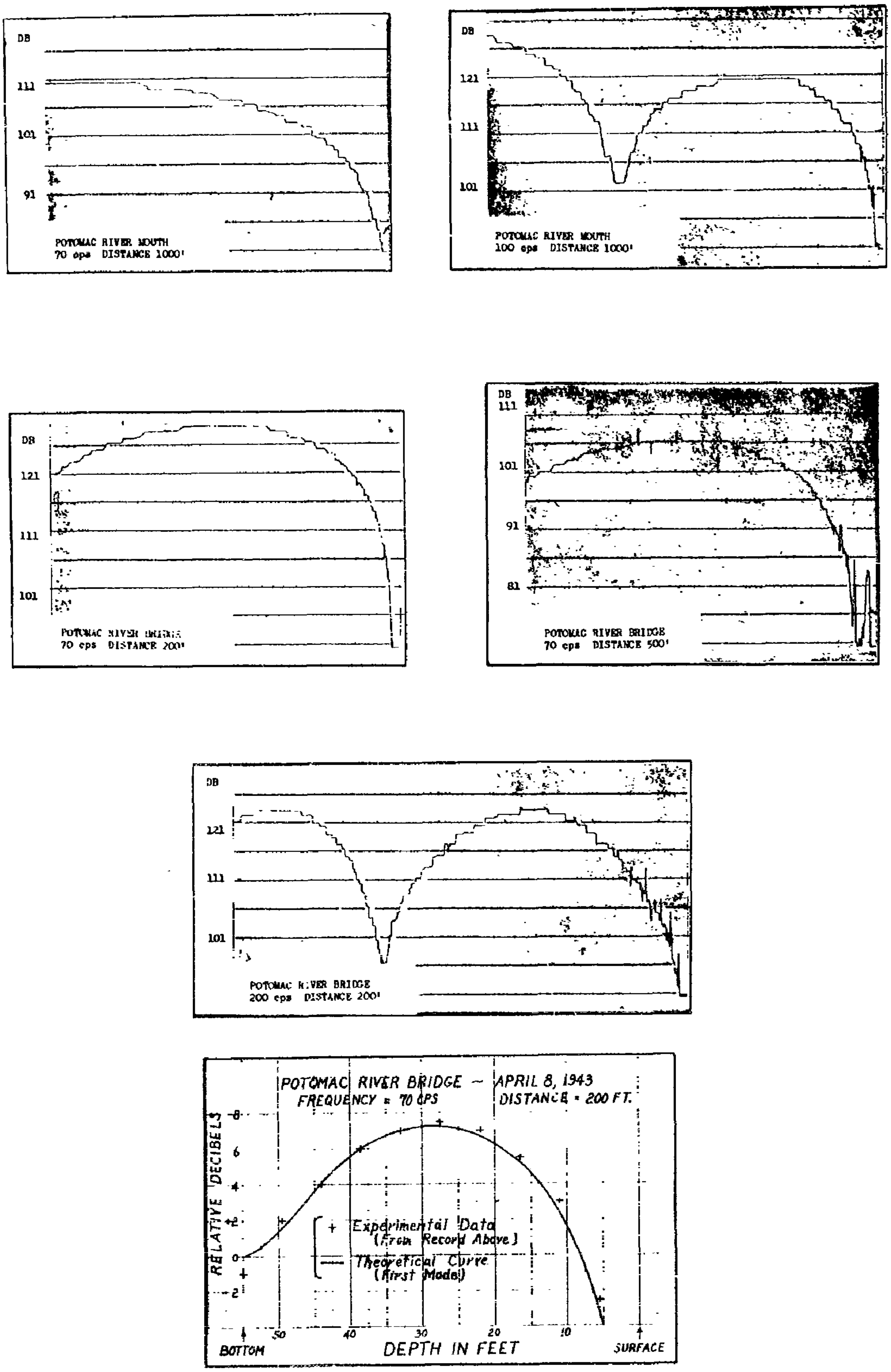
PLATE 9

TRANSMISSION OVER HARD AND SOFT BOTTOM



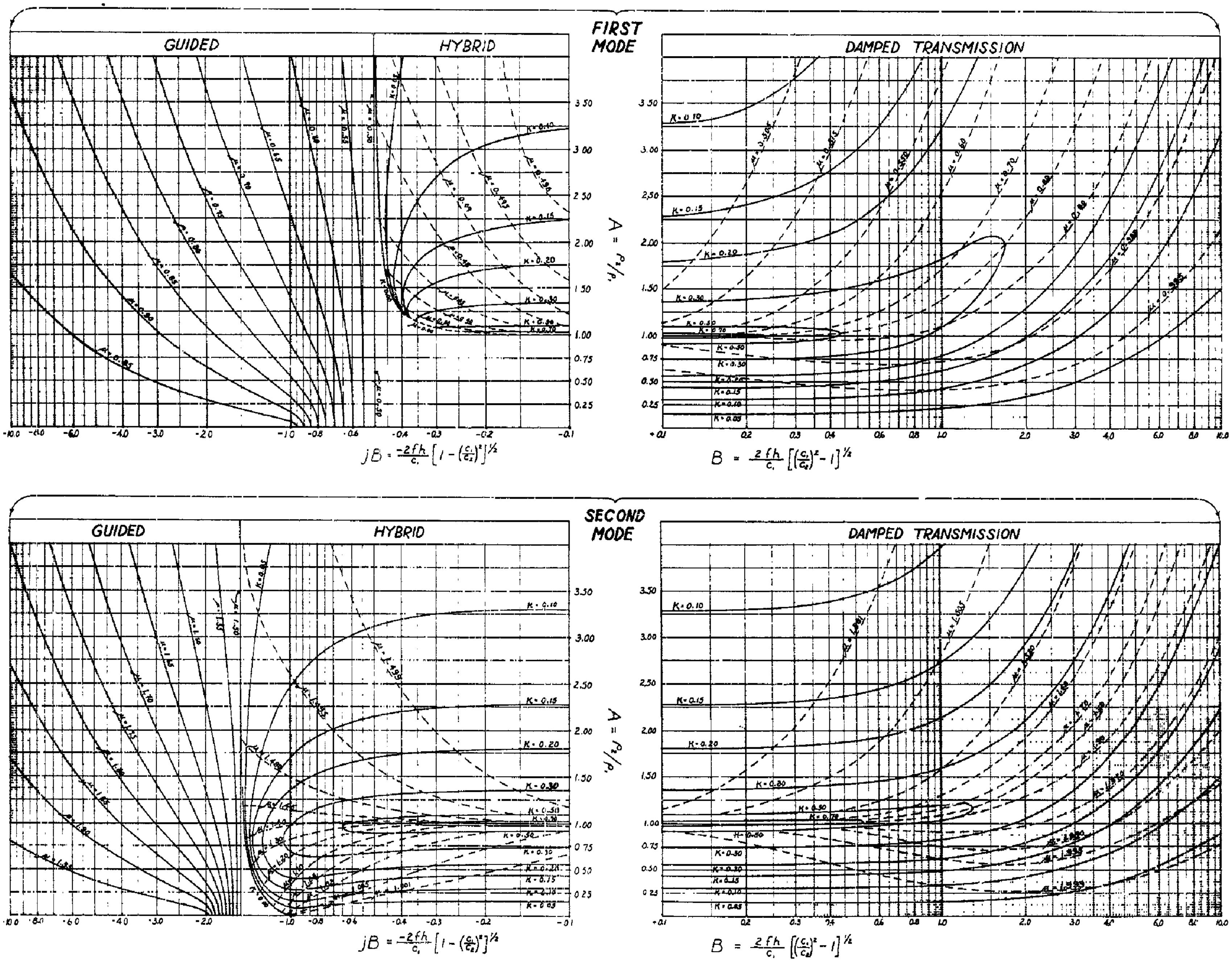
ENVELOPES OF RANGE RECORDS SHOWING CONSISTENCY OF DATA

PLATE 10

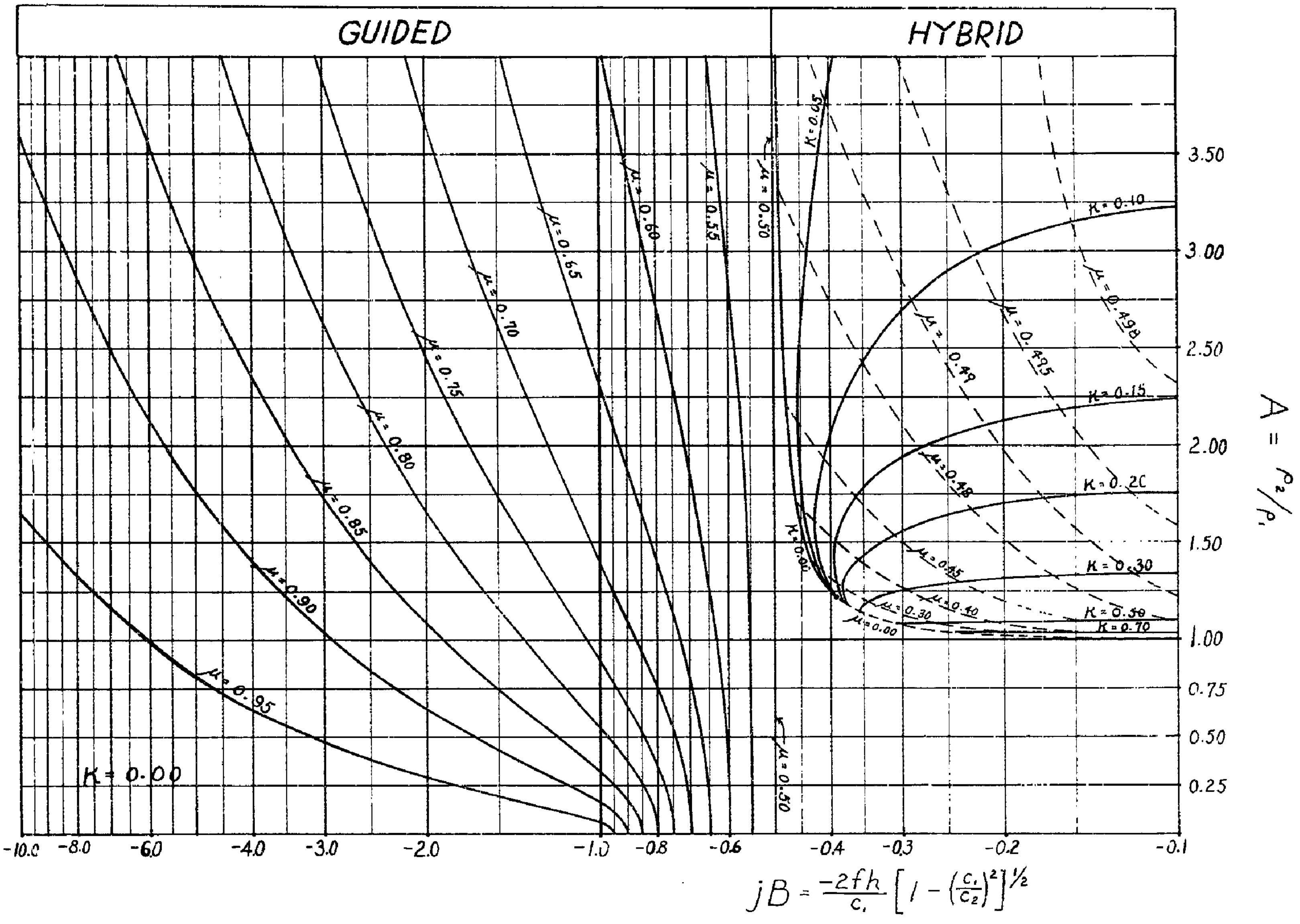


HYDROPHONE SOUNDINGS AT A DISTANCE FROM THE SOURCE

PLATE II

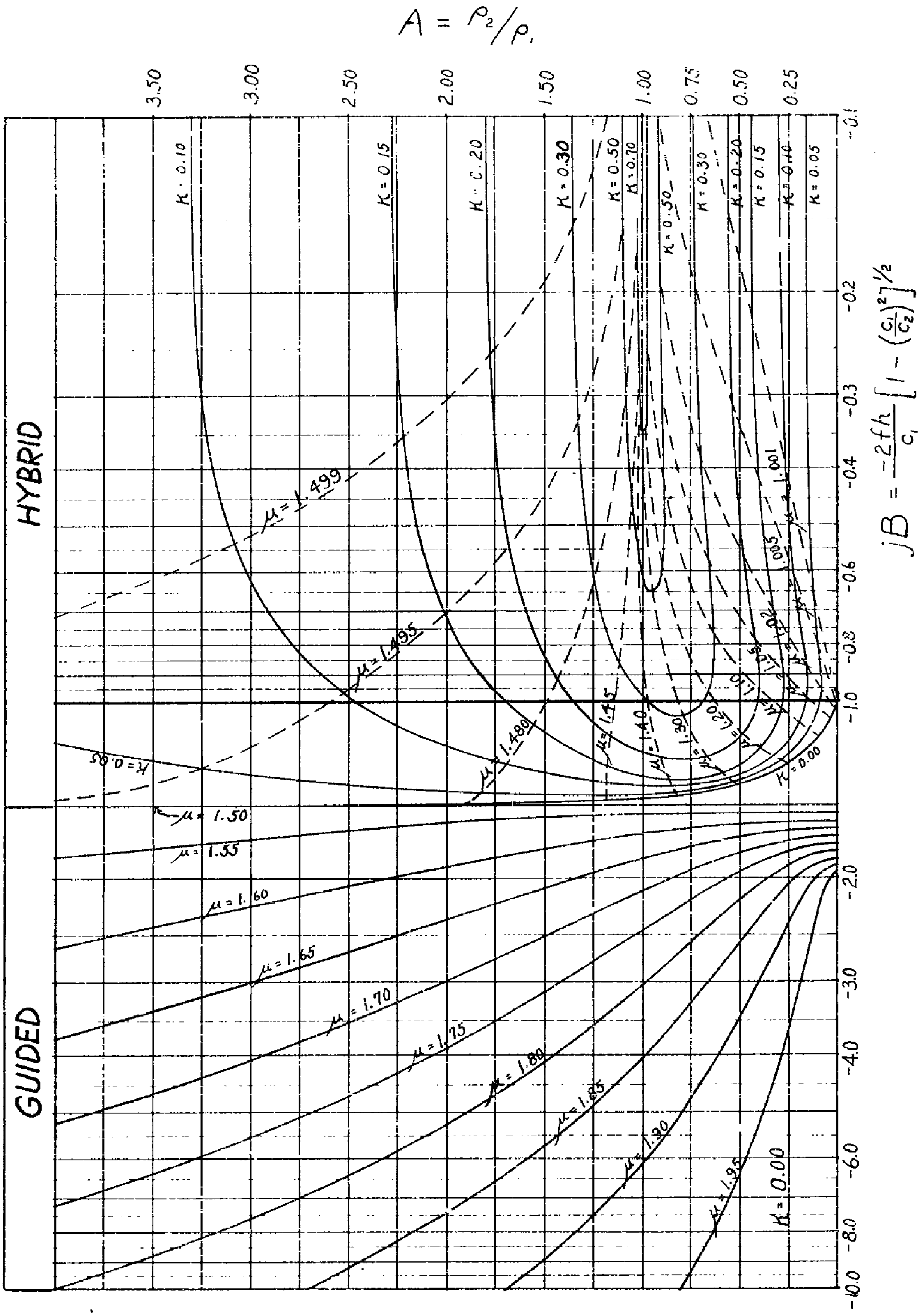


DISTRIBUTION CHARTS FOR THE FIRST AND SECOND MODES



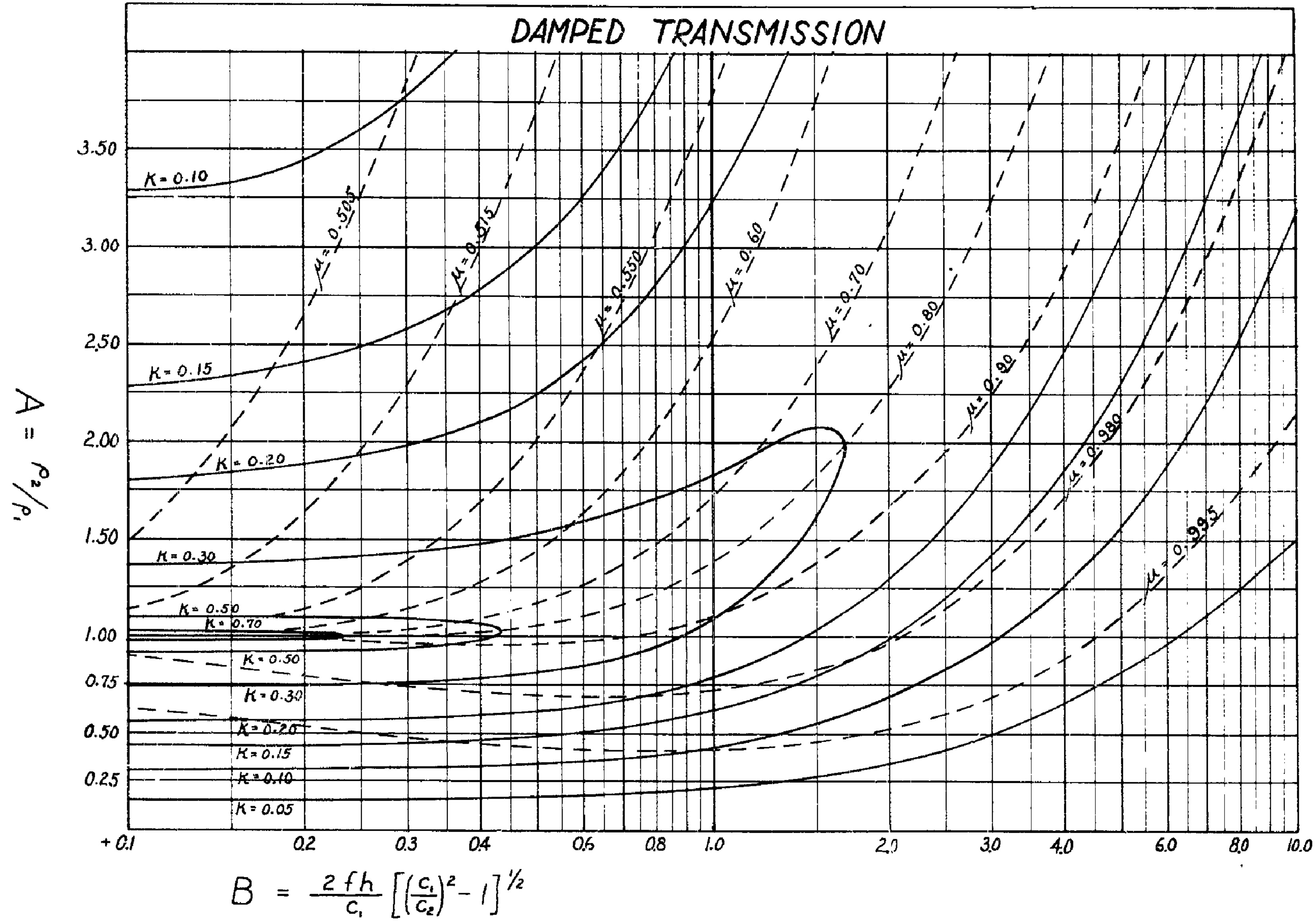
DISTRIBUTION CHART ~ FIRST MODE ~ HARD BOTTOM

PLATE 13



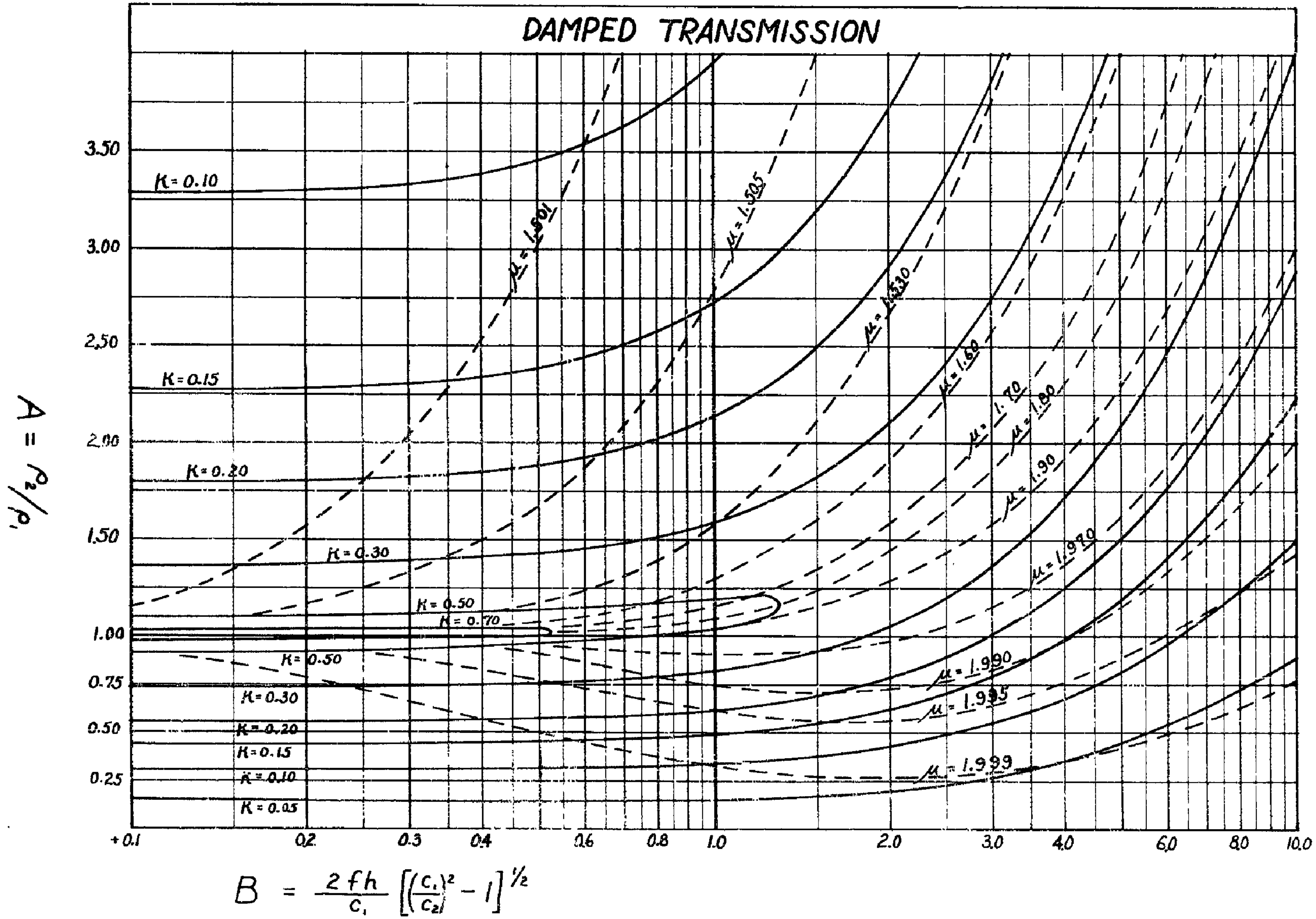
DISTRIBUTION CHART ~ SECOND MODE ~ HARD BOTTOM

PLATE 14



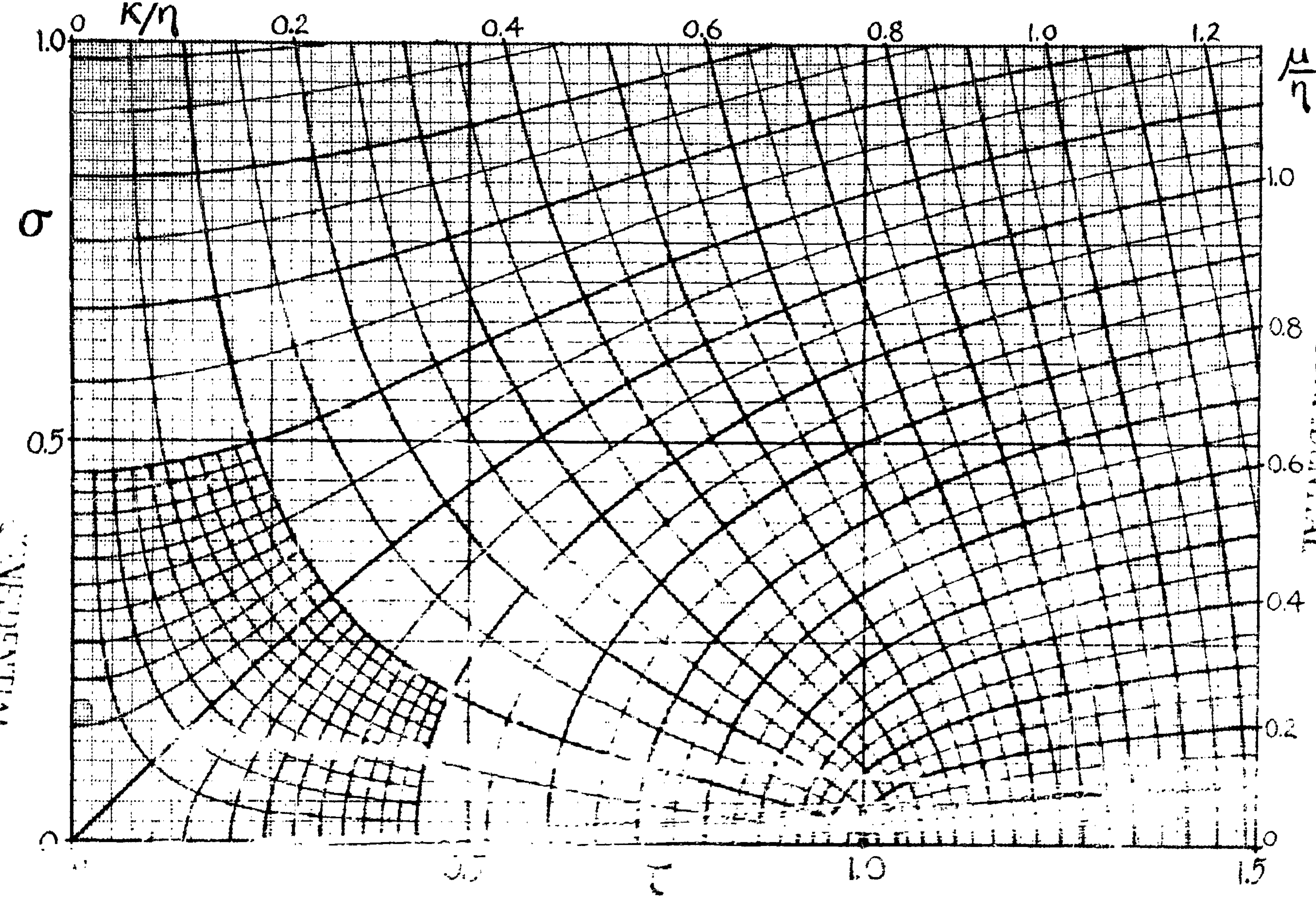
DISTRIBUTION CHART ~ FIRST MODE ~ SOFT BOTTOM

PLATE 15



DISTRIBUTION CHART ~ SECOND MODE ~ SOFT BOTTOM

PLATE 16



RELATIONS BETWEEN THE PROPAGATION AND DISTRIBUTION CONSTANTS

PLATE 17

AD-B801 494

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USGO: others to Director, Naval Research Lab., Wash., D.C.

(UNPUBLISHED CARD) ~~CONFIDENTIAL~~

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(OVER)

OFFICE OF NAVAL RESEARCH, NAVAL RESEARCH LAB., WASH., D.C.
(NRL REPORT NO. S-2113)

THE PROPAGATION OF UNDERWATER SOUND AT LOW FREQUENCIES, AS A FUNCTION OF THE ACOUSTIC PROPERTIES OF THE BOTTOM - AND APPENDIXES A-E - AND ADDENDA

JOHN M. IDE; RICHARD F. POST; WILLIAM J. FRY 15 AUG '43
188 PP. PHOTO, GRAPHS, DRWGS, MAP

SCIENCES, GENERAL ~~(33)~~ WAVES, SOUND - PROPAGATION IN FLUIDS
PHYSICS (2) ACOUSTICS, UNDER WATER

Underwater Acoustics

~~CONFIDENTIAL~~

~~SCR-3, AUTH: DOD DIR 5200.10, 29 JUNE 61~~

U Auth: USNRL ltr, 26 Feb 70

UNITED STATES GOVERNMENT

memorandum

7103-317:BGH:tec

DATE: 21 March 2001

FROM: Burton G. Hurdle (Code 7103)

SUBJECT: REVIEW OF REFS. (a) and (b) FOR REMOVAL OF RESTRICTIONS

TO: Code 1221.1 tL 3/22/01

VIA: Code 7100

REF: (a) NRL Report No. S-2113, "The Propagation of Underwater Sound at Low Frequencies, As a Function of The Acoustic Properties of The Bottom" by John M. Iede, Richard F. Post and William J. Fry; August 15,1943.

1. Reference (a) is a basic document on the development and use of normal mode theory in underwater acoustics.

2. The science and technology of this report have long been superseded. The current value of this reports is historical.

3. Based on the above, it is recommended that references (a) be available with no restrictions.

Burton G. Hurdle

BURTON G. HURDLE
Acoustics Division

CONCUR:

Edward R Franchi 3/21/01

E R. FRANCHI Date
Superintendent
Acoustics Division

OPTIONAL FORM NO. 19
(REV). 1-80)
GSA FPMR(41 CFR) 101-11.6
5010-114

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AN (1)  AD-B801 494/XAG
FG (2)  200100
CI (3)  (U)
CA (5)  NAVAL RESEARCH LAB  WASHINGTON DC
TI (6)  The Propagation of Underwater Sound at Low Frequencies, as a Function of the
        Acoustic Properties of the Bottom - and Appendixes A - E - and Addenda
AU (10) Ide, John M.
        Post, Richard F.
        Fry, William J.
RD (11) 15 Aug 1943
PG (12) 188 Pages
RS (14) NRL-S-2113
RN (18) XD-XD
RC (20) Unclassified report
AL (22) DISTRIBUTION AUTHORIZED TO DOD ONLY; ADMINISTRATIVE/OPERATIONAL USE; 24 FEB
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ID (25) Waves, Sound - Propagation in Fluids, Acoustics, Under Water, ATI-99 336
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DL (33) 04
CC (35) 251950
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