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REPORT NO. A-33 : PROGRESS REPORT

OSRD #380

# PRELIMINARY EXPERIMENTS ON THE PROPAGATION OF PLASTIC DEFORMATION

by

Pol. E. Duwez



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NATIONAL DEFENSE RESEARCH COMMITTEE

REPORT NO. A-33 : PROGRESS REPORT

# PRELIMINARY EXPERIMENTS ON THE PROPAGATION OF PLASTIC DEFORMATION

by

Pol. E. Duwez



Approved February 4, 1942
for submission to the Section Chairman

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Approved February 10, 1942
for submission to the Division Chairman

John E. Burchard, Chairman
Section B, Division A

Approved February 10, 1942
for submission to the Committee

Richard C. Tolman, Chairman,
Division A

## Preface

The work described in this report is pertinent to the projects designated by the War Department Liaison Officer as CE-5 and CE-6. The report is supplementary to NDRC Report A-29 (OSRD No.365), by Th. von Kármán. The work was done at the suggestion of Doctor von Kármán at a time when its possible military value was not clearly recognized; hence no formal arrangements for an NDRC project were made.

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# PRELIMINARY EXPERIMENTS ON THE PROPAGATION OF PLASTIC DEFORMATION


## Abstract

In a recent report,[1/] von Kármán develops a theory of the propagation of plastic deformation in solids that may open the way to a systematic interpretation of a great many impact and penetration problems in which plastic deformations of beams, plates and armor are involved. The present report describes experiments that have been made with the object of testing the assumptions of the theory and that provide data on (i) the existence of a plastic wave front of a given amplitude, (ii) the relation between the velocity of impact and the amplitude of the plastic wave front, and (iii) the shape of the plastic wave and the velocity of propagation of the plastic front. The experiments show that the theory is able to describe, along general lines, the process of the propagation of deformation in solids. In particular, the theoretically predicted relation between the velocity of impact and the resulting maximum plastic deformation checks quite well with the experiments. The actual shape of the plastic wave is found to be at some variance with the theoretically obtained curves and hence needs further explanation.


The object of the experiments described in this report is to check the formulas established in von Kármán's theory of the propagation of plastic deformation in solids.[1/] The verification deals chiefly with the following three points:

(i) The existence of a plastic wave front of a given amplitude.

(ii) The relation between the amplitude $\varepsilon_1$ of the plastic wave front and the velocity of impact $v_o$, namely,[2/]

$$v_o = \int_0^{\varepsilon_1} \sqrt{\frac{(d\sigma/d\varepsilon)}{\rho}}\, d\varepsilon, \qquad (11)$$

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1/ Th. von Kármán, On the propagation of plastic deformation in solids, NDRC Report A-29.

2/ This is Eq.(11) in von Kármán's report.

where $\sigma$ is the stress, $\varepsilon$ is the strain and $\rho$ is the density of the solid material.

(iii) The distribution of the plastic strain $\varepsilon$ between the plastic and the elastic front as given by the formula[3/]

$$T(\varepsilon) = \rho x^2/t^2, \qquad (9)$$

where $T$ [= $d\sigma/d\varepsilon$] is the modulus of deformation, $x$ is the distance from the end of the wire to the point under consideration, and $t$ is the time.

## 1. Experimental set-up

We were able to use the high velocity impact testing machine designed by Doctor D.S. Clark. In this testing machine, the impact is produced by a hammer that falls between two vertical rails and that is accelerated by pre-stretched rubber bands. The maximum velocity attainable is approximately 200 ft/sec. The velocity is measured by a suitable electric device.

The specimen used was an annealed copper wire about 100 in. long and 0.071 in. in diameter. On every specimen equidistant marks were made with 1-in. spacing. After the test, the plastic strain was determined by measuring the displacement of each mark. In order to observe the propagation of the plastic wave while it travels up the wire, a series of experiments was made, all with the same speed but with the impact stopped after different time intervals which varied from 0.4 to 4 millisec. In this way we

---

3/ Eq.(9) in von Kármán's report.

"freeze" the plastic strain along the wire, since the elastic deformation is negligibly small in comparison with the permanent set. For this purpose the following device was employed.

The bottom end of the wire is attached to a rigid piece A (Fig.1). At B is a vertical rod that rests on the bottom frame of the machine. The upper end of this rod fits loosely into the tubular part of A. When the hammer H hits the piece A, the specimen elongates until A reaches the rod B. The piece A contains a circular notch N, and the rim of A breaks off at this notch after A has traveled the distance D and comes to rest on the rod B. The purpose of this arrangement is to allow the hammer to continue to move downward and also to dissipate some of its remaining kinetic energy. However, no kinetic energy is transferred to the specimen after A reaches B. The time of impact is therefore the distance D divided by the velocity of the hammer during the process of elongation.

## 2. Static stress-strain curve for the specimen used

The static stress-strain curve for the copper used in the impact tests is shown in Fig.2. The specimen employed in the static test was a 20-in. length of the wire used for the impact experiments. The stress plotted in Fig.2 is the true stress calculated by means of the formula

$$\sigma_{true} = \sigma_{app.}/(1-\varepsilon),$$

where $\sigma_{app.}$ denotes the load divided by the initial cross sectional area.

From the aforementioned static stress-strain curve we calculated the value of $d\sigma/d\varepsilon$ as a function of $\varepsilon$ and then plotted the curve showing the quantity

$$\sqrt{\frac{(d\sigma/d\varepsilon)}{\rho}}$$

as a function of $\varepsilon$. This quantity, according to the theory, gives $v_1$, the velocity of propagation. In Fig.3 the corresponding values of the permanent set $\varepsilon$ and the quantity $\sqrt{(d\sigma/d\varepsilon)/\rho}$ derived from the static tests are plotted up to a value of 2000 ft/sec. Figure 4 represents the continuation of the curve up to the velocity of the elastic front. From these curves we can calculate, for each value of $\varepsilon_1$, the value of the quantity

$$v_o = \int_0^{\varepsilon_1} \sqrt{\frac{(d\sigma/d\varepsilon)}{\rho}}\, d\varepsilon, \qquad (11)$$

where $v_o$ is the velocity of impact corresponding to a plastic front of amplitude $\varepsilon_1$. The curve $v_o$ versus $\varepsilon_1$ is shown in Fig.5. This curve reaches an end at the point $\varepsilon_1$ = 16 percent, $v_o$ = 180 ft/sec. The impact velocity of 180 ft/sec is therefore the "critical velocity" for this material. An impact with a higher velocity must produce an instantaneous breakdown of the specimen.

## 3. Measurements of the distribution of the strain along the specimen

As mentioned in Sec.1, marks were made on the specimen at intervals of 1 in. The distance between the origin and

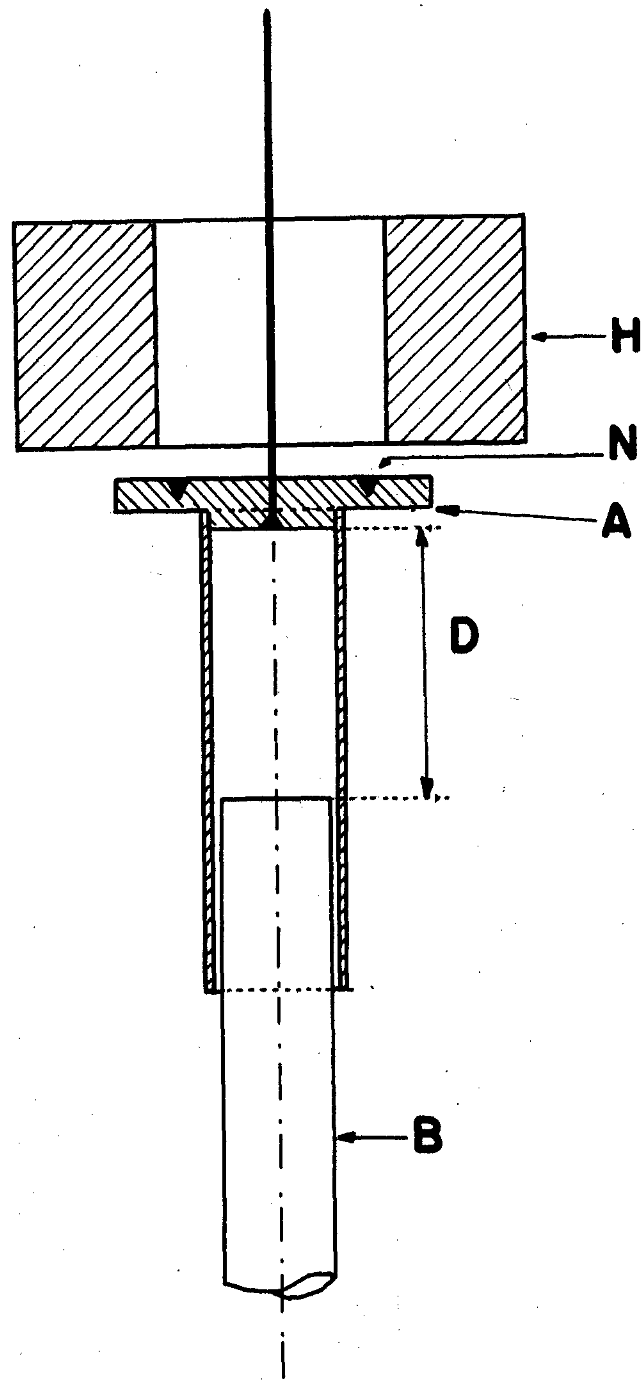


Fig. 1. Experimental device used to stop the impact after a given deformation of the specimen is reached.

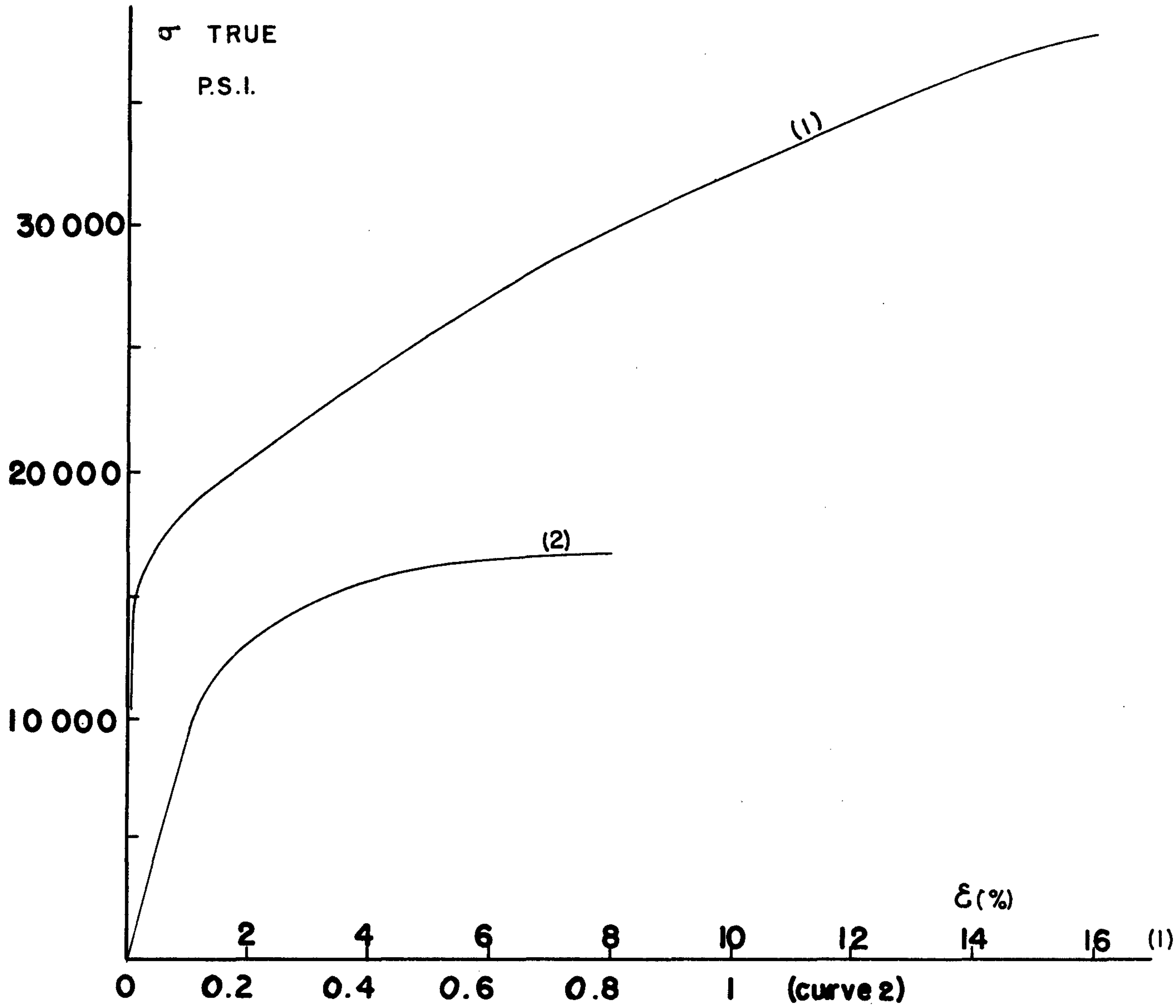


*Fig. 2. Static-strain curve for the specimen (copper wire, 0.071 in. diameter). Curve 2 is the portion of the same stress-strain curve with an enlarged scale for the strain.*

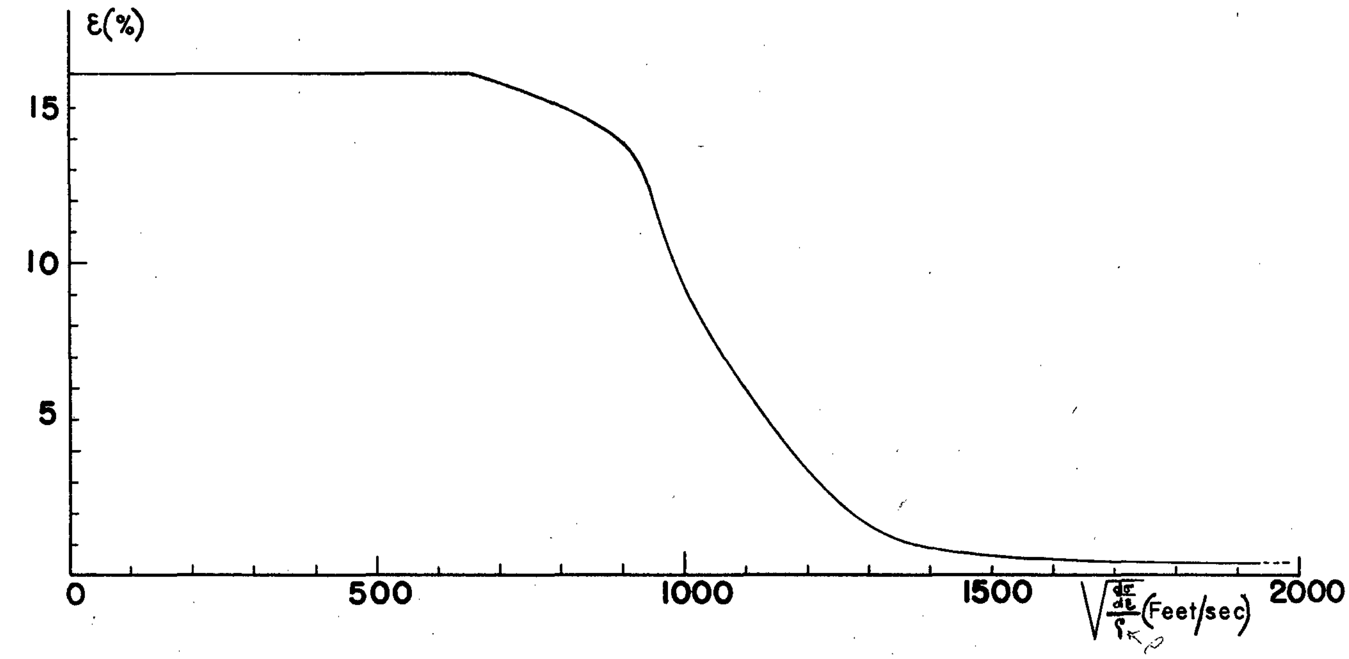


*Fig. 3. Permanent set ε versus speed of propagation, as calculated from the static stress-strain curve.*

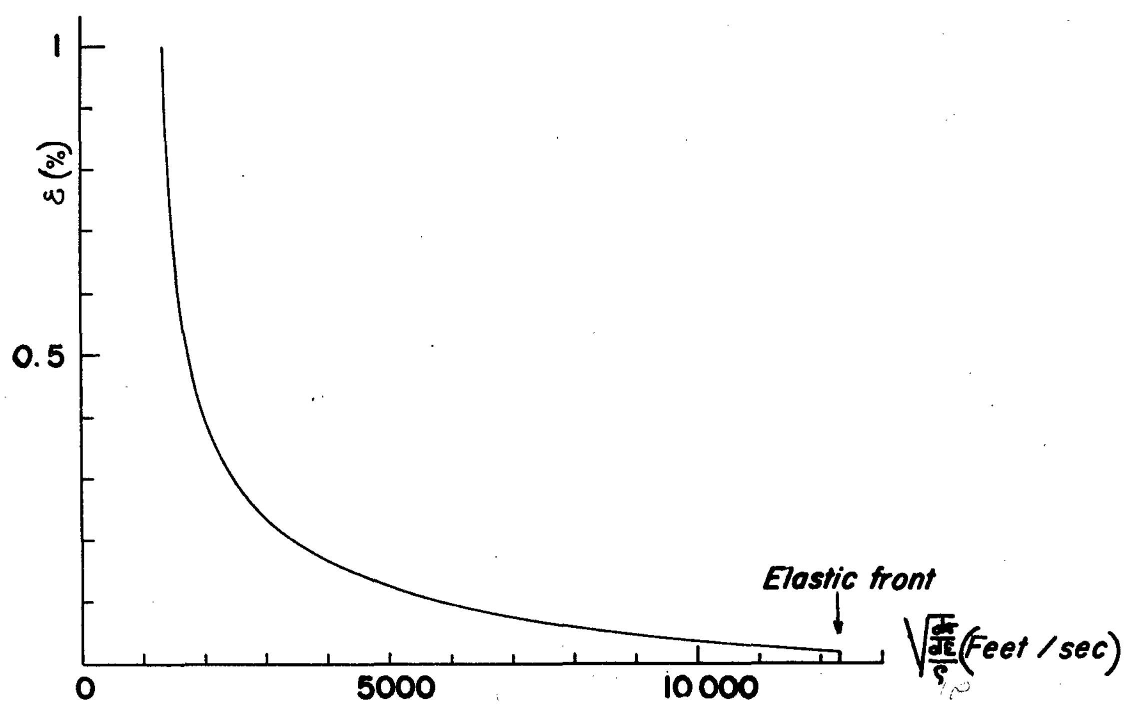


*Fig. 4. Continuation of the curve of Fig. 3 up to the elastic front.*

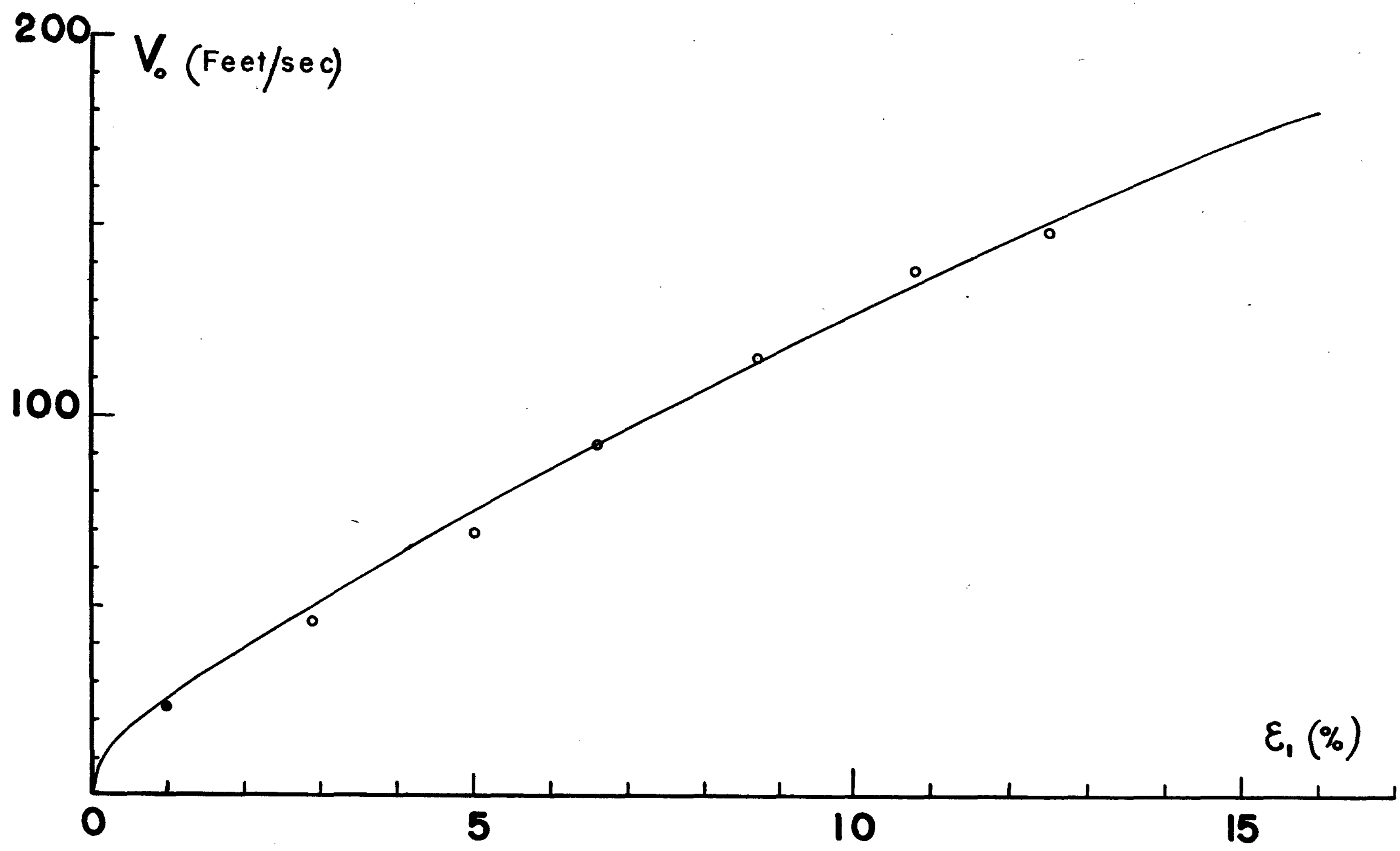


Fig. 5. Variation of the permanent set $\varepsilon_1$ with the speed of impact. Solid line computed theoretically; circles measured experimentally.

each mark is denoted by $x_1$, $x_2$, ... $x_n$, respectively. After the test the new distances between the origin and the marks were measured; these distances are denoted by $x_1'$, $x_2'$, ... $x_n'$. When the differences $x_n' - x_n$ are plotted as a function of $x$, curves such as those shown in Fig.6 are obtained. The sequence of curves corresponds to increasing time intervals. By taking the derivatives of the functions represented by the curves, we obtain the values of the strain $\varepsilon$ at each point of the wire. This way of measuring the strain has been found to be the most practical one.

## 4. Results of the high velocity impact tests

The results are given in three separate sub-sections -- (i), (ii) and (iii) -- in order to show to what extent they agree with the three principal theoretical results which we expressed at the beginning of this report.

(i) Existence of a plastic wave front. -- The first series of tests was made in order to show that the amplitude of the plastic front is a function of the velocity of impact alone. This amplitude remains constant while the elastic front and the plastic front travel along the specimen. In these particular experiments, the velocity of impact was always 92.50 ft/sec, but the durations of the impact were varied. To control the duration of impact, the two pieces A and B shown in Fig.1 must be placed at a proper distance apart. It is rather difficult to measure this distance accurately. For that reason we calculated the duration of impact $t$ from the total elongation $D$, measured on the specimen after the test, and the velocity $v_o$ of the hammer after impact; the formula used is $t = D/v_o$. The velocity after impact, $v_o$, was determined by measuring the velocity of the hammer immediately before the impact by means of an electric device and then multiplying this value by the ratio $m_H/(m_H + m_A)$, where $m_H$ and $m_A$ denote the masses of the hammer and the piece A, respectively.

Figure 6 shows the distribution of the elongation $x_n' - x_n$ along the wire. The duration of impact is indicated on each curve and is also given in Table I. All the curves have, near the origin, a straight portion whose slope is a constant. Figure 7 shows the actual readings and their deviations from a straight line for each test. The curves in Fig.8 -- obtained by computing the derivatives of the functions shown in Fig.6 -- give the distribution of the strain along the specimen. We may therefore conclude that a plastic front of a given amplitude $\varepsilon_1$ is revealed by the experiments as predicted from the theory.

Table I. Tests in which the velocity of impact was always 92.50 ft/sec but the durations of impact were different.

D, total elongation of the specimen;
d, distance traveled by the front of the plastic wave and measured on the curves of Fig.8;
$t[=D/v_o]$, calculated time of impact (millisec);
$v_1[=d/t]$, velocity of propagation of the plastic wave.

| D (in.) | d (in.) | t ($10^{-3}$sec) | $v_1$ (ft/sec) |
|---|---|---|---|
| 0.46 | 4.4 | 0.415 | 890 |
| .925 | 9.45 | .834 | 945 |
| 1.14 | 12.4 | 1.03 | 1000 |
| 1.81 | 17.8 | 1.63 | 910 |
| 2.50 | 26.4 | 2.255 | 975 |
| 4.17 | 41.5 | 3.76 | 920 |
| | | Mean Value | 940 |

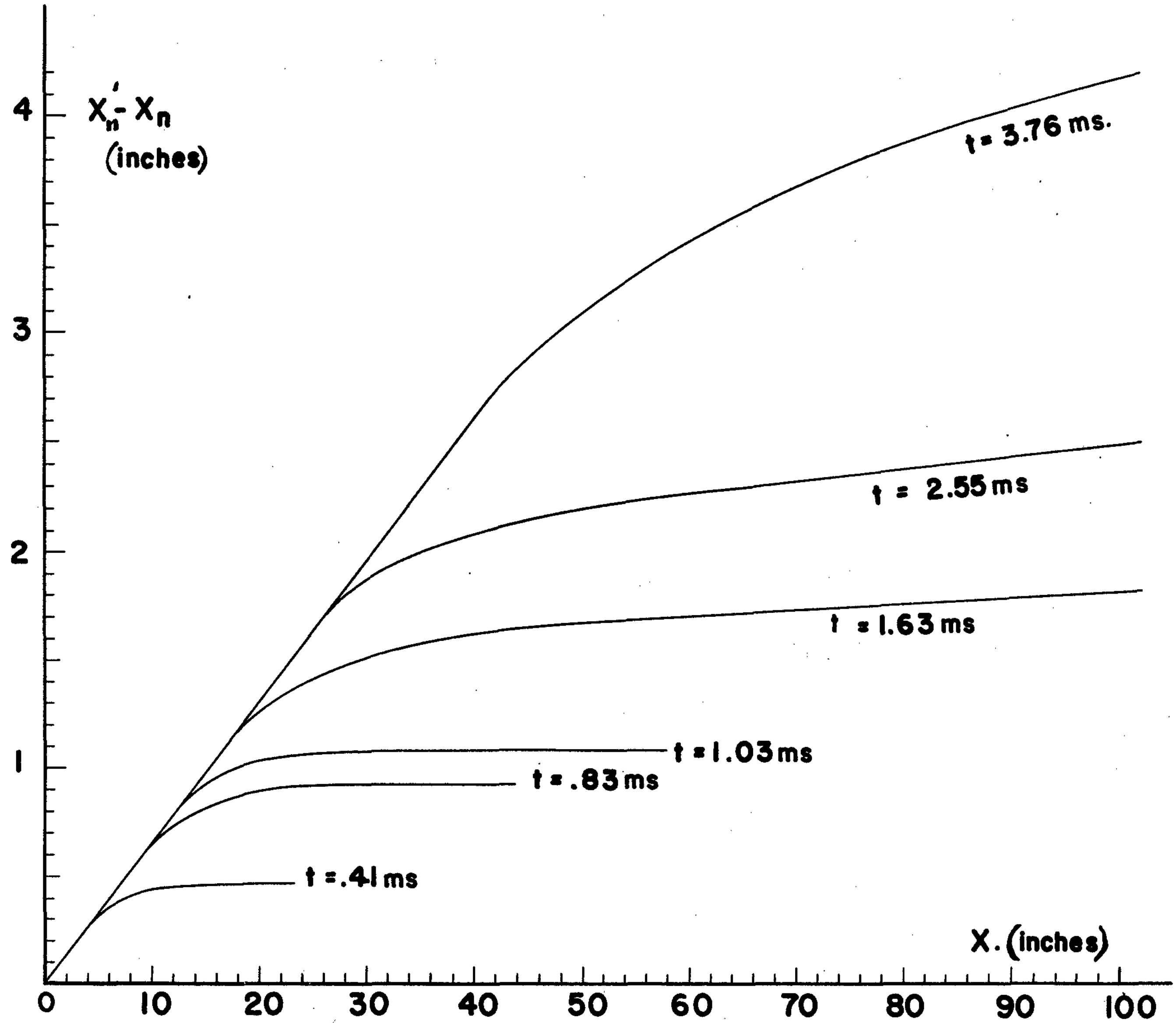


Fig. 6. Distribution of the elongation $X_n' - X_n$ along the specimen. Speed of impact, 92.5 ft/sec. The duration of impact (millisec) is indicated on each curve.

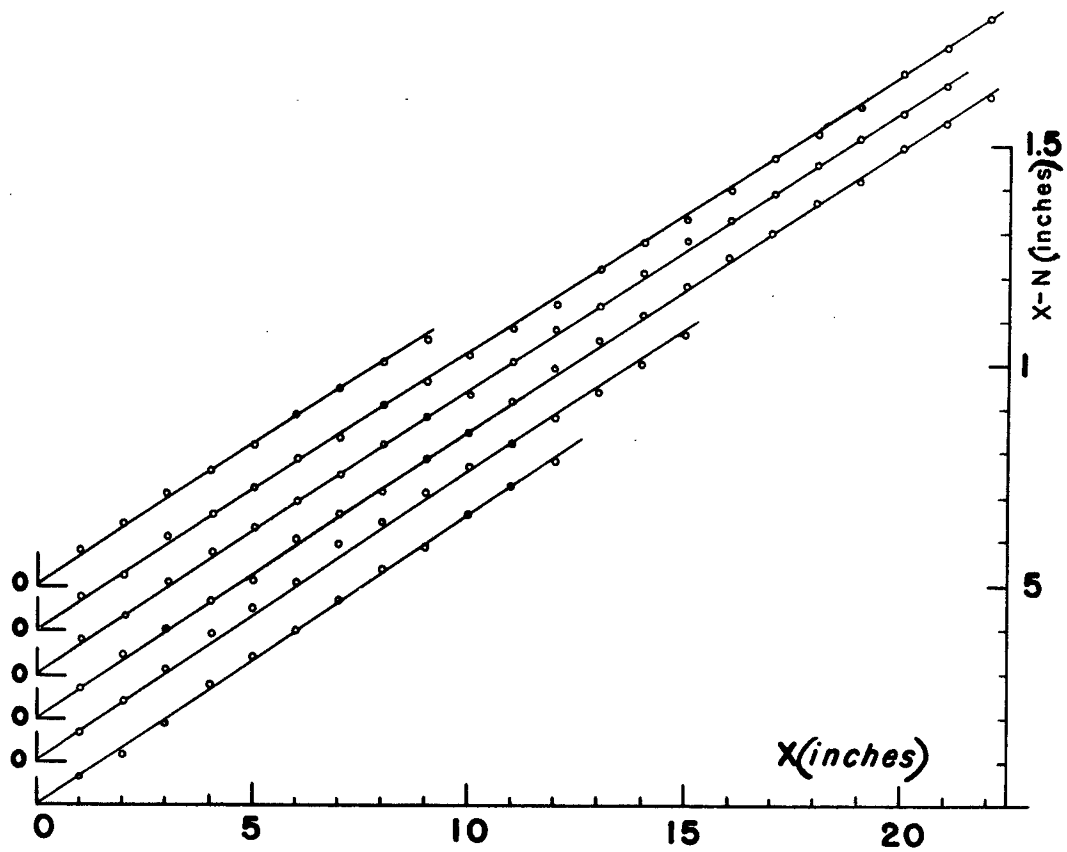


*Fig. 7. Linear portions of the curves of Fig. 6, showing the actual readings and their deviations from a straight line.*

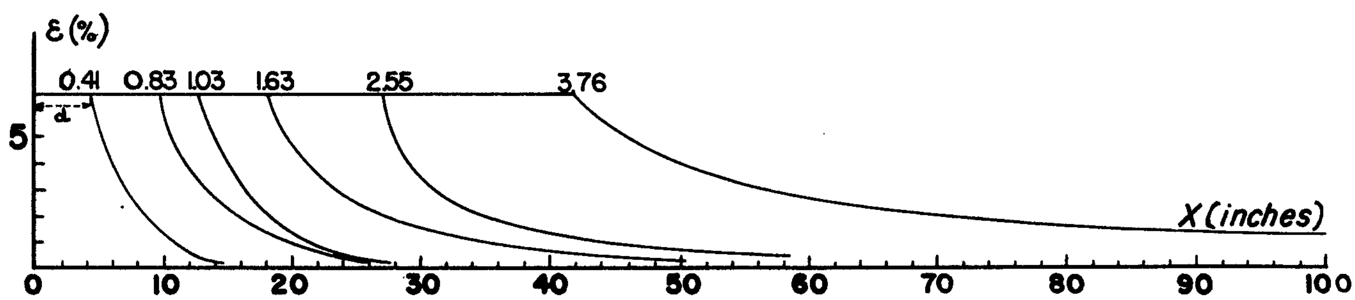


*Fig. 8. Distribution of the strain ε along the specimen. Speed of impact, 92.5 ft/sec. The duration of impact (millisec) is indicated on each curve.*

(ii) Relation between the velocity of impact and the amplitude of the plastic wave front. -- In this series of tests, the velocity of impact was varied from one test to the other. The total elongation was not necessarily the same for all the tests. The stopping device was adjusted in such a manner that, during the impact, the plastic front traveled a distance of between 20 and 40 in. Figure 9 again shows the elongations $x_n' - x_n$ at points on the specimen corresponding to the distances $x$ from the origin. The slope of the straight line portion of each curve gives the amplitude of the plastic front for each speed of impact. Figure 10 gives the distribution of the strain along the specimen in each case. The experimental values of $\varepsilon_1$ corresponding to each velocity tested are listed in Table II. For comparison with the theory, the experimental values are plotted as points in Fig.5, whereas the solid curve represents the result of the theoretical

Table II. Tests in which the velocity of impacts was varied.

$v_0$, velocity of impact;
$\varepsilon_1$, amplitude of the plastic wave (%);
$D$, total elongation of the specimen;
$d$, distance traveled by the front of the plastic wave and measured on the curves Fig.10;
$t[=D/v_0]$, calculated time of impact (millisec);
$v_1[=d/t]$, speed of propagation of the plastic wave.

| $v_0$ (ft/sec) | $\varepsilon_1$ (%) | $D$ (in.) | $d$ (in.) | $t$ ($10^{-3}$sec) | $v_1$ (ft/sec) |
|---|---|---|---|---|---|
| 23.1 | 1.0 | 0.47 | 25 | 1.70 | 1230 |
| 46.2 | 2.9 | .82 | 20 | 1.47 | 1130 |
| 69.4 | 5.0 | 2.81 | 40.5 | 3.37 | 1000 |
| 92.5 | 6.6 | 1.81 | 17.8 | 1.63 | 910 |
| 115 | 8.7 | 2.14 | 16 | 1.55 | 860 |
| 139 | 10.8 | 3.24 | 17 | 1.94 | 730 |
| 148 | 12.5 | 2.49 | 10.5 | 1.40 | 630 |
| 171 | | Rupture | | | |

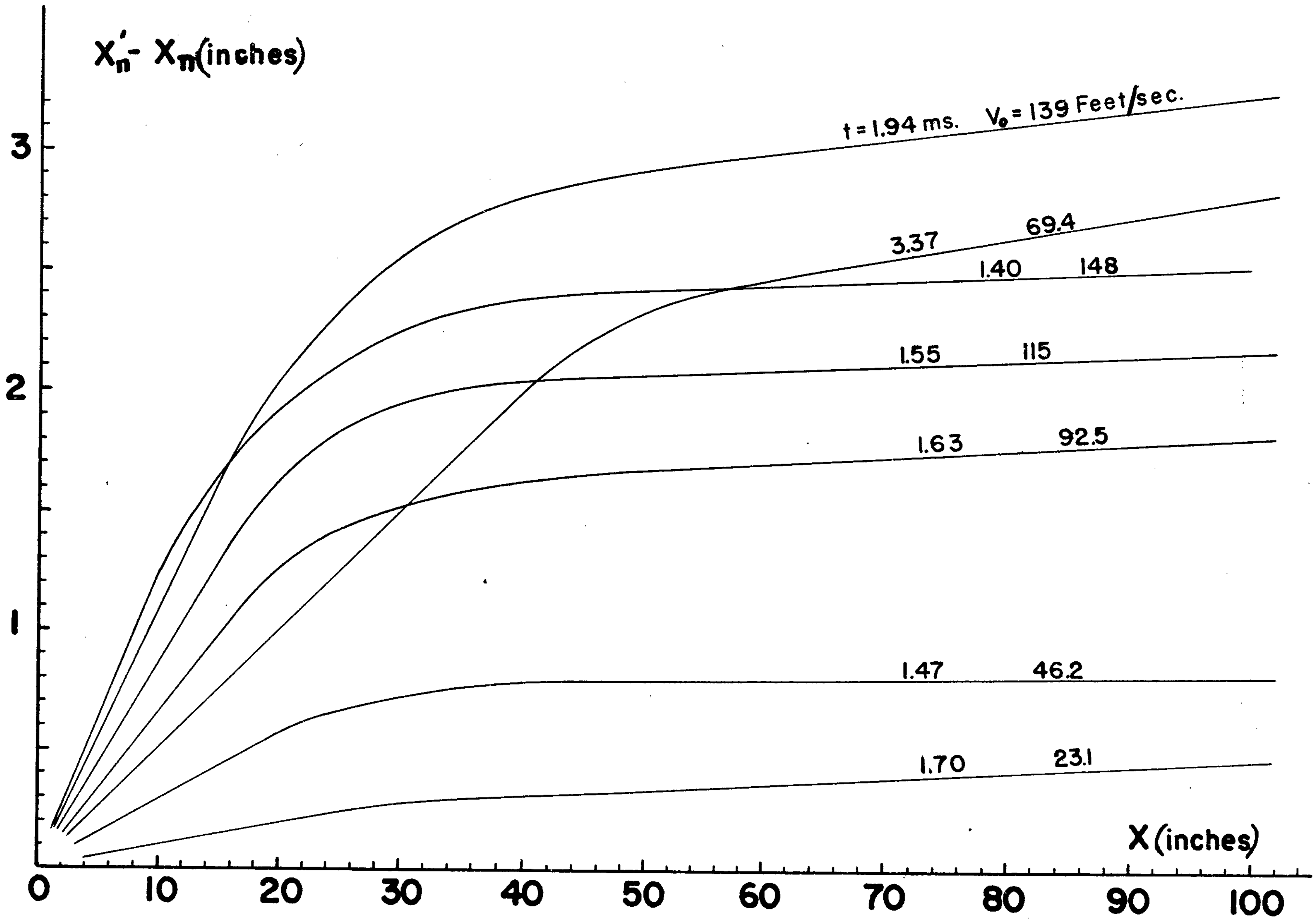


*Fig. 9. Distribution of the elongation $X_n' - X_n$ along the specimen. The velocity and duration of impact are indicated on each curve.*

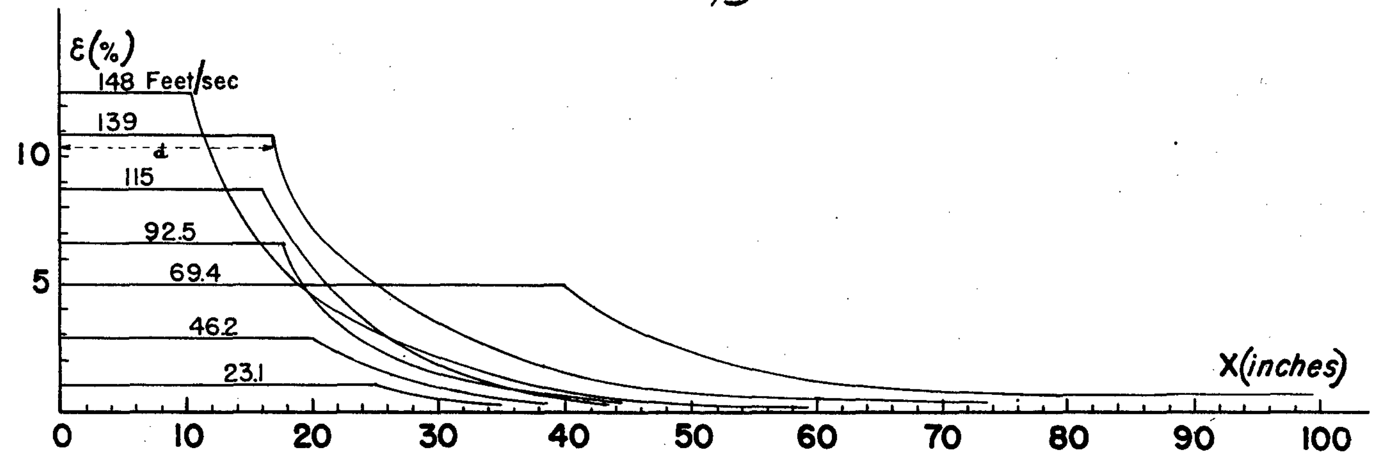


*Fig. 10. Distribution of the strain ε along the specimen. The speed of the impact (ft/sec) is indicated on each curve.*

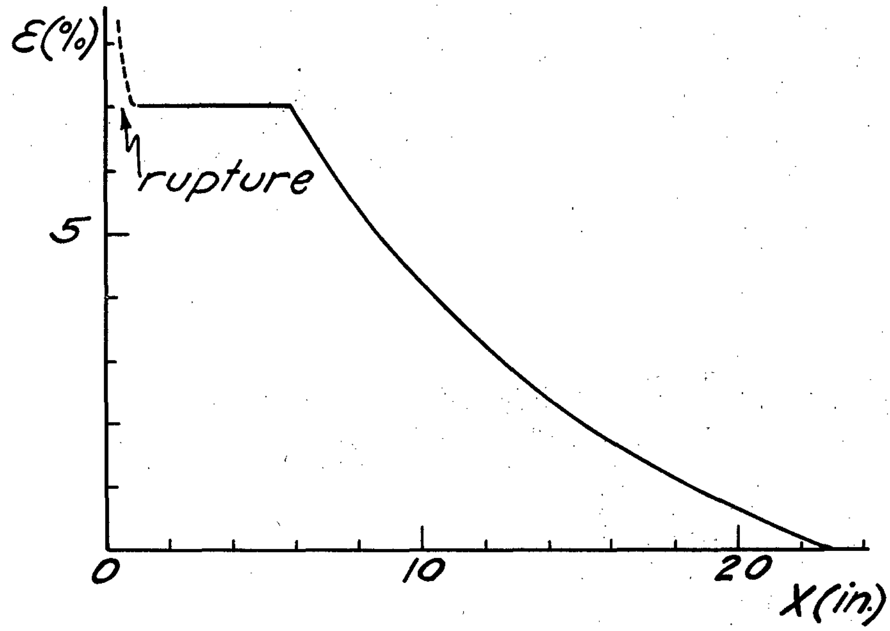


*Fig. 11. Distribution of the strain along the specimen broken at a speed of impact of 171 ft/sec. Rupture occured within 1 in. from the end of the specimen, where the strain is about 50 percent.*

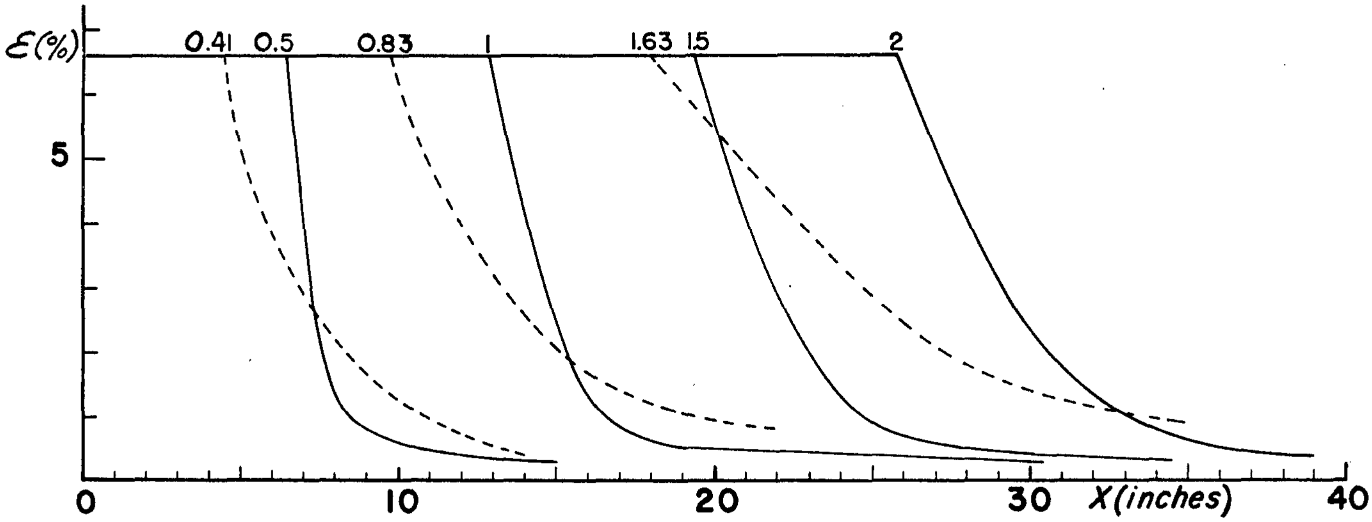


*Fig. 12. Plastic waves computed from theory (solid curves) and measured (broken curves). Speed of impact, 92.5 ft/sec. The duration of impact (millisec) is indicated on each curve.*

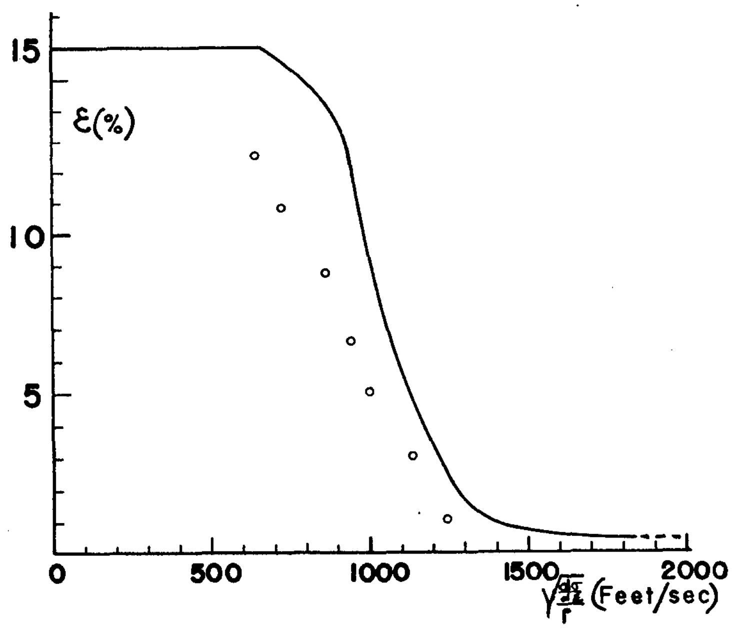


*Fig. 13. Same curve as Fig. 3 with experimental results (dots) added.*

computation. The agreement between the experimental results and the predetermined curve is fairly good. The highest speed of impact we used was 171 ft/sec. For that speed the specimen broke within the first inch. The distribution of the strain along the specimen is shown in Fig.11. The theoretical limiting velocity is 180 ft/sec, and therefore is slightly higher than the velocity for which we obtained a rupture. It must be pointed out also that, whereas the rupture occurs at the bottom end of the specimen, a plastic wave of relatively small intensity travels a considerable distance along the specimen, as shown by the curve in Fig.11. It appears that the rupture required a relatively large time interval. The first inch of the specimen showed an elongation of about 50 percent; this corresponds to a time interval of 0.24 millisec. More experiments on the critical impact velocity are necessary to elucidate this point.

(iii) Shape of the plastic wave and velocity of propagation of the plastic front. -- The distribution of the plastic strain $\varepsilon$ along the wire between the plastic and the elastic fronts may be calculated from Eq.(9), using the curves of Figs.3 and 4. The solid curves in Fig.12 represent theoretically computed amplitude distributions of a plastic wave after time intervals equal to 0.5, 1, 1.5 and 2 millisec; the assumed velocity of impact $v_o$ is 92.5 ft/sec and $\varepsilon_1$ is 6.6 percent. The dashed curves in Fig.12 represent plots of the distribution of the strain for three tests made with the velocity of impact of 92.5 ft/sec. It is seen that the experimental and theoretical curves deviate considerably. In general, the velocity of propagation of the plastic front and of the upper portion of the wave is smaller than the theory would lead us to expect. On the otherhand, the velocity of propagation of the small strains is greater than the theoretical velocity. The curve for an observed plastic wave is not as steep as the theoretical one. In Fig.13 the computed and measured velocities of propagation are compared. It is seen that the actual velocity of propagation is smaller than that given by the theory for the same value of the strain.

It was mentioned in sub-section (ii) that the theoretical relation between velocity of impact and maximum strain checks quite well. This is not at variance with the findings of the present sub-section, since the velocity of impact that produces a certain amplitude $\varepsilon_1$ is determined by the average value of the speed of propagation, which is almost the same for the theoretical and experimental curves shown in Fig.12.

## 5. Conclusions

It is seen that the theory is able to describe in general lines the process of the propagation of plastic deformation in solids. The theoretical relation established between the velocity of impact and the maximum plastic deformation produced by the same impact checks quite well with the experiments. The shape of the plastic wave is at some variance with the theoretically obtained curves. These deviations need further explanation. However, it is possible that they can be ascribed to the influence of the finite length of the specimens, the influence of the rate of strain on the stress-strain relation of the material (neglected in the theory) and to the fact that the impact is not perfectly instantaneous. It is obvious that further theoretical and experimental research is necessary to clear these aspects of the problem. As far as the influence of the finite length is concerned, a number of observations have already been made during the tests which clearly show that the plastic wave is reflected at the fixed end of the specimen, and this of course must influence the shape of the measured plastic wave.

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ABSTRACT:

Experiments have been made with the object of testing the assumptions of the von Karman theory of propagation of plastic deformation. They provide data on the existence of a plastic wave front of a given amplitude, the relation between the velocity of impact and the amplitude of the plastic wave front, and the shape of the plastic wave and the velocity of propagation of the plastic front. The experiments show that the theory is able to describe, along general lines, the process of the propagation of deformation in solids. In particular, the theoretically predicted relation between the velocity of impact and the resulting maximum plastic deformation checks well with the experiments. The actual shape of the plastic wave is found to be at some variance with the theoretically obtained curves and hence needs further explanation.


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