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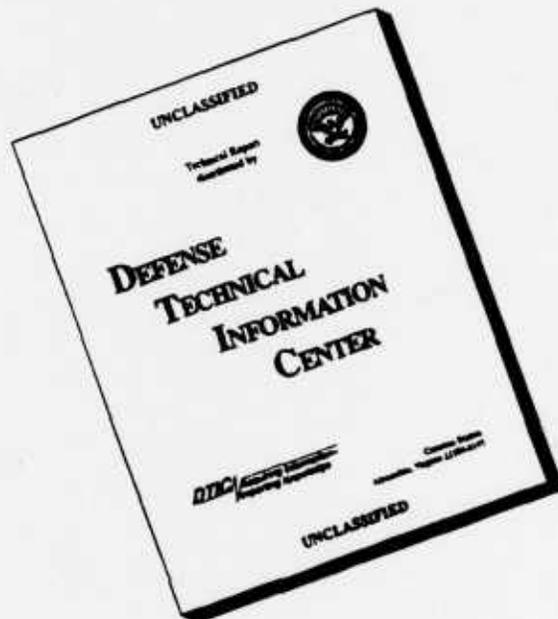
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ANISOPLANATISM IN ADAPTIVE OPTICS

by

Feng Yuezhong, Gong Zhiben, Song Zhengfang

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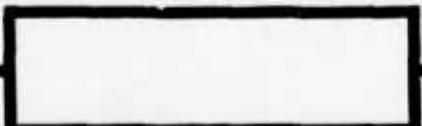
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ANISOPLANATISM IN ADAPTIVE OPTICS

By: Feng Yuezhong, Gong Zhiben, Song Zhengfang

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1

ANISOPLANATISM IN ADAPTIVE OPTICS

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Abstract: The paper discusses the influence of the
compensatory effect due to the anisoplanatic field in adaptive
optics, and proposes a new method, a phase gradient method to
reduce the error in anisoplanatism. As shown by analytical
results, this method can significantly increase the range of
angles for effective calibration; thus, the influence on the
anisoplanatic error is greatly reduced. The authors anticipate
that the phase gradient method can be expected to solve beacon
anisoplanatic problem in adaptive optics.

Key words: adaptive optics, isoplanatic angle, phase
gradient method.

I. Introduction

When calibrating the effect of turbulence in the atmosphere
by using adaptive optics, in the conventional work approach
measure the wavefront error information carried by the light beam

coming from the target, and apply the conjugate calibration
amount to the emitted light beam; thus, the phase distortion in
the channel is precisely cancelled out when the emitted light
beam traverses the same light path. The light beam providing the
wavefront error information is called the beacon light beam,
which comes from the target or other independent light sources.
Related to the beacon, an important parameter is the isoplanatic
angle, which forms a certain conical region in the atmosphere.
In the direction of the isoplanatic angle in the region,
turbulence induces phase fluctuations; these are correlated.
When the beacon conforms to the target direction, the wavefront
distortion of the beacon light beam will fully represent the
phase error along the transmission channel. When the beacon
light beam and the target light beam form an included angle not
exceeding the isoplanatic angle, the beacon light beam still can
provide precise phase information for the transmission channel.
However, when the included angle exceeds the isoplanatic angle,
the turbulent regions traversed by the beacon light beam and the
emitted light beam will be quite different; the phase information
of the beacon light beam cannot completely represent the phase
distortion along the emission path. Thus, the system performance
of the adaptive optics will be limited by the anisoplanatic
field. The calculation results in the literature [1,2] indicate
that the influence of the anisoplanatic field on the compensation
effect is quite serious. To overcome the anisoplanatism error,
some researcher [3] proposed the concept of a multilayer adaptive

optics system in calibrating atmospheric turbulence in layers at different heights. However, up to now there have been no further reports on theoretical work and experimental status. This paper will further discuss the compensation for the effect of atmospheric turbulence by using an adaptive optics system for calibration under anisoplanatic conditions; a new method is also proposed to overcome anisoplanatic errors.

II. Anisoplanatic Influence in Conventional Adaptive Optics

When the beacon deviates from the target, the beacon light beam and the emitted light beam will traverse two different turbulent paths. Strictly speaking, even if these two beams are within the same isoplanatic field, the phase distortion of the beacon channel and the phase distortion of the emission channel cannot be completely identical. In traditional adaptive optics, conjugated phase distortion in the beacon channel thus detected is applied to the initial emission field in order to calibrate the phase distortion in the emission channel. This compensation will certainly generate an error in the residual phase. Generally speaking, the error increases with increasing distance between beacon and target. Assume that the atmospheric equivalent distance between beacon and target is P, and the target lies on a symmetrical axis of the emission aperture, the effective thickness of the atmospheric layer is L (see Fig. 1).

3
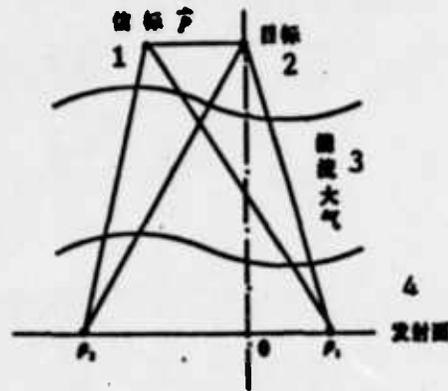
Fig. 1. Schematic diagram of the system
KEY: 1 - Beacon   2 - Target   3 - Turbulent
atmosphere   4 - Ground emission plane

Based on the expanded theory of Huygens-Fresnel [4], the average light intensity after calibration at the target is

$$\langle I(0)\rangle = \lambda^{-1}L^{-1}\iint d^2\rho_1 d^2\rho_2 u_0(\rho_1)u_0^*(\rho_2)\exp\left(ik\frac{\rho_1^2-\rho_2^2}{2L}\right)$$

$$\cdot\langle\exp\{\chi(\rho_1,0)+\chi(\rho_2,0)+i[S(\rho_1,0)-S(\rho_2,0)]-i[S(P,\rho_1)-S(P,\rho_2)]\}\rangle \quad (1)$$

Here, $u_0(\rho)$ is the initial-field distribution; λ is the wavelength of the emitted light beam; $\chi(\rho_1,0)$ and $S(\rho_1,0)$ are, respectively, logarithmic fluctuation of oscillation amplitude and phase fluctuation of the light field at the point $(0, L)$ after transmission through a turbulent atmosphere for a spherical-surface wave at point $(\rho_1,0)$; $S(P,\rho_1)$ is the phase fluctuation of the spherical-surface wave at point (L,P) arriving at point $(0,\rho_1)$. Based on the atmospheric interchange principle, phase fluctuation $S(\rho_1,P)$ at point (L,P) for spherical-surface waves of $S(P,\rho_1)$ and the spherical-surface wave at point $(0,\rho_1)$ are the same. Both theory and experiments indicate that the

4

fluctuations in the oscillation amplitude and phase fluctuations
satisfy the Gaussian distribution; these two fluctuations are
independent. Therefore, Eq. (1) can be rewritten as

$$\langle J(0)\rangle = \lambda^{-2}L^{-2}\iint d^2\rho_1 d^2\rho_2 u_0(\rho_1)u_0^*(\rho_2)\exp\left(ik\frac{\rho_1^2-\rho_2^2}{2L}\right)$$
$$\cdot \exp\left[-\frac{1}{2}D_x(\rho_1-\rho_2)-\frac{1}{2}D_{xs}(\rho_1-\rho_2,P)\right] \qquad (2)$$

Here $D_x(\rho_1-\rho_2)$ is the logarithmic structure function of
oscillation amplitude fluctuations; $D_{xs}(\rho_1-\rho_2,p)$ is the structure
function of the residual phase caused by anisoplanatism, and can
be expressed by the following equation:

$$D_{xs}(\rho_1-\rho_2,P)=\langle[S(\rho_1,0)-S(\rho_1,P)-S(\rho_2,0)+S(\rho_2,P)]^2\rangle \qquad (3)$$

Function $S(\rho,P)$ can be expanded into a Taylor series. The
authors proved that the higher-order terms of the expansion
equation can be neglected when satisfying condition $P < 0.3D$ (D
is the effective emission aperture) by only taking the single-
order approximation

$$S(\rho,P)=S(\rho,0)+\vec{p}\cdot\nabla_\rho S(\rho,P)|_{\rho=0} \qquad (4)$$

Here, $\nabla_\rho S(\rho,P)$ is the phase gradient along the direction of $\vec{p}$.
Thus the structure function in Eq. (3) can be expressed as

$$D_{xs}=\langle[\vec{p}\cdot(\nabla_\rho S(\rho_1,P)-\nabla_\rho S(\rho_2,P))|_{\rho=0}]^2\rangle \qquad (5)$$

For further computation, the authors expand a set of orthogonal
Zernike polynomials [5,6] for wavefront phase distortion

$$S(\rho,P)=\sum_{i=1}^{\infty}a_iF_i(2\vec{r}/D) \quad (\vec{r}=\vec{p}-\vec{p}) \qquad (6)$$

In the equation, coefficient a_i reveals the proportion of mode
$F_i(2\vec{r}/D)$ in phase distortion. By using Eq. (6), we can derive

5

that the phase gradient is

$$\nabla_\rho S(\rho, P) = \sum_{i=1}^{\infty} a_i \nabla_\rho F_i (2(\vec{p} - \vec{p})/D) \tag{7}$$

Substituting Eq. (7) in Eq. (5), if only the first six orders (manifesting the major function) are only retained in the expansion equation, we can obtain

$$D_{\Delta S} = \frac{2^4 \cdot 24 \cdot \langle a_1^2 \rangle}{D^2} (P^2 |\vec{p}_1 - \vec{p}_2|^2 + 2[(\vec{p}_1 - \vec{p}_2) \cdot \vec{P}]^2) \tag{8}$$

With Kolmogorov's turbulence spectrum, we have obtained the correlation among various order expansion coefficients; among them,

$$\langle a_1^2 \rangle = 2.32 \times 10^{-3} (\frac{3.18 D}{L \theta_0})^{5/3} \tag{9}$$

Here, θ_0 is defined as the isoplanatic angle of phase fluctuation

$$\theta_0 = [2.905 k^2 \int ds \, s^{5/3} C_n^2(s)]^{-3/5} \tag{10}$$

Substituting Eq. (9) in Eq. (8), we obtain

$$D_{\Delta S} = 8.87 \frac{P^2}{(\frac{L\theta_0}{3.18})^{5/3} D^{1/3}} |\vec{p}_1 - \vec{p}_2|^2 (1 + 2C^2) \tag{11}$$

The above equation can be rewritten as

$$D_{\Delta S} = 8.87 (\frac{3.18\theta}{\theta_0})^2 \frac{1}{D^{1/3}} (\frac{Lr_0}{3.18 L_0})^{5/3} |\vec{p}_1 - \vec{p}_2|^2 (1 + 2C^2) \tag{12}$$

In the equation, $C = \cos(\vec{p}, \vec{p}_1 - \vec{p}_2)$; r_0 is the coherent length of the spherical-surface wave; $\theta = P/L$ is the angle formed by the beacon and target.

$$r_0 = [\frac{2.905}{6.88} k^2 \int ds (1 - \frac{s}{L})^{5/3} C_n^2(s)]^{-3/5} \tag{13}$$

6

$$L_0 = \frac{r_0}{\theta_0} = \left[6.88 \frac{\int dz\, z^{5/3} C_n^2(z)}{\int dz (1 - \frac{z}{L})^{5/3} C_n^2(z)} \right]^{3/5} \tag{14}$$

Substituting Eq. (12) in Eq. (2), we obtain

$$\langle I(0) \rangle = \lambda^{-2} L^{-2} \iint d^2\rho_1 d^2\rho_2\, u_0(\rho_1) u_0^*(\rho_2) \exp\left(ik\, \frac{\rho_1^2 - \rho_2^2}{2L} \right)$$

$$\cdot \exp\left[-\frac{1}{2} D_s(\rho_1 - \rho_2) - 4.43 \left(\frac{3.18\theta}{\theta_0} \right)^2 \frac{1}{D^{1/3}} \left(\frac{Lr_0}{3.18 L_0} \right)^{1/3} |\rho_1 - \rho_2|^2 (1 + 2C^2) \right] \tag{15}$$

Assume that the initial emission field is of a gaussian distribution

$$u_0(\rho) = \exp\left[-\frac{4\rho^2}{D} - i\frac{k\rho^2}{2F} \right] \tag{16}$$

Here, F is the radius of curvature of the emission light beam. In the following discussion, we will consider the situation that the light beam is focussed at the target; that is, $F = L_0$. Eq. (15) can be simplified as

$$\langle I(0) \rangle = \frac{\pi D^2}{8\lambda^2 L^2} \exp(-\sigma_s^2) \iint d^2\rho \exp\left[-\frac{2\rho^2}{D^2} - 4.43 \left(\frac{3.18\theta}{\theta_0} \right)^2 \right.$$

$$\left. \cdot \frac{1}{D^{1/3}} \cdot \left(\frac{Lr_0}{3.18 L_0} \right)^{1/3} \rho^2 (1 + 2C^2) \right] \tag{17}$$

In the equation, σ_χ^2 is the variance of the logarithmic fluctuation of the oscillation amplitude. After simple integration, we can obtain

$$\langle I(0) \rangle = \frac{\pi^2 D^4}{16\lambda^2 L^2} \exp(-\sigma_s^2) \left\{ \left[1 + 2.21 \left(\frac{3.18\theta}{\theta_0} \right)^2 \left(\frac{Lr_0}{3.18 L_0 D} \right)^{1/3} \right] \right.$$

$$\left. \cdot \left[1 + 6.65 \left(\frac{3.18\theta}{\theta_0} \right)^2 \left(\frac{Lr_0}{3.18 L_0 D} \right)^{1/3} \right] \right\}^{-1/2} \tag{18}$$

7

Therefore, we can obtain the result that the stereo ratio R_S (characterizing light beam quality) is

$$R_{s_-} = \left\{ \left[1 + 2.21 \left(\frac{3.18\theta}{\theta_0} \right)^2 \left(\frac{Lr_0}{3.18L_0 D} \right)^{1/3} \right] \left[1 + 6.65 \left(\frac{3.18\theta}{\theta_0} \right)^2 \right. \right.$$
$$\left. \left. \cdot \left(\frac{Lr_0}{3.18L_0 D} \right)^{1/3} \right] \right\}^{1/2} \exp(-\sigma_i^2) \tag{19}$$

In reference [7], we discussed the influence on the compensatory effect due to fluctuations of oscillation amplitude. Here we discuss only the effect of anisoplanatism. From Eq. (19) the effect due to anisoplanatism intensifies with increase in   or decrease in r_0 ; however, the increase in aperture D can reduce the effect of anisoplanatism. From Eq. (19), we obtain the stereo ratios of two extreme cases:

$$R_s = 1 \qquad \text{when} \quad \left(\frac{\theta}{\theta_0} \right)^2 \left(\frac{Lr_0}{L_0 D} \right)^{1/3} \ll 1 \tag{20}$$

$$R_s = 0.12 \left(\frac{\theta_0}{\theta} \right)^2 \left(\frac{L_0 D}{Lr_0} \right)^{1/3} \qquad \text{when} \quad \left(\frac{\theta}{\theta_0} \right)^2 \left(\frac{Lr_0}{L_0 D} \right)^{1/3} \gg 1 \tag{21}$$

From this we can deduce that when the included angle formed by the beacon and target is relatively large, and if the aperture D or the coherent length r_0 is relatively small, the compensation for atmospheric turbulence for the case when conventional adaptive optics will be very poor; after compensation, light beam quality may possibly be worse than in the absence of compensation.

III. New Method of Reducing Anisoplanatic Error

When there is an included angle between the beacon light beam and the target light beam, wavefront distortion of the beacon light beam cannot fully represent wavefront distortion along the emission path. Therefore, the error of the residual phase will be produced if one simply substitutes $S(P,\rho)$ for $S(0,\ell)$. To reduce this anisoplanatic error, we can indicate the phase distortion on the target path by using the wavefront phase (and its gradient) of the beacon light beam; that is, by using

$$\tilde{S} = S(P,\rho) - \vec{P} \cdot \nabla_\rho S(P,\rho) \tag{22}$$

as the phase predistortion applied to the initial emission field. We call this method the phase gradient method. After the phase calibration shown in Eq. (22), we can find that the light intensity at the axis is

$$\langle I'(0) \rangle = \lambda^{-2} L^{-2} \iint d^2\rho_1 d^2\rho_2 u_0(\rho_1) u_0^*(\rho_2) \exp(ik \frac{\rho_1^2 - \rho_2^2}{2L})$$
$$\cdot \langle \exp\{\chi(\rho_1, 0) + \chi(\rho_2, 0) + i[S(\rho_1,0) - S(P,\rho_1) + \vec{P} \cdot \nabla_\rho S(P,\rho_1)]$$
$$- i[S(\rho_2, 0) - S(P,\rho_2) + \vec{P} \cdot \nabla_\rho S(P,\rho_2)]\} \rangle \tag{23}$$

The above equation can be simply rewritten as

$$\langle I'(0) \rangle = \lambda^{-2} L^{-2} \iint d^2\rho_1 d^2\rho_2 u_0(\rho_1) u_0^*(\rho_2) \exp(ik \frac{\rho_1^2 - \rho_2^2}{2L})$$
$$\cdot \exp[-\frac{1}{2} D_\chi(\rho_1 - \rho_2) - \frac{1}{2} D'_{\Delta S}(\rho_1 - \rho_2, P)] \tag{24}$$

In this equation, $D'_{\Delta S}$ is the residual-phase error under the phase gradient method; the error can be indicated by the second-order derivative of the phase

9

$$D'_{\Delta s} = \left\langle \left[\frac{1}{2} \left(P_x^2 \frac{\partial^2}{\partial P_x^2} + 2 P_x P_y \frac{\partial^2}{\partial P_x \partial P_y} + P_y^2 \frac{\partial^2}{\partial P_y^2} \right) \right.\right.$$

$$\left.\left. \cdot \{ S(P, \rho_1) - S(P, \rho_2) \} |_{\rho \to 0} \right]^2 \right\rangle \qquad (25)$$

The first ten terms are retained in the phase expansion equation, then the above equation becomes

$$D'_{\Delta s} = \frac{2^6 \cdot 144 \cdot \langle a_j^2 \rangle}{D^4} \cdot P^4 \cdot |\vec{\rho}_1 - \vec{\rho}_2|^2 \cdot (1 + 4C^2) \qquad (26)$$

At Kolmogorov's turbulence spectrum $\langle a_j^2 \rangle = 6.19 \times 10^{-3} \left(\frac{3.18 D}{L \theta_0} \right)^{5/3}$, therefore

$$D'_{\Delta s} = 57.05 \left(\frac{3.18 \theta}{\theta_0} \right)^4 \cdot \frac{1}{D^{13/3}} \cdot \left(\frac{L r_0}{3.18 L_0} \right)^{5/3} \cdot |\vec{\rho}_1 - \vec{\rho}_2|^2 \cdot (1 + 4C^2) \qquad (27)$$

Substituting Eq. (27) into Eq. (24) and assuming that the emission field is the focused gaussian light beam, we can obtain

$$\langle I'(0) \rangle = \frac{\pi D^4}{8 \lambda^2 L^2} \exp(-\sigma_z^2) \iint d^2 \rho \exp\left[- \frac{2\rho^2}{D} - 28.5 \cdot \left(\frac{3.18 \theta}{\theta_0} \right)^4 \right.$$

$$\left. \cdot \frac{1}{D^{13/3}} \cdot \left(\frac{L r_0}{3.18 L_0} \right)^{5/3} \rho^2 \cdot (1 + 4C^2) \right] \qquad (28)$$

After computation with integrating on the above equation, we obtain

$$\langle I'(0) \rangle = \frac{\pi^2 D^4}{16 \lambda^2 L^2} \exp(-\sigma_z^2) \cdot \left\{ \left[1 + 14.3 \left(\frac{3.18 \theta}{\theta_0} \right)^4 \left(\frac{L r_0}{3.18 L_0 D} \right)^{5/3} \right] \right.$$

$$\left. \cdot \left[1 + 71.5 \left(\frac{3.18 \theta}{\theta_0} \right) \left(\frac{L r_0}{3.18 L_0 D} \right)^{5/3} \right] \right\}^{-1/2} \qquad (29)$$

Thus, we can obtain the result that the stereo ratio in the phase gradient method is

$$R_s' = \{[1 + 14.3(\frac{3.18\theta}{\theta_0})^* \cdot (\frac{Lr_0}{3.18L_0 D})^{1/3}][1 + 71.5(\frac{3.18\theta}{\theta_0})^* (\frac{Lr_0}{3.18L_0 D})^{1/3}]\}^{-1/3}$$
$$\cdot \exp(-\sigma_z^2) \tag{30}$$

To verify how effective the phase gradient method is, we should find by how the angle is expanded for effective calibration after using the phase gradient method. With the conventional method, the range of angles for effective calibration is the isoplanatic angle θ_0. When the beam is at the fringe of the isoplanatic angle, the stereo ratio is

$$R_{s0} = \{[1 + 2.21 \times 3.18^{1/3} \cdot (\frac{Lr_0}{L_0 D})^{1/3}] \cdot [1 + 6.65 \times 3.18^{1/3} \cdot (\frac{Lr_0}{L_0 D})^{1/3}]\}^{-1/3} \tag{31}$$

We define the range of angles for effective calibration in the phase gradient method is realized when the stereo ratio R_S' is equal to the θ corresponding to R_{S0}. Therefore

$$R_{s0} = \{[1 + 14.3 \cdot (\frac{3.18\theta}{\theta_0})^* (\frac{Lr_0}{3.18L_0 D})^{1/3}]$$
$$\cdot [1 + 71.5(\frac{3.18\theta}{\theta_0})^* (\frac{Lr_0}{3.18L_0 D})^{1/3}]\}^{1/3} \tag{32}$$

R_{S0} is given by in Eq. (31). Therefore, we can obtain the number of times required for expanding the angle range with effective calibration after using the phase gradient method:

$$G = \frac{\theta}{\theta_0} \tag{33}$$

$$= \left(\frac{\sqrt{7361.6(\frac{Lr_0}{L_0 D})^* + 28125.6(\frac{Lr_0}{L_0 D})^{13/3}[101.1(\frac{Lr_0}{L_0 D})^{1/3} + 8.86]} - 85.8(\frac{Lr_0}{L_0 D})^2}{14062.8(\frac{Lr_0}{L_0 D})^{13/3}}\right)^{1/3}$$

11

In the following, we consider the results of homogeneous and inhomogeneous turbulence conditions. For a horizontal path, turbulence is homogeneous. In this case, the atmospheric equivalent distance P between beacon and target is the actual distance between beacon and target. L is the distance between target and emission plane. According to the definition of L_0, we can obtain the result that $L_0 = 3.18L$. Therefore, on the horizontal path, G is

$$G = \left(\frac{\sqrt{71.99(\frac{r_A}{D})^4 + 187.03(\frac{r_A}{D})^{11/3}[68.74(\frac{r_A}{D})^{1/3} + 8.86]} - 8.48(\frac{r_A}{D})^2}{93.52(\frac{r_A}{D})^{11/3}} \right)^{1/2} \quad (34)$$

On the slant path, the turbulence intensity along the path is a function of height. Generally speaking, turbulence intensity weakens with increase in height. As indicated in the experimental results, C_n^2 at a height of 20km is lower by three or four orders of magnitude than that on the ground. Therefore, the major function of anisoplanatism is the atmosphere below the height of 20km. Above 20km, θ_0 basically does not vary. Therefore, when the target is outside the atmospheric layer at a height H from ground surface (H > 20km), we can consider that the effective thickness of the atmospheric layer is 20km. Therefore, if the zenith angle of the target is ψ, then L=20,000 sec ψ (m); the atmospheric effective distance P=20,000 $\cdot$ θ sec ψ(m) between beacon and target; however, the actual distance (between them) is H $\cdot$ θ $\cdot$ sec ψ. To find the value of L_0, we should know the distribution of the turbulence intensity C_n^2 with height. Based

12

on the accumulated experimental data, reference [8] gives the distribution mode of turbulence height adaptable to different situations

$$C_n^2(H) = A \cdot H^{-2/3} \cdot \exp(-H/H_0) \qquad (35)$$

In the equation, H_0 is the characteristic height. In the situation of weak turbulence, $A=4 \cdot 10^{-14}$ and $H_0=2500m$. In the situation of intensive turbulence, $A=10^{-12}$ and $H_0=4000m$. Substituting Eq. (35) into Eq. (14) and after computation, we obtain

$$L_0 = 3.18 \sec\psi \cdot H_0 \cdot [\gamma(\frac{1}{2}, \frac{H}{H_0})]^{-2/3} \qquad (36)$$

Here, $\gamma(b,x)$ is an incomplete Gamma function. When H is sufficiently large, the above equation becomes $L_0 = a \cdot \sec \psi$. In the conditions of weak and strong turbulence, a is 4428m and 7095m, respectively. Upon comparison with reference [5], these values are basically consistent with the values for day and night. Therefore, we can obtain that the amount of amplification for the range of angles in which effective calibration can be made under the phase gradient method is

$$G = \left(\frac{\sqrt{7361.6(\frac{20000r_0}{aD})^2 + 28125.6(\frac{20000r_0}{aD})^{2/3} \cdot [101.1(\frac{20000r_0}{aD})^{1/3} + 8.86] - 85.8(\frac{20000r_0}{aD})^{2/3}}}{14062.8(\frac{20000r_0}{aD})^{1/3}} \right)^{1/4} \qquad (37)$$

From Eqs. (34) and (37), we can see that whether horizontal or slant path, the amount of amplification G for increasing the range of angles with effective calibration is related to D/r_0. In Fig. 2, the authors plot the calculation results of G with

13

variation in D/r_0. Obviously, the greater the D/r_0, the greater G is. For identical D/r_0 values, the amount of amplification in a horizontal path is larger than for a slant path. For example, when D=4m and r_0=0.1m, G on the horizontal path is approximately 6.6; however, on the slant path corresponding to conditions of weak and strong turbulence, G is 1.7 and 2.2, respectively. We should point out, in addition, that when D/r_0 is sufficiently reduced to a certain value (under conditions of weak and strong turbulence, D/r_0 is 14 and 8.5, respectively), the amount of amplification G will be smaller than 1. In other words, the range of angles with effective calibration shrinks under the phase gradient method; this is because we only revised the first-order terms of phase gradient but neglected the higher-order terms. Therefore, when D/r_0 is relatively small, the second-order terms and even higher-order terms of phase gradient should be further considered. Thus, the amount of amplification G will increase; we are carrying this research.
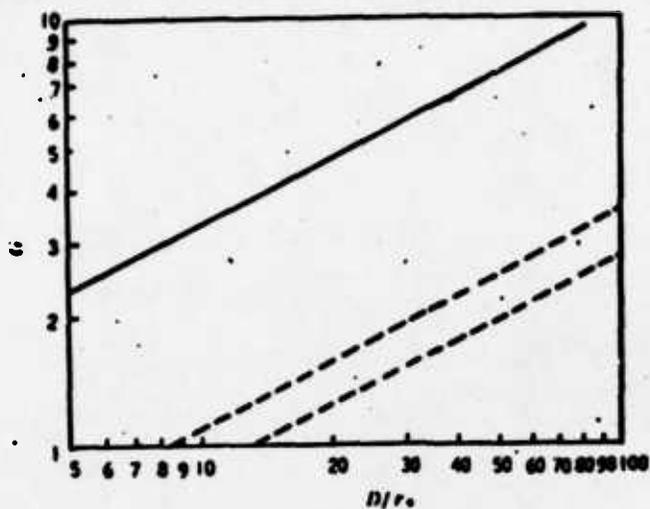
Fig. 2. Variation in amount of amplification
G with D/r$_0$
_____ horizontal path _ _ _ _ slant path
1 - Weak turbulence   2 - Strong turbulence

IV. Preliminary Conclusions

Anisoplanatism of a beacon is an important factor
restricting adaptive optics as a technique. To effectively
calibrate the effects due to atmospheric turbulence, the beacon
should lie within the range of the isoplanatic angle; otherwise,
the compensatory effect will be degraded. Generally, the
isoplanatic beam is very small (only about several seconds in
angular measure); this imposes rigorous requirements on the
beacon. The authors conducted some improvements on aspects of
the principle to significantly increase the range of angles for
effective calibration, thus the effect of anisoplanatic error is

15

greatly reduced. As shown by the analytical results: along the horizontal path, the effective calibration range of several seconds of original angular measure is expanded to tens of seconds by using the phase gradient method with 4m aperture. Thus, the anisoplanatic problem of the beacon is solved. Along the slant path, the range of angles with effective calibration can also be increased by about twofold. By using an even larger aperture, the problem of beacon anisoplanatism can be solved. In particular, in the infrared waveband, the problem can be basically solved by a factor of 2 taken as the amount of amplification. In practical execution, only with a beacon light source (such as a moving target or multistar system) can we provide for phase gradient of real time measurements, so it is not difficult to carry out the calibration u der the phase gradient method.

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16

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