AFFDL-TM-FBR-74- 28

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A GENERALIZED DEVELOPMENT OF YIELD ZONE MODELS

prepared by

J. P. Gallagher

Technical Memorandum FBR-74-28

January 1974

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE January 1974	3. REPORT TYPE AND DATES COVERED Final

4. TITLE AND SUBTITLE A GENERALIZED DEVELOPMENT OF YIELD ZONE MODELS	5. FUNDING NUMBERS Work Unit 14670313
6. AUTHOR(S) J. P. Gallagher	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Air Force Flight Dynamics Laboratory Structures Division Solid Mechanics Branch Wright-Patterson AFB, Ohio 45433	8. PERFORMING ORGANIZATION REPORT NUMBER AFFDL-TM-FBR-74-28
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Same as above	10. SPONSORING/MONITORING AGENCY REPORT NUMBER
11. SUPPLEMENTARY NOTES	
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for Public Release; Distribution is Unlimited	12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The purpose of this report is to present the concept of load interaction zones and to detail the reformulation of the Wheeler and Willenborg et. al. models in a stress intensity factor format.

14. SUBJECT TERMS	15. NUMBER OF PAGES 20
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
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FOREWORD

This report is the result of an in-house effort under Project 1467, "Analysis Methods for Military Flight Vehicle Structures," Work Unit 14670313, "Experimental Studies in Fatigue Crack Propagation in USAF Structures."

The manuscript was released by the author in January 1974.

This Technical Memorandum has been reviewed and approved.

FRANCIS J. JANIK, JR.
Chief, Solid Mechanics Branch
Structures Division

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

a	crack length
a_i	crack length with tip position defined in Fig. 1 for $i = 1$ thru 5
a^*	overload affected crack length
Δa	crack movement following overload application
C, C_W, C_E, n, p, q, b	crack growth rate constants
da/dN	crack growth rate
K, K^∞	remote stress intensity factor
K_{eff}	effective (local) stress intensity factor
$K^\infty_{max}, K^\infty_{min}$	maximum and minimum remote stress intensity factor
K^{max}_{eff}, K^{min}_{eff}	maximum and minimum effective stress intensity factor
K^{OL}_{max}	K_{max} for application of overload
K^*_{max}	no retardation - stress intensity factor
K_{RED}	difference between K^*_{max} and K_{max}
ΔK	range of stress intensity factor$(K^\infty_{max} - K^\infty_{min})$
m	Wheeler shaping parameter
P	load
P_i	loads defined in Figure 1 for $i - 1,2,3$
R	stress (load) ratio
R_{eff}	effective (local) stress ratio
r_y	plane stress plastic zone radius

LIST OF SYMBOLS (CONTINUED)

r_y^*	plane strain plastic zone radius
z	load interaction zone
z_i	load interaction zones defined in Figure 1 for $i = 1,2,3$
z_{OL}	load interaction zone created by overload
z^*	no retardation - load interaction zone
	a constant, linearly relating z to r_y
σ, σ^{∞}	remote (gross) stress
σ^{∞}_{max}	maximum remote stress
σ_{eff}	Willenborg et. al. defined effective stress
σ_{RED}	Willenborg et. al. defined reducing stress
σ^*	Willenborg et. al. defined no retardation - stress
σ_{ys}	yield strength of material
γ	generalized yield zone constant

ABSTRACT

The purpose of this report is to present the concept of load interaction zones and to detail the reformulation of the Wheeler and Willenborg et. al. models in a stress intensity factor format.

SECTION I

INTRODUCTION

A viable fatigue life prediction capability must provide for a proper characterization of the localized residual stress state generated by prior loading so that the influence of this residual stress state on subsequent fatigue behavior might be determined. The crack growth retardation models proposed by Wheeler [1]* and Willenborg et. al. [2] have been used to account for both the magnitude and extent of the crack tip residual stress state generated by high spectrum loads. In general, the maximum extent of any load generated residual stress state, i.e., its zone of influence on subsequent crack growth behavior, can be defined with a parameter z, called the load interaction zone.

* See List of References for numbers in brackets.

SECTION II

LOAD INTERACTION ZONE CONCEPT

A schematic showing the relationship between applied load and its corresponding load interaction zone is shown in Figure 1. Load P_1 applied at crack length a_1 develops a zone z_1 which extends to some future crack length position a_3. Load P_2 applied at crack length a_2 develops a load interaction zone which spans the distance between a_2 and either a_3 or a_4. One basic assumption of both the Wheeler and Willenborg et. al. models is that if the load P_2 develops a load interaction zone which extends out to or past the furthest extent of a previously developed interaction zone, i.e., $a_2 + z_2 \geq a_1 + z_1$, the growth increment associated with the P_2 loading is calculated using the steady state equation. Conversely, a crack growth rate reduction is assumed when the load P_3 applied at crack length position a_2 develops a load interaction zone z_3 which is smaller than that required to reach the furthest extent of any previously developed load interaction zone boundary, i.e., $a_2 + z_3 < (a_1 + z_1)$ or $(a_2 + z_2)$ whichever is largest. Both models assume that the difference between the load interaction zone boundaries is related to the amount of crack growth rate reduction.

SECTION III

PLASTIC ZONE ASSUMPTION

Wheeler [1] assumed that the load interaction zone z was equal to the plastic zone radius size created under plane strain loading:

$$z = \frac{1}{4\pi\sqrt{2}}\left(\frac{K_{max}^{\infty}}{\sigma_{ys}}\right)^2 = r_y^* \qquad (1)$$

while Willenborg et. al. [2] suggested that the plane stress plastic zone size (radius) might be more appropriate, i.e.

$$z = \frac{1}{2\pi}\left(\frac{K_{max}^{\infty}}{\sigma_{ys}}\right)^2 = r_y \qquad (2)$$

In general, it might be assumed that the load interaction zone is a function of the maximum stress intensity factor, K_{max}^{∞}, (associated with remote loading) and yield strength, σ_{ys}.

Consider Figure 2 which shows the relationship for a remotely calculated stress intensity factor which is required for continuous decay in the assumed load interaction zone such that the furthest extremity of each zone calculated is coincident. This stress intensity factor relationship is the minimum one assumed by both Wheeler and Willenborg et. al. models to give no retardation. It is derived using the following equations (see Figure 2 for nomenclature):

$$z^* = z_{OL} - \Delta a \qquad (3)$$

is related to stress intensity using a generalization of Equations 1 and 2.

$$\gamma\left(\frac{K_{max}^{*}}{\sigma_{ys}}\right)^2 = \gamma\left(\frac{K_{max}^{OL}}{\sigma_{ys}}\right)^2 - \Delta a \text{ , } \gamma = \text{a constant} \qquad (4)$$

which can be rearranged in terms of K^*_{max}, K^{OL}_{MAX}, Δa, and z_{OL} to give

$$K^*_{max} = K^{OL}_{max} \sqrt{1 - \frac{\Delta a}{z_{OL}}} \quad . \tag{5}$$

Stating the Wheeler-Willenborg et. al. retardation concept in a stress intensity factor format implies that if $K^{\infty}_{max} < K^*_{max}$, a crack growth rate below steady state predictions can be expected for those cycles associated with K^{∞}_{max}. If $K^{\infty}_{max} \geq K^*_{max}$, no reduction in crack growth rates below steady state predictions is expected for these cycles.

SECTION IV

OVERLOAD AFFECTED CRACK LENGTH

Both Wheeler and Willenborg et. al. models predict a return to steady state cracking rates subsequent to the application of the overload as soon as K_{max}^{∞} and K_{max}^{*} are equal. The crack length increment defined by the condition $K_{max}^{\infty} = K_{max}^{*}$ and Equation 5 is termed the overload affected crack length (a^*) because it is the growth increment which experiences the influence of the overload created residual stress field. The overload affected crack length is predicted by Equation 6:

$$a^* = z_{OL} \left(1 - \left(K_{max}^{\infty} / K_{max}^{OL} \right)^2 \right) \qquad (6)$$

SECTION V

CRACK GROWTH REDUCTION SCHEMES

The Wheeler and Willenborg et. al. models diverge when consideration is given to how the crack growth rates associated with $K_{max}^{\infty} \geq K_{max}^{*}$ are reduced below their steady state level. Shown in Figure 3 is a schematic diagram which compares K_{max}^{*} to K_{max}^{∞} for the case of a single large load cycle followed by a large number of low level constant amplitude load cycles. The Wheeler model considers the ratio of remotely applied stress intensity factor to the no retardation stress intensity factor as the driving force for retardation, while the Willenborg et. al. model utilizes the difference between these same two stress intensity factors for its assessment of the amount of retardation applied to the low amplitude load induced crack growth rates.

The Wheeler model was [illegible]ved in a stress intensity factor format but such a [illegible] easily be developed. The Wheeler crack growth model can be expressed for each cycle of crack growth

$$\frac{da}{dN} = \left(\frac{z}{z^*} \right)^m \cdot C \quad \Delta K^p \quad \text{for } z < z^* \tag{7a}$$

or

$$= C \; \Delta K^{\,p} \qquad \text{for } z \geq z^* \tag{7b}$$

where the m values found from data derived from several spectra ranged between 1.3 and 2.0.

If the load interaction zones are assumed to be linearly related to the plastic zone size parameters, i.e., if

$$z = \gamma \left(\frac{K^{\infty}_{max}}{\sigma_{ys}} \right)^2 , \quad z^* = \gamma \left(\frac{K^*_{max}}{\sigma_{ys}} \right)^2 \tag{8}$$

then a direct substitution into Equation 7a gives

$$\frac{da}{dN} = C \left(\left(\frac{K^{\infty}_{max}}{K^*_{max}} \right)^{\frac{2m}{p}} \cdot \Delta K \right)^p \tag{9}$$

for $K^{\infty}_{max} \leq K^*_{max}$. Equation 9 shows the assignment of the Wheeler crack growth reduction factor $(K^{\infty}_{max}/K^*_{max})$ directly applied to the stress intensity range. It should be noted that the Wheeler model reduces growth rates by reducing the applied stress intensity range.

While the Willenborg et. al. model was derived using stress intensity factor concepts, it has to date been described using a stress format: the stress used to calculate the effective (local) stress intensity factor was given by

$$\sigma_{eff} = \sigma^{\infty} - \sigma_{RED} \tag{10}$$

with

$$\sigma_{RED} = \sigma^* - \sigma^{\infty}_{max} \tag{11}$$

where

σ^{∞} = remotely applied stress

σ^* = stress required to achieve K^*_{max} in Figure 3

σ^{∞}_{max} = remotely applied maximum cyclic stress related directly to K^{∞}_{max} in Figure 3.

Dividing the stresses in Equations 10 and 11 by the characterizing stress intensity factor coefficient (K/σ) for a given geometric structure and crack length yields a stress intensity factor format

$$K_{eff} = K^{\infty} - K_{RED} \tag{12}$$

with

$$K_{RED} = K^{*}_{max} - K^{\infty}_{max} \ . \tag{13}$$

The effective (local) stress intensity factor, K_{eff}, is reduced below the applied stress intensity factor by a constant (for a given crack length) as long as $K^{*}_{max} \geq K^{\infty}_{max}$. Since the local stress intensity factor range is the difference between the local maximum and minimum stress intensity factors, one finds that

$$\Delta K_{eff} = (K^{\infty}_{max} - K_{RED}) - (K^{\infty}_{min} - K_{RED}) = \Delta K \tag{14}$$

i.e., the effective (local) stress intensity range equals the remote stress intensity range but the local stress ratio R_{eff} is depressed below that of the remote stress ratio R^{∞}. The crack growth rates are calculated using an equation which interrelates the influence of stress intensity range and stress ratio such as the Walker Equation [3]

$$\frac{da}{dN} = C_W \left[K^{max}_{eff} \cdot (1 - R_{eff})^n \right]^p \tag{15}$$

or Krause-Crooker Equation [4]

$$\frac{da}{dN} = C_W \left[K^{max}_{eff} \cdot (1 - b\, R_{eff})^n \right]^p \tag{16}$$

or Elber Equation [5]

$$\frac{da}{dN} = C_E \left[\Delta K \cdot (1 + q\, R_{eff}) \right]^p \tag{17}$$

Constants for Equations 15,16, and 17 should be established for positive and negative stress ratio data separately for reasons detailed by Mayle [6].

SUMMARY

The load interaction zone concept was presented to generalize the approach employed by Wheeler [1] and Willenborg et. al [2] in the development of crack growth retardation models. A stress intensity factor format was provided for the Wheeler crack growth retardation model with Equation 9 and for the Willenborg et. al. reducing stress intensity factor model with Equations 12 and 13. The (overload generated) no retardation intensity factor (K^*_{max}) for both models was defined by Equation 5.

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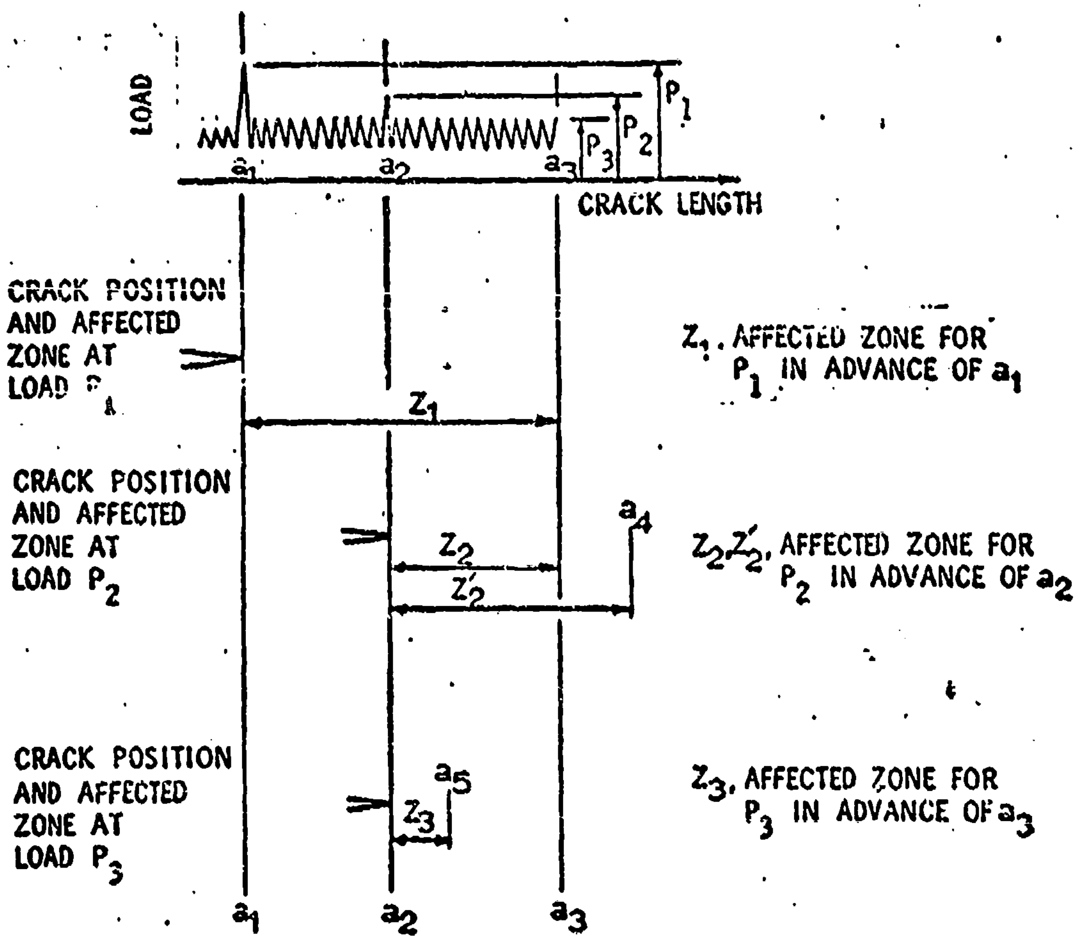

FIGURE. 1 SCHEMATIC ILLUSTRATING THE LOAD INTERACTION ZONE CONCEPT

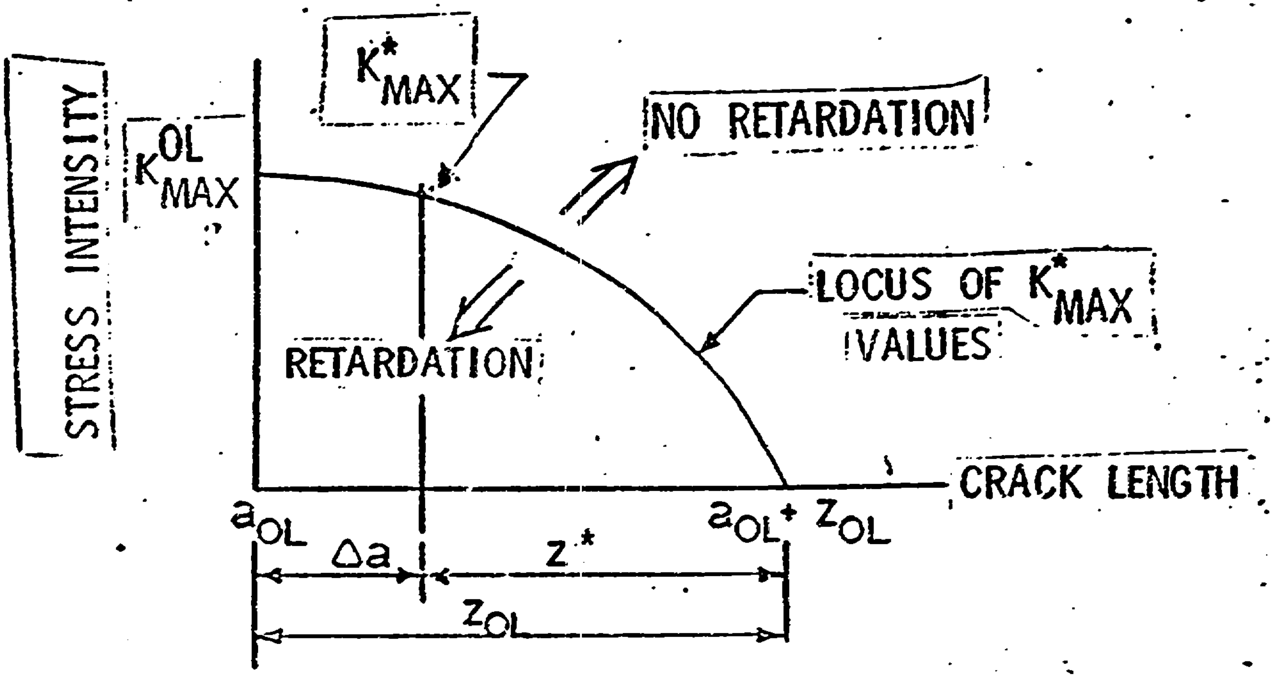

FIGURE 2 MAXIMUM STRESS INTENSITY FACTOR (K^*_{MAX}) REQUIRED FOR COINCIDENT LOAD INTERACTION BOUNDARIES

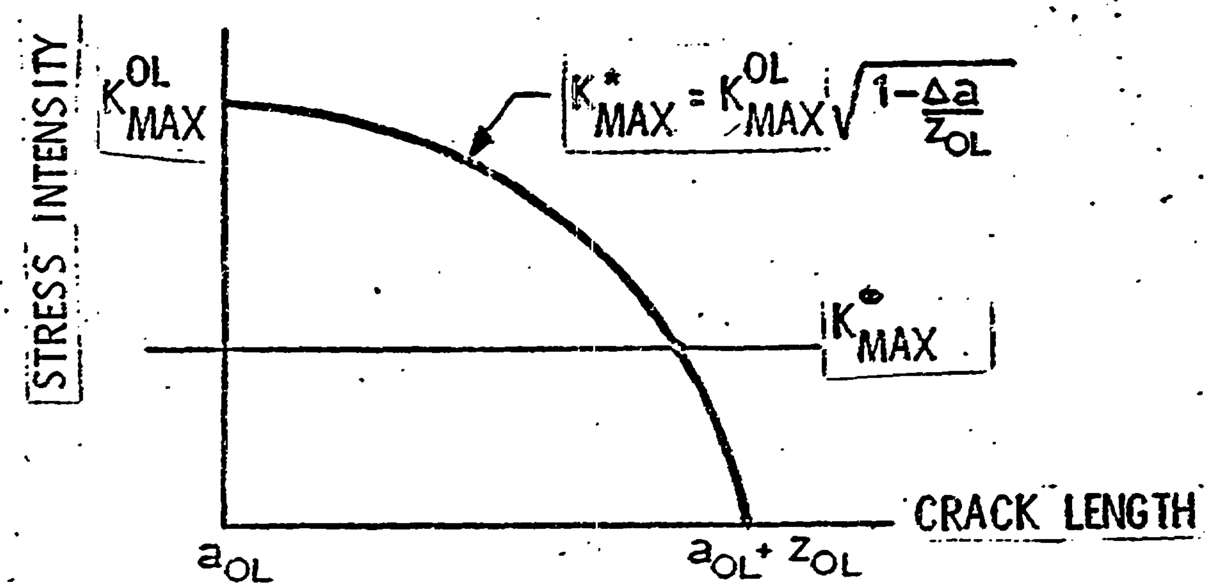

FIGURE 3 COMPARISON BETWEEN REMOTELY APPLIED STRESS INTENSITY FACTOR AND NO RETARDATION STRESS INTENSITY FACTOR