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VIRTUAL MODE SURFACE DUCT MODEL FOR FACT

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by

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SECTION 1
INTRODUCTION

Since its introduction in April 1973, the FACT Model[1] has proven to be a useful tool for predicting and analyzing long-range low-frequency propagation in range-independent environments. The original development emphasized the incorporation of diffraction and surface-image effects in a fully automated, highly efficient model. For short-range high-frequency surface-ducted propagation, Clay augmented the ray model by an approximate surface-duct model already in use in FACT's predecessor, RP70[2]. While the Clay Model was recognized to have certain deficiencies, an efficient alternative was not available. This report documents such an alternative which is proposed for incorporation in FACT.

1.1 BACKGROUND

The Clay Model underwent a series of reviews (e.g., Labianca[3]), and was considered in a comparison by Morris of several models with data from the Surface Duct Sonar (SUDS) experiments[4]. Labianca's analysis showed that the model had an oversimplified depth dependence and the wrong dependence of leakage on environmental parameters. The leakage error resulted from assigning leakage losses to partial reflection from the gradient discontinuity at the bottom of the duct rather than to the correct quantum-mechanical "tunneling" phenomenon. The depth dependence was characterized by a single 10-dB drop in level from that in the duct if one of either the source or receiver were moved below the duct ("cross-layer"). In a number of cases the lack of environmental sensitivity proved critical.

Morris' comparisons showed that the Clay Model, AMOS[5] and the Bucker surface-duct model[6] were in fair agreement in a number of cases with none particularly better than the others. Since both the Clay Model and AMOS were largely based on experimental data this result was not altogether surprising. The data set was not a severe test of environmental sensitivity and was apparently contaminated by a range dependence in the sound-speed profile that none of the models were developed to include. Bucker's model clearly had the strongest theoretical basis, however it was

much longer running than the others and based on the comparisons a change was not justified.

In the period following these initial evaluations, considerable progress was made in understanding the basic mechanisms of surface ducted propagation, largely as a result of Labianca's[7] work on virtual modes. Subsequently, Spofford[8] identified Labianca's asymptotic expressions for deeply trapped virtual modes with mode bundle ray-equivalents ("smodes") and was able to incorporate rough surface scattering effects. A prototype surface duct propagation code was developed by Spofford, using the smode techniques. While showing improvement over the Clay Model, Spofford's model displayed poor performance for some propagative scenarios.

1.2 MODEL REQUIREMENTS

As FACT saw more and more use, dissatisfaction with the Clay Model increased, and LRAPP authorized the development of a better model subject to the following constraints:

1. increased accuracy for a wide range of environmental parameters;
2. continuity of transmission-loss in depth (including acoustic reciprocity);
3. range-smoothed transmission-loss of the approximate form $A + B\log R + \alpha R$; and
4. sufficient speed to have a minimal impact on overall FACT running time (presently about 4 CPU seconds per 100 nmi).

SAI has developed such a model as reported herein.

1.3 TECHNICAL SUMMARY

The approach used in the new model is a further refinement and extension of Labianca's virtual mode results to include modes near cutoff as well as leaky modes. This permits intensity calculations in ducts below cutoff and/or at short ranges where the leaky modes make important contributions. The WKB work of Spofford was extended to the rough surface losses. A brief description of the model theory is provided here, with detailed discussions postponed till Section 2.

The surface duct model is designed to compute the relative incoherent intensity level from a point source in a canonical bilinear duct. Labianca's formulation of the complex pressure field as a Hankel transform over the continuum eigenspectrum is the theoretical starting point. An approximate, yet highly accurate, evaluation of the transform results in the relative incoherent intensity $\tilde{I}$ being given as a finite sum of virtual modes

$$\tilde{I} \simeq \frac{2\pi}{r} \sum_{m} e^{-\alpha_m r} I_m(z_s) I_m(z_r)$$

where r is the range and $z_s(z_r)$ is the source (receiver) depth. The I_m are modal intensity amplitudes and the α_m are modal attenuation coefficients. Individual terms in the sum arise from resonances in the kernel of the Hankel transform due to the proximity of Regge poles in the complex energy plane. The resonances in the transform occur at locations along the real energy axis corresponding to the real part of the Regge pole E_m.

The intensity amplitudes I_m are related to normalized depth functions and are expressible in terms of Airy functions. Efficient expansions for the Airy functions provide a rapid means for evaluation of the virtual mode sum, while preserving numerical accuracy. For the deeply trapped modes, Labianca's results are obtained as limiting cases.

The modal attenuation coefficient α_m incorporates volume, surface and diffractive losses. Volume losses are obtained from standard absorption loss formulas presently included in FACT. The diffractive portion of α_m is obtained from the imaginary component of the Regge pole and physically arises from leakage of acoustic energy into the region below the duct. The surface loss portion of α_m is computed by using conservation of energy applied to the virtual modes' ray-equivalent reflecting from the rough ocean surface. Each interaction of the ray-equivalent with the surface yields an effective reflection loss which is then accumulated continuously over the ray's cycle distance. The rough surface reflection losses are computed in a separate module as a function of the ray's surface grazing angle, sea state and frequency.

The virtual mode model was compared to a series of benchmarks generated using both the parabolic-equation (PE) code[16] and the NOSC n-layer normal mode (NM) code[12] for a representative surface duct. The model shows excellent agreement with both PE and NM calculations. Details of the comparisons are described in Section 3.

1.4 ACKNOWLEDGEMENTS

This work was supported by the Ocean Programs office of NORDA (Code 500), under the program management of Mr. Bob Wheatley. The support of technical staff at the Naval Ocean Systems Center (NOSC), particularly Mr. M. A. Pedersen and Dr. D. F. Gordon, in making the normal mode computer runs, is gratefully acknowledged. At SAI, prior work by Dr. H. L. Wilson on modeling surface scattering using the transport equation and by Mr. C. W. Spofford and Ms. R. E. Keenan on rough surface loss was incorporated into the surface duct model. Ms. E. Holmes assisted in making the parabolic equation runs used in the model evaluation. The author is grateful to Mr. C. W. Spofford for initially suggesting this problem and for discussions related to the virtual mode model.

SECTION 2
TECHNICAL APPROACH

This section describes the theoretical basis for a new surface duct model proposed for use in FACT. The model is an extension of previous work by Labianca on virtual mode representations and by Spofford on ray-mode equivalents. The model is designed to compute the incoherent acoustic intensity from a point source in a range independent ocean surface duct as a function of source/receiver geometry. The ocean surface duct is approximated by a canonical bilinear model first analyzed by Furry[9] in connection with the transmission of electromagnetic waves in the atmosphere.

2.1 FURRY MODEL

Furry's model approximates the square of the refractive index n^2 with the bilinear form[Fig. (2.1)]

$$n^2(z) = \left[c_0/c(z)\right]^2 = 1 - g_- z \quad , \quad 0 \le z < d$$

$$= 1 - g_- d + g_+(z-d) \quad , \quad z \geq d$$

unit: ?

g_+, g_- are gradient of n^2 so have units m^{-1}

where d is the depth of the surface duct, $g_-(g_+)$ is the gradient within (below) the duct, c_0 is the surface sound speed, and $c(z)$ is the depth dependent sound speed. The Furry model has been employed by a number of authors in the context of ocean acoustics, with the conventional Fourier-Bessel or longitudinal wavenumber (λ) representation being used by Marsh,[10] Hall[11] and Pedersen and Gordon,[12] while the transverse wavenumber (κ) representation was employed by Labianca.[7] The latter is used in this report.

2.2 VIRTUAL MODE SOLUTION

In polar cylindrical coordinates, the complex pressure field $\underline{P}$ from a unit strength monopole source satisfies the Helmholtz equation

$$\left[\frac{1}{r}\frac{d}{dr}\, r\,\frac{d}{dr} + \frac{d^2}{dz^2} + k_0^2 n^2(z)\right] \underline{P}(r,z_r,z_s) = \frac{-1}{2\pi r}\,\delta(r)\delta(z_r-z_s) \quad .$$

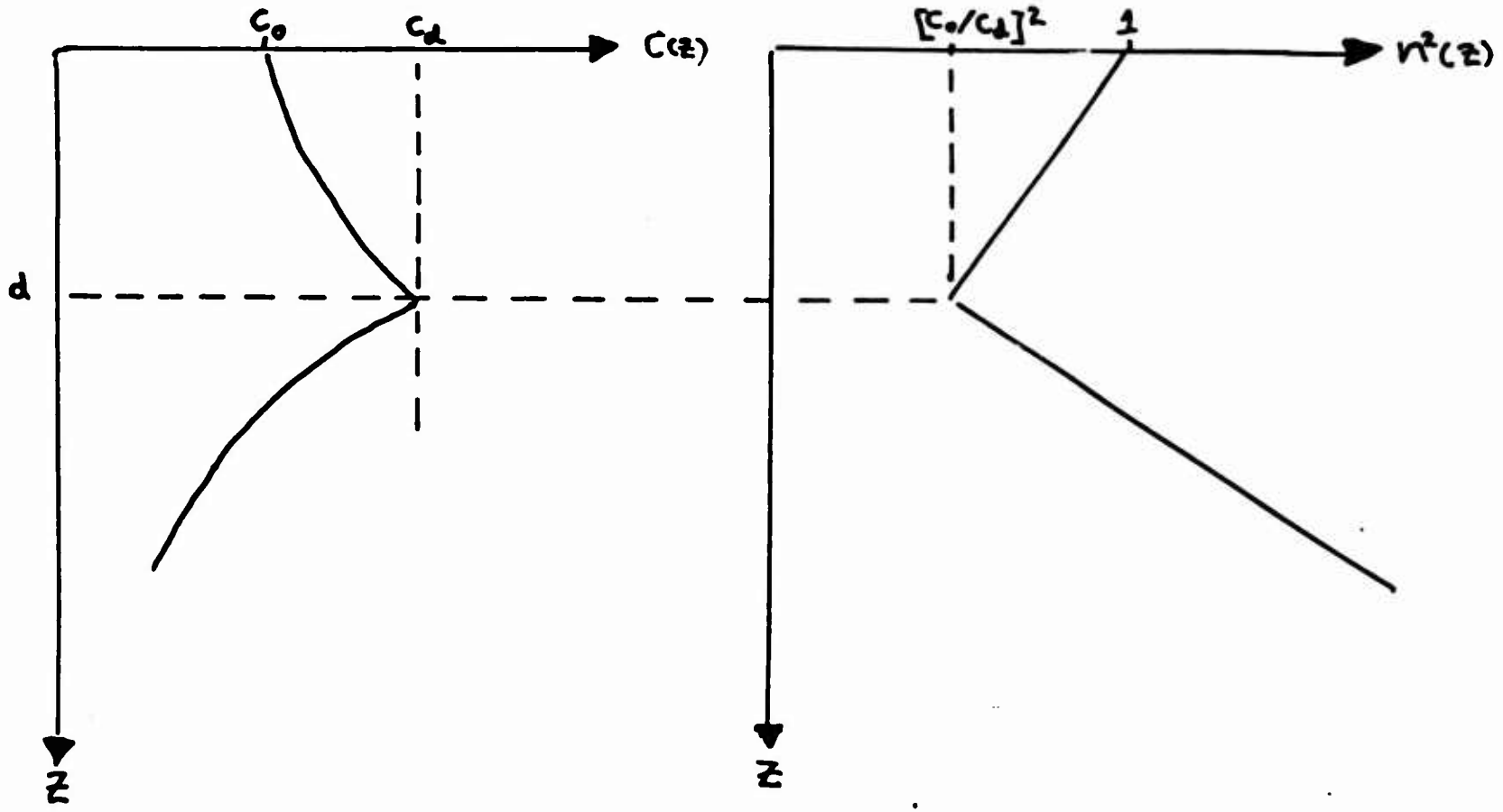


Figure 2-1. Furry Model Profile.

A harmonic time dependence, $\exp(-i\omega t)$, is suppressed with ω the angular frequency and $k_0 = \omega/c_0$. The source is located at $\vec{r} = (o, z_s)$, the receiver at $\vec{r} = (r, z_r)$ and $\underline{P}$ is to satisfy a pressure release boundary condition (P=0) at the surface and a radiation condition at deep depths.

Employing standard separation of variables techniques, the pressure is represented by the convolution

$$P(r,z_r,z_s) = \frac{-1}{8\pi}\int_{\Gamma} H_0^{(1)}\left[r(k_0^2 - \kappa^2)^{\frac{1}{2}}\right] g(\kappa^2,z_r,z_s)d\kappa^2 \tag{2.1}$$

where the depth Green's function g satisfies

$$\left[\frac{d^2}{dz^2} + k_0^2\,(n^2 - 1) + \kappa^2\right] g = -\,\delta(z_r - z_s)$$

with κ the transverse wavenumber $(k_0^2 = \lambda^2 + \kappa^2)$.

The depth Green's function is expanded in a complete set of orthonormal eigenfunctions $\hat{\phi}$ of the homogeneous equation

$$\left[\frac{d^2}{dz^2} + k_0^2\,(n^2 - 1) + \kappa^2\right]\hat{\phi} = 0 \quad , \tag{2.2}$$

subject to the pressure release boundary condition. For the Furry model the eigenfunctions $\hat{\phi}$ form a continuous spectrum and the integral in Eq. (2.1) becomes

$$P(r,z_r,z_s) = \frac{i}{4\pi}\int_{-\infty}^{\infty} H_0^{(1)}\left[r(k_0^2 - \kappa^2)^{\frac{1}{2}}\right]\hat{\phi}(z_r,\kappa^2)\hat{\phi}(z_s,\kappa^2)d\kappa^2 \quad .$$

Analytic solutions for the eigenfunctions are readily obtained by introducing a natural unit of length, $H = (g_+k_0^2)^{-\frac{1}{3}}$, whereby Eq. (2.2) becomes

$$\left[\frac{d^2}{dx^2} + E - Q(x)\right]\psi(x,E) = 0 \quad . \tag{2.3}$$

The new independent variable is $x = (z - d)/H$, and the "energy" E and "potential" Q are

$$E = H^2\kappa^2 - s^3 D \quad ,$$

and

$$Q(x) = s^3 x \quad , \quad x \le 0$$
$$= - x \quad , \quad x > 0$$

with $s^3 = g_-/g_+$ and $D = d/H$. Solutions of Eq. (2.3) may be expressed in terms of the Airy functions. Within the duct,

$$\psi(x,E) = \frac{\pi}{s}\left[Ai(-q)Bi(-q_o) - Ai(-q_o)Bi(-q)\right] \ , \ x \le 0 \tag{2.4a}$$

with $q_o = w + sD$, $q = w - sx$ and $w = E/s^2$, and below the duct

$$\psi(x,E) = \frac{\pi^{\frac{1}{2}}}{2}\left[T_+(E)h_1(p) + T_-(E)h_2(p)\right] \ , \ x > 0 \tag{2.4b}$$

with $p = E + x$. The h functions are

$$h_1(p) = Ai(-p) - i\ Bi(-p)$$

and

$$h_2(p) = Ai(-p) + i\ Bi(-p) \quad .$$

Continuity of ψ and $d\psi/dx$ at the duct bottom fixes the T coefficients, leading to

$$T_+(E) = - i\pi^{\frac{1}{2}}\left[\psi'(o,E)h_2(E) - \psi(o,E)h_2'(E)\right] \quad , \tag{2.5a}$$

and

$$T_-(E) = i\pi^{\frac{1}{2}}\left[\psi'(o,E)h_1(E) - \psi(o,E)h_1'(E)\right] \quad . \tag{2.5b}$$

correction

The pressure field integral, Eq. (2.1), is then given in terms of the new variables x, ψ as[13]

$$P(r,x_r,x_s) = \frac{i}{4\pi H} \int_{-\infty}^{\infty} H_0^{(1)} \left[r(k_0^2 - \kappa^2)^{\frac{1}{2}}\right] \frac{\psi(x_r,E)\psi(x_s,E)}{T_+(E)T_-(E)} \, dE \quad , \tag{2.6}$$

with $\kappa^2 = (E + s^3 D)/H^2$.

The kernel of the Hankel transform in Eq. (2.6) has resonances along the real E axis from Regge poles in the lower half E plane. The Regge poles are the complex roots of $T_-(E_m) = 0$, with the real part of E_m corresponding to the resonance position and the imaginary part of E_m proportional to the resonance width. An approximate, yet highly accurate, evaluation of the integral in Eq. (2.6) is obtained by evaluating the integrand in the region of the resonances. The incoherent intensity, $\tilde{I} = |P|^2$, is then given in the far field $\left[r(k_0^2 - \kappa^2)^{\frac{1}{2}} >> 1\right]$ as[14]

$$\tilde{I} \simeq \frac{2\pi}{r} \sum_m e^{-r\alpha_m^L} \frac{I_m(x_s)I_m(x_r)}{|k_0^2 - \kappa_m^2|^{\frac{1}{2}}} \tag{2.7}$$

with $\kappa_m^2 = (E_m + s^3 D)/H^2$,

$$\alpha_m^L = 2 \, \mathrm{Im} \left[(k_0^2 - \kappa_m^2)^{\frac{1}{2}}\right] \quad ,$$

$$I_m(x) = |\psi^2(x,E_m)/N_m|$$

and

$$N_m = H \int_{-D}^{\infty} \psi_m^2(x) \, dx \quad . \tag{2.8}$$

The ψ_m have been termed virtual modes by Labianca.

When the imaginary part of E_m is small (low attenuation) the depth function $\psi_m(x)$ is contained or "trapped" within the duct. The trapped modes

are capable of propagating acoustic energy to long ranges. Conversely, modes with large imaginary part are not localized in the duct and are termed "leaky".

Trapped modes have $E_r = \mathrm{Re}(E_m)$ negative, and there are two zeroes (turning points) of $E_r - Q(x)$ denoted by x_1 and x_2 (Figure 2.2). When $x < x_1$, or $x > x_2$ ψ_m is oscillatory; when $x_1 < x < x_2$ ψ_m is exponential. The region between the turning points acts as a barrier, confining the depth function to the duct. ψ_m has an imaginary component that is essentially zero until below the barrier ($x > x_2$) and then spirals about the origin of the complex plane in a counter-clockwise sense with increasing magnitude as x increases.

Leaky modes have positive E_r and complex turning points. ψ_m is oscillatory and complex everywhere with no enhancement in the duct. While of lesser importance at long ranges, the leaky modes may make significant contributions at short ranges and need to be included in the residue series.

A ray-equivalent may be associated with ψ_m by defining a surface grazing angle θ_m through

$$\sin\theta_m = \left[g_+ H(E_r + s^3 D)\right]^{1/2} \quad . = \left[g_+ t^3 \alpha^2 - g_- d \right]^{1/2}$$

The trapped modes are thus seen to have ray-equivalents which turn within the duct, while the leaky modes correspond to ray-equivalents with large grazing angle.

The major effort in using the virtual mode series is in finding the Regge poles E_m. The roots of $T_-(E_m) = 0$ are generally obtained using complicated numerical methods as was done by Pedersen and Gordon.[12] A different approach will now be discussed which provides accurate approximate expressions for the E_m, yet is very efficient.

2.3 TRAPPED MODES

The Regge pole E_m satisfies $T_-(E_m) = 0$, which upon using Eq. (2.5b) becomes the transcendental equation

$$\psi'(o,E_m)h_1(E_m) = \psi(o,E_m)h_1'(E_m) \quad .$$

Pedersen use numerical solution Newton-Raffson technique

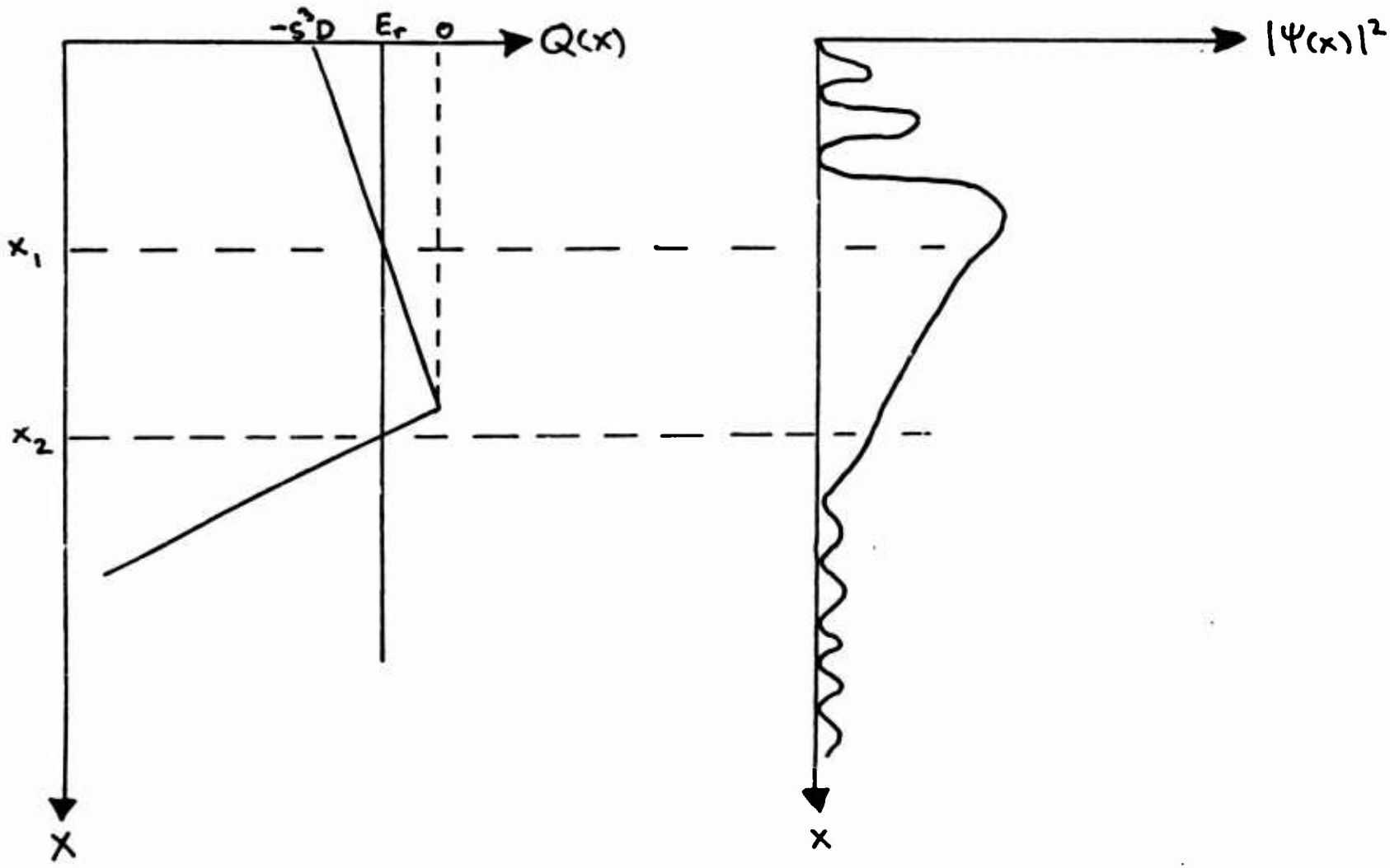


Figure 2-2. Potential Q(x) and Schematic of Trapped Eigenfunction.

Using ψ from Eq. (2.4a), the above eigenvalue equation can be recast as (the mode subscript will be suppressed on E)

$$\frac{Ai(-q_0)}{Bi(-q_0)} = \frac{Ai'(-w) - Z(E)Ai(-w)s}{Bi'(-w) - Z(E)Bi(-w)s} \quad . \tag{2.9}$$

with the effective "impedance" Z of the duct bottom expressed as

$$Z(E) = h_1'(E)/h_1(E) \tag{2.10}$$

and where $w = E/s^2$ and $q_0 = w + sD$.

If phase integral (WKB) methods were used to solve for the trapped eigenvalues, the m-th mode quantization condition would be

$$\int_{-D}^{x_1} (E - s^3x)^{\frac{1}{2}} dx = \frac{2}{3} q_0^{\frac{3}{2}} = (m - \tfrac{1}{4})\pi \tag{2.11}$$

where x_1 (complex) is the upper turning point. Since the imaginary part of E is small for the trapped modes, a good approximation for x_1 is to assume it to be real. This corresponds to q_0 in Eq. (2.11) being an approximate root of $Ai(-q_0) = 0$, and suggests a zeroth order approximation to E as $E \sim E_0 = (q_0^* - sD)s^2$ where q_0^* is a real zero of Ai. Zeroes of Ai lie along the negative real axis and are tabulated in Abramowitz and Stegun.[15]

Next, expand the left side of Eq.(2.9) around q_0^* with $q_0 = q_0^* + \Delta q$ and use the Wronskian relation of the Ai and Bi to give

$$\left.\frac{Ai}{Bi}\right|_{-q_0} \simeq \Delta q \; \pi \; Ai'(-q_0^*)^2 + O(\Delta q^2) \tag{2.12}$$

automatically get imaginary term $E_r + iE_i$

$$= \Delta q \; \alpha_0 \; , \; \alpha_0 = \pi \; Ai'(-q_0^*)^2 \quad .$$

Finally, expanding the right side of Eq. (2.9) around E_0 yields the linearized form

$$\alpha_0 \Delta q = \left. \frac{Ai' - Z_0 Ai}{Bi' - Z_0 Bi} \right|_{-E_0/s^2} , \quad Z_0 = Z(E_0) \tag{2.13}$$

from which the perturbed eigenvalue is $E = E_0 + \Delta q\, s^2$. This perturbation solution for E, using Δq from Eq. (2.13), is fast and direct unlike the complicated iterative solution of the original equation (2.9). Furthermore, since E_0 is real, the Airy functions have real arguments greatly simplifying numerical calculations.

For well trapped modes ($E_0 << 0$), asymptotic expansion of the Airy functions gives

$$\alpha_0 \Delta q = - \left[\frac{(1+s^3)e^{-\zeta}}{16(-E_0)^{3/2}} + \frac{i}{4} e^{-\zeta(1+s^3)} \right] \tag{2.14}$$

with $\zeta = 4(-E_0)^{3/2}/3s^3$.

The expression for Δq in Eq. (2.14) is based on large argument asymptotic expansions of Ai and Bi. Careful analysis of these expansions indicate that this form is not valid if $w,E \gtrsim -3.5$. Instead, when E_0 is not large and negative (or s^3 not small), Z_0 is evaluated from Eq. (2.10) and Δq computed by direct methods from Eq. (2.13). Compared with the numerical calculations of Pedersen and Gordon, this procedure yields excellent agreement for the trapped modes [Re(E) < 0].

The perturbation expression for Δq [Eq. (2.13)] is only valid if q_0 is near a zero of Ai. Thus a "cutoff" condition is $sD > 2.338$, the first zero of Ai. Ducts with smaller values of sD cannot support trapped modes. Using the asymptotic formulas for zeroes of Ai, an approximate expression for the number of trapped modes N_{vm} is

$$N_{vm} \simeq .25 + (sD)^{3/2}/1.5\pi \quad .$$

2.4 LEAKY MODES

The leaky wave [$Re(E_m) > 0$] Regge poles are difficult to calculate by solving for roots of $T_-(E)$ as was done for the trapped modes. The imaginary part of the pole is comparable to the real part and recourse is usually made to iterative numerical solutions of Eq. (2.9). Instead a different approach will be used for the leaky modes, which gives an approximate, yet accurate, solution.

As discussed previously, the integrand in Eq. (2.6) resonates for real values of E corresponding to the real part of E_m. Expanding the product $T_+(E)T_-(E)$ using modulus and phase representations for the Airy functions (Appendix A) yields

$$T_+(E)T_-(E) = C(E)G^2(E)$$

where

$$G^2(E) = \beta^2\sin^2\chi_+ + \sin^2\chi_- + 2\beta\sin\chi_+\sin\chi_-\cos\Delta \quad . \tag{2.15}$$

The envelope $C(E)$ is a slowly varying function of the energy while G^2 has resonances. The various terms in Eq. (2.15) are

$$\beta = M(w)N(E)/sN(w)M(E) \quad ,$$

$$\Delta = \phi(E) - \theta(E) \quad , \quad \delta = \phi(w) - \theta(w) \quad ,$$

$$\chi_\pm = (\chi \pm \delta)/2 \quad ,$$

and

$$\chi = 2\theta(q_0) - \phi(w) - \theta(w) \quad .$$

The M, N, θ, and ϕ functions are the moduli and phases of the various Airy functions and their derivatives.

All the terms in Eq. (2.15) are slowly varying functions of E with the exception of the angle χ. Analysis shows that minima in G^2 occur at those energy values E_r where χ is a negative multiple of 2π

$$\chi(E_r) = -2m\pi \quad , \quad m = 1, 2, \ldots \tag{2.16}$$

When $E_r >> 0$, the asymptotic expansions for the phase functions θ and ϕ yield

$$\chi(E_r) \sim \pi/2 - \frac{4}{3} q_0^{3/2} - \pi + \frac{4}{3} w^{3/2} = - \frac{\pi}{2} - \frac{4}{3} (q_0^{3/2} - w^{3/2}) \tag{2.17}$$

and thus the eigenvalue condition is

$$\frac{4}{3} (q_0^{3/2} - w^{3/2}) \Big|_{E_r} = \frac{4}{3s^3} \left[(E_r + s^3 D)^{3/2} - E_r^{3/2} \right] = 2\pi(m - 1/4) \quad . \tag{2.18}$$

But, the phase integral between the surface and duct bottom is just

$$J(E_r) = \int_{-D}^{0} (E_r - s^3 x)^{1/2} dx = \frac{2}{3} (q_0^{3/2} - w^{3/2}) \quad , \tag{2.19}$$

phase integral action integral

so the eigenvalue condition for highly leaky modes is just $J(E_r) = \pi(m - 1/4)$, or the usual WKB quantization condition. Equation (2.17) in practice proves to be a good approximation even for small values of E_r, and is used to solve Eq. (2.16) numerically.

Once the real part E_r of the Regge pole is obtained from the extremum in G^2, the imaginary portion E_I is found from conservation of energy. From the form Eq. (2.7), E_I gives rise to an attenuation coefficient α^L:

$$\alpha^L = 2 \, I_m \left\{ \left[k_0^2 - (E_r - iE_I + s^3 D)/H^2 \right]^{1/2} \right\} \quad .$$

Using the ray-equivalent to the leaky mode, α^L is

$$\alpha^L = - \ln\left[R(\theta_m)\right]/D(\theta_m) \tag{2.20}$$

where θ_m is the surface grazing angle of the ray-equivalent defined by

$$\sin^2\theta_m = g_+ H(E_r + s^3 D) \quad ,$$

$R(\theta_m)$ is the reflection coefficient of the ray equivalent by the gradient discontinuity in refractive index at the duct bottom, and where $D(\theta_m)$ is the ray's cycle distance. The cycle distance $D(\theta_m)$ is given by

$$D(\theta_m) = -2k_0H^2\frac{\partial}{\partial E}\chi(E_r) \sim \underbrace{4k_0H^2(q_0^{1/2} - w^{1/2})}_{\text{cws}} \quad , \tag{2.21}$$

cycle distance (need trapped modes) within 35%. only valid extreme limit

k_0 should be K_m but they are close

where χ from Eq. (2.17) has been used.

R should be a $\frac{1}{s^2}$ in approximation

The reflection coefficient $R(\theta_m)$ is found to be

$$R(\theta_m) = R(E_r) = \frac{\beta^2 + 1 + 2\beta\cos(\delta + \Delta)}{\beta^2 + 1 + 2\beta\cos(\delta - \Delta)} \quad , \tag{2.22}$$

No approximation

with limiting values

$$R = (1 + s^3)/(1 + s)^3 \quad , \quad E_r \to 0$$

$$R = \frac{(1 + s^3)^2}{64\,E_r^3} \quad , \quad E_r \to \infty \quad .$$

The results [Eqs. (2.18) and (2.20)] obtained for the leaky modes are suggestive of WKB formulas. As pointed out by Furry, however, (Ref. 9) phase integral solutions for the bilinear profile are not strictly valid due to the non-analytic nature of the refractive index profile. In particular the phase integral methods do not give the proper above barrier ($E_r > 0$) reflection coefficient Eq. (2.22) arising from gradient discontinuities in n at the duct bottom.

2.5 EIGENFUNCTIONS

Once the eigenvalues E_m are known, the virtual mode depth functions $\psi_m = \psi(x,E_m)$ are obtained from Eq. (2.4a) or Eq. (2.4b). The latter can be simplified, by making use of continuity across the duct bottom, to

$$\psi_m(x) = \psi_m(o)h_1(p)/h_1(E_m) \quad , \quad x > 0 \tag{2.23}$$

with $p = E_m + x$. The normalization N_m is easily obtained from properties of the Airy functions and Eqs. (2.4a) and (2.23) as

$$N_m = H \int_{-D}^{\infty} \psi_m^2(x)dx = \frac{H}{s^3}\left[1 + (1 + s^3)\psi_m^2(o)\,\frac{d}{dE_m}(h_1'/h_1)\right] \quad . \tag{2.24}$$

The calculation of ψ_m and N_m involves computing Airy functions of complex arguments (due to the complex nature of the Regge pole E_m), which tends to be time consuming. Instead various approximations are used to significantly reduce the numerical calculations.

2.5a TRAPPED MODES

Trapped modes have imaginary components E_I of the Regge pole that are small. Since the virtual mode depth functions are analytic with respect to the energy, this suggests a Taylor series expansion about $E_r = \mathrm{Re}\, E_m$:

$$\psi(x,E) \simeq \psi(x,E_r) + i\, E_I\, \partial\psi(x,E_r)/\partial E \quad .$$

For positions within the duct, this yields

$$\psi(x) \sim \frac{\pi}{s}\, Bi(-q_o^*)\left[(Ai - \alpha_o\Delta q_r Bi) + i\,\Delta q_i(Ai' - \alpha_o\Delta q_r Bi')\right]\Big|_{-q_r}$$

where $\alpha_o = \pi\, Ai'(-q_o^*)^2$, $\Delta q = \Delta q_r + i\Delta q_i$, and $q_r = E_r/s^2 - sx$. The squared amplitude is thus

$$|\psi(x)|^2 \sim \frac{\pi}{\alpha_o s^2}\left[(Ai - \alpha_o\Delta q_r Bi)^2 + \Delta q_i^2\,(Ai' - \alpha_o\Delta q_r Bi')^2\right] \quad . \tag{2.25}$$

Strongly trapped modes have $|\Delta q| << 1$ and therefore

$$|\psi(x)|^2 \sim \frac{\pi}{\alpha_0 s^2} Ai^2(1 - \alpha_0 \Delta q_r \; Bi/Ai)^2 \quad , \tag{2.26}$$

where the Ai,Bi are evaluated at $q_r = E_0/s^2 - sx$.

Similar considerations for the below duct region $(x > 0)$ lead to

$$h_1(p) \sim h_1(p_r) + i \; E_i h_1'(p_r) \quad , \quad p_r = x + E_r \quad , \quad E_i = \mathrm{Im}(E)$$

and thus

$$\psi(x) \sim \psi(o) \frac{h_1(p_r) + i \; E_i h_1'(p_r)}{h_1(E_r) + i \; E_i h_1'(E_r)} \quad .$$

The squared amplitude of u then is

$$|\psi(x)|^2 \sim |\psi(o)|^2 \cdot \frac{M^2(p_r) + N^2(p_r)E_i^2 - 2E_i/\pi}{M^2(E_r) + N^2(E_r)E_i^2 - 2E_i/\pi}$$

where $M^2(x) = Ai^2(-x) + Bi^2(-x)$, and $N^2(x) = Ai'(-x)^2 + Bi'(-x)^2$. For those modes with $|\Delta q| << 1$, this becomes

$$|\psi(x)|^2 \sim |\psi(o)|^2 \; \frac{M^2(E_o + x)}{M^2(E_o)} \quad . \tag{2.27}$$

The normalization N_m is

$$N_m = \frac{H}{s^3} \left[1 + (1+s^3) \; \psi^2(o) \; \frac{d}{dE} \left(\frac{h_1'}{h_1} \right) \Bigg|_E \right] \quad ,$$

and for well trapped modes becomes

$$N_m \sim \frac{H}{s^3}\left[1 + \frac{(1 + s^3)e^{-\zeta}}{4w_0\alpha_0 s^2}\right] \longrightarrow \frac{H}{s^3} \tag{2.28}$$

where $w_0 = |E_0|/s^2$ and $\zeta = 4/3\ w_0^{3/2}$.

2.5b LEAKY MODES

Leaky virtual modes have positive E_r with E_i not small. Asymptotic expansion of the Airy functions with arguments in the sector $|\arg z| < 2/3\ \pi$ yield for the in duct form (Eq. 2.4a) of ψ

$$\psi(x,E) \sim s^{-1}(qq_0)^{-1/4}\ [\sin(\zeta - \zeta_0) - \cos(\zeta - \zeta_0)5/72\zeta]$$

where $\zeta = 2/3\ q^{3/2}$, $\zeta_0 = 2/3\ q_0^{3/2}$ ($|\arg\{q,q_0\}| < 2/3\ \pi$). The squared amplitude of ψ is then

$$|\psi(x,E)|^2 = s^{-2}|qq_0|^{-1/2}\ |\sin(\zeta - \zeta_0) - \cos(\zeta - \zeta_0)5/72\zeta|^2$$

$$= s^{-2}|qq_0|^{-1/2}\ |\sin(\zeta - \zeta_0)|^2 \quad , \quad |\zeta| >> 1$$

or

$$|\psi(x)|^2 = \frac{1}{2}\ s^{-2}|qq_0|^{-1/2}\ (\cosh 2\Delta\zeta_i - \cos 2\Delta\zeta_r) \quad , \quad x < 0 \tag{2.29}$$

with $\Delta\zeta = \Delta\zeta_r + i\ \Delta\zeta_i = \zeta - \zeta_0$.

Below the duct, similar expansions are made for $h_1(p)$ in Eq. (2.23) to give

$$h_1(p) = Ai(-p) - i\ Bi(-p)$$

$$\simeq \sqrt{\pi}\ p^{-1/4}\ e^{i(t-\pi/4)}\ (1 - i5/72t) \quad , \quad |p| >> 1$$

with $p = E + x$, and $t = t_r + i\ t_i = 2/3\ p^{3/2}$.

The squared modulus of h_1 is

$$|h_1(p)|^2 \simeq \pi\, p^{-\frac{1}{2}}\, e^{-2t_i}\, |1 - i5/72t|^2 \quad ,$$

$$\rightarrow \pi\, p^{-\frac{1}{2}}\, e^{-2t_i} \quad , \tag{2.30}$$

which leads to ψ having the form

$$|\psi(x)|^2 \sim \frac{|\psi(o)|^2}{|h_1(E_m)|^2}\, \pi\, p^{-\frac{1}{2}}\, e^{-2t_i} \tag{2.31}$$

below the duct. Note that since $E_i < 0$, Eq. (2.31) predicts an exponential <u>increase</u> in the virtual mode amplitude with increasing depth below the duct. This behavior is in precise agreement with the numerical results of Pedersen and Gordon.

2.5c TURNING POINTS

The expressions for the virtual mode depth function ψ_m developed thus far are based on large argument expansions of the Airy functions which fail near turning points (i.e., regions where q or p are zero). However, ϕ_m is an analytic function of its arguments and thus can be expanded in a Taylor series about the turning point.

Concentrating on the in-duct form [Eq. (2.4a)] of ψ , a series expansion about the turning point is $(q = E_m/s^2 - sx)$

$$\psi_m(x) = \psi(q) = \psi_o + q\,\psi_o' + \frac{q^2}{2}\,\psi_o'' + \frac{q^3}{6}\,\psi_o''' + \ldots \tag{2.32}$$

where $\psi_o = \psi(q=o)$, $\psi_o' = \partial\psi(q=o)/\partial q$, etc. The series can be greatly simplified by using the differential equation satisfied by ψ

$$\partial^2\psi/\partial q^2 = -\,q\psi$$

to yield

$$\psi(q) = \psi_0(1 - q^3/3! + 4q^4/6! - \ldots)$$

$$+ q\psi_0' (1 - 2q^3/4! + 10q^6/7! - \ldots) \quad . \tag{2.33}$$

The depth function and its derivative at the turning point are .

$$\psi = c_1 \frac{\pi}{s} \left[Bi(-q_0) - \sqrt{3}\, Ai(-q_0)\right] \quad , \tag{2.34}$$

$$\psi_0' = c_2 \frac{\pi}{s} \left[Bi(-q_0) + \sqrt{3}\, Ai(-q_0)\right] \quad , \tag{2.35}$$

where $c_1 = Ai(o)$ and $c_2 = -Ai'(o)$. Separate asymptotic expressions for ψ_0, ψ_0' are obtained depending on whether the mode is trapped or leaky (i.e., depending on q_0).

Trapped modes have q_0 near a zero of Ai, and the perturbation expansion of Eq. (2.12) leads to

$$|\psi_0|^2 \sim \frac{\pi c_1^2}{\alpha_0 s^2} \left[(1 - \alpha_0 \sqrt{3}\, \Delta q_r)^2 + 3\alpha_0^2\, \Delta q_i^2\right] \tag{2.36}$$

and

$$\psi_0'/\psi_0 \sim \frac{c_2(1 + \alpha_0 \sqrt{3}\, \Delta q)}{c_1(1 - \alpha_0 \sqrt{3}\, \Delta q)} \quad , \quad \Delta q = \Delta q_r + i\Delta q_i \quad . \tag{2.37}$$

Similar results for leaky modes are obtained by using the complex asymptotic forms of $Ai(-q_0)$ and $Bi(-q_0)$:

$$\psi_0'/\psi_0 \sim - \frac{c_2 \cos(\zeta_0 - \pi/12)}{c_1 \sin(\zeta_0 + \pi/12)} \quad , \quad \zeta_0 = \frac{2}{3} q_0^{\frac{3}{2}} \tag{2.38}$$

and

$$|\psi_0|^2 \sim \frac{4\pi c_1^2}{s^2} |q_0|^{-\frac{1}{2}} |\sin(\zeta_0 + \pi/12)|^2 \quad . \tag{2.39}$$

For both trapped and leaky modes, the amplitude of $|\psi|^2$ is obtained from Eq. (2.33) as

$$|\psi(q)|^2 \sim |\psi_0|^2 \left|(1 - q^3/3! + 4q^4/6!) + q\frac{\psi_0'}{\psi_0}(1 - 2q^3/4! + 10q^6/7!)\right|^2 \quad . \tag{2.40}$$

2.5d NEAR SURFACE

Near the surface ($x \sim -D$) the virtual mode depth function can be expanded in a Taylor series as was done for the turning point region

$$\psi(q) = \psi(q_0) + \Delta q\psi'(q_0) + \frac{\Delta q^2}{2}\psi''(q_0) + \ldots \tag{2.41}$$

with $\Delta q = q - q_0$ and $q_0 = E/s^2 + sD$ the surface value for q. At the surface $\psi(q_0) = 0$ and $\psi'(q_0) = 1/s$ so Eq. (2.41) becomes

$$\psi(q) = \frac{\Delta q}{s}(1 - q_0\Delta q^2/s3! - 2\Delta q^3/s4! + \ldots) \tag{2.42}$$

where $\Delta q = q - q_0 = -s(x+D)$. The amplitude of ψ is then

$$|\psi|^2 \sim (x+D)^2|1 - q_0 s(x+D)^2/6 - s^3(x+D)^3/12 + \ldots|^2 \tag{2.43}$$

which displays the proper quadratic depth dependence and pressure-release condition at $x = -D$.

2.6 ROUGH SURFACE EFFECTS

The presence of a non-planar ocean surface will modify the Regge poles E_m leading to a larger imaginary component. The virtual mode intensity decays with range at an increased rate due to an additional attenuation term arising from rough surface scattering. The resultant attenuation is the sum

of the previously computed leakage term α^L and the surface scattering term α^S

$$\alpha = \alpha^L + \alpha^S \quad .$$

The surface scattering term is found by applying energy conservation to the modal ray-equivalent interacting with the ocean surface.

A ray-equivalent may be identified with each virtual mode through a surface grazing angle θ_m defined by

$$\sin^2\theta_m = (E_r + s^3 D)/(k_o H)^2$$

correction previously $\sin^2\theta_m = g_r H (E_r + s^3 D)$

this is correct

and a corresponding cycle distance $D(\theta_m)$

$$D(\theta_m) = 2H \cos\theta_m \int_{-D}^{x_1} \left[E_r - Q(x)\right]^{-\frac{1}{2}} dx \quad .$$

x_1 is the upper turning point (real) of the mode and for leaky modes is taken as the duct bottom ($x = 0$). Each interaction of the ray-equivalent with the rough surface is treated as a specular reflection with corresponding reflection coefficient $R^S(\theta_m)$. The intensity decay with range is then modeled as a "loss per bounce" applied continuously over the ray's cycle distance, leading to

$$\alpha^S = - \ln\left[R^S(\theta_m)\right]/D(\theta_m) \quad .$$

The rough surface specular reflection coefficient R^S is computed as a function of frequency, grazing angle and sea-state from previous work by Spofford and Keenan.[17] Strictly speaking, the rough surface effects should be included as a modification of the surface impedance condition and the Regge poles found in a self consistent manner. The error made in adopting the present approach is, however, consistent with present understanding of surface scattering phenomena and within the approximation of replacing the true index of refraction profile by the Furry model.

SECTION 3
MODEL EVALUATION

The virtual mode surface duct model described earlier was evaluated by comparisons with output from the parabolic equation (PE) code[16] and the NOSC n-layer normal mode (NM) code. The test environment was a bilinear n^2 profile for a deep (1,000 foot) surface duct with a corresponding sound speed profile having a pressure gradient ($\sim .018\ sec^{-1}$) within the duct and a moderate ($\sim -\ .1\ sec^{-1}$) gradient below the duct. The ocean was assumed infinitely deep with a smooth, pressure-release surface and no surface loss. Incoherent transmission loss, for a monochromatic point source, was computed for a variety of source/receiver geometries.

Source frequencies of 25, 50, 100, 250, 500, and 1,000 Hz were considered which yielded propagation conditions corresponding to zero and 21 trapped modes respectively. This effectively simulated results for thinner ducts at higher frequencies. Source-receiver combinations for all depth pairs between 250 and 1,500 feet in 250-foot increments (21 distinct pairs) were chosen to cover a wide range of in-layer, cross-layer and below-layer geometries for the various source frequencies. In total, 126 different frequency/geometry propagation scenarios were examined in the comparisons.

Two types of surface duct model evaluations were made: (1) incoherent transmission-loss for all 126 scenarios, and (2) individual comparisons of modal attenuation coefficients and intensity amplitudes.

The transmission-loss evaluations were made by direct comparison of the virtual mode surface duct model results with those of the PE and NM codes. The PE code actually outputs coherent transmission-loss values, displaying characteristic interference features. These oscillations were removed by intensity averaging the PE output with an eight (8) nautical mile moving window to simulate incoherent levels.

The NM code, in addition to providing TL output, also output the individual modes used in the TL calculation. Modal attenuation coefficients and eigenfunction intensity amplitudes were available for direct comparison with the surface duct model.

3.1 PE COMPARISONS

The AESD PE code was run at six source frequencies (25, 50, 100, 250, 500, and 1,000 Hz) and for six receiver depths (250, 500, 750, 1000, 1250, and 1500 feet). The PE code is a finite depth ocean model, requiring an absorbing bottom to effectively simulate an infinitely deep ocean. The bottom was placed at 8,000 feet for the 25 Hz run and at 2,000 feet for all other frequencies. The FFT transform size used by the PE code was 2^8 for 25, 500, and 1,000 Hz runs, and 2^7 for the 50, 100, and 250 Hz runs. Due to the use of a running intensity average, PE results at ranges closer than 8 nautical miles are not reliable and are ignored. The PE code implicitly includes the leaky as well as the trapped waves and thus provides a check on the validity of truncating the virtual mode spectrum after three leaky modes.

Comparisons between the virtual mode surface duct model and PE are shown in Figures (3.1) to (3.36) for the various propagation scenarios. Good agreement with the PE code is indicated for within-, cross-, and below-layer geometries.

The 25 Hz run had no trapped modes and the three lowest leaky virtual modes were included in the virtual mode sum. Good agreement with the PE results are seen for ranges exceeding 10-15 nautical miles. Deviation between PE and model results for shorter ranges are possibly an artifact of the PE smoother and/or due to insufficient terms in the modal sum.

The 50 Hz run case corresponds to one trapped mode. The PE code predicts a higher intensity (by ~1-2 dB) at 50 nautical miles. This is probably an artifact of the PE run due to the bottom being too shallow. Such an effect occurred on earlier 25 Hz runs with shallower bottoms and arises from the non-radiation boundary condition used in the PE code. The PE code boundary condition effectively forces an increase in the pressure field within the water column, leading to higher intensities, hence less TL. Additional deviation between the PE code and the surface duct model occur at intermediate ranges (10 - 20 nm) where PE shows more loss. This trend persists for other frequencies. A plausible explanation for this, is phase cancellation leading to a deep null in this region. Since the surface duct model is not designed

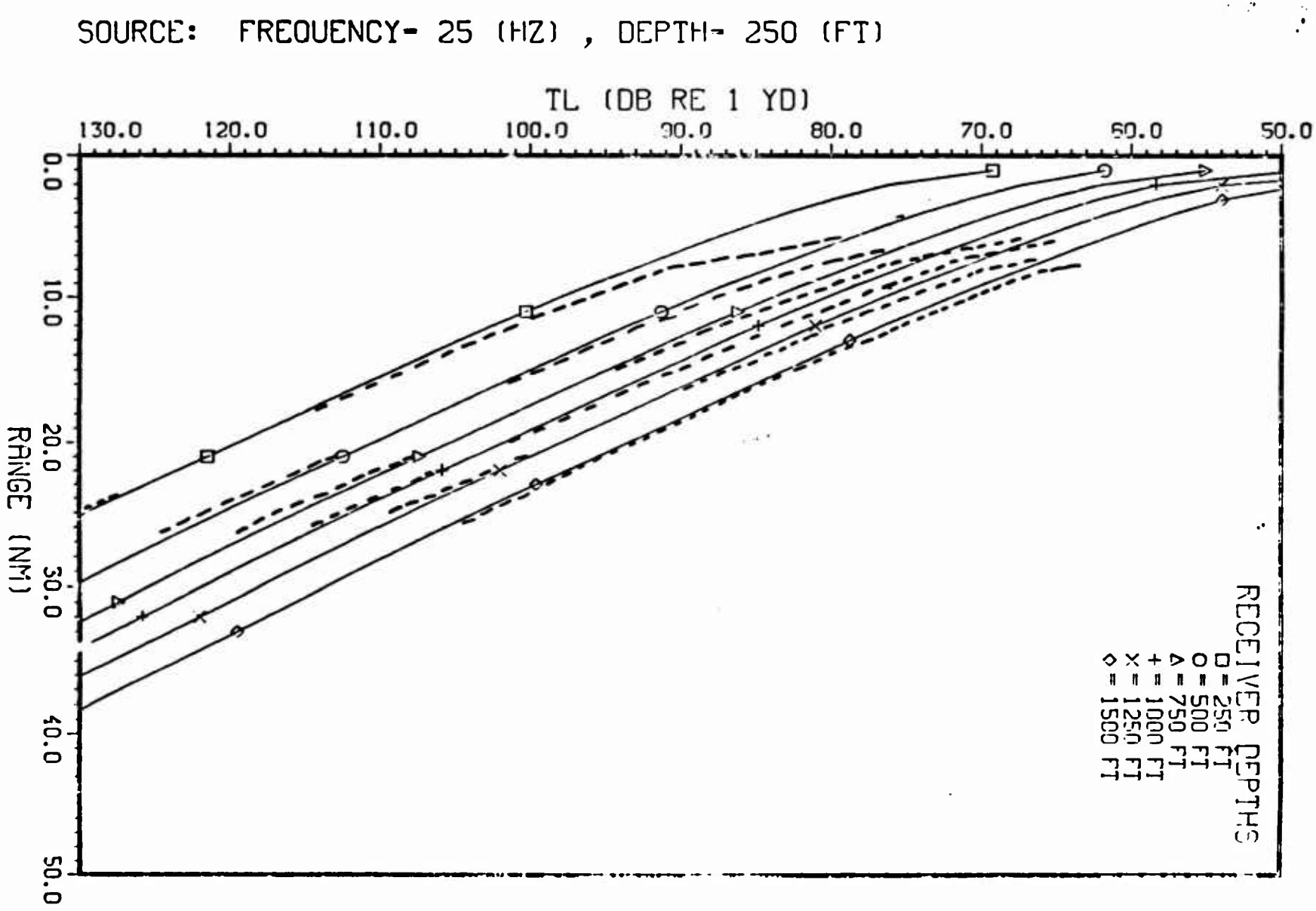
correction

to compute phased TL, such differences are to be expected. In any case, the maximum deviation is ~ 2 dB.

The 100 Hz runs exhibit the same general trends as the 50 Hz data, with the exception of much smaller offset at 50 nm. This is due to the decreasing effect of the finite depth bottom used by the PE code, and a corresponding decrease in intensity from "ghost" reflections off the bottom.

The 250, 500, and 1,000 Hz runs can be grouped as a whole for comparisons. Generally the agreement between PE and the virtual mode code is good at longer ranges. For short ranges ($r \lesssim 10$ nm) possible spectrum truncation and/or PE smoother effects are noticed. Medium ($10 \lesssim r \lesssim 25$ nm) ranges exhibit a weak null in the PE results which is not seen in the virtual mode data. This is most probably caused by phase cancellation which the virtual mode model is not treating. For sources within the duct, the PE results display a characteristic interference with range that the smoother has not completely removed. This effect is more pronounced with increasing frequency.

A volume absorption loss term α_v, of the form $\alpha_v = .125\ (f/1000)^2$ dB/nm (f the frequency in Hertz), has been added to the virtual mode model results shown in Figures (3.1) to (3.36) to comply with the PE predictions.



FACT SURFACE-DUCT MODEL

Figure 3-1 . Comparison of Surface-Duct Model (solid) with PE (dash). Consecutive curves displaced 5 db up .

FACT SURFACE-DUCT MODEL

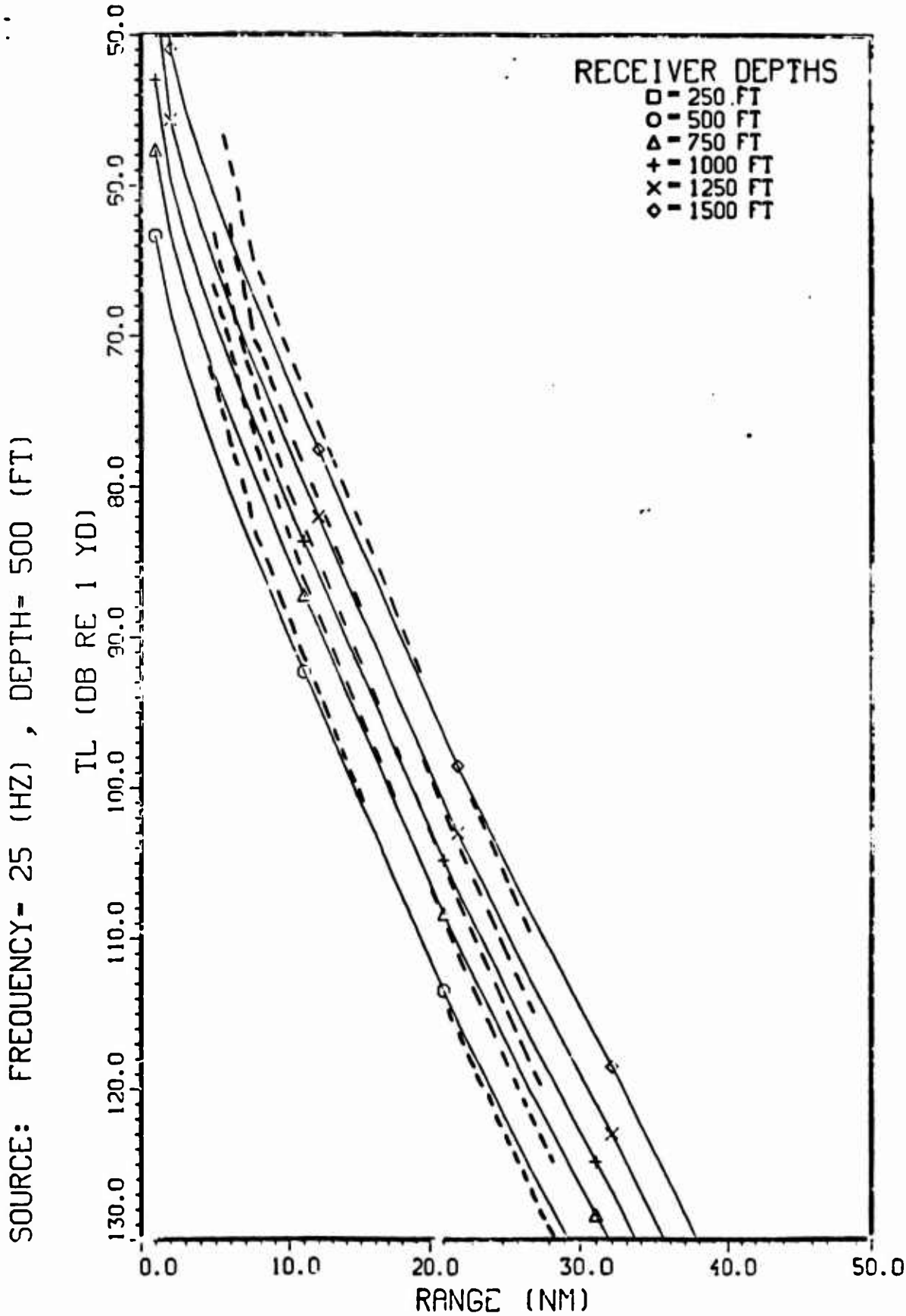


Figure 3-2. Comparison of Surface-Duct Model (solid) with PE (dash). Consecutive curves displaced 5 db up.

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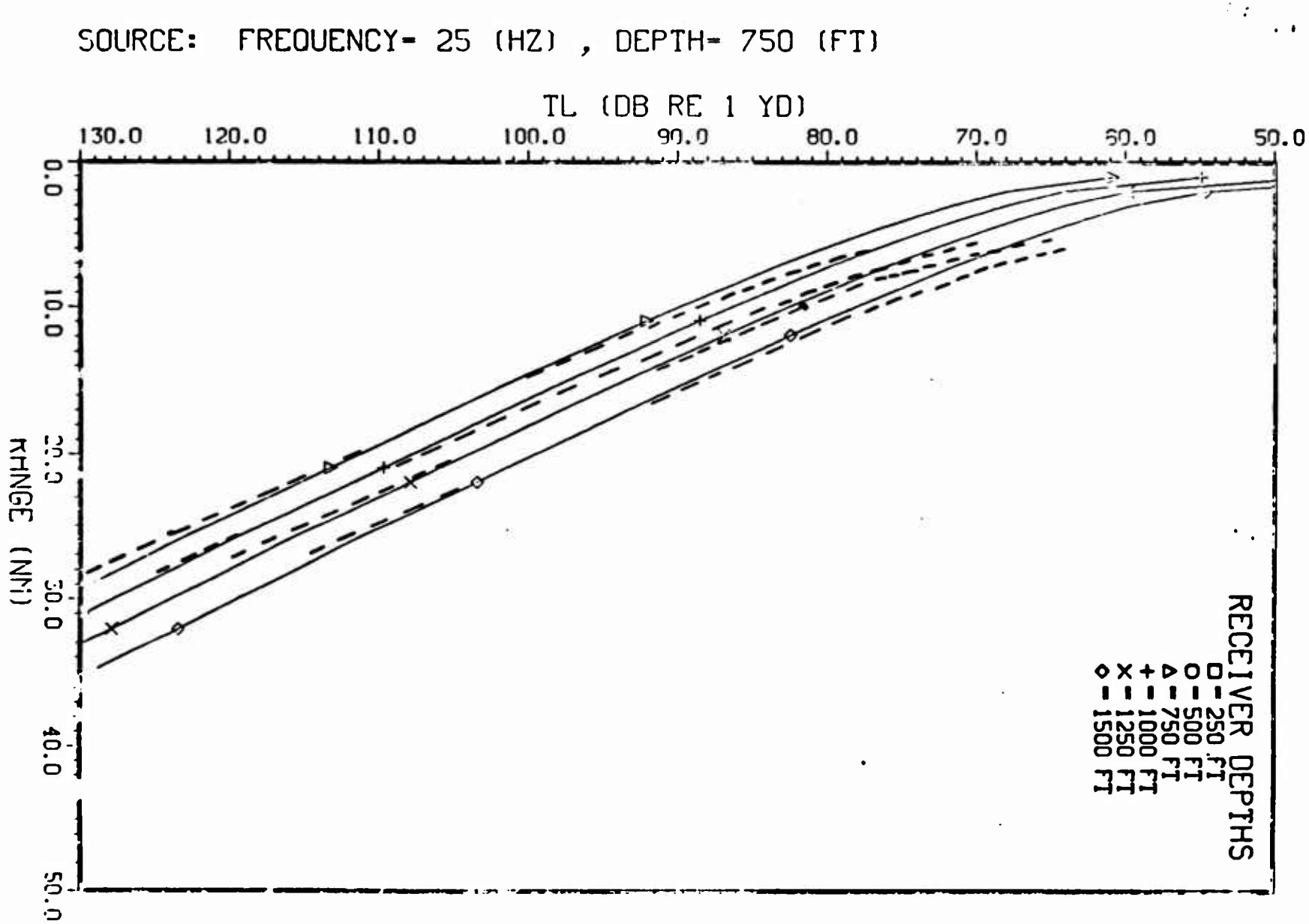
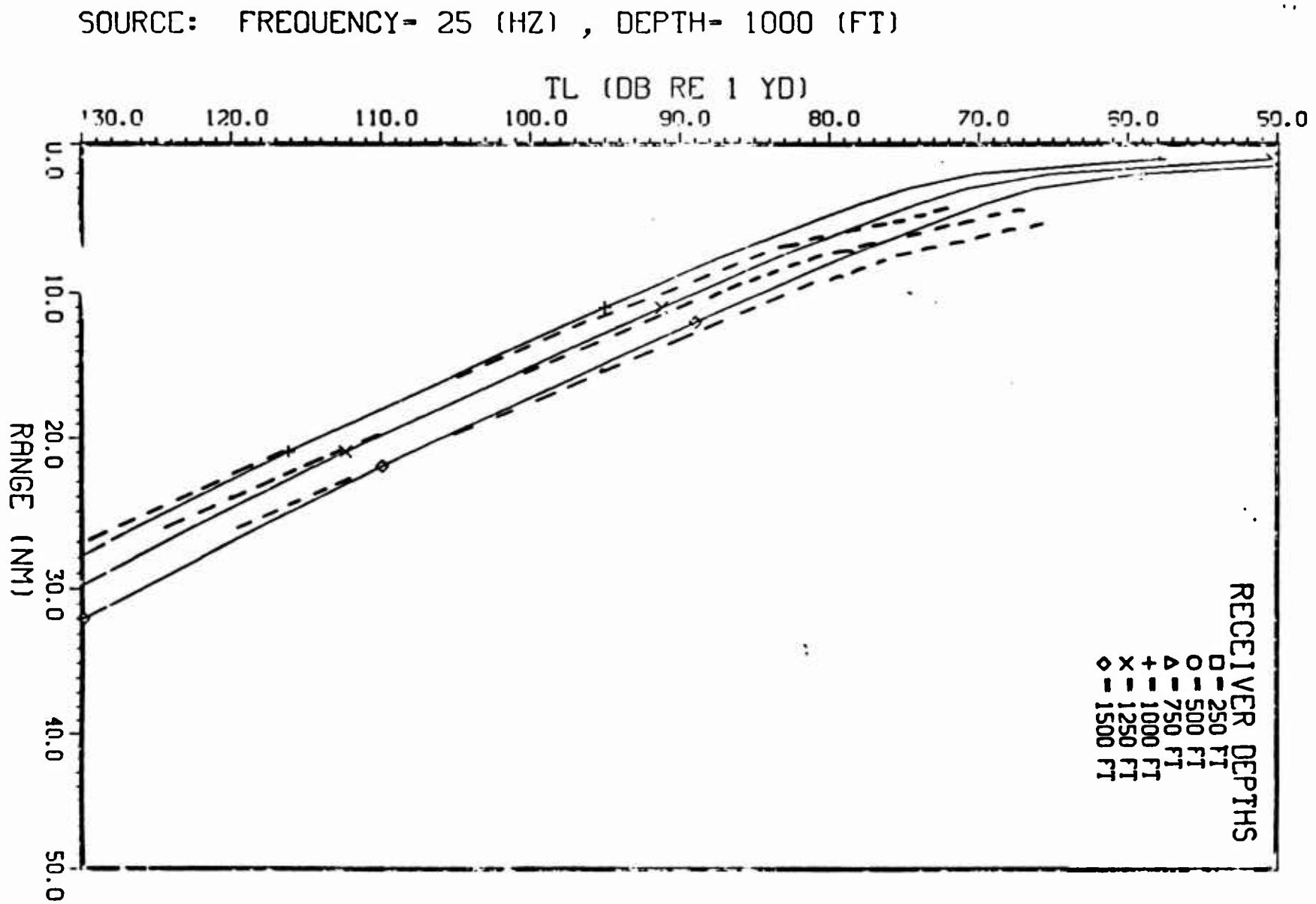


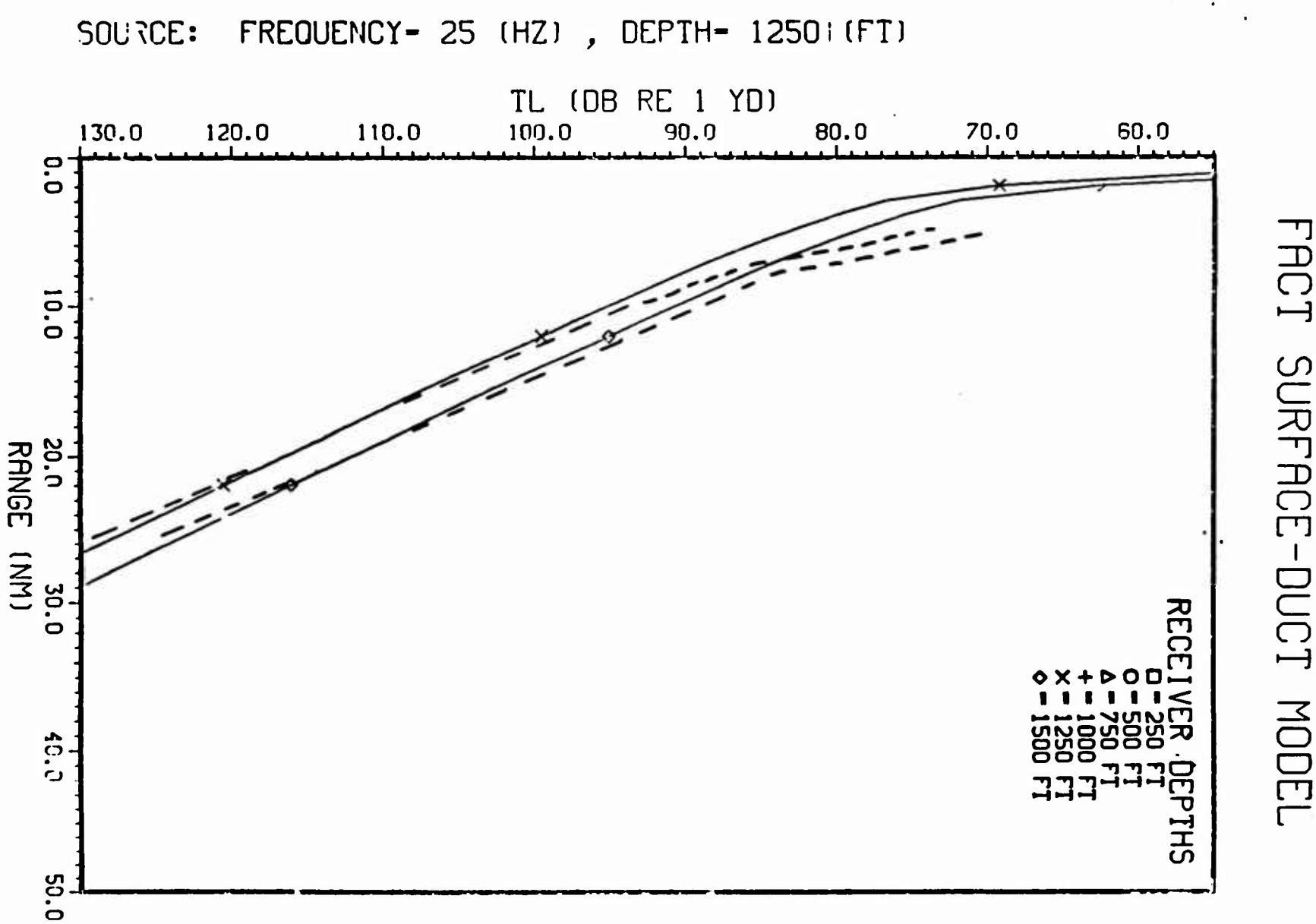
FACT SURFACE-DUCT MODEL

Figure 3-3 . Comparison of Surface-Duct Model (solid) with PE (dash). Consecutive curves displaced 5 db up .



FACT SURFACE-DUCT MODEL

Figure 3-4 . Comparison of Surface-Duct Model (solid) with PE (dash). Consecutive curves displaced 5 db up .



FACT SURFACE-DUCT MODEL

Figure 3-5 . Comparison of Surface-Duct Model (solid) with PE (dash). Consecutive curves displaced 5 db up .

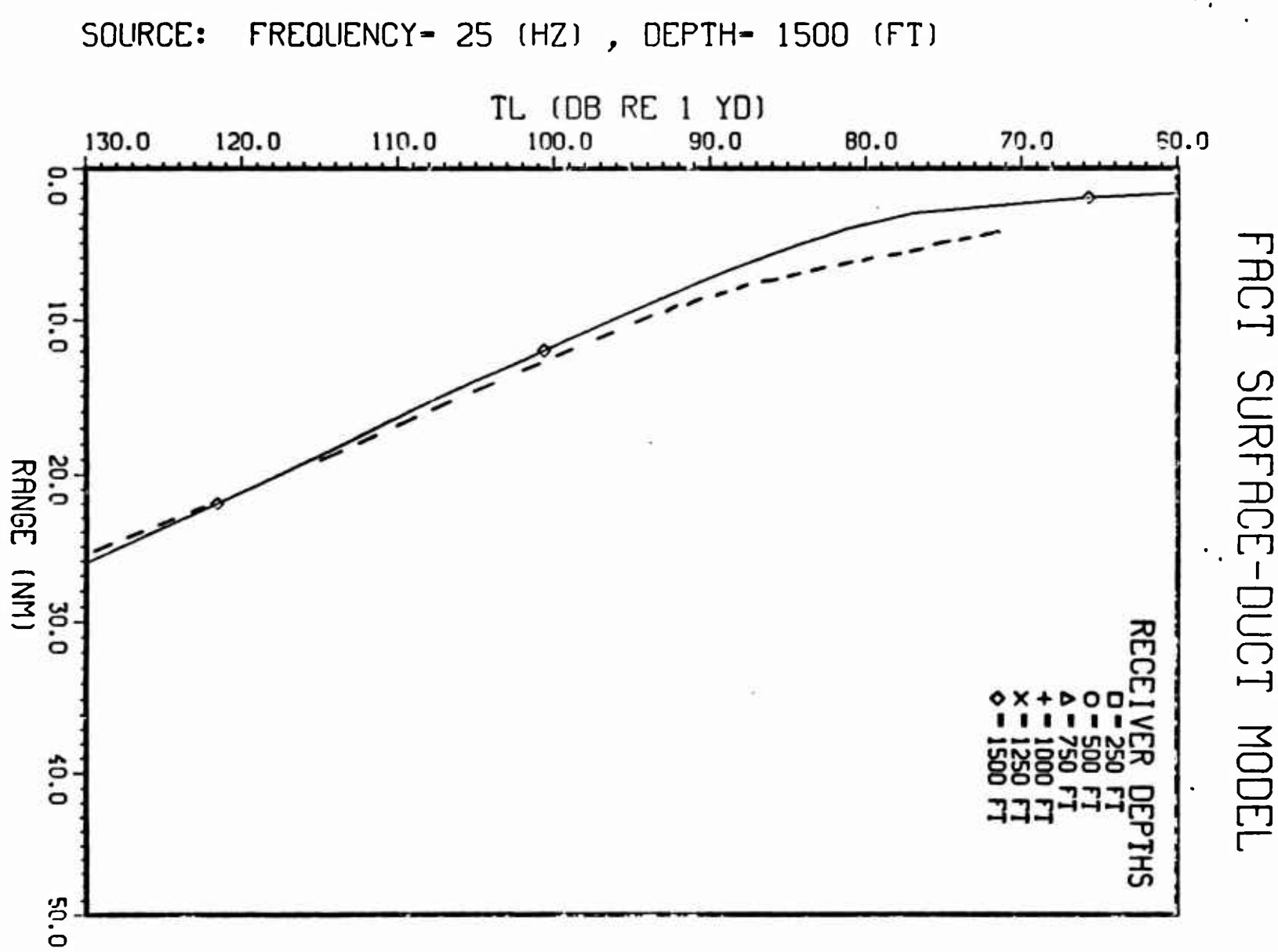


Figure 3-6 . Comparison of Surface-Duct Model (solid) with PE (dash).

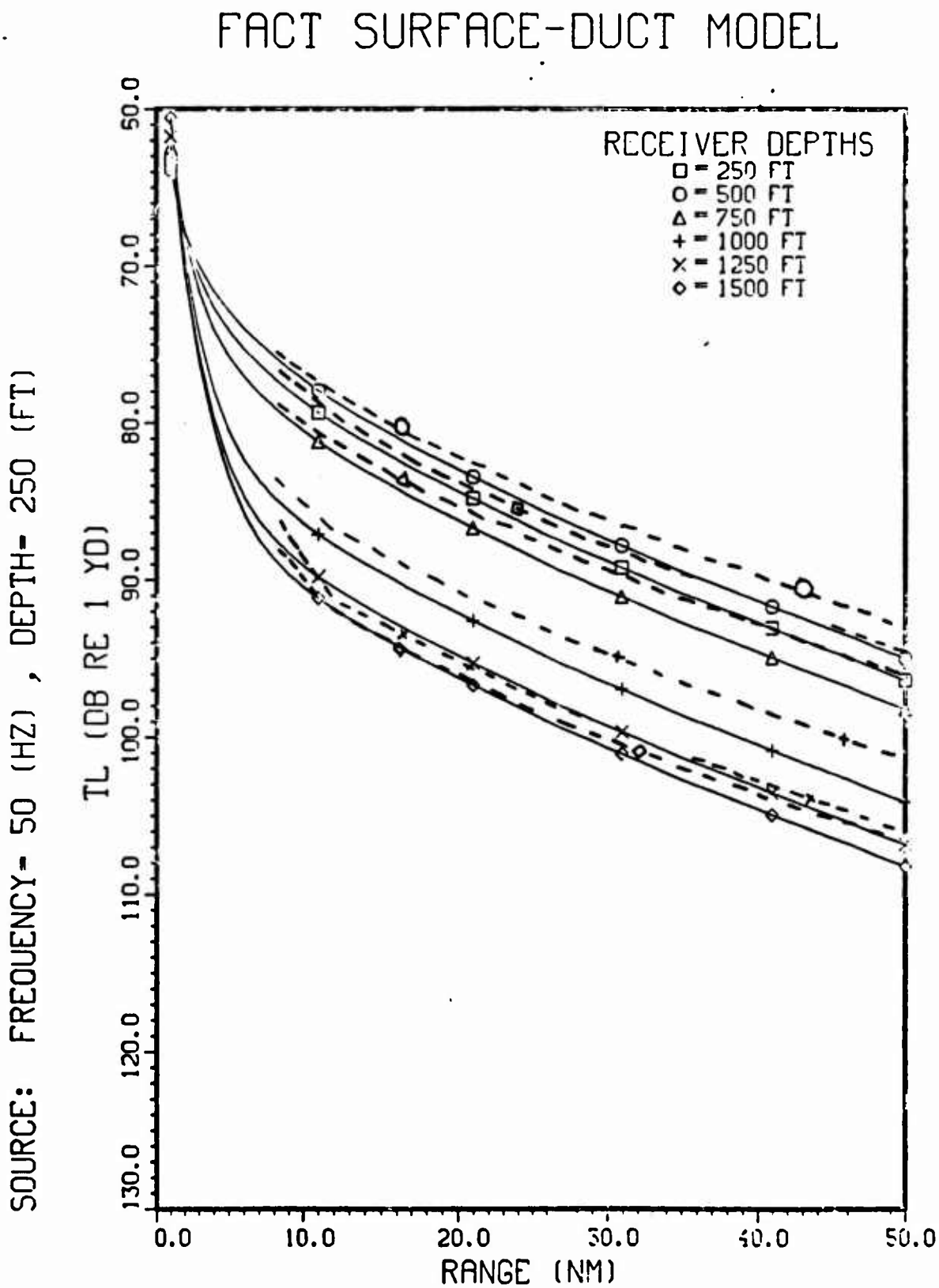


Figure 3-7. Comparison of Surface-Duct Model (solid) with PE (dash).

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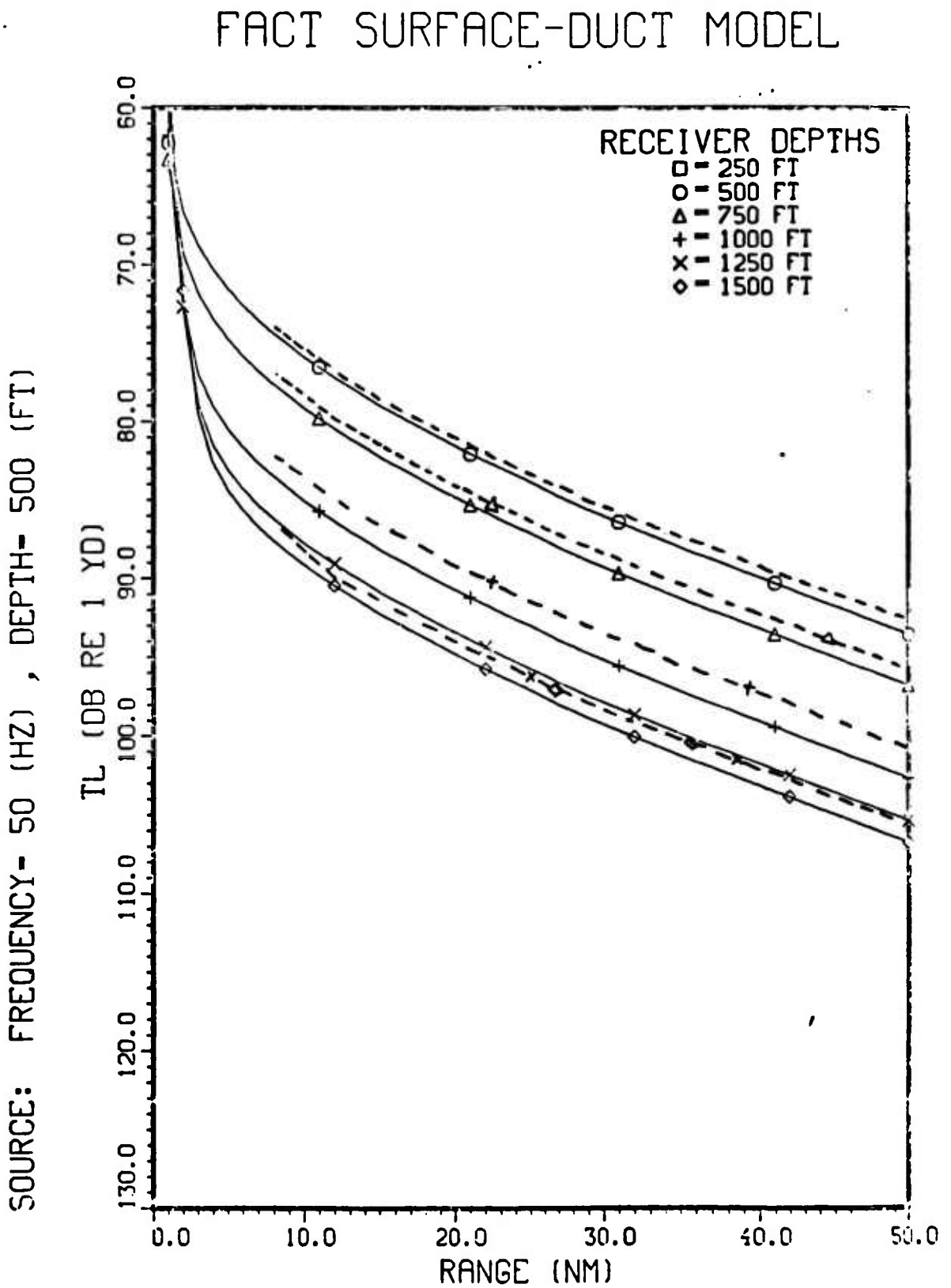

Figure 3-8. Comparison of Surface-Duct Model (solid) with PE (dash).

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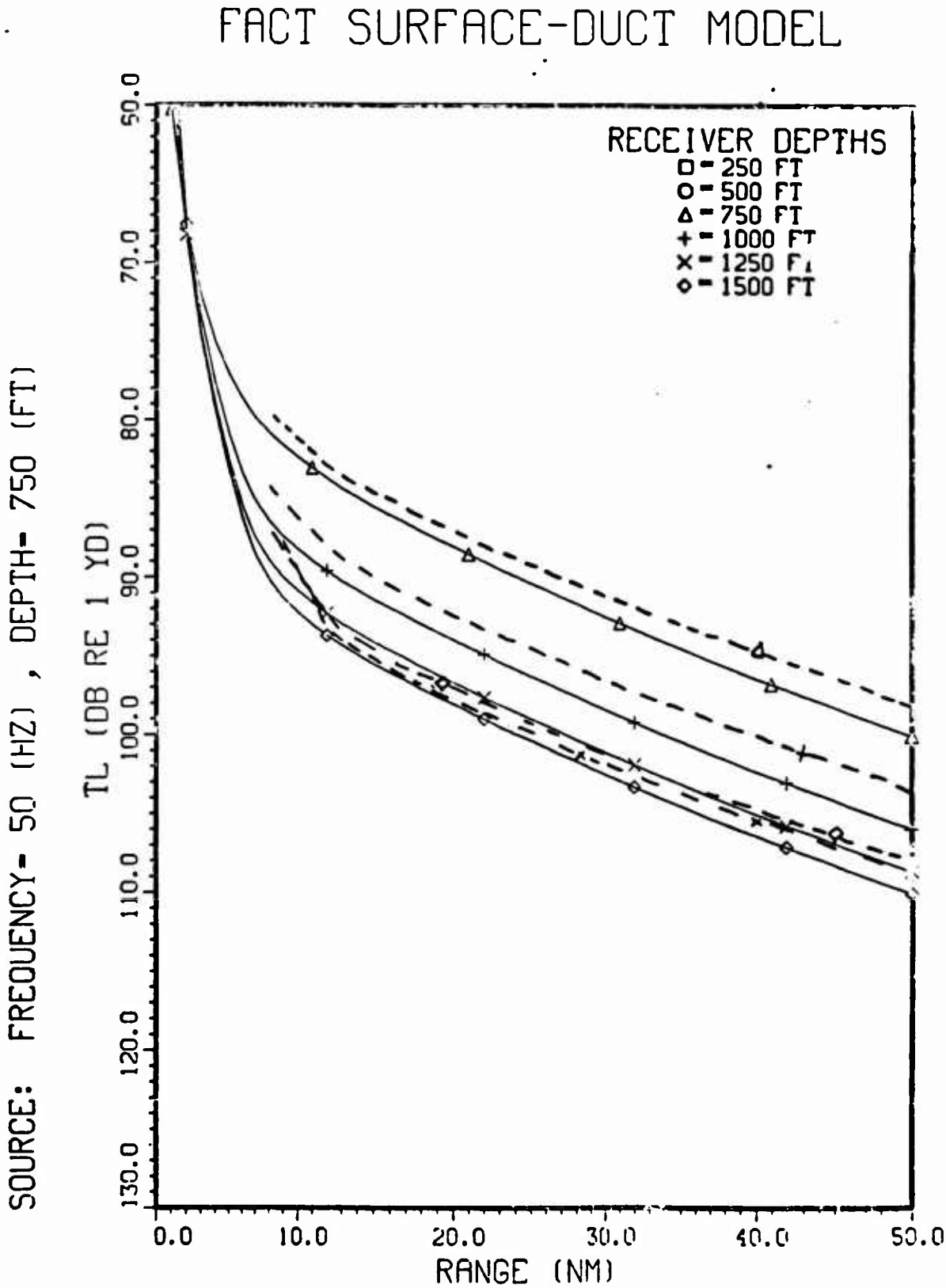

Figure 3-9. Comparison of Surface-Duct Model (solid) with PE (dash).

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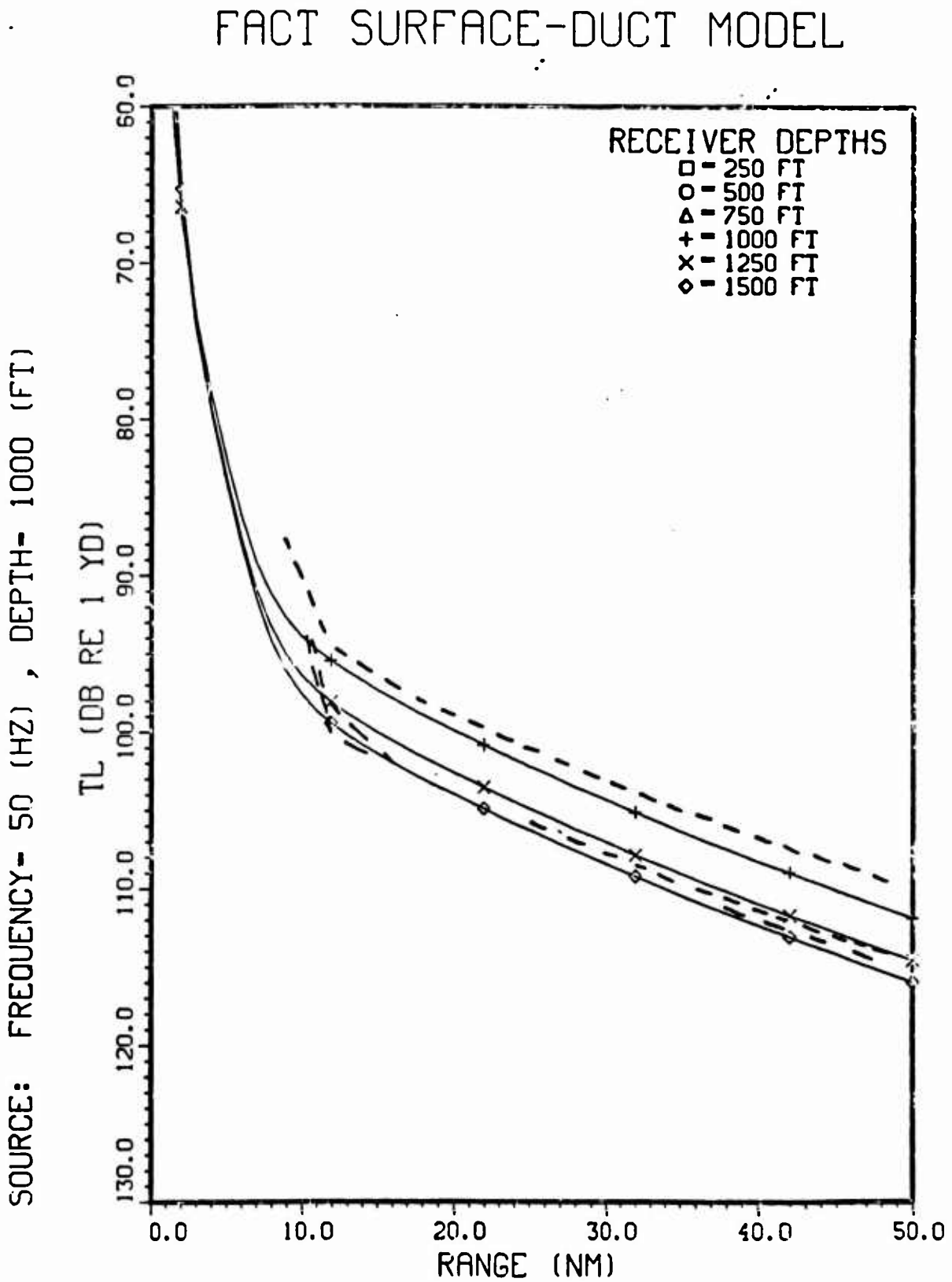

Figure 3-10 . Comparison of Surface-Duct Model (solid) with PE (dash).

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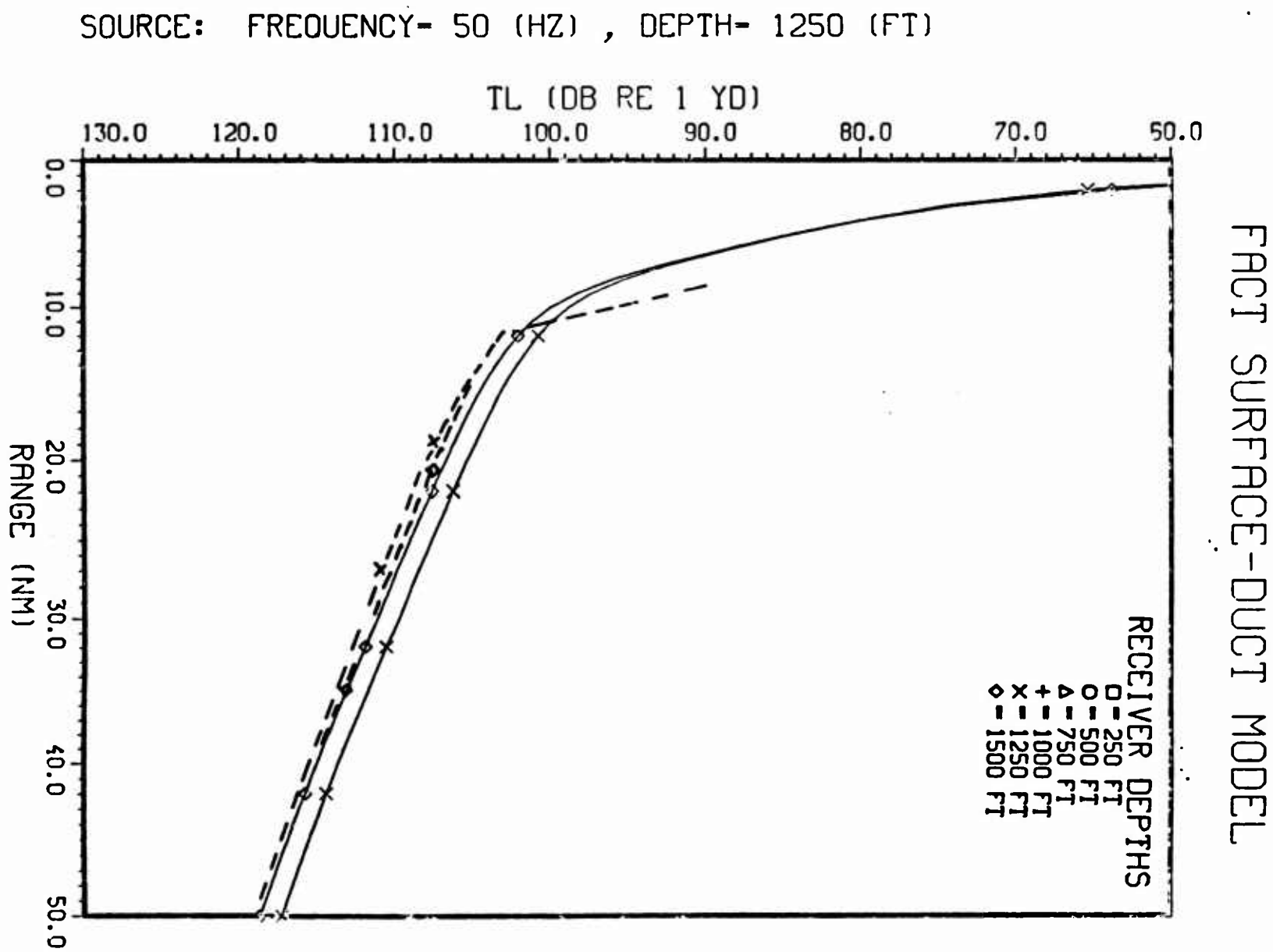

Figure 3-11 . Comparison of Surface-Duct Model (solid) with PE (dash).

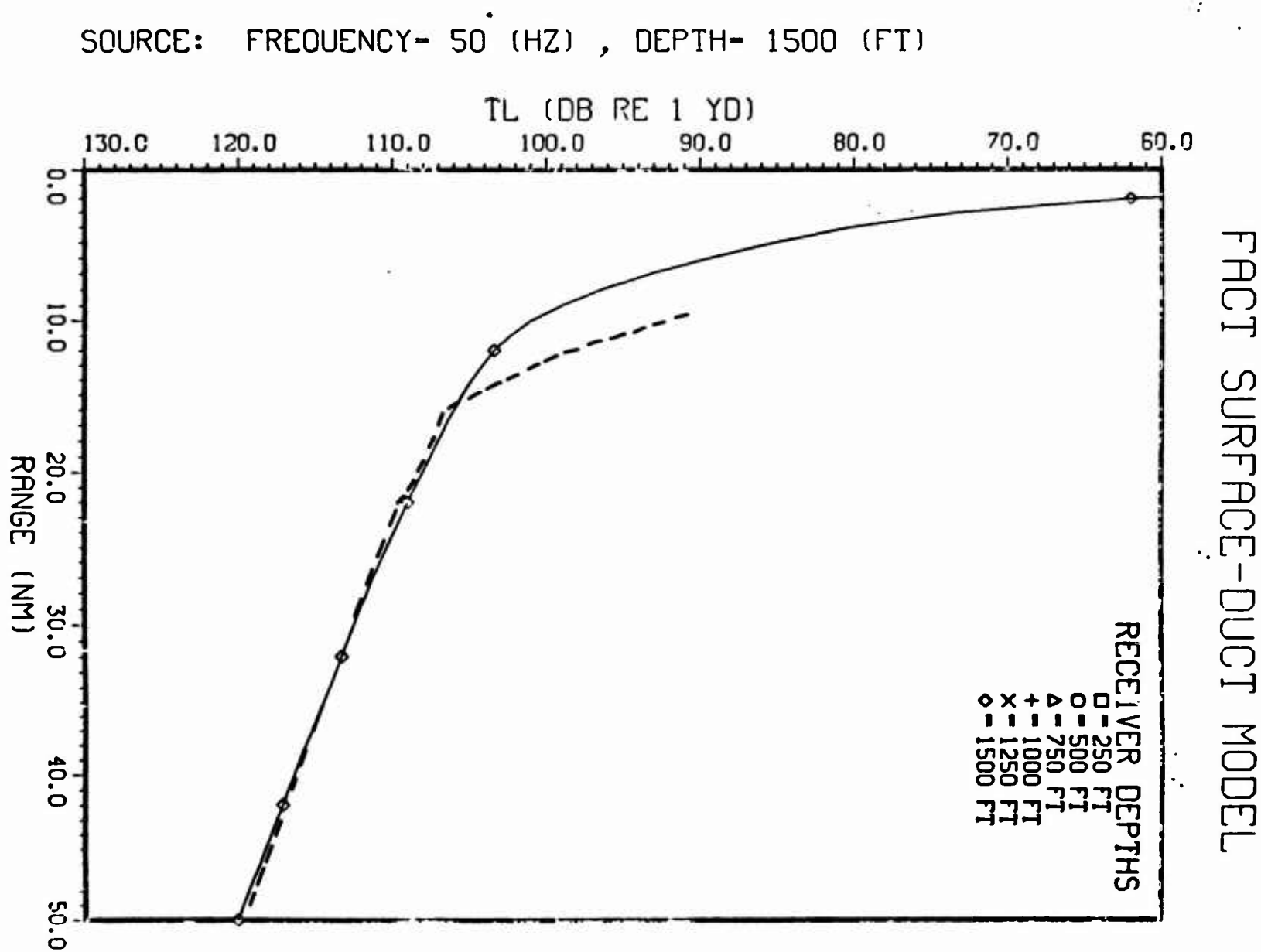

Figure 3-12 . Comparison of Surface-Duct Model (solid) with PE (dash).

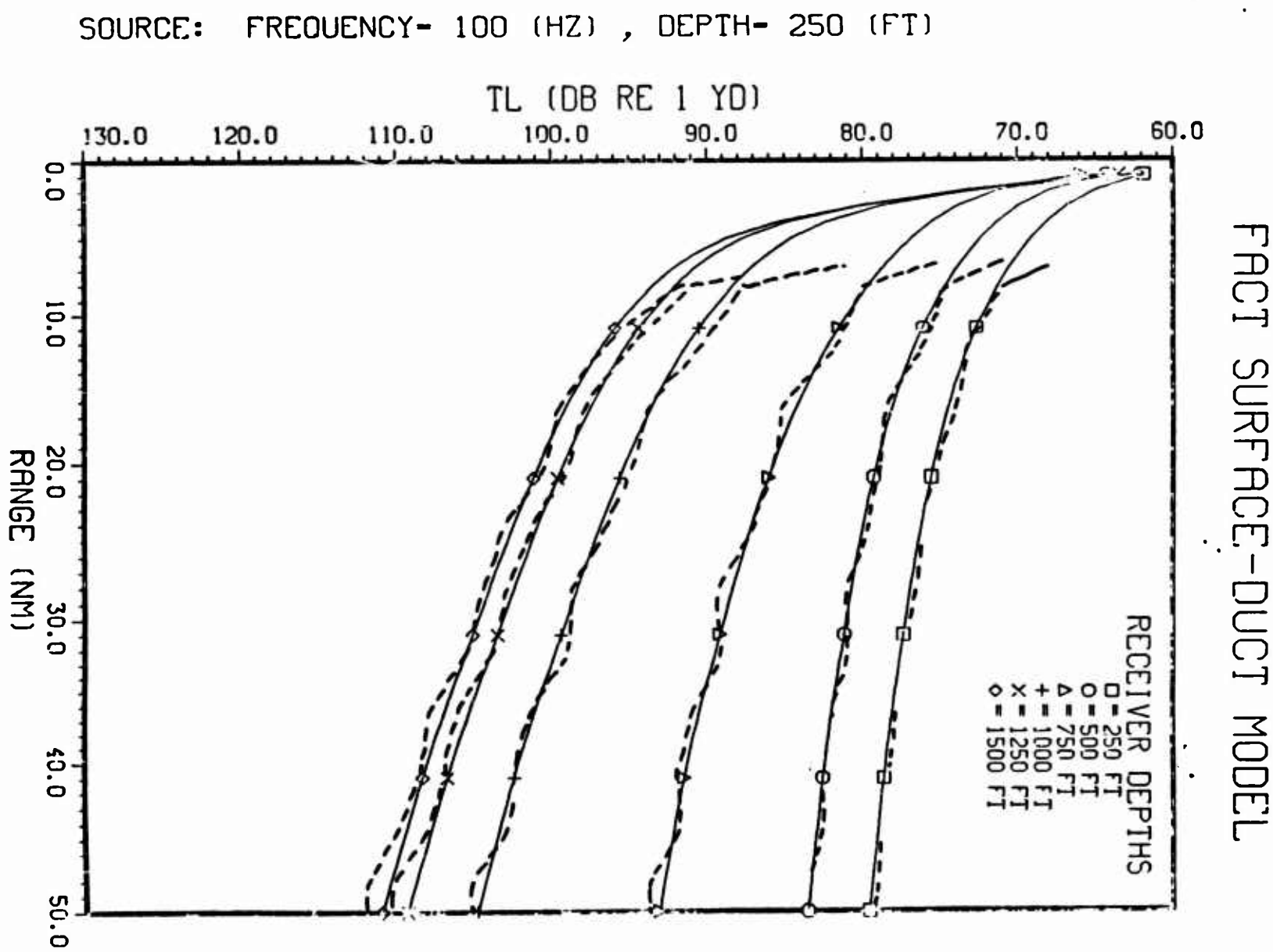

Figure 3-13 . Comparison of Surface-Duct Model (solid) with PE (dash).

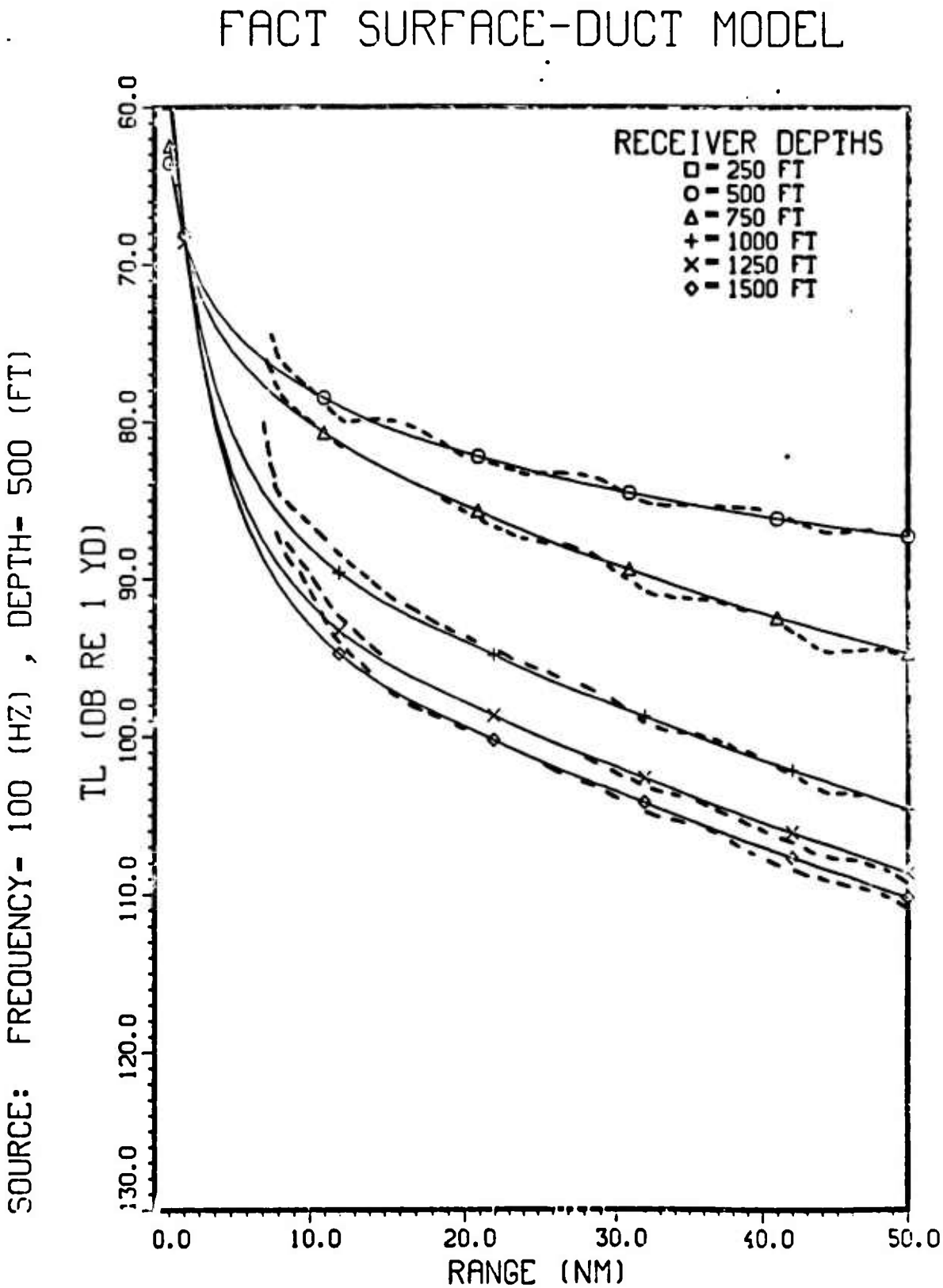

Figure 3-14. Comparison of Surface-Duct Model (solid) with PE (dash).

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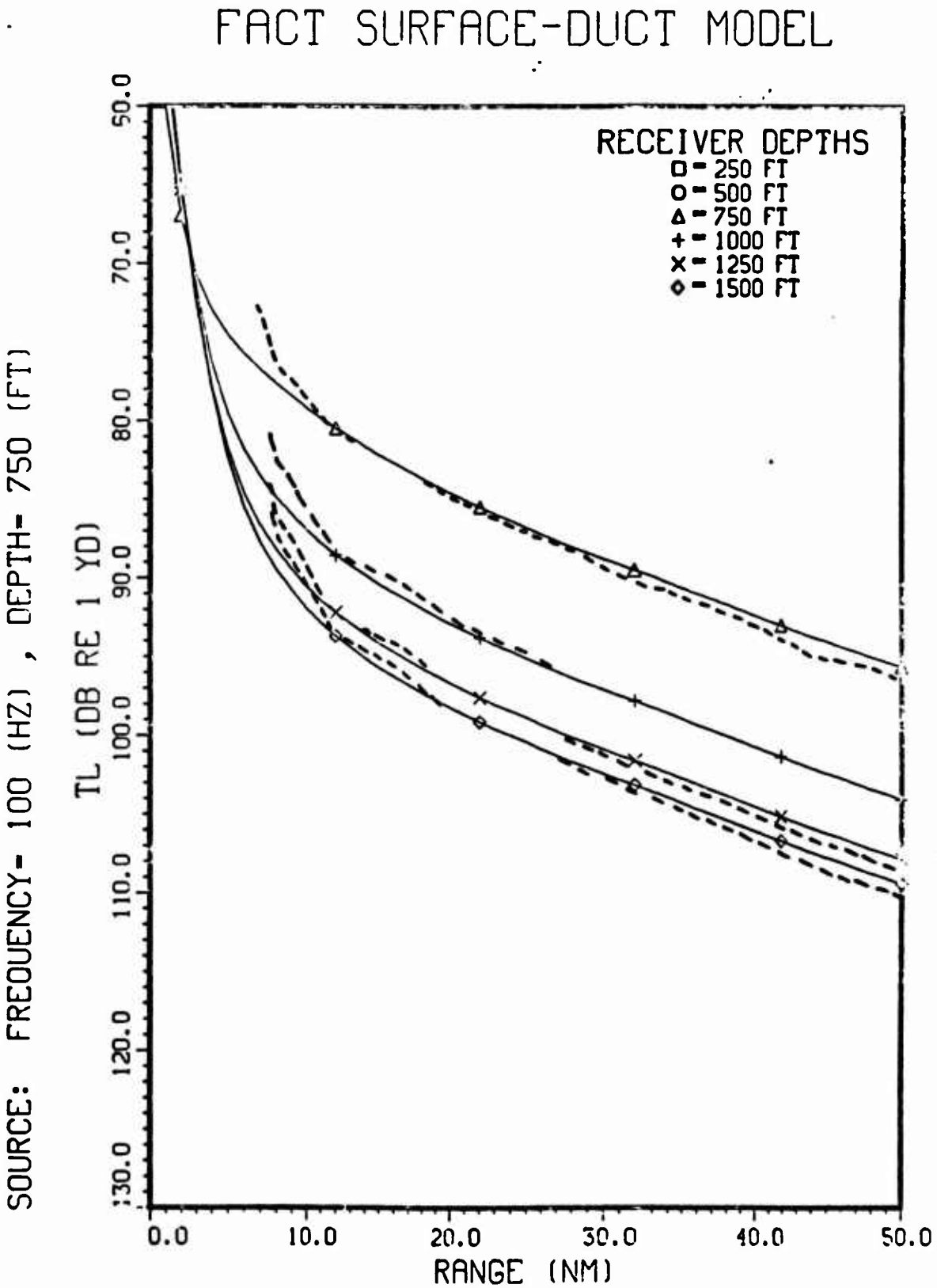

Figure 3-15. Comparison of Surface-Duct Model (solid) with PE (dash).

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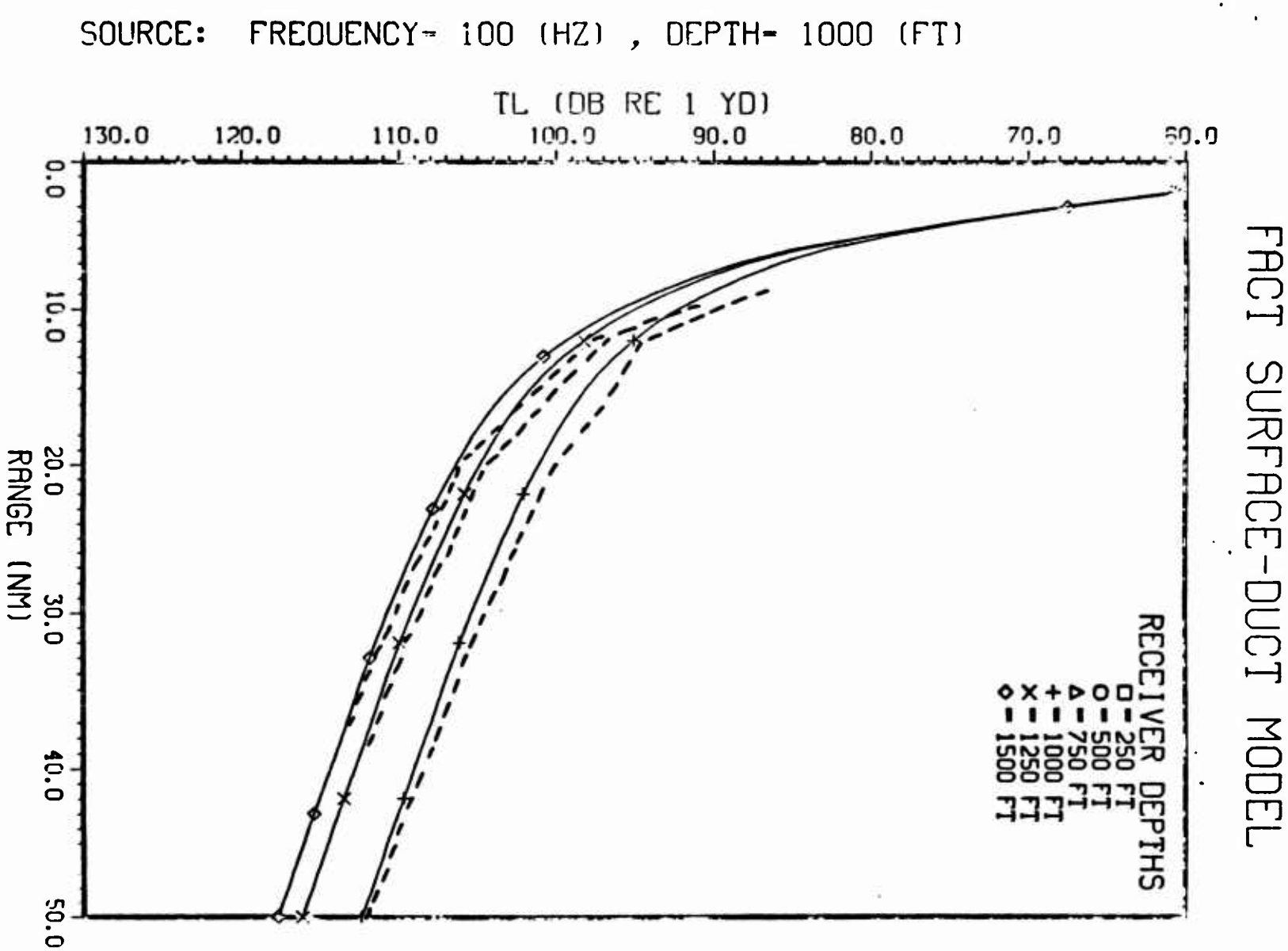


FACT SURFACE-DUCT MODEL

Figure 3-16 . Comparison of Surface-Duct Model (solid) with PE (dash).

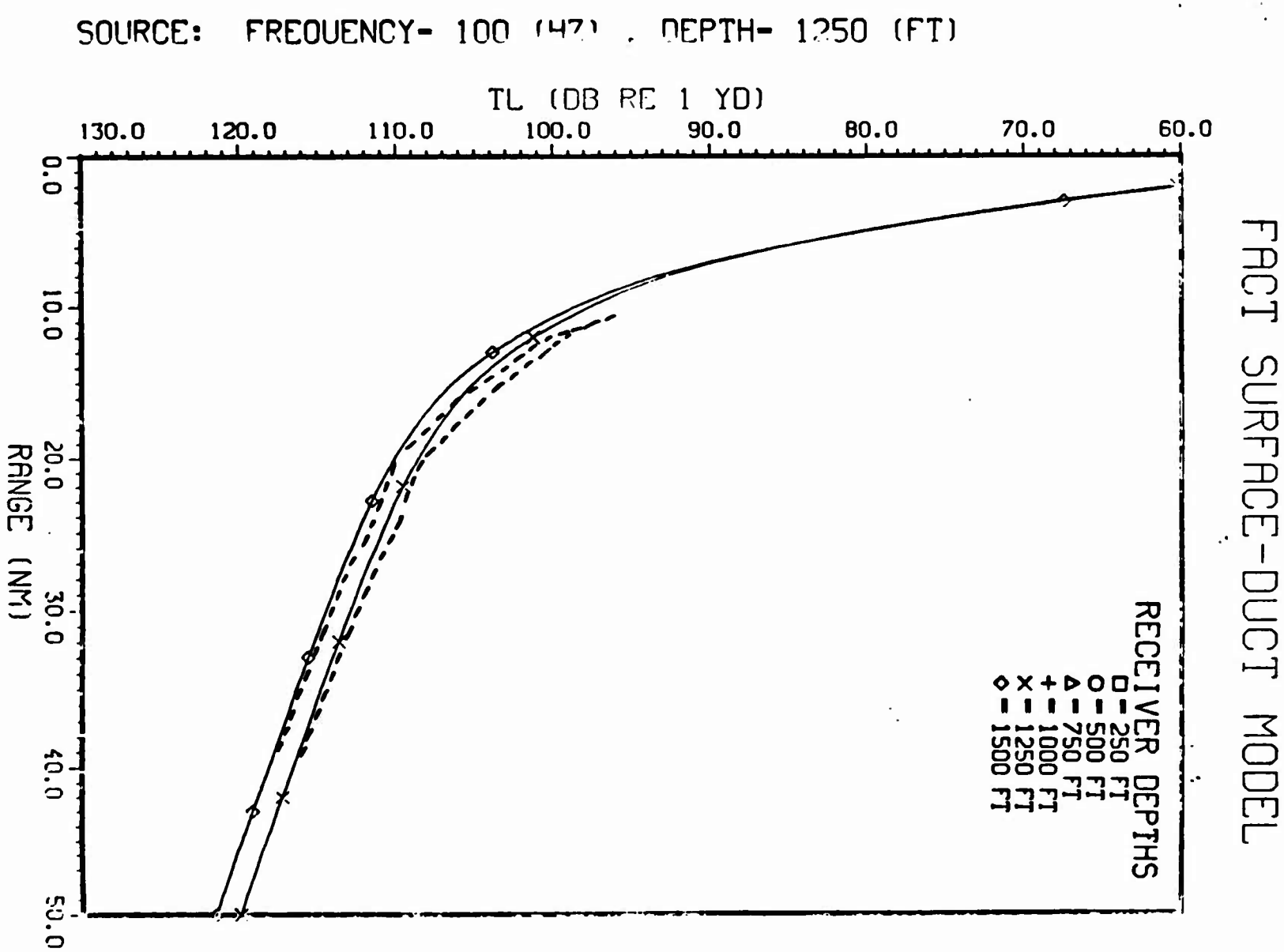


Figure 3-17 . Comparison of Surface-Duct Model (solid) with PE (dash).

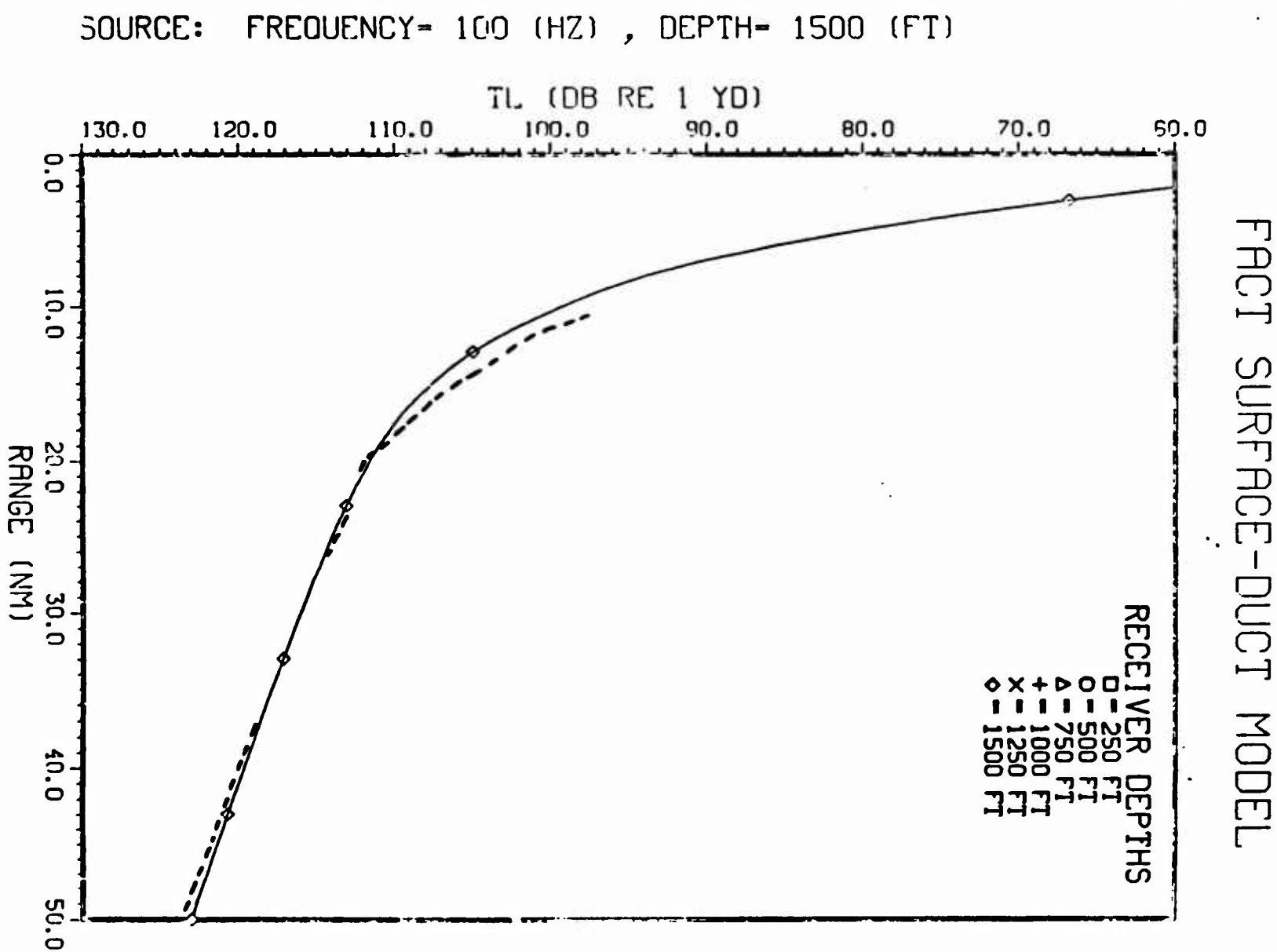


Figure 3-18 . Comparison of Surface-Duct Model (solid) with PE (dash).

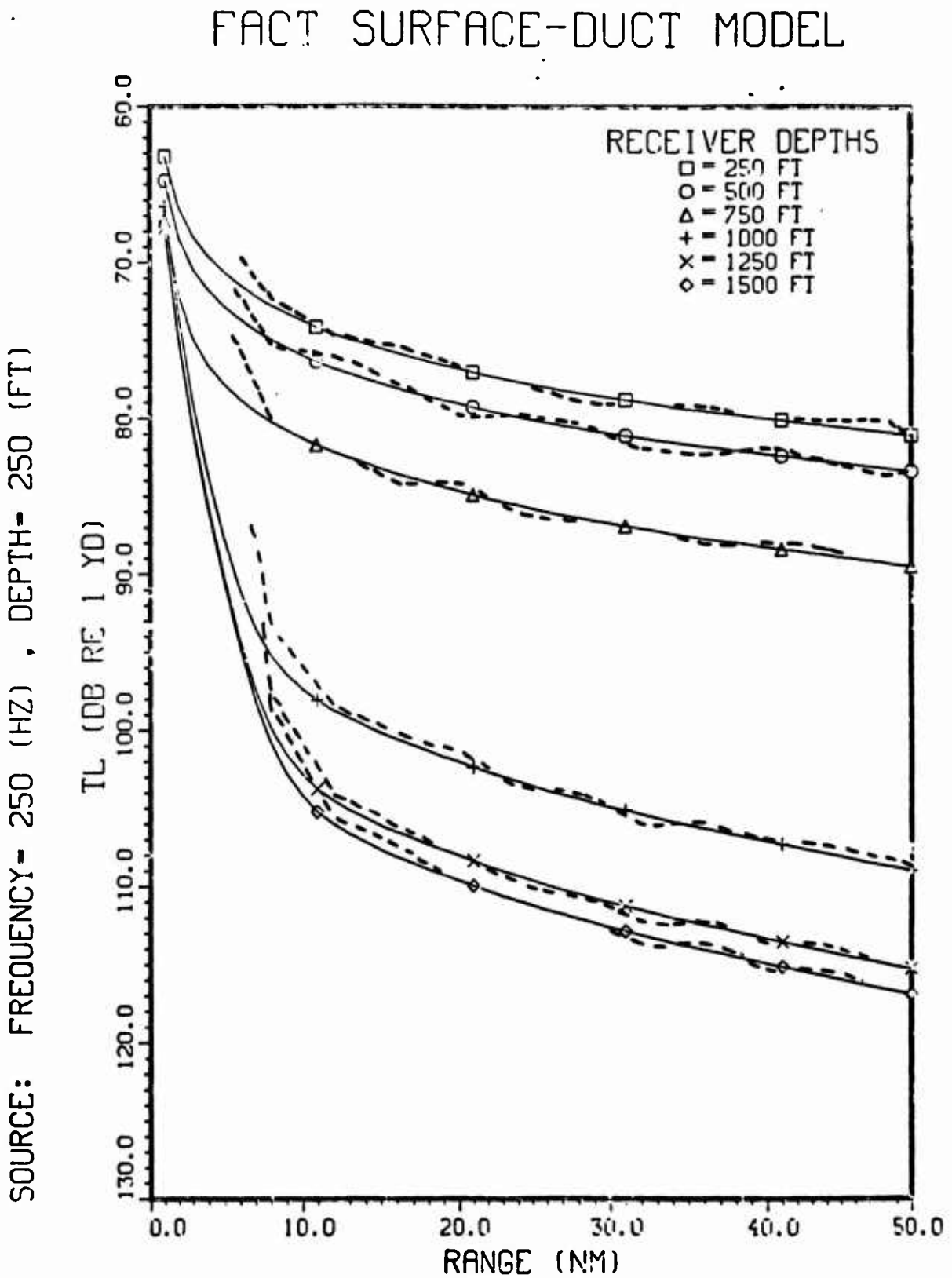


Figure 3-19. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 19 20.10.00 WED 1 OCT, 1980 , SAI DISSPLA VER 8.2

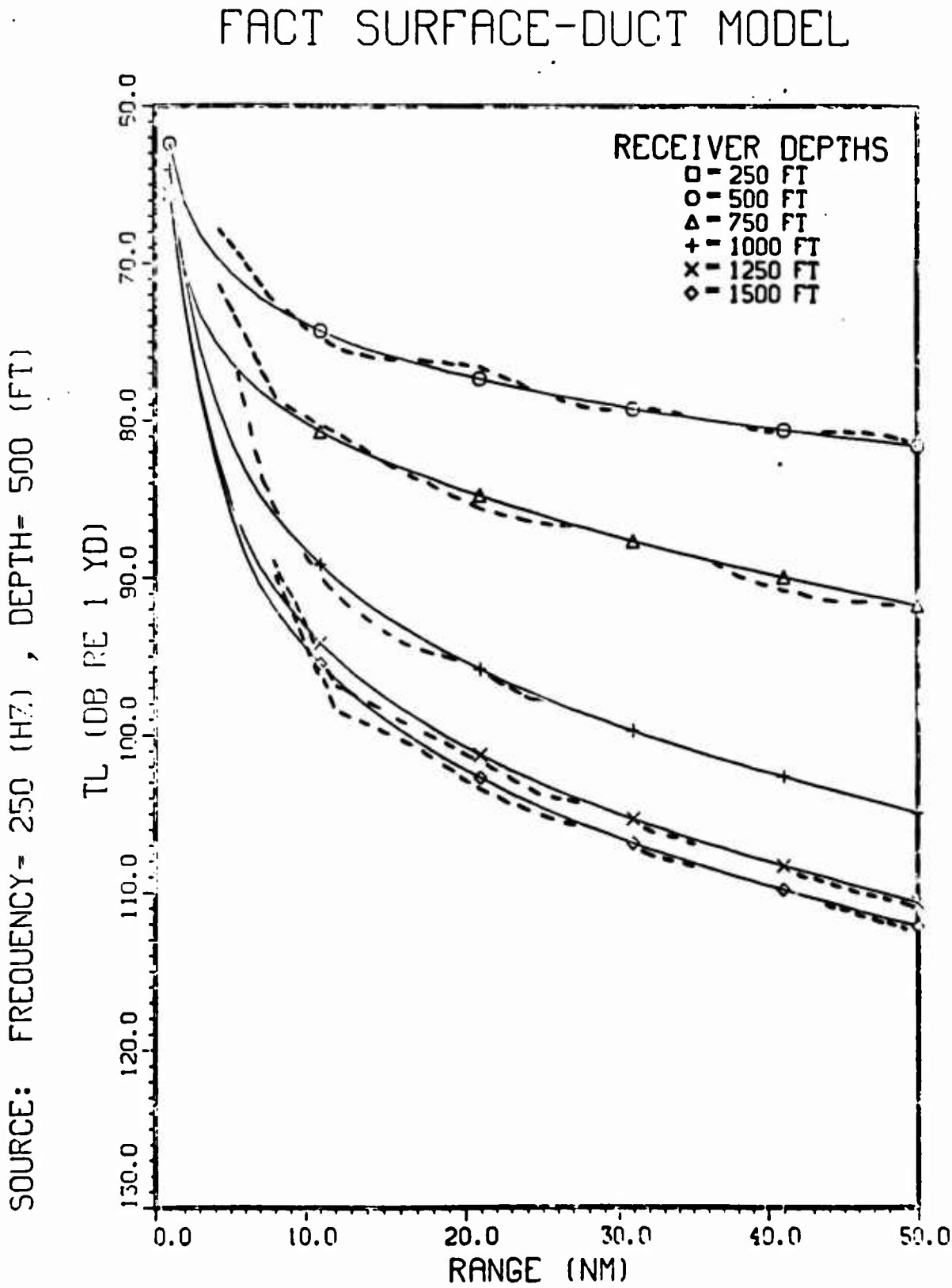


Figure 3-20. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 20 20.10.80 WED : OCT, 1980 , SNI DISSPLA VER 8.2

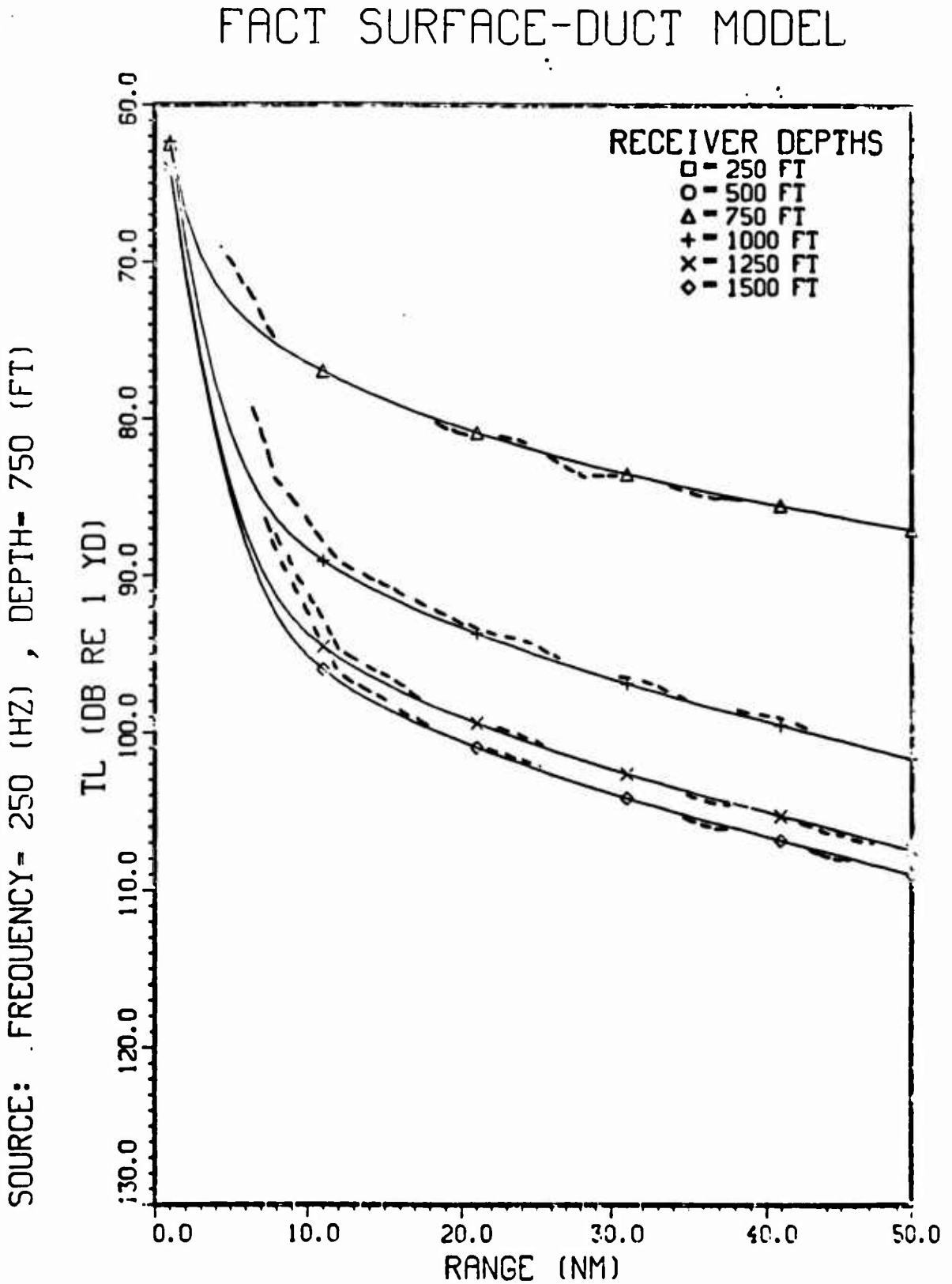


Figure 3-21. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 21 30.10.00 WED 1 OCT, 1980 , SAI DISSPLA VER 8.2

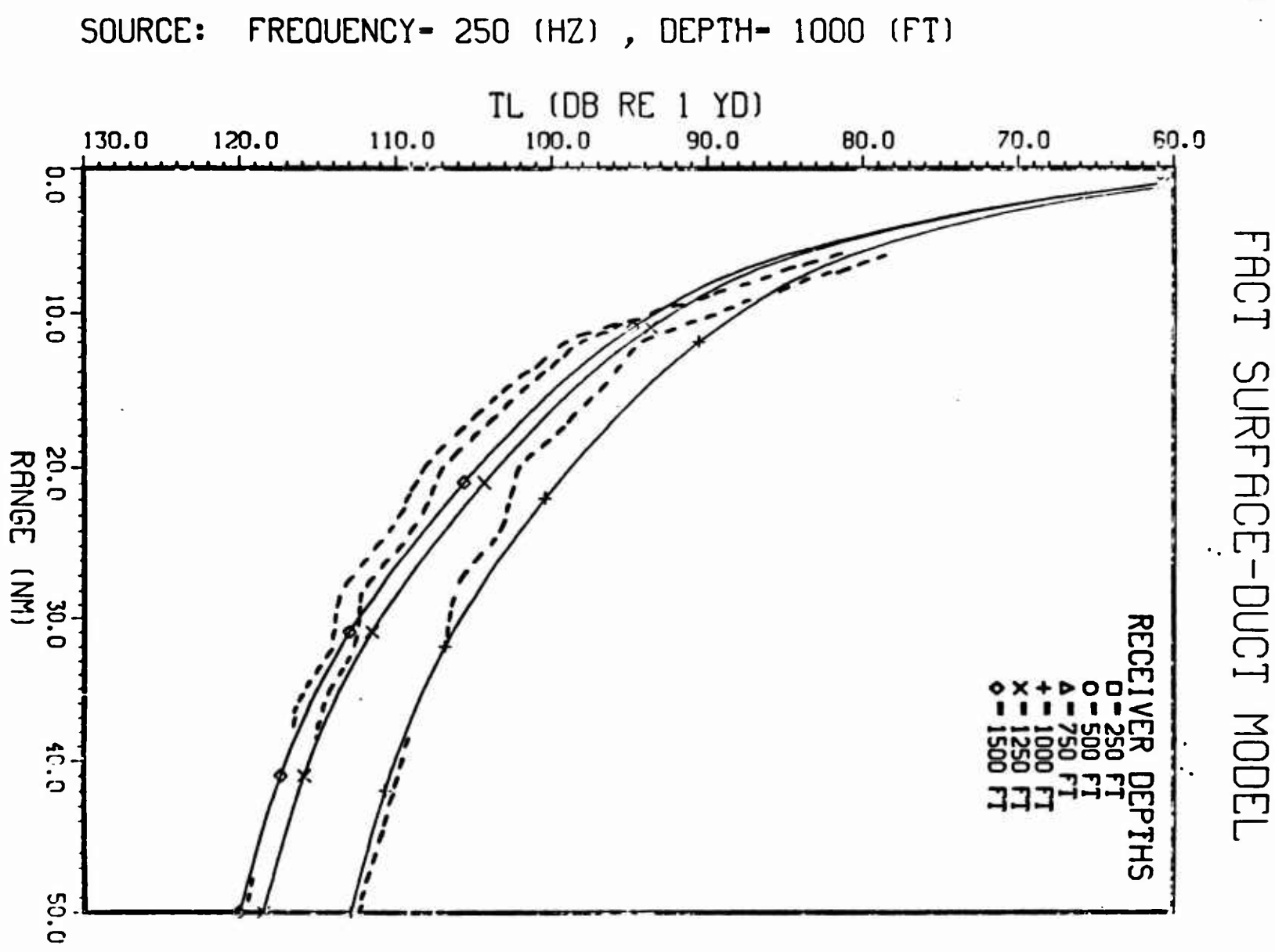


Figure 3-22 . Comparison of Surface-Duct Model (solid) with PE (dash).

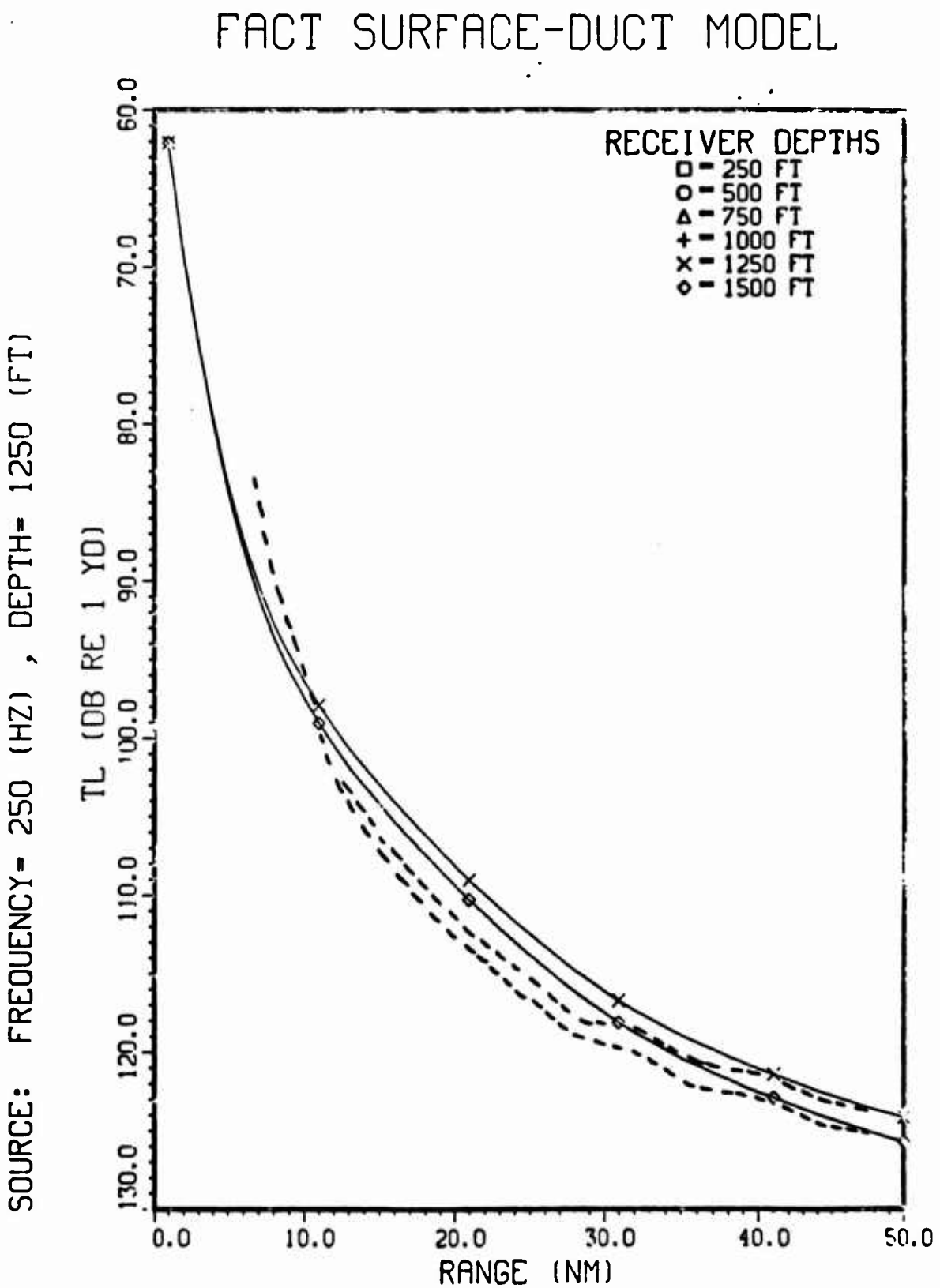


Figure 3-23. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 23 10.10.00 WED 1 OCT, 1980 , SAI DISSPLA VER 8.2

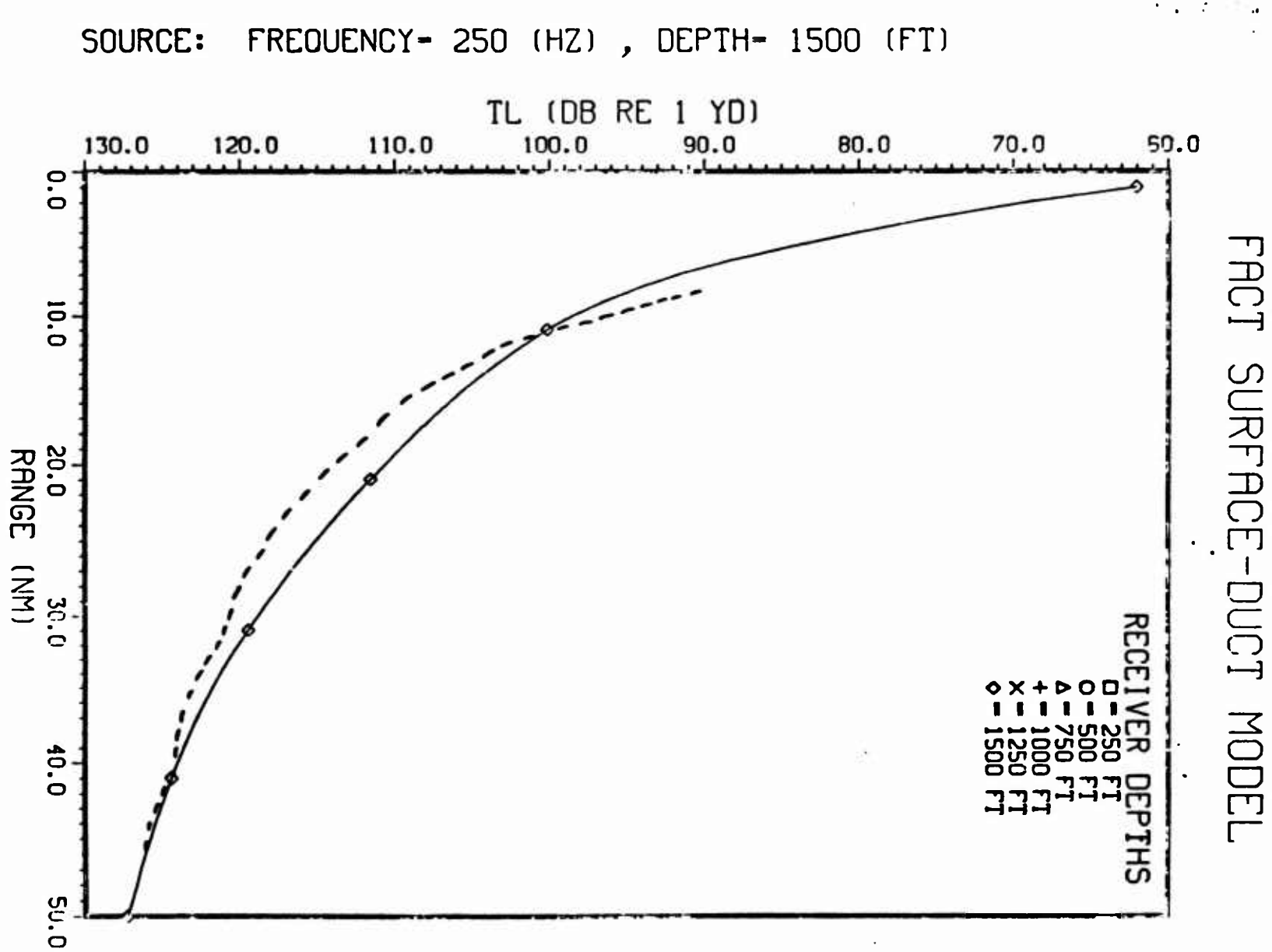


Figure 3-24 . Comparison of Surface-Duct Model (solid) with PE (dash).

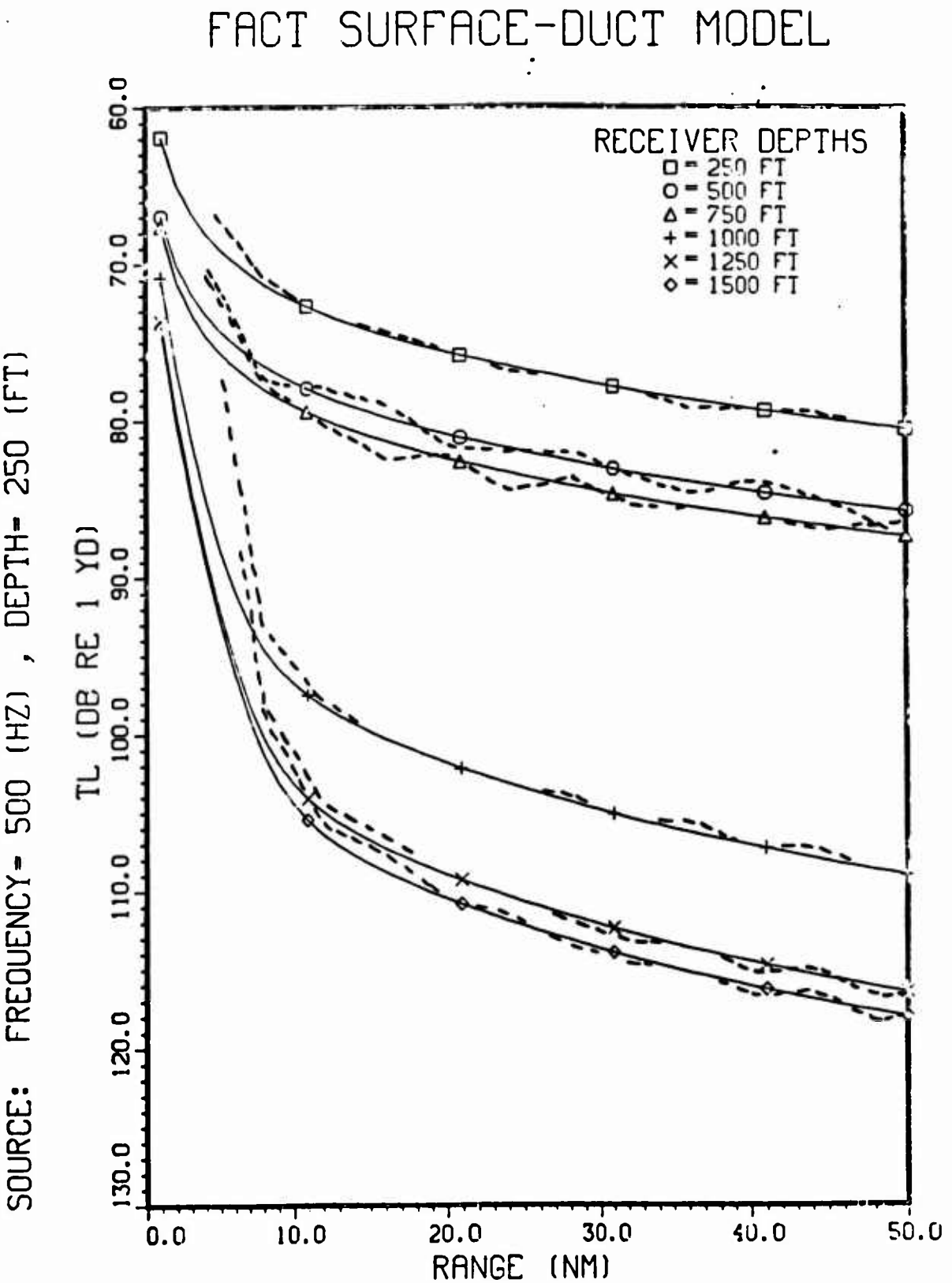


Figure 3-25. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 25 20.10.00 WED 1 OCT, 1980 , SAI DISSPLA VER 8.2

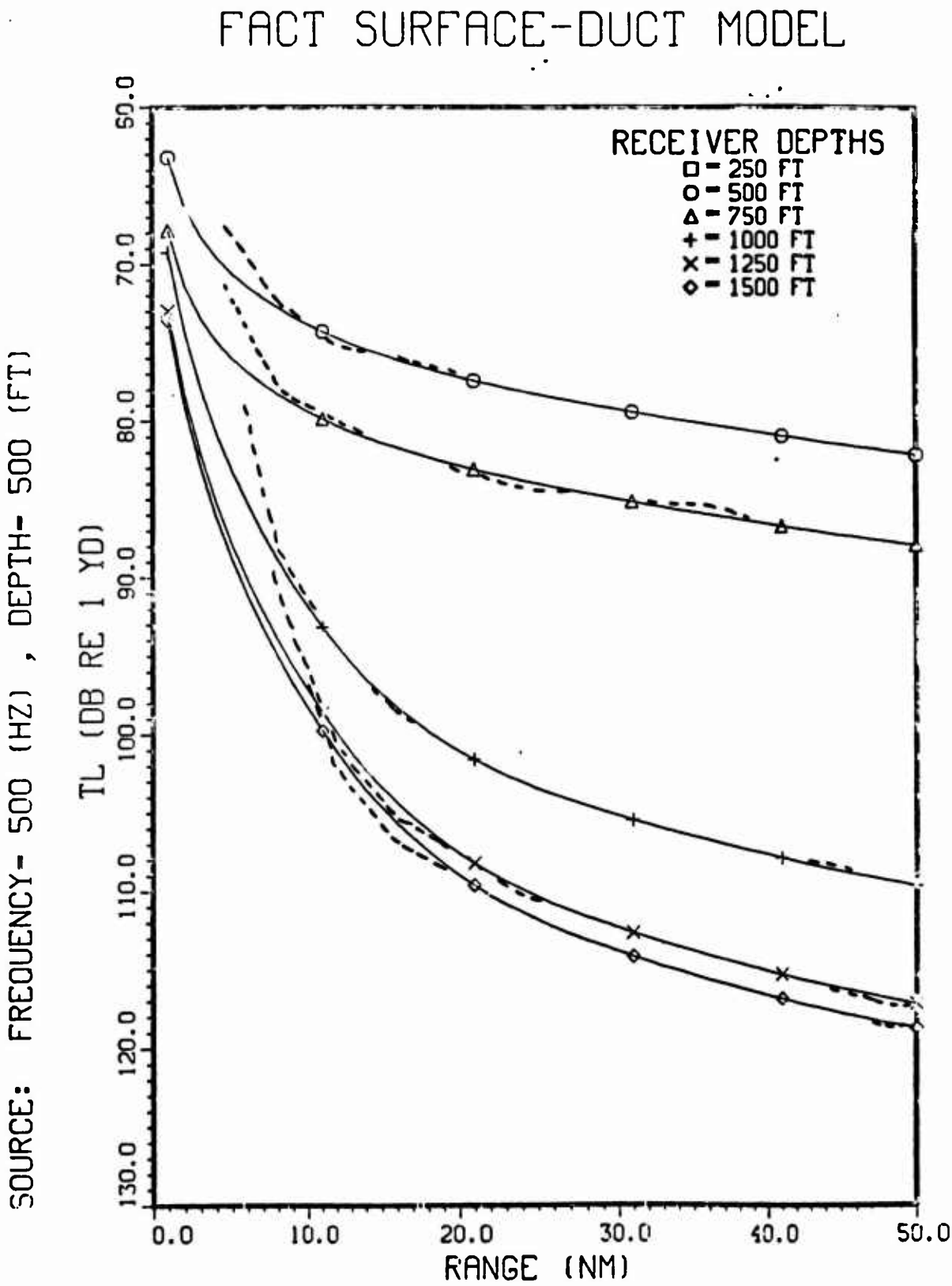


Figure 3-26. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 26 20.10.00 WED 1 OCT, 1980 , SAI DISSPLA VER 8.2

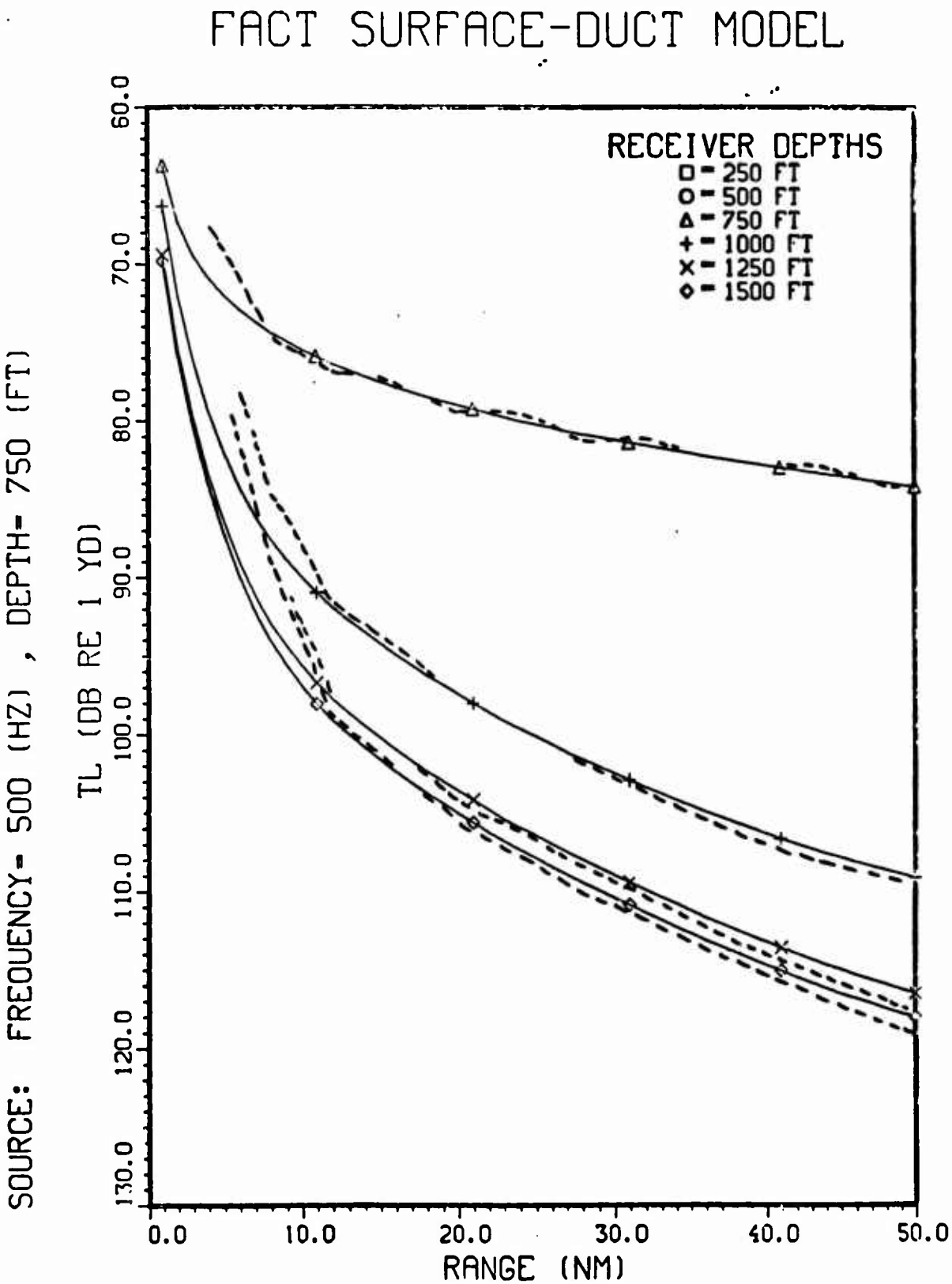


Figure 3-27. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 27 20.10.00 WED 1 OCT, 1980 , SRI DISSPLA VER 8.2

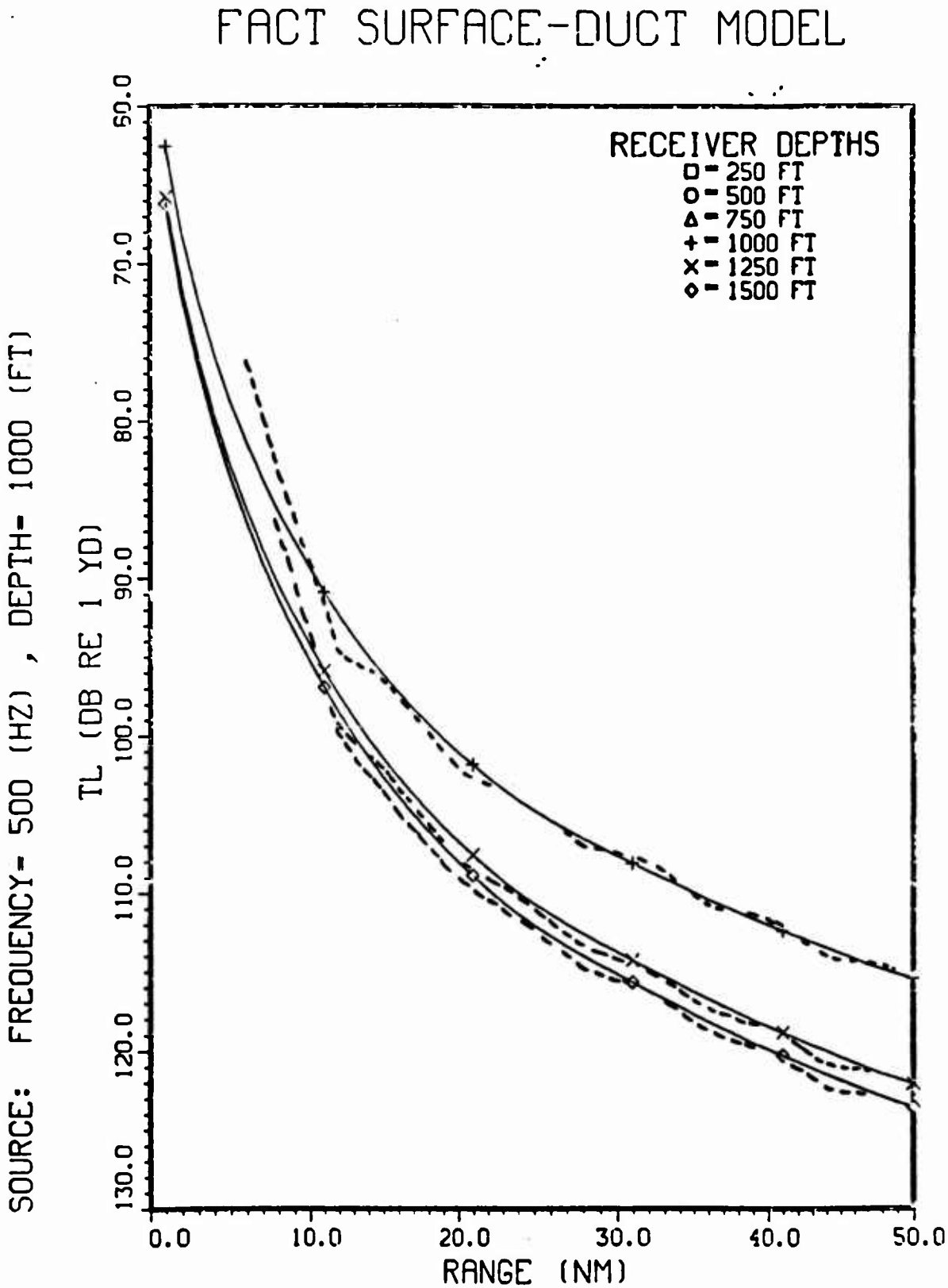


Figure 3-28. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 20 20.10.00 WED 1 OCT, 1980 , SAI DISSPLA VER 8.2

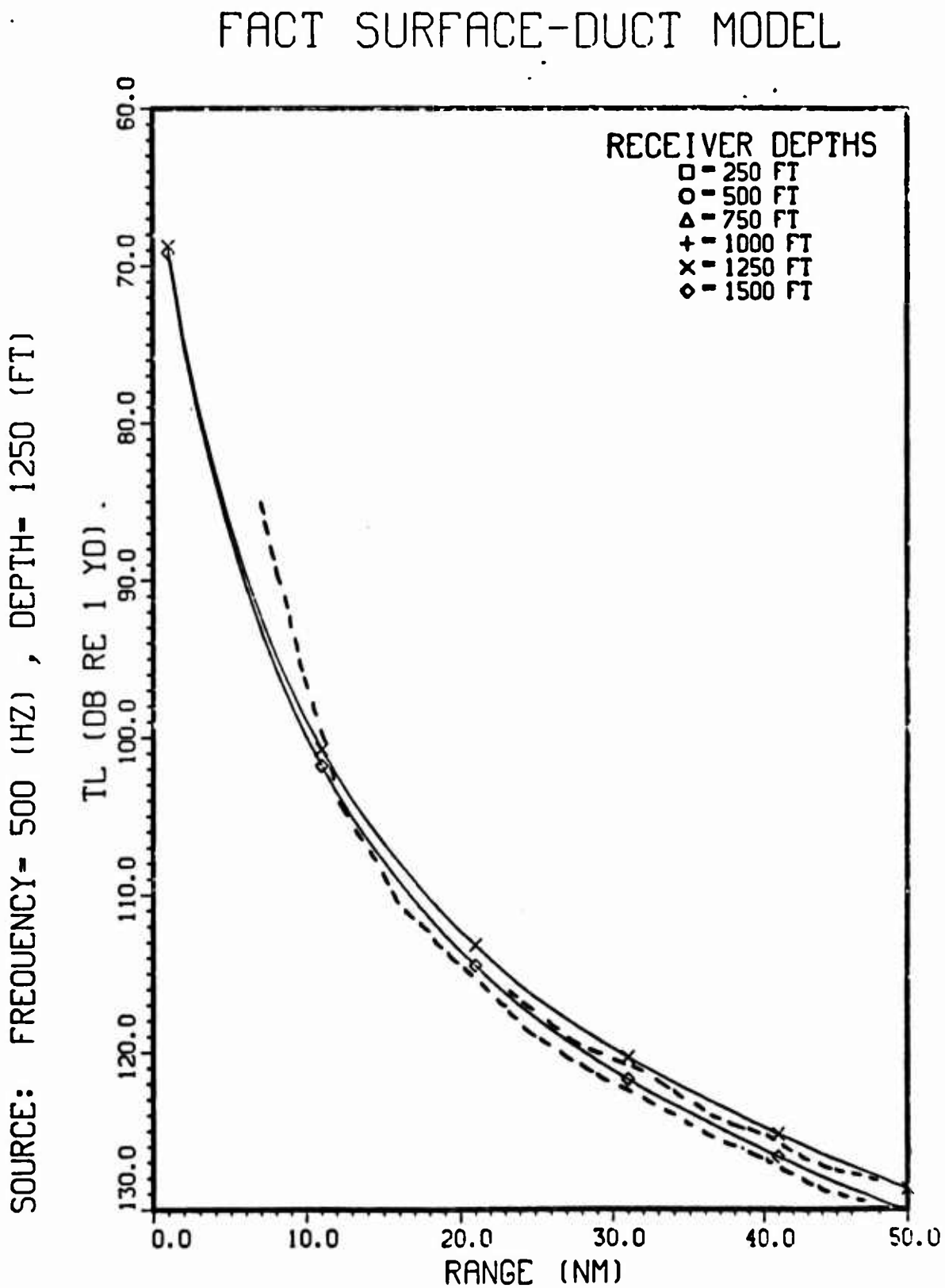
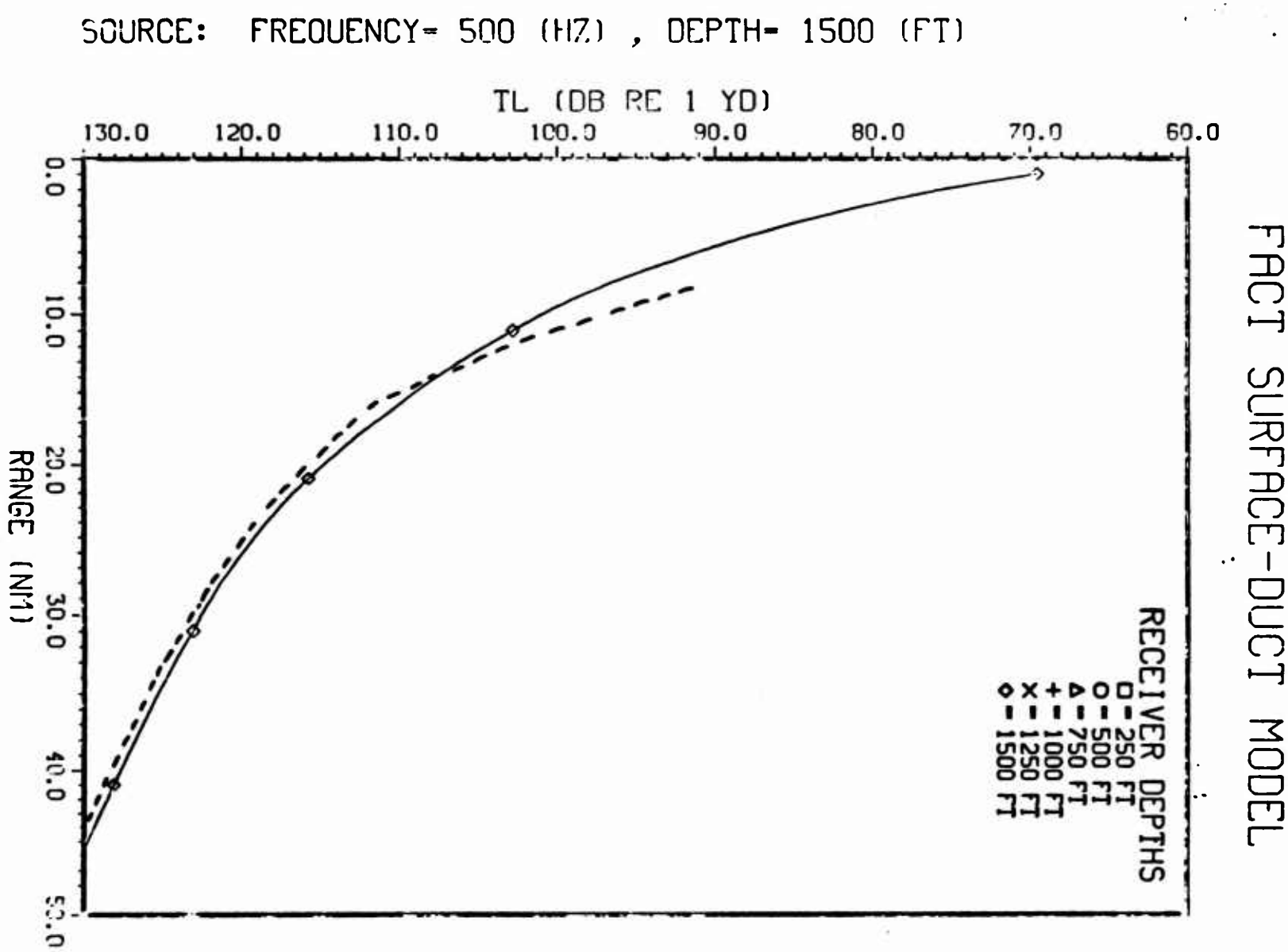


Figure 3-29. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 29 20.11.00 WED 1 OCT, 1980 , SRI DISSPLA VER 8.2



FACT SURFACE-DUCT MODEL

Figure 3-30 . Comparison of Surface-Duct Model (solid) with PE (dash).

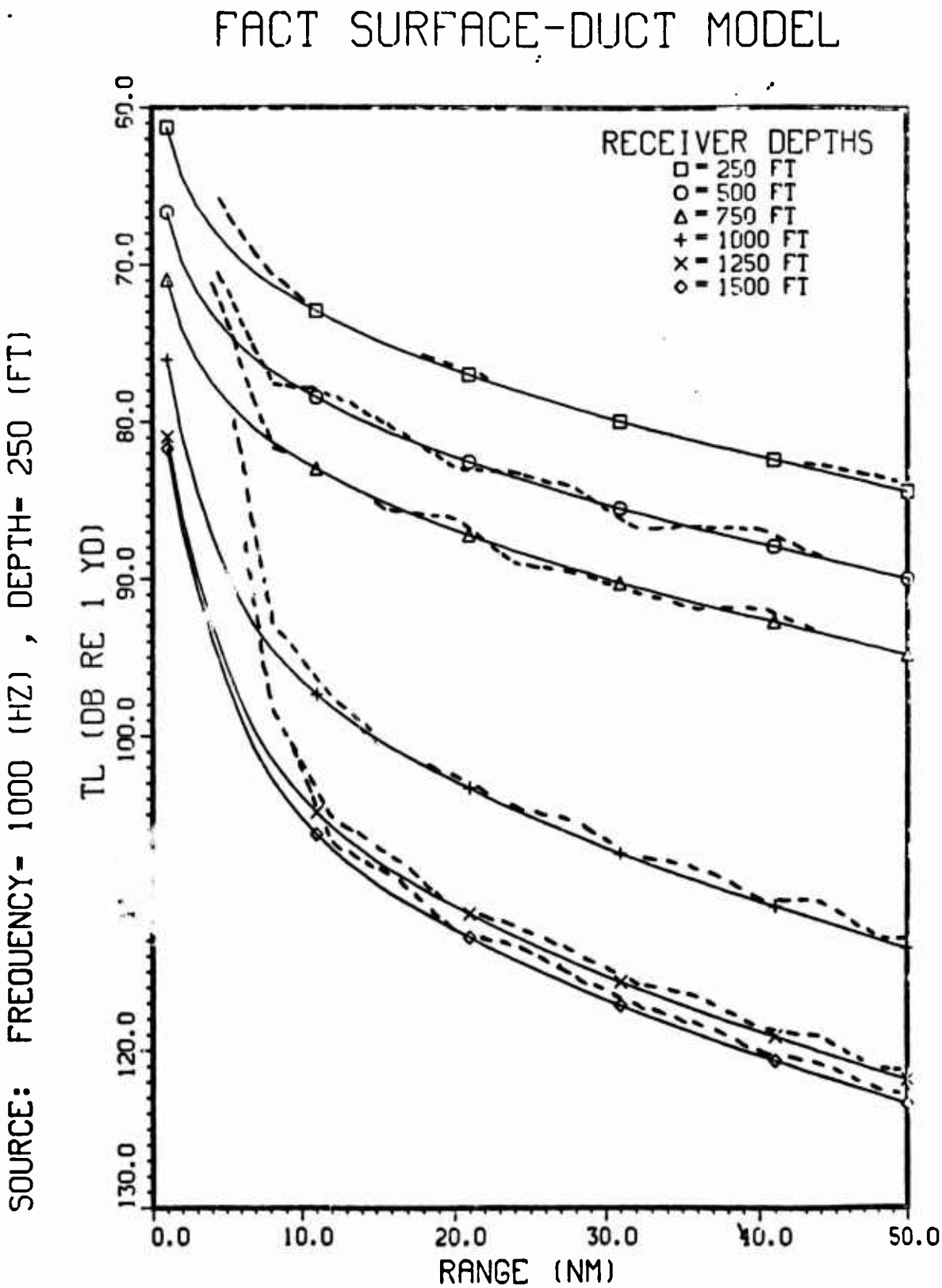


Figure 3-31 . Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 31 10.11.00 WED 1 OCT, 1980 , SRI DISSPLA VER 8.2

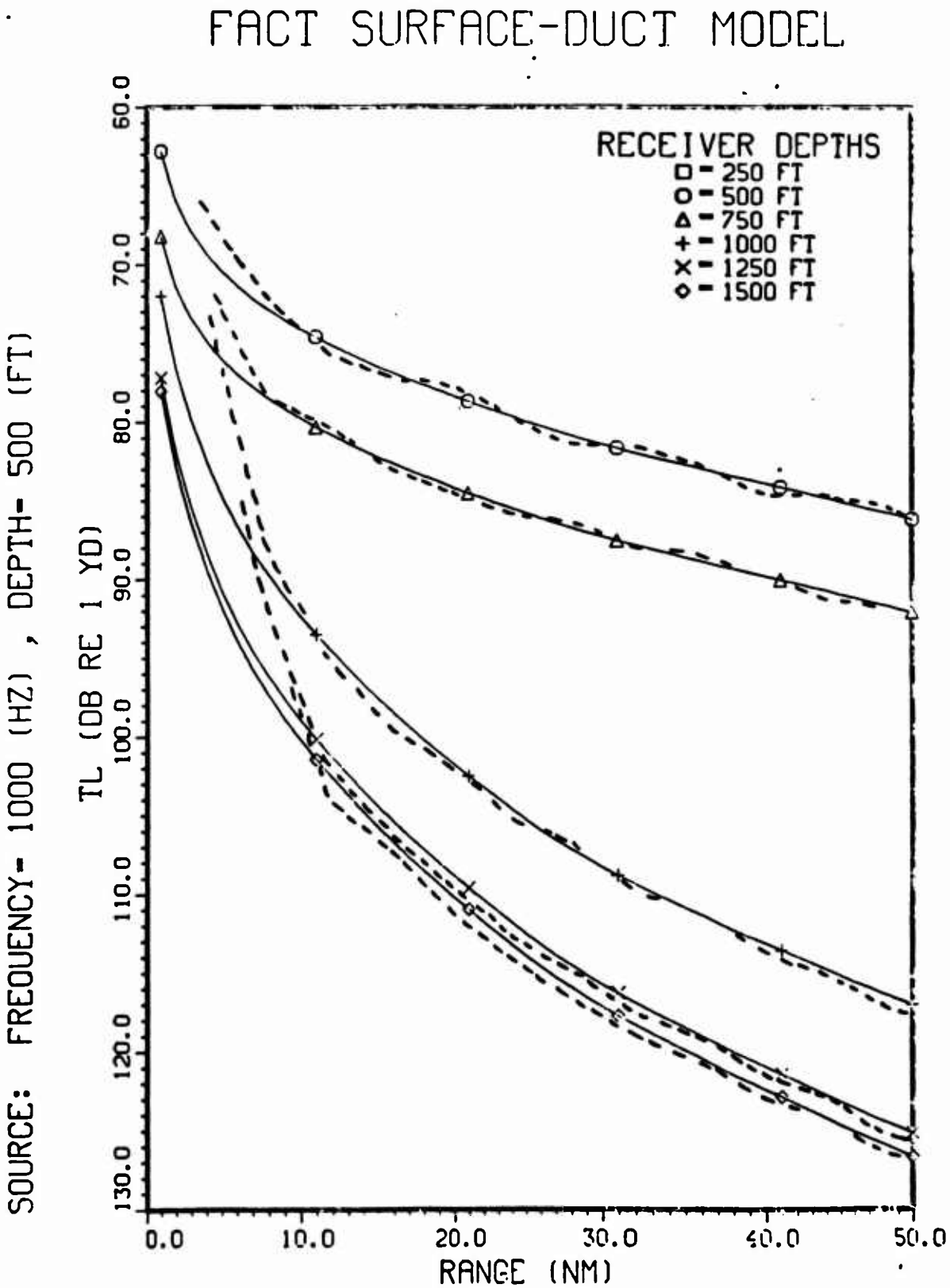


Figure 3-32. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 32 20.11.00 WED 1 OCT, 1980 , SRI DISSPLA VER 8.2

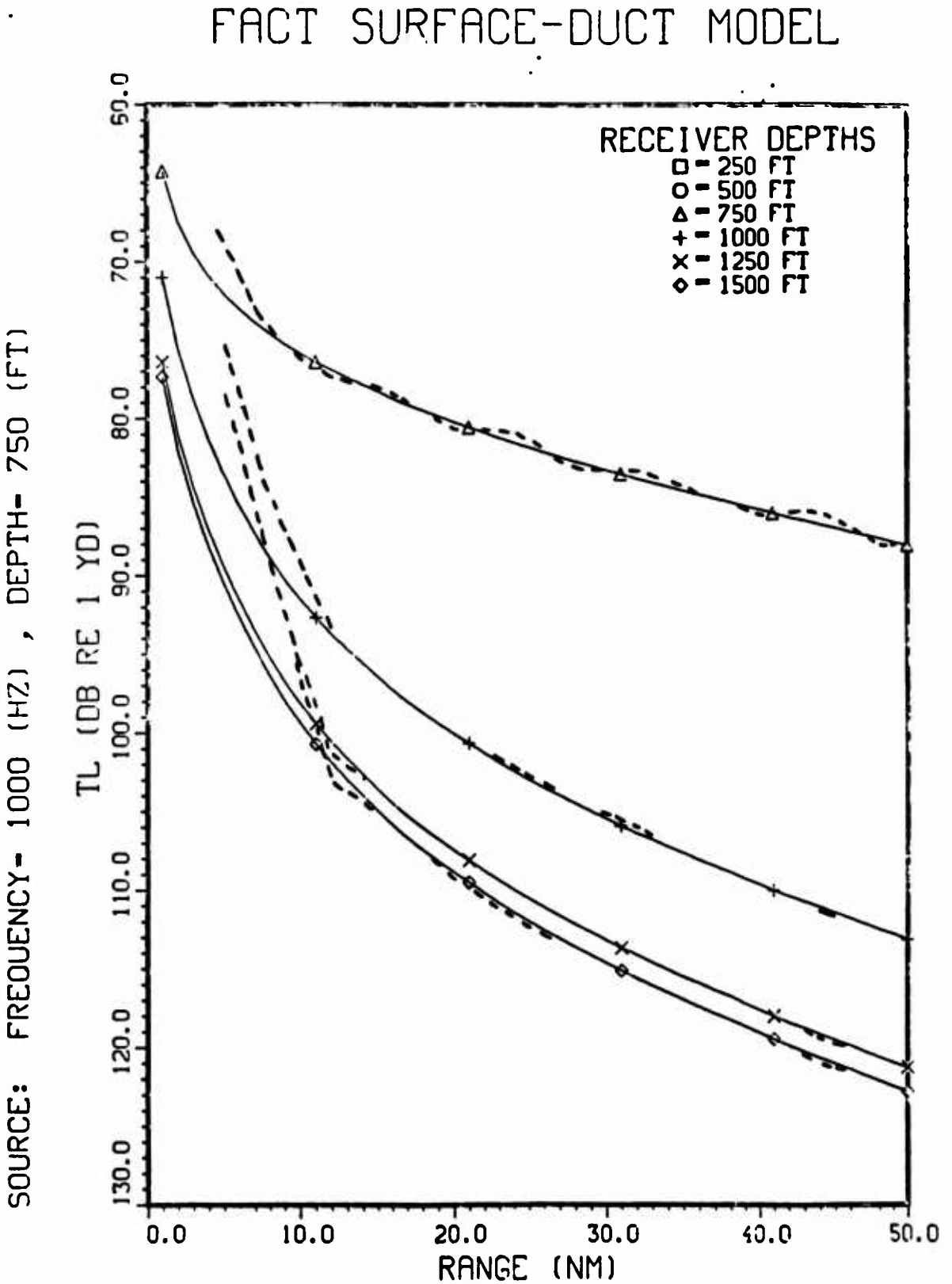


Figure 3-33. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 33 20.11.00 WED 1 OCT, 1980 , SRI DISSPLA VER 8.2

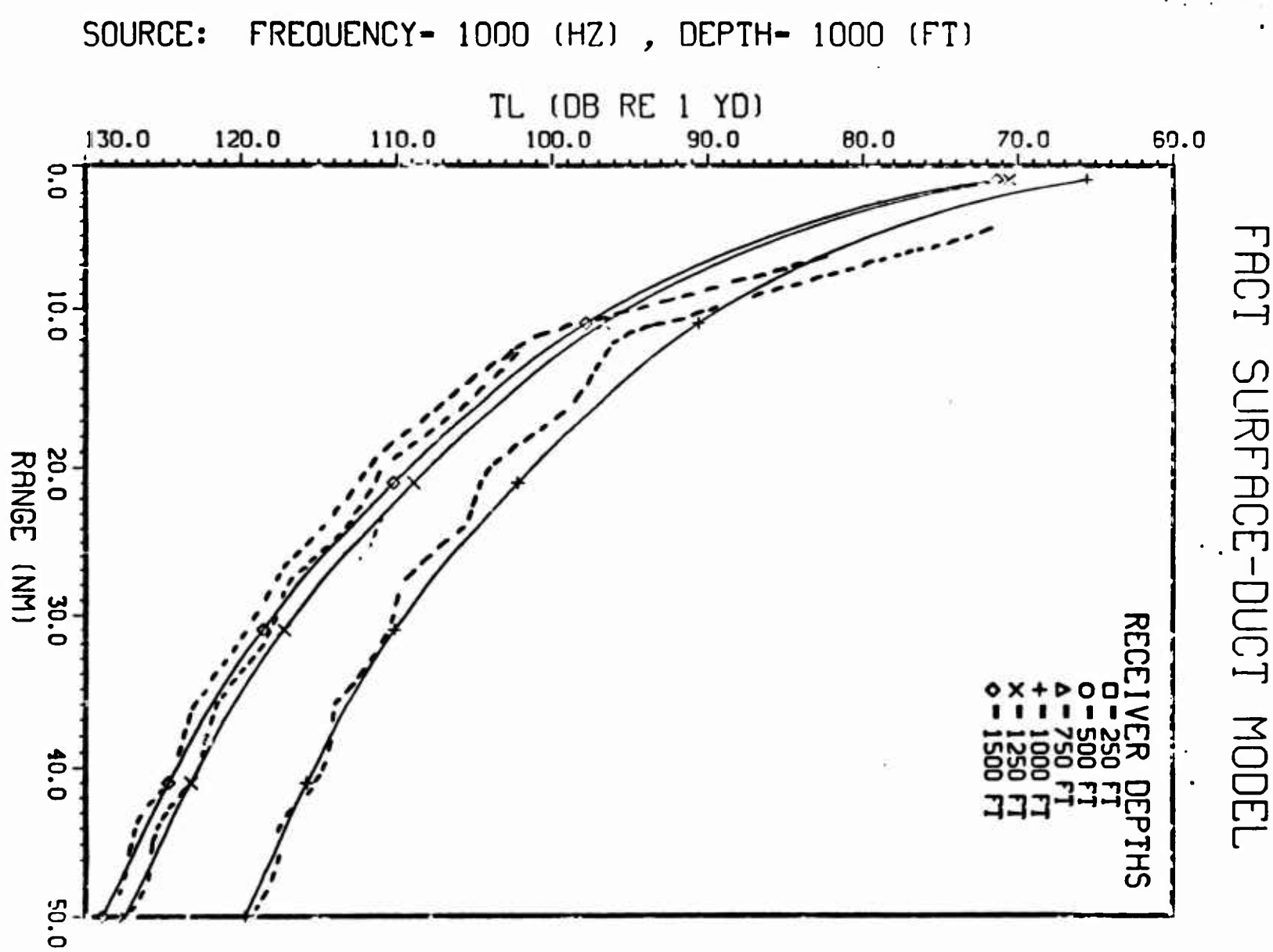


Figure 3-34 . Comparison of Surface-Duct Model (solid) with PE (dash).

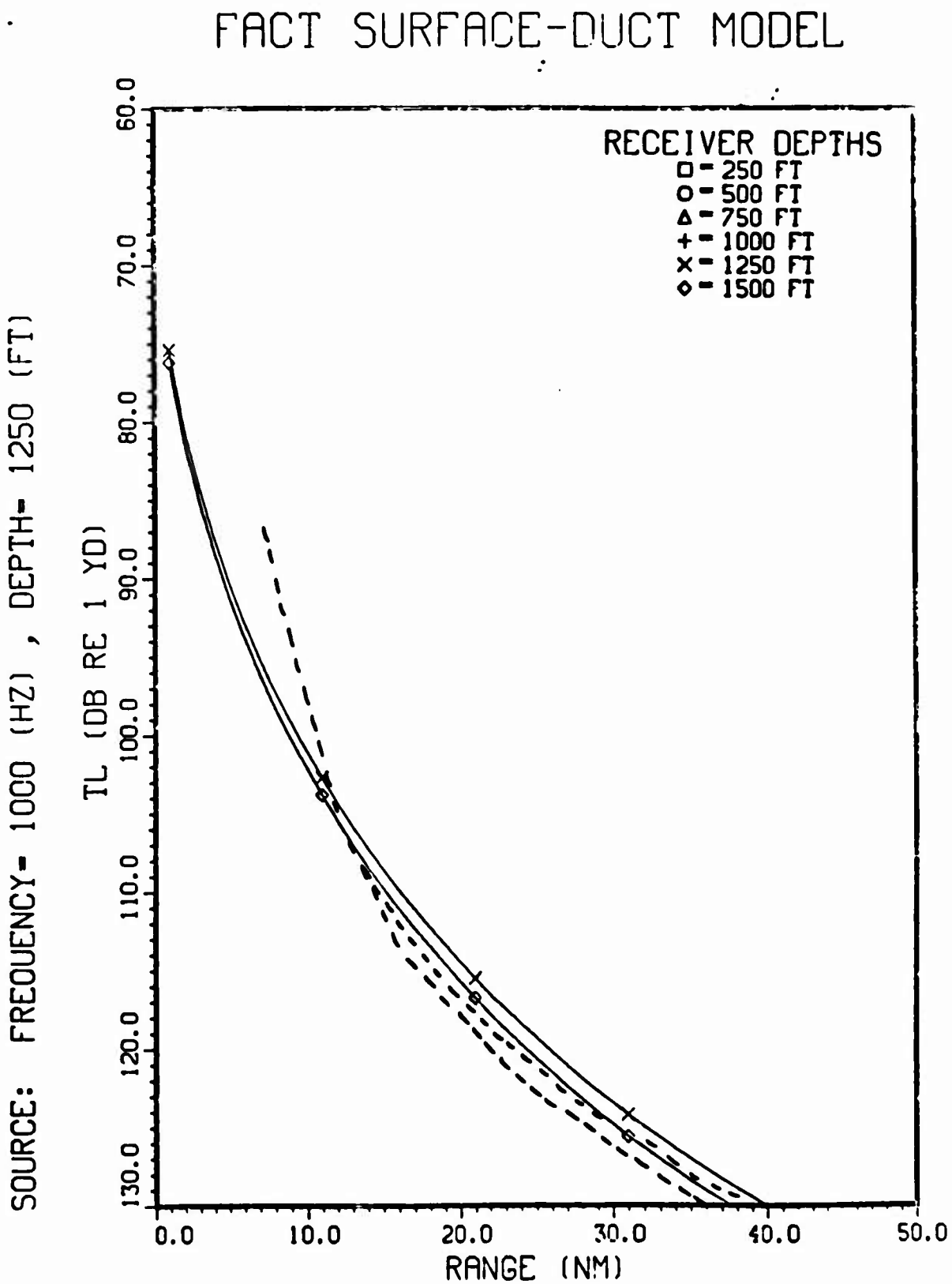


Figure 3-35. Comparison of Surface-Duct Model (solid) with PE (dash).

PLOT 35 20.11.00 WED 1 OCT, 1990 , SRI DISSPLA VER 8.2

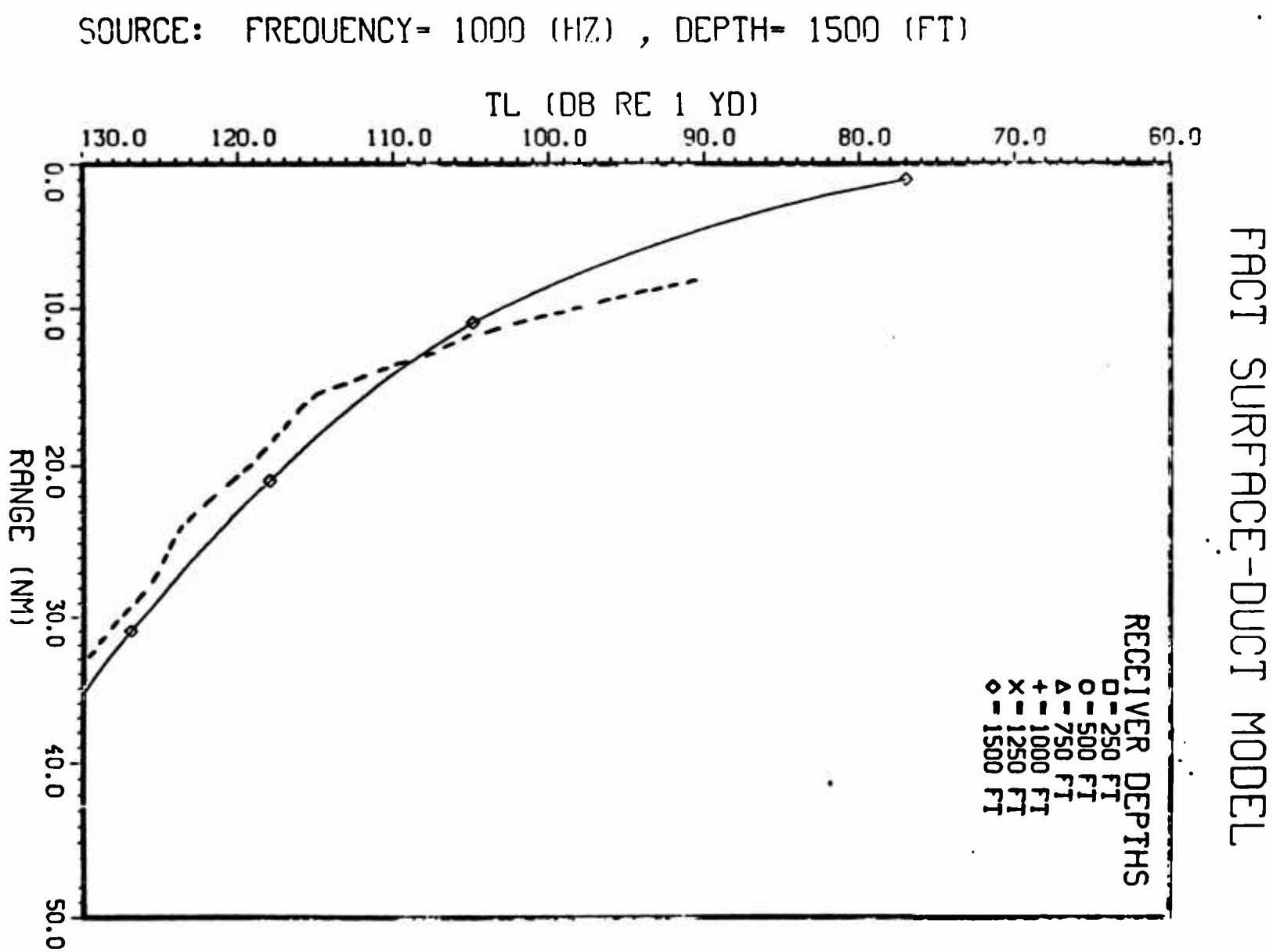


Figure 3-36 . Comparison of Surface-Duct Model (solid) with PE (dash).

3.2 NORMAL MODE COMPARISONS

A more detailed evaluation of the virtual mode surface duct model was made by comparing the predicted modal longitudinal wavenumbers λ_m and intensity amplitudes ψ_m with corresponding values computed using the NOSC n-layer normal mode code. The NM code was run with the same bilinear profile used by the surface duct model and the ocean modeled as infinitely deep. Both models (surface duct and NM) truncated the modal spectrum, hence near field TL values may be in error due to an insufficient number of modes. The imaginary part of λ_m is related to the attenuation coefficient α_m by $\alpha_m = 2\ I_m(\lambda_m)$.

The λ_m computed by the virtual mode surface duct model agree quite well with the NM results: relative error in $|I_m(\lambda_m)| \lesssim 10^{-1}$ and $|Re(\lambda_m)| \lesssim 10^{-4}$. Agreement is better for the trapped modes and gets worse for the strongly leaky waves. However, the strongly leaky modes have large attenuation coefficients and contribute little to the intensity at ranges greater than a few nautical miles.

Intensity amplitudes ψ_m are close to the NM predictions with two exceptions. First, the true ψ_m may exhibit deep nulls which are clipped in the surface duct model. The second exception involves deeply trapped modes where ψ_m is computed within or below the barrier. In the NOSC NM code, numerical difficulties in computing Hankel functions lead to a modification of the bilinear profile. The virtual mode model uses a different formulation of the ψ's and avoids this problem. Thus computed intensity amplitudes are expected to be lower than those predicted by the NM code. This is in fact observed.

Detailed comparison of λ_m for the surface duct model predictions and the NM code results are provided for 25, 100, and 500 Hz. This corresponds to cases with zero, two, and eleven trapped modes, respectively. The λ_m are shown in Tables (3-1) with values in parenthesis being the NM results.

Table 3-1. Comparison of λ_m (NM results in parenthesis).

Mode	$Re(\lambda_m)$	$I_m(\lambda_m)$
	** 25 Hz **	
1	3.126790 E-2 (3.126250)	3.492409 E-5 (3.778848)
2	3.087473 E-2 (3.097519)	2.877695 E-4 (2.453416)
3	3.015320 E-2 (3.035346)	6.646130 E-4 (6.215050)
	** 100 Hz **	
1	.1254460 (.1254452)	3.310623 E-8 (3.177127)
2	.1252816 (.1252807)	4.514341 E-6 (4.761233)
3	.1251256 (.1251152)	3.782473 E-5 (3.949632)
4	.1248763 (.1248825)	1.075746 E-4 (1.041099)
5	.1245431 (.1245658)	1.917165 E-4 (1.856100)
	** 500 Hz **	
1	.6279464 (.6279451)	5.030603 E-33 (0.)
2	.6276678 (.6276654)	3.230407 E-27 (0.)
3	.6274396 (.6274364)	8.766610 E-23 (0.)
4	.6272378 (.6272339)	3.685180 E-16 (0.)
5	.6270533 (.6270487)	4.186859 E-14 (3.466829)
6	.6268813 (.6268761)	1.684570 E-13 (1.430650)
7	.6267190 (.6267133)	2.801780 E-11 (2.433916)
8	.6265645 (.6265582)	2.104337 E-9 (1.868979)
9	.6264166 (.6264097)	7.414939 E-8 (6.730201)
10	.6262741 (.6262668)	1.181578 E-6 (1.103277)
11	.6261333 (.6261259)	7.110299 E-6 (7.500457)
12	.6259948 (.6259753)	2.183201 E-5 (2.295352)
13	.6258225 (.6258053)	4.323009 E-5 (4.329292)
14	.6256280 (N/A)	6.543505 E-5 (N/A)

3.3 EVALUATION SUMMARY

The surface-duct model transmission-loss (TL) results were found to be in good agreement with PE and NM computations for a bilinear test case. The virtual mode model predicted the proper frequency dependence of the TL and exhibited excellent performance in predicting the depth dependence of the TL for within-, cross-, and below-layer propagation geometries. Comparison with the NM results shows good agreement for mode attenuation coefficients, giving the proper range decay of intensity.

The execution time of the surface duct model for the 126 scenarios was about 12 CPU seconds on a Digital DEC-10, including loading and I/O. For comparison, a CDC CYBER-76/175, or UNIVAC 1110 are approximately eight (8) times faster than the DEC-10, leading to projected execution times of ~ 2 seconds - well within the time constraint of the FACT model.

Deviations between model computed TL's and PE results are small (2 - 3 dB max) and generally occur at short ranges. These ranges are within the direct-path regime and can be adequately modeled using ray theory as is presently being done in FACT. Systematic deviations, not related to the surface duct model, are probably present in the PE data arising from the smoother and/or "ghost" bottom reflections. In any event, larger variations in the computed TL are more apt to arise from use of the bilinear approximation to the environmental index of refraction.

SECTION 4
SURFACE DUCT PROGRAM

The virtual mode surface duct model described earlier has been implemented into a FORTRAN program suitable for inclusion into the FACT model. The surface duct program is written in ANSI standard FORTRAN-77, compatible with compilers on CDC, UNIVAC and DEC computers. The surface duct model is designed to operate as a self-contained module needing only environmental specifications and desired propagation scenarios. The virtual mode calculations are automatic and require no initial conditions from the user.

The surface duct code is configured as a main control sub-program, SDUCT, and six ancillary subroutines: TRAPED, LEAKY, SMODE, AIRY, H1SQ, and REFLKT. To facilitate use as a stand-alone module, a separate driver program DRIVER is supplied which automatically calls SDUCT. A FORTRAN source listing of the surface duct model and the stand-alone driver are provided in Appendix B.

User inputs to DRIVER are provided via free-format NAMELIST inputs. The surface duct model is configured to handle up to six frequencies and six distinct source/receiver depths for a total of 6 x 21 = 126 distinct transmission-loss scenarios. The surface duct program is capable of using either metric or english units and automatically adjusts outputs accordingly. Transmission-loss (TL) is computed as a function of range, and optional debug flags allow individual modal values of attenuation coefficient α_m and intensity amplitude ψ_m to be printed. A sample execution, corresponding to the deep duct test case used in Section 3, is shown in Appendix C.

SECTION 5

CONCLUSIONS

The virtual mode surface duct model described in this report is proposed as a replacement to the current FACT surface duct model. The virtual mode model was compared with parabolic equations and normal mode calculations for a bilinear duct over a wide range of propagation scenarios and found to be in good agreement with each. The virtual mode model exhibited the proper frequency and depth dependence displayed by the PE code results. Model execution times are well within FACT constraints and no user interaction is required other than environmental inputs and specification of frequency/depth scenarios.

The major limitation of the surface duct model is in approximating the environmental refractive index profile by the Furry model, and future efforts should be directed toward removing this limitation. Additional work in the area of surface loss mechanisms is also warranted.

APPENDIX A

EVALUATION OF T_+T_-

For real arguments, the Airy functions are real valued hence the expressions for T_+ and T_- [Eqs. (2.5)] are complex conjugates of one another and

$$T_+(E)T_-(E) = |T(E)|^2 = \pi|\psi'(o,E)h_1(E) - \psi(o,E)h_1'(E)|^2 \quad .$$

Now $\psi(o,E)$ and $\psi'(o,E)$ are found from Eq. (2.4a) to be

$$\psi(o,E) = \frac{\pi}{s}\left[Ai(-w)Bi(-q_o) - Ai(-q_o)Bi(-w)\right] \quad ,$$

and

$$\psi'(o,E) = \frac{d\psi(o,E)}{dx} = \pi\left[Ai'(-w)Bi(-q_o) - Ai(-q_o)Bi'(-w)\right] \quad .$$

Since w and q_o are positive for leaky modes, the Airy functions and their derivatives can be expressed in terms of modulus and phase as

$$Ai(-w) = M(w)\cos\theta(w) \quad , \quad Bi(-w) = M(w)\sin\theta(w) \quad ,$$

and

$$Ai'(-w) = N(w)\cos\phi(w) \quad , \quad Bi'(-w) = N(w)\sin\phi(w) \quad ,$$

with similar expressions for $Ai(-q_o)$, $Bi(-q_o)$. Thus

$$\psi(o,E) = \frac{\pi}{s} M(w)M(q_o)\sin\left[\theta(q_o) - \theta(w)\right] \quad ,$$

and

$$\psi'(o,E) = \pi\, M(q_o)N(w)\sin\left[\theta(q_o) - \phi(w)\right] \quad .$$

The function $h_1(E)$ and its derivative $h_1'(E)$ are

$$h_1(E) = Ai(-E) - i\,Bi(-E) = M(E)e^{-i\theta(E)}$$

and

$$h_1'(E) = -[Ai'(-E) - i\,Bi'(-E)] = -N(E)e^{-i\phi(E)} \quad .$$

Thus the expression for $|T(E)|^2$ becomes

$$|T(E)|^2 = \frac{\pi^3}{s^2} M^2(q_0)\Big|sN(w)\sin\Big[\theta(q_0) - \phi(w)\Big]M(E)e^{-i\theta(E)}$$

$$+ M(w)\sin\Big[\theta(q_0) - \theta(w)\Big]N(E)e^{-i\phi(E)}\Big|^2$$

with the definitions

$$\beta = N(E)M(w)/sM(E)N(w) \quad ,$$

$$\Delta = \phi(E) - \theta(E) \quad ,$$

$$\delta = \phi(w) - \theta(w) \quad ,$$

$$\chi = 2\theta(q_0) - \phi(w) - \theta(w) \quad ,$$

and

$$\chi_\pm = (\chi \pm \delta)/2 \quad ,$$

$$|T(E)|^2 = \pi^3 M^2(q_0)M^2(E)N^2(w)G^2(E)$$

where

$$G^2(E) = \beta^2\sin^2(\chi_+) + \sin^2\chi_- + 2\beta\,\sin\chi_+\sin\chi_-\cos\Delta \quad .$$

APPENDIX B

SURFACE-DUCT PROGRAM LISTING

The following is a FORTRAN listing of the virtual mode surface program and stand-alone driver.

```
      INTEGER ERROR,PERMUT(21),ISHEAD(21),IRHEAD(21)
C
C*****************************************************************
C
C                    STAND-ALONE DRIVER FOR SURFACE DUCT MODEL
C
C
C        AUTHOR:     F.J. RYAN
C                    OCEAN ACOUSTICS DIVISION
C                    SCIENCE APPLICATIONS, INC.
C                    LA JOLLA, CA.
C
C        PURPOSE:    PROVIDES STAND ALONE CAPABILITY FOR VIRTUAL MODE
C                    SURFACE DUCT MODEL BY CALLING CONTROL SUBROUTINE -SDUCT-
C
C
C
C*****************************************************************
C
      LOGICAL METRIC,PLOT,DUMP
C.......NOTE: THE DRIVER IS PRESENTLY CONFIGURED FOR 6 FREQUENCIES/DEPTHS
C               AND 50 RANGE STEPS. CHANGE DIMENSIONS OF FOLLOWING ARRAYS
C               IF OTHER VALUES DESIRED.
      DIMENSION FREQ(6),DEPTHS(6),TTL(21),RELINT(50,21)
      DIMENSION TO(3),R(50),TL(50,21)
      EQUIVALENCE (TTL,RELINT)
C
C.........ALL USER INPUTS TO DRIVER ARE THRU /FUPRY/ NAMELIST
C          MULTIPLE NAMELISTS ARE FEASIBLE IN A SINGLE RUN
      NAMELIST/FUPRY/CONVRT,C0,CD,Z0,ZD,WAVEHT,NFREQ,ND,NRNGS,NP,
     1            METRIC,TO,FREQ,DEPTHS,PERMUT,PLOT,DUMP
C
C*****************************************************************
C
C                          INPUT PARAMETERS
C
C        CONVRT.........CONVERSION FACTOR (DB'S) SUBTRACTED FROM MODEL
C                       TL VALUES TO GET FINAL TL.
C                       20.5 TO CONVERT FROM DB RE 1 FT TO DB RE 1 YD.
C        C0.............SURFACE SOUND SPEED
C                       IF C0 LE 0., INDICATES END OF NAMELIST DATA.
C                       USED AS UNITS FLAG:
C                            C0 < 2000....METRIC UNITS
C                            C0 > 4500....ENGLISH UNITS
C
C        CD.............SOUND SPEED AT DUCT BOTTOM
C        Z0.............COUPLING DEPTH OF C0
C        ZD.............DEPTH OF DUCT BOTTOM  Z0 < ZD
C        WAVEHT.........SURFACE WAVE HEIGHT
C                       (IF ZERO, NO SURFACE LOSSES COMPUTED)
C        NFREQ..........NUMBER OF SOURCE FREQUENCIES (MAX 6)
C        ND.............NUMBER OF DISTINCT SOURCE/RECEIVER DEPTHS (MAX 6)
C        NRNGS..........NUMBER OF RANGE POINTS FOR TL CALCULATION (MAX 50)
C        NP.............NUMBER OF DISTINCT SOURCE-RECEIVER PATHS FOR
C                       WHICH TL CALCULATIONS ARE TO BE MADE.
C                       IF NP=0, ALL POSSIBLE PATHS CONSISTENT WITH ND ARE
```

```
C                          FORMED.
C        RNGINC..........RANGE INCREMENT FOR TL CALCULATIONS.
C        IO(3)...........I/O ARRAY CONTAINING UNIT NUMBERS FOR OUTPUT
C                          IO(1)=...UNIT FOR MINIMAL PRINT-OUT
C                          IO(2)=...UNIT FOR DETAILED PRINT-OUT
C                          IO(3)=...BINARY UNIT FOR INTENSITY OUTPUT
C        FREQ(6)........SOURCE FREQUENCIES (HZ)
C        DEPTHS(6)......DISTINCT SOURCE/RECEIVER DEPTHS
C        PERMUT(21).....ARRAY CONTAINING THE NP DISTINCT SOURCE/RECEIVER
C                          PAIRS FOR WHICH TL CALCULATIONS ARE TO BE MADE.
C                          FORMAT:
C                                PERMUT(I) = ISIR , WHERE ISIR IS A TWO DIGIT
C                          NUMBER WITH IS,IR BETWEEN 1 AND ND. MAXIMUM SIZE
C                          IS NP= ND*(ND+1)/2
C
C                           NOTE: UNITS
C        METRIC
C                  DEPTHS IN METERS, SOUND SPEEDS IN M/SEC, RANGES IN KM.
C                  INTENSITY COMPUTED RELATIVE TO POINT SOURCE AT 1 METER.
C
C        ENGLISH
C                  DEPTHS IN FEET, SOUND SPEEDS IN FT/SEC, RANGES IN NM.
C                  INTENSITY COMPUTED RELATIVE TO A POINT SOURCE AT 1 FOOT.
C
C.......................................................................
      DATA C0,CD,Z0,ZD,RNGINC,NRNGS/5000.,5018.,1180.,1000.,1.,50/
      DATA NFREQ,FREQ/6,25.,50.,100.,250.,500.,1000./
      DATA IO,WAVEHT/6,0,0,0./,CONVRT/9.54/,PLOT/.FALSE./,DUMP/.FALSE./
      DATA ND,DEPTHS/6,250.,500.,750.,1000.,1250.,1500./
      FORMAT=0.
   10 READ(5,FORMAT)
      IF(C0.LE.0.) GO TO 1000
      IF(DUMP) OPEN(UNIT=9,DEVICE='DSK',MODE='BINARY',DISPOSE='SAVE')
      WRITE(6,FORMAT)
      IF(LE.GT.0) GO TO 20
C..........DETERMINE SOURCE/RECEIVER COMBINATIONS
      IJ=0
      DO 15 I=1,ND
      DO 15 J=I,ND
      IJ=IJ+1
      PERMUT(IJ)= 10*I+J
   15 CONTINUE
      NP=IJ
C..........FORM HEADER LINES FOR TL OUTPUT
   20 IF(C0.LT.2000.) METRIC=.TRUE.
      IF(C0.GT.4500.) METRIC=.FALSE.
      DO 30 I=1,NP
      IS=PERMUT(I)/10
      IR=PERMUT(I)- 10*IS
      ISHEAD(I)= DEPTHS(IS)+.5
      IRHEAD(I)= DEPTHS(IR)+.5
   30 CONTINUE
C.......LOOP ON FREQUENCIES
      DO 200 IF=1,NFREQ
C
      CALL SDUCT(C0,CD,Z0,ZD,WAVEHT,FREQ(IF),1,DEPTHS,ND,PERMUT,NP,
```

```
00113          1              NRNGS,NRNGS,RELINT,IO,EWROW)
00114            IF(DUMP) WRITE(IO(3)) NRNGS,NP,((RELINT(I,J),I=1,NRNGS),J=1,NP).
00115    C..........COMPUTE TL VALUES
00116            WAVLEN=0.
00117            PRINT 70,FREQ(IF),WAVFHT,CONVRT
00118         70 FORMAT(1H1,20X,'TRANSMISSION-LOSS....FREQ= ',F7.1,' WAVFHT,CONVRT=
00119           1 ',2E15.4)
00120            DO 100 I=1,NRNGS
00121            IF(MOD(I-1,50).NE.1) GO TO 100
00122            PRINT 40,(ISHAD(I),I=1,NP)
00123            IF(METRIC) PRINT 82
00124            IF(.NOT. METRIC) PRINT 83
00125         80 FORMAT(1H0,'75',A4,21I5)
00126         82 FORMAT(1H0,5X,8H( M))
00127         83 FORMAT(1H0,5X,8H(FT))
00128            PRINT 90,(IRMLAN(I),I=1,NP)
00129            IF(METRIC) PRINT 86
00130            IF(.NOT. METRIC) PRINT 86
00131         86 FORMAT(1H0,5X,8H( M)/1X,6HR (KM))
00132         88 FORMAT(1H0,5X,8H(FT)/1X,6HR (NM))
00133         90 FORMAT(1H,'ZP',8X,21I5)
00134        100 RANGE=(IR-1)*DELRNG
00135            DO 120 IP=1,NP
00136            NPTS=NPTS+1
00137            TL(IR,IP)= -10.*ALOG10(RELINT(IR,IP)+1.E-35)-CONVRT
00138            ITL(IP)=10.*TL(IP,IP)+.5
00139        120 CONTINUE
00140            PRINT 130,RANGE,(ITL(I),I=1,NP)
00141        130 FORMAT(F4.2,21I5,F6.1)
00142            PRINT 135,RANGE
00143        135 FORMAT(1H0,113X,F7.2)
00144        180 CONTINUE
00145        200 CONTINUE
00146            GO TO 10
00147       1000 CONTINUE
00148            STOP
00149            END
```

EQUIVALENCED VARIABLES

```
( TL    1          RELINT 1          )
```

SUBPROGRAMS CALLED

```
POD.     ALG10.   SDUCT
```

SCALARS AND ARRAYS (*** NO EXPLICIT DEFINITION - *** NOT REFERENCED)

```
*CO      2033     *IF      2034     RELINT 1      *IO      2035     TL       1
*IP      2036     *CONVRT  2037     *NPTS    2040 *IJ      2041     *WAVFHT  2042
 DUMP    2043      METRIC  2044     *NRNGS   2045 *J       2046     *CO      2047
 IO      2050      DEPTHS  2053      ISHAD   2061  .80007 2106      .80004 2107
 .80005  2110      .80004  2111      .80003  2112  PERMUT 2113      .80002 2140
```

```
 .S0001 2131    .S0001 2142    *NP     2143    ERROR  2144    *ZD     2145
*WAVEHT 2146    ISSCAD 2147    *ND     2174    %F             .90012 2175
*RANGE  2176    .S0011 2177    .S0010 2200    *IS     2201    *I      2202
 PLOT   2203    FWD    2204    ITL    2212    *RNGINC 2237    *IP     2240

TEMPORARIES

*FUNRY  2320    .00000 2321

MAIN.   [ NO ERRORS DETECTED ]
```

```
      SUBROUTINE SDUCT(CO,CD,ZO,ZD,WAVEHT,FREQ,NFREQ,DEPTHS,
     1                 ND,PERMUT,NP,RNGINC,NRNGS,RELINT,IO,ERROR)
C
C*********************************************************************
C
C     PURPOSE: COMPUTES THE INCOHERENT TL FIELD FOR A BI-LINEAR SURFACE
C              DUCT AS A FUNCTION OF RANGE/FREQUENCY AND SOURCE/RECEIVER
C              GEOMETRY.
C     METHOD:  THE VIRTUAL MODE THEORY OF LARIANCA IS USED FOR TRAPPED MODES
C              WITH INCLUSION OF TWO LEAKY MODES USING "SMODE" METHODS.
C
C     AUTHOR:  F.J. RYAN
C              OCEAN ACOUSTICS DIVISION
C              SCIENCE APPLICATIONS,INC.
C
C     ARGUMENTS:
C                CO........SURFACE SOUND SPEED
C                CD........SOUND SPEED AT DUCT BOTTOM  (CD>CO)
C                ZO........CONJUGATE DEPTH OF CO
C                ZD........DEPTH OF DUCT BOTTOM
C                DEPTHS....ARRAYCONTAININGNDISTINCT DEPTHS AT WHICH
C                DEPTHS....ARRAY CONTAINING DISTINCT DEPTHS AT WHICH
C                          MODAL INTENSITY AMPLITUDES ARE TO BE
C                          CALCULATED. MAX(6)
C                ND........NUMBER OF VALUES IN DEPTHS ARRAY
C                PERMUT....SOURCE/RECEIVER COMBINATIONS FOR WHICH
C                          TL COMPUTATIONS MADE. FORMAT IS
C                          PERMUT(I)=ISIR, WHERE IS AND IR ARE BETWEEN
C                          1 AND ND
C                NP........NUMBER OF ENTRIES IN PERMUT ARRAY MAX(15)
C                RELINT....RELATIVE INTENSITIES.
C                FREQ......FREQUENCY ARRAY
C                RNGINC....RANGE INCREMENT FOR TL CALCULATIONS
C               WAVEHT...SURFACE WAVE HEIGHTS
C                NFREQ.....NO. OF FREQUENCIES
C                NRNGS.....NO. OF RANGE STEPS
C                IOL.......FORTRAN OUTPUT UNIT FOR DEBUG PRINTING
C                IARTYP....FORTRAN UNIT FOR TL'S
C                ERROR.....ERROR FLAG   =0 IF OK
C                          =1   BAD INPUT DATA
C                          =2   ERROR IN TRAPED
C                          =3   ERROR IN LEAKY
C                          =4   ERROR IN SMODE
C                          =5   VMOD .GT. 1
C
C                   ******** NOTE: INPUT UNITS ********
C
C                   METRIC
C                          DEPTHS IN METERS, SOUND SPEEDS IN M/SEC, RANGES
C                          IN KILOMETERS. INTENSITY COMPUTED RELATIVE TO
C                          POINT SOURCE AT 1 METER.
C                   ENGLISH
C                          DEPTHS IN FEET,SOUND SPEEDS IN FT/SEC, RANGES
C                          IN NAUTICAL MILES. INTENSITY COMPUTED RELATIVE
C                          TO POINT SOURCE AT 1 FOOT.
C
```

```
C
C       I/O UNITS USED:
C                       IO1......DEBUG PRINT
C                       IARTVP...OPTIONAL TL INTENSITIES
C
C       COMMON BLOCKS USED:
C
C       SUBPROGRAMS CALLED:
C
C
C*****************************************************************************
      REAL KO,KOM,NM
      INTEGER ERROR,PERMUT
      COMPLEX EM,LAMBDA,DQ
      LOGICAL CUTOFF,DEBUG,METRIC
      DIMENSION FREQ(NFREQ),DEPTHS(MD),PERMUT(NP),RELINT(NRNGS,NP,NFREQ)
      DIMENSION XD(6),NMODE(6),REALEM(35),XMDE(35),PSI(6),IO(1),
     1          AMP(35,21)
      COMMON/DUCT/S,S2,S3,KOM,S3D,D,GPLUS,GMINUS,M,KO
      COMMON/MODES/NEM,ALPHA0,EM,DQ,NM,PSI0,U0S0
      COMMON/CONST/PI,TWOPI,C1,C2,OVRFLO,SQRT3,SQRTPI
      DATA PI,TWOPI/3.14159265,6.29318531/,NMAX/25/,OVRFLO/80./
      DATA C1,C2,SQRT3,SQRTPI/.35502 8054, .25881 9404, 1.73205 0808,
     1     1.77245 3851/
C
C......BLOCK 1- CONSTANTS AND FREQUENCY INDEPENDENT DUCT PARAMETERS.
C
      ERROR=1
      DEBUG=.FALSE.
C.........SET METRIC FLAG BY EXAMINING VALUE FOR CO
      IF(IO(1).GT.0) DEBUG=.TRUE.
      IF(CO.LT.2000.) METRIC=.TRUE.
      IF(CO.GT.4500.) METRIC=.FALSE.
      IF(METRIC) DELTAR=RNGINC*1000.
      IF(.NOT.METRIC) DELTAR=RNGINC*6076.12
      S3= (ZO-ZD)/ZD
      S2= S3**(2./3.)
      S=SQRT(S2)
      EPSD= (CO+CD)*(CD-CO)/CD**2
      GMINUS= EPSD/ZD
      GPLUS=  EPSD/(ZO-ZD)
      IF(.NOT.DEBUG) GO TO 100
      WRITE(IO(1),50) CO,CD,ZO,ZD,WAVEHT,NRNGS,RNGINC
   50 FORMAT(1H1,20X,20(1H*),5X,'ENTER SURFACE DUCT MODULE',5X,10(1H*)/
     1       5X,'DUCT PARAMETERS...CO,CD=',2F7.1,10X,'ZO,ZD=',2F7.1,
     2       5X,'WAVEHT=',F7.1,5X,'NRNGS,RNGINC=',I4,F7.2)
      IF(METRIC) WRITE(IO(1),52)
      IF(.NOT.METRIC) WRITE(IO(1),53)
   52 FORMAT(1H+,43X,6H M/SEC,24X,2H M,17X,2H M,27X,2HKM)
   53 FORMAT(1H+,43X,6HFT/SEC,24X,2HFT,17X,2HFT,27X,2HNM)
      WRITE(IO(1),55) S,S2,S3,EPSD,GPLUS,GMINUS,IO
   55 FORMAT(5X,'S,S2,S3=',3(1PE13.6),' EPSD,G+,G- =',3E13.6,4X IO=',3I3)
      IF(METRIC) WRITE(IO(1),57)
      IF(.NOT.METRIC) WRITE(IO(1),58)
   57 FORMAT(1H+,104X,4H M-1)
   58 FORMAT(1H+,104X,4HFT-1)
```

```
      WRITE(IO(1),60) NFREQ,(FREQ(I),I=1,NFREQ)
   60 FORMAT(5X,I1,' SOURCE FREQUENCIES',6H (HZ)=,6F7.1)
      WRITE(IO(1),65) ND,(DEPTHS(I),I=1,ND)
   65 FORMAT(1H+,75X,I1,' DEPTHS= ',6F7.1)
      IF(METRIC) WRITE(IO(1),67)
      IF(.NOT.METRIC) WRITE(IO(1),68)
   67 FORMAT(1H+,127X,2H M)
   68 FORMAT(1H+,127X,2HFT)
      WRITE(IO(1),70) KP,(PERMUT(I),I=1,KP)
   70 FORMAT(5X,I3,' SOURCE/RECEIVER PAIRS= ',21I3)
C
C************ BLOCK 2: FREQUENCY DEPENDENT PARAMETERS
C
C     LOOPING ON INPUT FREQUENCIES
  100 DO 1000 IF= 1,NFREQ
      IF(DEBUG) WRITE(IO(1),135) FREQ(IF)
  135 FORMAT(1H0,15X,'*******  BEGIN  COMPUTATIONS FOR FREQUENCY=
     1       F8.2,' HZ','   *******')
C
C           COMPUTE NATURAL UNIT OF LENGTH FOR DUCT -H-, AND
C           OTHER FREQUENCY DEPENDENT CONSTANTS. TRANSFORM ALL
C           DEPTHS TO DUCT UNITS AND DETERMINE NUMBER OF TRAPPED
C           MODES FOR FREQ(IF). ADD 3 LEAKY MODES. IF TOTAL NUMBER
C           OF MODES -NMODES- IS MORE THAN MAXIMUM, TRUNCATE SPECTRUM.
C
      K0= TWOPI*FREQ(IF)/C0
      WAVELN=C0/FREQ(IF)
      H= (K0**2*CPLNS)**(-1./3.)
      K0H= K0*H
C           CONVERT FIELD POINT DEPTHS TO DUCT UNITS.
      DO 120 I=1,ND
      ZD(I)= (DEPTHS(I)-ZD)/H
  120 CONTINUE
      S3D= S3*ZD/H
      SD=S*ZD/H
      D=ZD/H
C           DETERMINE NUMBER OF TRAPED MODES -NVM-
      NVM= .25+ 2.*SD*SQRT(SD)/(3.*PI)
      IF(NVM.GT.NMAX.AND.DEBUG) WRITE(IO(1),150) NVM,NMAX
  150 FORMAT(' SDUCT   TRUNCATION OF VIRTUAL MODE SPECTRUM',2I4)
      NMODES=NVM+3
      NLEAKY=3
      NMODES=MIN0(NMODES,NMAX)
      NMODE(IF)=NMODES
      IF(.NOT.DEBUG) GO TO 200
      WRITE(IO(1),160) K0,K0H,H,ND,(ZD(I),I=1,ND)
      WRITE(IO(1),165) S3D,SD,D,NVM,NLEAKY,NMODES
  160 FORMAT($X,'K0,K0H,H=X',3(1PE13.6),' ND,ZD= ',I2,2X,3E13.6)
  165 FORMAT(5X,'S3D,SD,D= ',3(1PE13.6),' VIRTUAL MODES...TRAPPED=',I3,
     2       ' LEAKY=',I2,' TOTAL=',I3)
C
C**************BLOCK 3:  LOOP OVER TRAPPED  AND LEAKY MODES, COMPUTING
C                 AMPLITUDES, ATTENUATION AND RAY-EQUIVALENT PARAMETERS.
C
  200 DO 300 IMODE=1,NMODES
      MODE=IMODE
```

```
C
C          COMPUTE MODE ENERGIES -EM-,NORMALIZATION -NM-, AND
C          RAY-EQUIVALENT PERIOD -DM-. TRAPED MODES HAVE SMALL IMAGINARY
C          PARTS AND NEGATIVE REAL PARTS (ENERGY BELOW BARRIER).
C
      IF(MODE.LE.NVM) CALL TRAPED(MODE,IO(2))
      IF(MODE.GT.NVM) CALL LEAKY(MODE,IO(2),REALEM)
      REALEM(MODE)=REAL(EM)
      LAMBDA= CSQRT(1.-CPLUS*M*(EM+S3D))*KO
C
C       COMPUTE LEAKAGE DECAY COEFFICIENT
      EMR=REAL(EM)
      EMI=AIMAG(EM)
      ALPHAL= -KOM*CPLUS*EMI
      IF(DEBUG) WRITE(IO(1),220) MODE,EM,NM,LAMBDA,ALPHAL
  220 FORMAT(1X,'LEAKAGE LOSSES MODE=',I3,' EM=',2(1PE14.6),' NM=',
     1        E13.6,' LAMBDA=',2E13.6,' ALPHAL=',E13.6)
      ERROR=0
C
C     COMPUTE RAY-EQUIVALENT ANGLE (RADIANS) AT SURFACE
C
      DM=SQRT(EMR+S3D)
      IF(EMR.GT.0.) DM=(DM-SQRT(EMR))
      DM= 4.*DM*SQRT(1.-CPLUS**(EMR+S3D))/(KOM*CMINUS)
      IF(.NOT.METRIC) PERIOD=DM/6076.12
      IF(METRIC) PERIOD=DM/1000.
      THETAM= ASIN(SQRT(EMR+S3D)/KOM)
      THDEG= THETAM*180./PI
      IF(DEBUG) WRITE(IO(1),230) EMR,THETAM,THDEG,PERIOD,DM
  230 FORMAT(' RAY STUFF ...EMR,THETAM,THDEG,PERIOD,DM= ',5(1PE14.6))

      RSLOSS=1.
C
C          ADD SURFACE LOSS, ASSUMING RAY-EQUIVALENT ANGLE AT SURFACE=THETAM.
C          LOSS IS COMPUTED AS EQUIVALENT OF SINGLE REFLECTION LOSS
C          APPLIED CONTINUOUSLY PER RAY'S CYCLE DISTANCE DM. THE ROUGH
C          SURFACE REFLECTION COEFFICIENT IS COMPUTED IN SUBROUTINE 'SLOSS1'
C
C
C       IF(WAVENT.GT.0.) RSLOSS=SLOSS1(THETAM,IF)
      ALPHAS=-ALOG(RSLOSS)/DM
      ALPHA=ALPHAL+ALPHAS
      DRLOSS= 8685.*ALPHA
      DECAY=EXP(-DELTAR*ALPHA)
      VMOD(MODE)=DECAY
      IF(DEBUG) WRITE(IO(1),240) RSLOSS,ALPHAS,ALPHA,DECAY,DRLOSS
  240 FORMAT(5X,'SURFACE LOSSES...RSLOSS,ALPHAS=',2(1PE13.6),2X,
     1'ALPHA,DECAY,DRLOSS=',3E13.6)
      PSIO=-1.
      IF(IO(2).LE.0) GO TO 255
C
C            OPTIONAL COMPUTATION
C            NORMALIZED PRESSURE AMPLITUDE AS FUNCTION OF DEPTH
C
      ZS=-.5*D
      SQRTPS= SQRT(SMODE(MODE,ZS,IO(2)))
```

```
      WRITE(IO(2),242) MODE
  242 FORMAT('0 SDUCT....DEPTH DEPENDENT PRESSURE FOR MODE=',I3/
     1  5X,'Z',11X,'Y',10X,'QR',12X,'QI',9X,'SMODE',6X,'PSI',9X,
     2   7HMM/PSIO,7X,'UOSQ',6X,'PRESSURE'/)
      NY=40
      DZ=2.*ZD/FLOAT(NY)
      DY=DZ/4
      Y=-ZD/4
      DO 250 IY=1,NY
      Y=Y+DY
      P=SQRTPS*SQRT(SMODE(MODE,Y,-IO(2)))
      WRITE(IO(2),245) P
  245 FORMAT(1H+,95X,1PE13.5)
  250 CONTINUE
C
C           COMPUTE INTENSITY AMPLITUDES AT FIELD DEPTHS -XD-
C
  255 DO 260 I=1,ND
      PSI(I)=SMODE(MODE,XD(I),IO(2))
  260 CONTINUE
C
C           NOW FORM MODAL AMPLITUDES FOR EACH SOURCE/RECEIVER PAIR
C
      DO 270 I=1,NP
      IS=PERMUT(I)/10
      IR=PERMUT(I)-10*IS
      AMP(MODE,I)= WAVELN*PSI(IS)*PSI(IR)
      IF(DEBUG) WRITE(IO(1),265) I,PERMUT(I),IS,IR,PSI(IS),PSI(IR),
     1          AMP(MODE,I)
  265 FORMAT(' INTENSITY AMPLITUDES...IP,PERMUT,IS,IR=',4I3,' PSIS,PSIR,
     1AMP= ',3(1PE13.6))
  270 CONTINUE
  300 CONTINUE
C
C************BLOCK 4:  STEP OUT IN RANGE, COMPUTING RELATIVE INTENSITY LEVELS
C                      IF METRIC UNITS USED, INTENSITY RELATIVE TO
C                      POINT SOURCE AT 1 METER. IF ENGLISH UNITS USED,
C                      INTENSITY RELATIVE TO POINT SOURCE AT 1 FOOT.
C
C
      RANGE=0.
      DO 500 IR=1,NRNGS
      RANGE= RANGE+ DELTAR
      DO 490 IP=1,NP
      SUM=0.
      DO 450 MODE=1,NMODES
      AMP(MODE,IP)=AMP(MODE,IP)*VMDF(MODE)
      SUM=SUM+AMP(MODE,IP)
  450 CONTINUE
      RELINT(IR,IP,IF)= SUM/RANGE
  490 CONTINUE
  500 CONTINUE
 1000 CONTINUE
      ERROR=0
      RETURN
      END
```

```
COMMON BLOCKS

/DUCT/(+12)
S       +0      S2      +1      S3      +2      KOH     +3      S3D     +4
D       +5      GPLUS   +6      GMINUS  +7      H       +10     KO      +11

/MODES/(+11)
NVM     +0      ALPHAO  +1      EM      +2      DQ      +4      NM      +6
PSIO    +7      UOSQ    +10

/CONST/(+7)
PI      +0      TWOPI   +1      C1      +2      C2      +3      OVRFLO  +4
SQRT3   +5      SQRTPI  +6


SUBPROGRAMS CALLED

SMODE   REAL.   LEAKY   SQRT.   CSQRT.  ALOG.
ASIN.   FLOAT.  EXP.    TRAPED  AIMAG.  MINO.


SCALARS AND ARRAYS [ "*" NO EXPLICIT DEFINITION - "&" NOT REFERENCED ]

*THDEG  1       DEBUG  2       .I0011 3       .I0010 4       *SD     5
*CO     6       *IF     7       &CUTOFF        *PERIOD 10      *FMR    11
 RELINT 12      *NLEAKY 13      *ZO     14      *NX     15      REALEM 16
*DZ     61      *SQRTPS 62      *IP     63      XO      64      *ALPHAL 72
*NMODES 73      *NFREQ  74      *FPSD   75      *P      76      *ALPHAS 77
*IMODE  100     METRIC 101      *NRNGS  102     IO      103     DEPTHS 104
*CD     105     .S0007 106      .S0006 107      *MODE   110     .S0005 111
*SUM    112     .S0004 113      *DM     114     .S0003 115     PERMUT 116
 .S0002 117     *DBLOSS 120     .S0001 121     .S0000 122     *DX     123
*NP     124     ERROR  125      *EMI    126     *ZD     127     *XS     130
*WAVENT 131     *THETAM 132     *X      133     *DELTAR 134     .S0014 135
 NMODE  136     .I0007 144      *ND     145     .S0013 146     .I0006 147
 .S0012 150     *RANGE  151     .I0005 152     .S0011 153     .I0004 154
*IS     155     .S0010 156      *ALPHA  157     *RSLOSS 160     .I0003 161
 .I0002 162     PSI    163      .I0001 171     *I      172     *NMAX   173
 .I0000 174     *WAVELN 175     *RXO    176     *DECAY  177     *IX     200
 VMOD   201     LAMBDA 244      *RNGINC 246     AMP     247     *IR     1606


TEMPORARIES

 .A0016 2203    .Q0000 2204    .Q0001 2205

SDUCT   [ NO ERRORS DETECTED ]
```

```
      SUBROUTINE TRAPEN(INODE,IDI)
C
C           COMPUTES EIGENVALUE AND NORMALIZATION INTEGRAL FOR STRONGLY
C           TRAPPED AND WEAK LEAKY MODES. ZERO-TH ORDER GUESS FOR
C           EIGENVALUE IS PROVIDED BY PHASE INTEGRAL (WKB) TECHNIQUES.
C
C           AUTHOR: W.J RYAN
C                   OCEAN ACOUSTICS DIVISION/SAI
C
C           ARGUMENTS:
C                     INODE.....MODE NUMBER
C                     EN........EIGENVALUE (COMPLEX)
C                     DQ........PERTURBATION IN Q  (= ALPHAO)
C                     NN........MODE NORMALIZATION
C                     ALPHAO....PI*AIP(-AS)**2
C                     NVM.......NO OF VIRTUAL MODES PRESENT
C                     IDI.......DEBUG PRINT/OUT CONTROL
C
C
C*****************************************************************
      REAL FO,FOM,FM,LOOKUP(10)
      COMPLEX EN,DQ,Z,DQ2,DRDE,ZETA,ZETAO,ZT
      DIMENSION AS(10)
      COMMON/DUCT/S1,S2,S3,FOM,SQO,CHEO,CPLUS,CHIMUS,H,RO
      COMMON/MODES/NVM,ALPHAO,EN,DQ,FM,DELO,NOSO
      COMMON/CONST/PI,TWOPI,C1,C2,OTRPI,O,SQRT3,SQRTPI
C       ZEROES OF AI(Y)
      DATA AS/2.338107, 4.087940, 5.52055, 6.786708, 7.944134,
     1     9.022651, 10.040174, 11.009524, 11.935016, 12.828777/
C       PI*AI'(AS)**2
      DATA LOOKUP/1.544710471, 2.026289162, 2.351777188, 2.606419475,
     1     2.819405454, 3.004408797, 3.159110925, 3.318297234,
     2     3.455179779,3.591902922/
C
      DATA TOL/3./
C
C
C       QOSTAR= ZERO OF AI
      IF(INODE.GT.10) GO TO 30
C       TABLE LOOK-UP
       QOSTAR= AS(INODE)
      ALPHAO=LOOKUP(INODE)
      GO TO 40
   30 CONTINUE
C         ASYMPTOTIC EXP FOR ZEROES OF AI AND AI'
      QOSTAR= (.375*PI*(4*INODE-1))**(2./3.)
      ALPHAO= SQRT(QOSTAR)
   40 CONTINUE
C
C         DETERMINE ZERO-TH ORDER EIGENVALUE (QO*=ZERO OF AI)
C
      EN0= S2*QOSTAR-S1U
      Q0= EN0/S2
      ETA=ABS(EN0)*SQRT(ABS(EN0))/(.75*S3)
      T=ETA*S3
      Z=(0.,0.)
```

```
      W=(0.,0.)
      W02=(0.,0.)
      DPDE=(0.,0.)
      NW=1.
      IF(IO1.GT.0) WRITE(IO1,55) IMODE,QOSTAR,ALPHA0,EM0,W0,FTA,T
   55 FORMAT(' TRAPED....MODE=',I3,' QO*,ALPHA0,EM0,W0,FTA,T=',
     1        6(1PE14.6))
      IF(-EM0.LT.TOL.OR.-W0.LT.TOL) GO TO 60
C
C         LARGE ARGUMENT REGION FOR AIRY FUNCTIONS. USE ASYMPTOTIC
C         EXPANSIONS FOR COMPUTING DQ.
C
      DEP=0.
      GAMMA=0.
      F=EXP(AMIN1(FTA,OVRFLO))
      IF(FTA.GE.OVRFLO) GO TO 58
      U= (1.+S3)/T
      DEP= U/(12.*(1.-U/9.))
      GAMMA= -.25*EXP(-T)*(1.-5.*U/36.)/(1.-U/36.)
   58 DQ=CMPLX(DEP,GAMMA)/F
      EM=DQ*S2/ALPHA0 +EM0
      W02= (1.-DQ*F*2.)**2/(S*ALPHA0*4.*W*SQRT(-EM0))
      GO TO 100
   60 CONTINUE
      CALL AIRY(-EM0,AI,BI,AIP,BIP,3)
      Z=-CMPLX(AIP,-BIP)/(S*CMPLX(AI,-BI))
      IF(IO1.GT.0) WRITE(IO1,75) EM0,AI,BI,AIP,BIP
      CALL AIRY(-W0,AI,BI,AIP,BIP,3)
      IF(-W0.LE.TOL) DQ=(AIP-Z*AI)/(BIP-Z*BI)
      IF(-W0.GT.TOL) DQ= (AIP/BIP)*(1.-Z*(AI/AIP))/(1.-Z*(BI/BIP))
      IF(IO1.GT.0) WRITE(IO1,75) W0,AI,BI,AIP,BIP
   75 FORMAT(' TRAPED....-E,AI,BI,AIP,BIP=',5(1PE14.6))
      EM=DQ*S2/ALPHA0 +EM0
C
C       VIRTUAL MODE NORMALIZATION -NW-
C
      IF(-W0.LE.TOL) W02= PI*(AI-DQ*BI)**2/(S2*ALPHA0)
      IF(-W0.GT.TOL) W02= PI*(BI*(1.-DQ*BI/AI))**2/(S2*ALPHA0)
      DRDE= EM*S2**2
   95 NW= CABS(1.-(1.+S3)*W02*DRDE)
  100 NW=  NW*W/S3
      IF(IO1.GT.0) WRITE(IO1,110) Z,DQ,EM,NW,W02,DPDE
  110 FORMAT(' TRAPED... Z,DQ,EM,NW=',7(1PE14.6)/10X,'W02,DPDE=',
     1        4(1PE14.6))
      DOSQ= CABS(W02)/NW
      RETURN
      END

COMMON BLOCKS

/DUC*/(*12)
S       *0      S2      *1      S3      *2      KOH     *3      S3D     *4
GMO     *5      GPLUS   *6      GMINUS  *7      H       *10     KO      *11

/MODES/(*11)
```

```
NVM     +0      ALPHA0  +1      EM      +2      DQ      +4      NM      +6
PS10    +7      U0S0    +10

/CONST/(+7)
PI      +0      TWOPI   +1      C1      +2      C2      +3      OVRFLO  +4
SQRT2   +5      SQRTPI  +6


SUBPROGRAMS CALLED

CABS.   SQRT.   CMPLX.  EXP.    AIRY    ABS.
AMIN1.

SCALARS AND ARRAYS [ "*" NO EXPLICIT DEFINITION - "%" NOT REFERENCED ]

*TOL    1       Z       2       *T      4       DRDE    5       *GAMMA  7
*ETA    10      *W0     11      *IO1    12      *AI     13      *BIP    14
*QOSTAR 15      *INODE  16      AS      17      LOOKUP  31      *AIP    43
*FW0    44      *U      45      %ZT             *F      46      %ZETA0
*DER    47      U02     50      %ZETA           *BI     52

TEMPORARIES

 .00016 116      .00000 117      .00001 120      .00002 121

TRAPED  [ NO ERRORS DETECTED ]
```

```
      SUBROUTINE LEAKY(MODE,IO1,REALEM)
C
C       COMPUITES PARAMETERS ASSOCIATED WITH LEAKY MODES.
C       ENERGY -EM-, DECAY COEFFICIENT -ALPHAM-, AND
C       RAY EQUIVALENT PERIOD -RM-.
C
C*************************************************************
C
      DIMENSION REALEM(1)
      INTEGER ERROR
      COMPLEX EM,Z,J0Z,A,A2,Z2,DE,U02,DZETA,DQ,ZETA0,E0,R0,W,DRDE
      REAL K0,K0H,NM
      LOGICAL DEBUG
      COMMON/DUCT/S,S2,S3,K0H,S3D,D,CPLUS,CMINUS,H,K0
      COMMON/MODES/NVM,ALPHA0,EM,DQ,NM,PSI0,U0SQ
      COMMON/CONST/PI,TWOPI,C1,C2,OVRFLO,SQRT3,SQRTPI
      DATA R0/(-.36450537,.6313421608)/
C
C
      DEBUG=.FALSE.
      IF(IO1.GT.0) DEBUG=.TRUE.
      DQ=(1.,1.)
      SD=S*D
C
      IF(S3D.LT.1.E-2) GO TO 300
C
C      SIGNIFICANT DUCT IS PRESENT
C
C
C........DETERMINE REAL PART OF ENERGY EIGENVALUE -EM-.
C         FOUND BY SATISFYING PHASE INTEGRAL CONDITION
C
      B=SD*SQRT(SD)/(PI*.75)
      T=(4*MODE-1)/(3.*B)
      IF(T.LE.1.) A=1.5*T-1.
      IF(T.GT.1.) A=.25*(T+SQRT(T*T-1.))**2
      EOLD=A*S3D
C
C          DO NEWTON/RAPHESON ITERATES TO FIND -EM-
C
      DO 50 I=1,5
      AP= A-(((A+1.)*SQRT(A+1.)-A*SQRT(A))/1.5-T)/(SQRT(A+1.)-
     1        SQRT(A))
      IF(ABS(AP-A).LT.ABS(A)*1.E-3) GO TO 55
      A=AP
   50 CONTINUE
   55 EMR=AP*S3D
      IF(EMR.LE.0.) RETURN
      CHI= -.5 -B*((AP+1.)*SQRT(AP+1.)-AP*SQRT(AP))
      DCHIDE= -B*(SQRT(AP+1.)-SQRT(AP))
      IF(DEBUG) WRITE(IO1,80) EMR,EOLD,T,AP,CHI,DCHIDE
   80 FORMAT(' LEAKY ....EM,EOLD,T,AP,CHI,DCHIDE= ',6(1PE14.6))
C
C           COMPUTE RAY-EQUIVALENT PERIOD -RM-. ASSUME RAY TURNS AT
C           DUCT BOTTOM, AND NORMALIZATION -NM-
C
```

```
      DM= (SQRT(EMR+S3D)-SQRT(EMR))*H/S3
      RM= 4.*KOH*DM
      NM=.5*DM
C
C.........COMPUTE AMPLITUDE DECAY TERM FOR LEAKY MODES.
C         MODEL  DECAY AS REFLECTION LOSS PER RAY-EQUIVALENT
C         PERIOD APPLIED CONTINUOUSLY.
C
      RCOEFF= REFLKT(EMR,S2,IO1)
      IF(RCOEFF.GT.1.) RETURN
      ALPHAM= -ALOG(RCOEFF)/RM
      EMI=-ALPHAM/(KOH*CPLUS)
      EM=CMPLX(EMR,EMI)
      IF(EMR.GT.1.) DRDE= -.5/(-EM)**.5 +.25/EM**2
       IF(EMR.LE.1.) DRDE= -(R0**2 +EM*(.2251235 +EM*(.16231472*R0+
     1         EM*(.1169137*R0**2+ .0319463474*EM))))
      IF(CABS(EM).GT.1.5*S2) GO TO 150
C
C             SMALL ARGUMENT REGION. EXPAND IN POWER SERIES ABOUT
C             EM=0.
C
      W=-EM/S2
      Z=W**3/6.
      ZETA0= (-W+SD)**1.5/1.5 +PI/12.
      U02= 4.*PI*(C1*CSIN(ZETA0)*(1.+Z*(1.+.2*Z))+ C2*CSIN(ZETA0+PI/3.)*
     1      W*(1.+.5*Z*(1.+Z/7.)))*-2/(S*(EM+S3D)**.5)
      GO TO 160
  150 DZETA= (EM**1.5- (EM+S3D)**1.5)/(1.5*S3)
      U02= (CSIN(DZETA)- 5.*S3*CCOS(DZETA)/(48.*EM**1.5))**2/
     1      (EM*(EM+S3D))**.5
  160 CONTINUE
      Z=1.+(1.+S3)*U02*DRDE
      NM=CABS(Z)*H/S3
      IF(EMR.GT.S3D) NM=DM/2.
      U0SQ= CABS(U02)/NM
      IF(DEBUG) WRITE(IO1,200) EM,RCOEFF,RM,NM,U02,U0SQ,DRDE
  200 FORMAT(' LEAKY ...EM,RCOEFF,RM=',4E15.7/10X,
     1       'NM,U02,U0SQ,DRDE=',6(1PE14.6))
      GO TO 500
C
C*************************************
C             WEAK DUCT CASES.
C                1)  ZERO GRADIENT IN DUCT (S=0)
C                2)  SMALL DUCT THICKNESS  (SD SMALL)
C
C        METHOD: USE PERTURBATION THEORY AND EXPAND AROUND S=0 CASE.
C
C
  300 CONTINUE
C
C       FIRST FIND ASYMPTOTIC S=0 EIGENVALUE (COMPLEX)
C
      A=CMPLX(-.68586059,-1.1879454)
      A2=CMPLX(-.9408095,1.6295299)
      Z=A/D
      Z2=A2/D**2
```

```
00113          T= (2.5810567+ .27929556*(PI*FLOAT(MODE))**2)/D**3
00114          E0= (PI*FLOAT(MODE)/D)**2*(T*(1.+5.*Z)+ 7.74317/D**3+ 3.*Z2+
00115         1     2.*Z+1.)
00116          ER=REAL(E0)
00117          EI=AIMAG(E0)
00118          IF(DEBUG) WRITE(IO1,320) MODE,T,Z,Z2,E0
00119      320 FORMAT('0LEVEL 2 LEAKY...M:DE,T,Z,Z2,E0=',I4,6E14.7)
00120          IF(S.LT.1.E-6) GO TO 400
00121    C
00122    C      GRADIENT IS NON-ZERO, BUT EFFECTIVE THICKNESS OF DUCT IS SMALL.
00123    C      COMPUTE FIRST ORDER PERTURBATION TO S=0 ENERGY.
00124    C
00125          Z= CSQRT(E0)*D
00126          J0Z= CSIN(Z)/Z
00127          DE= (S3D*D/4.)*(J0Z*J0Z-1.)
00128          EM=E0+DE
00129          IF(DEBUG) WRITE(IO1,340) Z,J0Z,DE
00130      340 FORMAT(' LEVEL 2 LEAKY...Z,J0Z,DE=',6E15.7)
00131    C
00132    C      COMPUTE RAY-EQUIVALENT PERIOD -RM-.
00133    C      IF ENERGY IS POSITIVE (ABOVE BARRIER), ASSUME RAY TURNS AT
00134    C      BOTTOM OF DUCT.
00135    C
00136      400 RM=0.
00137          IF(S.EQ.0.) GO TO 500
00138          EMR=REAL(EM)
00139          RM=SQRT(EMR+S3D)
00140          IF(EMR.GE.0.) RM=(RM-SQRT(EMR))*4.*SQRT(1.-GPLUS*H*(EMR+S3D))/
00141         1                    (K0H*GMINUS)
00142      500 CONTINUE
00143          ERROR=0
00144          RETURN
00145          END
```


COMMON BLOCKS

```
/DUCT/(+12)
S        +0     S2       +1     S3       +2     K0H      +3     S3D      +4
D        +5     GPLUS    +6     GMINUS   +7     H        +10    K0       +11

/MODES/(+11)
NVM      +0     ALPHA0   +1     EM       +2     DQ       +4     NM       +6
PSI0     +7     U0SQ     +10

/CONST/(+7)
PI       +0     TWOPI    +1     C1       +2     C2       +3     OVRFLO   +4
SQRT3    +5     SQRTPI   +6
```

SUBPROGRAMS CALLED

```
REAL.    CABS.    SQRT.    CMPLX.   CSQRT.   ALOG.
CCOS.    FLOAT.   AIMAG.   REFLKT   CSIN.
```

SCALARS AND ARRAYS ["*" NO EXPLICIT DEFINITION - "%" NOT REFERENCED]

DE	1	DEBUG	3	Z	4	*SD	6	W	7
*AP	11	*T	12	DRDE	13	*EMR	15	*CHI	16
*B	17	*IO1	20	%REALEM		*RCOEFF	21	*ER	22
*ALPHAM	23	E0	24	A2	26	DZETA	30	*MODE	32
*DM	33	A	34	.S0000	36	ERROR	37	*EMI	40
*EOLD	41	*RM	42	JOZ	43	*I	45	Z2	46
*DCHIDE	50	ZETA0	51	U02	53	R0	55	*EI	57

TEMPORARIES

.A0016	133	.Q0000	134	.Q0001	135	.Q0002	136	.Q0003	137	.Q0004	140
.Q0005	141										

LEAKY [NO ERRORS DETECTED]

```
      FUNCTION SMODE(MODE,X,IO1)
C
C                 COMPUTES NORMALIZED INTENSITY FUNCTIONS ("SMODES")
C                 FOR ENV -EM-,AND POSITION -X- .
C
C*****************************************************************
      LOGICAL DEBUG
      COMPLEX D1,DZETA0,Q,QQ,ZETA0,DZETA,ZM,P,U
      REAL OVRFLO,ZM
      COMMON/DUCT/S,S2,S3,KO",S3D,D,GPLUS,GMINUS,H,KO
      COMMON/MODES/NYM,ALPHA0,EM,DQ,NM,PSI0,QQSQ
      COMMON/CONST/PI,TWOPI,C1,C2,OVRFLO,SQRT3,SQRTPI
C
C
C
      DEBUG=.FALSE.
      IF(IO1.GT.0) DEBUG=.TRUE.
      IF(IO1.LT.0) IOU=-IO1
      Z=H*(Y+D)
      IF(Y.GT.1.E-4) GO TO 200
C
C          TREAT NEAR SURFACE REGION USING POWER SERIES.
C
      ETA= Y+D
      Q=EM/S2-S*Y
      IF(ETA.GT..2) GO TO 30
      U= S* (ETA*ETA/6.)*(.5*ETA*S3+ (EM+S3D)*(-1.+.05*(EM+S3D)*
     1   ETA*ETA))
      PSI=ETA*CABS(U)**2/S2
      GO TO 100
   30 SMP=REAL(EM)
      QQ=Q+S*ETA
      ABSQ=CABS(Q)
      IF(ABSQ.LT.1.5) GO TO 70
      ABSQQ=CABS(QQ)
      IF(ABSQQ.GT..05.OR.EMM.GT.0.) GO TO 60
      QR=REAL(Q)
      QQR=REAL(QQ)
      SQRTQ=SQRT(ABSQ)
      ETA=ABSQ*SQRTQ/1.5
      IF(QR.LT.0.) GO TO 40
C.....OSCILLATORY REGION OF AIRY FUNCTIONS.  USE ASYMPTOTIC EXPANSIONS
      T=5./(72.*ETA**2)
      PSI=(1.-T)*(.01+COS(PI/4.-ETA*(1.-T))**2)/(SQRTQ*S2*ALPHA0)
      GO TO 100
C.....EXPONENTIAL REGION OF AIRY FUNCTIONS.
   40 CONTINUE
      C= 5./(72.*ETA)
      E=EXP(AMIN1(2.*ETA,OVRFLO))
      PSI= (1.-C-2.*QQR*E*(1.+C))**2/(4.*E*S2*SQRTQ*ALPHA0)
      GO TO 100
   60 CONTINUE
      DZETA=Q**1.5*(1.- (QQ/Q)**1.5)/1.5
      PSI= (.5/S2)*(COSH(2.*AIMAG(DZETA))-COS(2.*REAL(DZETA)))/SQRT(
     1        CABS(Q*QQ))
      GO TO 100
```

```
C
C           SMALL Q REGION. EXPAND WAVEFUNCTION ABOUT T.P. VALUE (Q=0)
C
   70 CONTINUE
      IF(EMP.LT.0.) GO TO 75
C
C           LEAKY MODES
      ZETA0= Q0**1.5/1.5 +PI/12.
      H02= TWOPI*C1**2*(COSH(2.*AIMAG(ZETA0))- COS(2.*REAL(ZETA0)))/
     1  (S2*SQRT(CABS(Q0)))
      DLOGH= -(C2/C1)*CCOS(ZETA0-PI/6.)/CSIN(ZETA0)
      GO TO 80
C
C           TRAPPED MODES
   75 H02=PI*C1**2*((1.-SQRT3*REAL(Q0))**2 +3.*AIMAG(Q0)**2)/(S2*ALPHA0)
      DLOGH= (C2/C1)*(1.+SQRT3*Q0)/(1.-SQRT3*Q0)
   80 H=Q**3/6.
      PSI= H02*CABS(1.+Q*(-1.+.2*H)+ Q*DLOGH*(1.+.5*Q*(-1.+H/7.)))**2
      GO TO 100
  100 CONTINUE
      SMODE= PSI/NM
      IF(DEBUG) WRITE(IO1,120) X,Q,PSI,NM,SMODE
  120 FORMAT(' SMODE .... X,Q,PSI,NM,SMODE ',6E15.7)
      IF(IO1.LT.0) WRITE(IOH,130) Z,X,Q,SMODE,PSI,NM
  130 FORMAT(1X,10(1PE12.4))
      RETURN
C*****************************************
C
C           BELOW DUCT REGION.  NORMALIZED INTENSITY FUNCTION
C           IS A TRAVELING WAVE PROPORTIONAL TO H1**2.
C
  200 CONTINUE
      IF(PSI0.LE.0.) PSI0=H1SQ(EM)
      P=EM+X
      PSI=H1SQ(P)
      SMODE=H0SQ*(PSI/PSI0)
      IF(DEBUG) WRITE(IO1,310) X,P,PSI0,H0SQ,PSI,SMODE
  310 FORMAT(' SMODE....X,P=',2(1PE14.6),' PSI0,H0SQ,PSI=',
     1      3(1PE14.6),' SMODE=',1PE14.6)
      IF(IO1.LT.0) WRITE(IOH,130) Z,X,P,SMODE,PSI,PSI0,H0SQ
      RETURN
      END
```

COMMON BLOCKS

```
/DUCT/(+12)
S       +0      S2      +1      S3      +2      KOH     +3      S3D     +4
D       +5      GPLUS   +6      GMINUS  +7      H       +10     KO      +11

/MODES/(+11)
NVM     +0      ALPHA0  +1      EM      +2      Q0      +4      NM      +6
PSI0    +7      H0SQ    +10

/CONST/(+7)
PI      +0      TWOPI   +1      C1      +2      C2      +3      OVRFLO  +4
```

```
SQRT?   *5      SQRTPI  *6


SUBPROGRAMS CALLED

COSH.   REAL.   SQRT.   CABS.   CCOS.   EXP.
AIMAG.  CSIN.   COS.    MISQ    AMINT.

SCALARS AND ARRAYS [ "*" NO EXPLICIT DEFINITION - "*" NOT REFERENCED ]

 DEBUG  1       *Z      2       *SMODE  3       *T      4       Q       5
*EMK    7       *ETA    10      *QR     11      *IO1    12      *IOU    13
 QO     14      *ABSQ   16       DZETA  17       P      21      *SQRTQ  23
*MODE           *X      24       U      25      *DQR    27      *PSI    30
 DLOGU  31      *F      33      *C      34       ZETAO  35      *U02    37
*ABSDQ  40

TEMPORARIES

 .A0010 75      .A0011 76      .A0016 77      .A0002 100     .A0003 101     .A0004 102
 .A0005 103     .A0006 104     .A0007 105     .Q0000 106     .Q0001 107     .Q0002 110
 .Q0003 111     .Q0004 112

SMODE   [ NO ERRORS DETECTED ]
```

```
      SUBROUTINE AIRY(X,AI,BI,AIP,BIP,MODE)
C
C         PURPOSE:  COMPUTES AIRY FUNCTIONS AND DERIVATIVES FOR REAL
C                      ARGUMENTS.
C
C         METHOD:   A COMBINATION OF CONVERGENT POWER SERIES FOR SMALL ARGU-
C                   MENTS, WITH ASYMPTOTIC EXPANSIONS FOR LARGE POSITIVE
C                   AND NEGATIVE ARGUMENTS
C                   REF= ABRAMOWITZ AND STEGUN, HANDBOOK OF MATHEMATICAL
C                              FUNCTIONS.
C
C         AUTHOR:   F.J. RYAN
C                   OCEAN ACOUSTICS DIV
C                   SCIENCE APPLICATIONS, INC.
C                   LA JOLLA   CA
C
C         SUBPROGRAMS CALLED:  NONE
C
C         ARGUMENTS:
C                   X........ARGUMENT OF AIRY FUNCTION
C                   AI,BI....AIRY FUNCTION (OUTPUT)
C                   AIP,BIP..AIRY FUNCTION DERIVATIVES (OUTPUT)
C                   MODE......=1  ONLY AI,BI COMPUTED
C                             =2  ONLY AIP,BIP COMPUTED
C                             =3  AI,BI,AIP,BIP COMPUTED
C
C
C
C*************************************************************
      REAL OVRFLO,N,M
      DATA C1,C2,SQRT3/.35502 80539, .25881 94038, 1.732050808/
      DATA PI,OVRFLO/3.1415926, 80./
C
C......INLINE FUNCTIONS
      F1(Q)= (1.+5.*Q*(1.+38.5*Q*(1.+221.*(Q/3.)*(1.+109.25*Q))))/
     1       1.772453851
      G1(Q)=(1.-7.*Q*(1.+32.5*Q*(1.+209.*(Q/3.)*(1.+106.25*Q))))/
     1       1.772453851
      F2(Q)= (1.+5.*Q*(1.+38.5*Q))/1.772453851
      G2(Q)= (1.-7.*Q*(1.+32.5*Q))/1.772453851
      IF(X.LT.-2.OR.X.GT.2.) GO TO 50
C
C         POWER SERIES REGION
C
      Z= X**3/6.
      IF(MODE.EQ.2) GO TO 30
C......AI,BI COMPUTATION
      F= (1.+Z*(1.+.2*Z*(1.+(Z/12.)*(1.+(Z/22.)*(1.+Z/35.)))))*C1
      G=X*(1.+.5*Z*(1.+(Z/7.)*(1.+(Z/15.)*(1.+(Z/26.)*(1.+.025*Z)))))*C2
      AI= F- G
      BI= SQRT3*(F+ G)
   50 IF(MODE.EQ.1) RETURN
      FP= X*X*(.5+.2*Z*(1.+.125*Z*(1.+(Z/16.5)*(1.+Z/28))))*C1
      GP=(1.+ 2.*Z*(1.+.25*Z*(1.+(Z/10.5)*(1.+.05*Z*(1.+Z/32.5)))))*C2
      AIP= FP-GP
```

```
00057          HIP= SQRT5*(FP*GP)
00058          RETURN
00059       50 Z=ABS(X)
00060          SQRTZ=SQRT(Z)
00061          ETA= Z*SQRTZ/1.5
00062          IF(X.GT.0.) GO TO 100
00063   C
00064   C       OSCILLATORY REGION
00065   C
00066          T=5./(32.*Z**3)
00067          IF(IODE.EQ.2) GO TO 60
00068          N= (1.-T*(1.-23.1*T))/(PI*SQRTZ)
00069          N= SQRT(N)
00070          THETA= -ETA*(1.-T*(1.-22.1*T/3.))+ .785398163*
00071          AI= N*COS(THETA)
00072          BI= N*SIN(THETA)
00073          IF(IODE.EQ.1) RETURN
00074       60 T=1.*T
00075          N= (1.+T*(1.-195.*T/14.))*SQRTZ/PI
00076          N= SQRT(N)
00077          PHI= -ETA*(1.+T*(1.-209.*T/42.))+ 2.35619449
00078          AIP= N*COS(PHI)
00079          BIP= N*SIN(PHI)
00080          RETURN
00081   C
00082   C       EXPONENTIAL REGION
00083   C
00084      100 T= 1./(72.*ETA)
00085          SQRTZ=SQRT(SQRTZ)
00086          F=1.
00087          IF(ETA.LT.OVRFLO) F=EXP(ETA)
00088          IF(IODE.EQ.2) GO TO 120
00089          IF(ETA.GT.4.) GO TO 110
00090          AI=F4(-T)
00091          BI=F4(T)
00092          GO TO 115
00093      110 AI= F2(-T)
00094          BI=F2(T)
00095      115 AI= .5*AI/(F*SQRTZ)
00096          BI= BI*F/SQRTZ
00097          IF(IODE.EQ.1) RETURN
00098      120 IF(ETA.GT.4.) GO TO 130
00099          AIP= G4(-T)
00100          BIP=G4(T)
00101          GO TO 140
00102      130 AIP=G2(-T)
00103          BIP=G2(T)
00104      140 AIP= -.5*SQRTZ*AIP/F
00105          BIP= SQRTZ*BIP*F
00106          RETURN
00107          END
```

SUBPROGRAMS CALLED

SQRT.
F4 G2 EXP. ABS. COS. F2

```
SIN.     64

SCALARS AND ARRAYS ( "*" NO EXPLICIT DEFINITION - "X" NOT REFERENCED )

*GP      1        */       2        *T       3        .F0003 4        .F0002 5
 .F0001 6        .F0000 7        *THETA   10       N       11       *SQRT3  12
*ETA     13       *PI      14       *AI      15       *HIP    16       *FP     17
 M       20       *G       21       *MODE    22       OVRFLO 23       *AIP    24
*C2      25       *X       26       *SQRT2   27       *PHI    30       *F      31
*C1      32       *PI      33

TEMPORARIES

 .A0016 34        .00000 35

AIRY     ( NO ERRORS DETECTED )
```

```
      FUNCTION REFLKT(EM,S2,IO1)
C
C           COMPUTES REFLECTION COEFFICIENT FOR MODES WITH ENERGIES
C           ABOVE BARRIER (E.GT.0).
C
      REAL N2,M2
      DOUBLE PRECISION P
      DIMENSION RHO(2),DELTA(2)
      LOGICAL DEBUG
      DATA PI/3.1415925/,C1,C2/.35502 80539,.25881 94038/
      REFLKT=10.
      DEBUG=.FALSE.
      IF(IO1.GT.0) DEBUG=.TRUE.
      IF(EM.LT.0.) RETURN
      IF(EM.GT.3.) GO TO 80
      IF(EM.GT..01) GO TO 20
C
C                 ENERGY VERY NEAR ZERO. USE LIMITING CASE (E=0)
C
      S=SQRT(S2)
      REFLKT= (1.+S*S2)*(1.+S)**-3
      RETURN
C
C                 ENERGY ABOVE BARRIER, BUT TOO LOW FOR ASYMPTOTIC
C                 EXPRESSIONS. USE EXACT FORM OF REFLECTION COEF
C
   20 CONTINUE
      E=-EM
      DO 40 I=1,2
      CALL AIRY(E,AI,BI,AIP,BIP,3)
      IF(DEBUG) WRITE(IO1,30) E,AI,AIP,BI,BIP
   30 FORMAT(' REFLKT #30...E,AI,AIP,BI,BIP=',6E15.7)
      M2= AI*AI+ BI*BI
      N2= AIP*AIP+ BIP*BIP
      RHO(I)= N2/M2
      DELTA(I)= ASIN(1./(PI*SQRT(M2*N2)))
      E=E/S2
   40 CONTINUE
      BETA= SQRT(RHO(1)/(S2*RHO(2)))
      R= (BETA+ 1./BETA+ 2.*COS(DELTA(1)+DELTA(2)))/(BETA+ 1./BETA+
     1            2.*COS(DELTA(1)-DELTA(2)))
      REFLKT=R
      IF(DEBUG) WRITE(IO1,50) RHO,DELTA,BETA,REFLKT
   50 FORMAT(' REFLKT #50...RHO,DELTA,BETA,REFLKT=',6E15.7)
      RETURN
C
C                 ENERGY WELL ABOVE BARRIER. USE ASYMPTOTIC
C                 FORM FOR REFLECTION COEF.
C
   80 CONTINUE
      S= SQRT(S2)
      V2= (1.+S*S2)**2/(16.*EM**3)
      REFLKT= V2/(4.+V2)
      RETURN
      END
```

```
SUBPROGRAMS CALLED

SQRT.
ASIN.   ATAN    COS.

SCALARS AND ARRAYS ( *** NO EXPLICIT DEFINITION - *** NOT REFERENCED )

 DEBUG  1        DELTA  2        *BETA   4        *PI     5        *W2     6
*E      7        *S2    10       *FN     11       *IO1    12        N2     13
*AT     14       *RIP   15       *S      16       *REFLKT 17        .50000 20
*AIP    21       *C2    22        RHO    23        R      25       *I      27
*C1     30        N2    31       *B1     32

TEMPORARIES

 .A0010 56       .A0011 57       .A0012 60       .A0013 61       .A0014 62       .A0015 63
 .A0016 64       .A0002 65       .A0003 66       .A0004 67       .A0005 70       .A0006 71
 .A0007 72       .00000 73       .00001 74

REFLKT  ( NO ERRORS DETECTED )
```

```
00001          FUNCTION HIS0(P)
00002          COMPLEX P,Q,R,ZETA
00003          COMMON/CONST/PI,TWOPI,C1,C2,OVRFLO,SQRT3,SQRTPI
00004          PR=REAL(P)
00005          PIMAG=AIMAG(P)
00006          ABSP=CABS(P)
00007          IF(ABSP.GT.1.5) GO TO 50
00008    C
00009    C       SMALL ARGUMENT REGION. EXPAND IN POWER SERIES ABOUT ORIGIN
00010    C
00011          Q=P**3/6.
00012          R=P*CMPLX(-.3645055665,.6313421509)
00013          HIS0=.5041795759*CABS(1.+Q*(-1.+.2*Q) +R*(1.+.5*Q*(-1.+Q/7.)))**2
00014          RETURN
00015       50 IF(PR.LT.0.) GO TO 100
00016    C
00017    C       LARGE ARGUMENT (COMPLEX) ASYMPTOTIC REGION
00018    C
00019          ZETA= (P**1.5)/1.5
00020          HIS0= EXP(-2.*AIMAG(ZETA))/(PI*SQRT(ABSP))
00021    C     IF(ABSP.LT.2.5) HIS0=HIS0*CABS(1.-CMPLX(0.,5./72.)/ZETA)**2
00022          RETURN
00023    C
00024    C
00025    C          SMALL COMPLEX P .  EXPAND AROUND REAL P VALUE.
00026    C
00027      100 CONTINUE
00028          SQRTP=SQRT(ABSP)
00029          ETA=ABSP*SQRTP/.75
00030          T=5./(36.*ETA)
00031          F=EXP(AMIN1(ETA,OVRFLO))
00032          HIS0=F/(PI*SQRTP)
00033          IF(-PR.LT.2.) HIS0=HIS0*(.25*(1.-T)**2/F+ (1.+T)**2)
00034      200 HIS0= HIS0- .6366198*PIMAG
00035          RETURN
00036          END
```

COMMON BLOCKS

```
/CONST/(+7)
PI      +0      TWOPI   +1      C1      +2      C2      +3      OVRFLO  +4
SQRT3   +5      SQRTPI  +6
```

SUBPROGRAMS CALLED

```
REAL.   SQRT.   CABS.   CMPLX.  EXP.    AIMAG.
AMIN1.
```

SCALARS AND ARRAYS (*** NO EXPLICIT DEFINITION - *** NOT REFERENCED)

```
*T      1       Q       2       *ETA    4       *PIMAG  5       P       6
*ABSP   10      *PR     11      *SQRTP  12      R       13      *P      15
*HIS0   16      ZETA    17
```

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```
TEMPORARIES

 .A0016 21        .A0002 22        .A0003 23        .A0004 24        .A0005 25        .Q0000 26
 .Q0001 27

H150     [ NO ERRORS DETECTED ]
```

APPENDIX C

SAMPLE EXECUTION

The following is transmission-loss (TL) output for the test case considered in Section 3. The values have been scaled by a factor of 10 and volume attentuation [$\alpha = .125\ (f/1000)^2$ dB/nm] subtracted.*

* The virtual-mode model does not include volume loss. However, the PE runs used in section 3 did, so I added volume loss to the values shown in the following paper to get the results plotted in section 3. The TL values in this appendix have been converted to dB re 1 yd.

TRANSMISSION-LOSS (DB RE 1 FT) FREQUENCY= 25.0 HZ WAVEHT= 0.0 FT CONVRT= 9.540000E+00 DB

ZS (FT)	250	250	250	250	250	250	500	500	500	500	500	750	750	750	750	1000	1000	1000	1250	1250	1500	
ZR (FT)	250	500	750	1000	1250	1500	500	750	1000	1250	1500	750	1000	1250	1500	1000	1250	1500	1250	1500	1500	
R (NM)																						
1.00	692	654	650	611	623	601	633	626	630	622	605	608	559	579	556	579	553	527	524	496	466	1.00
2.00	762	723	721	733	740	737	684	692	696	707	700	680	692	699	696	701	701	691	692	677	657	2.00
3.00	799	760	759	773	784	780	720	719	733	745	750	718	732	743	749	746	757	761	767	769	769	3.00
4.00	830	791	789	804	816	821	751	750	764	776	781	745	753	775	780	778	789	795	801	806	811	4.00
5.00	858	819	818	833	844	849	779	779	792	804	809	777	791	803	809	806	817	823	829	834	840	5.00
6.00	865	845	844	859	870	875	805	804	819	830	836	803	817	829	835	832	844	849	855	861	866	6.00
7.00	910	870	869	883	895	900	830	829	844	855	861	828	842	854	860	857	869	874	880	886	891	7.00
8.00	934	894	893	907	919	925	854	853	868	879	885	852	867	878	884	881	893	898	905	910	915	8.00
9.00	957	918	916	931	943	948	878	877	891	903	908	875	890	902	907	905	916	922	928	933	939	9.00
10.00	980	940	939	954	966	971	901	900	914	926	931	898	913	925	930	928	939	945	951	956	962	10.00
11.00	1003	963	962	976	988	993	923	922	937	948	954	921	935	947	953	950	962	967	973	979	984	11.00
12.00	1025	985	984	998	1010	1015	945	944	959	970	976	943	958	969	975	972	984	989	995	1001	1006	12.00
13.00	1047	1007	1006	1020	1032	1037	967	966	981	992	998	965	979	991	997	994	1006	1011	1017	1023	1028	13.00
14.00	1068	1028	1027	1042	1054	1059	989	988	1002	1014	1019	986	1001	1013	1018	1016	1027	1033	1039	1044	1050	14.00
15.00	1090	1050	1049	1063	1075	1080	1010	1009	1023	1035	1041	1008	1022	1034	1039	1037	1049	1054	1060	1066	1071	15.00
16.00	1111	1071	1070	1084	1096	1101	1031	1030	1045	1056	1062	1029	1043	1055	1061	1058	1070	1075	1081	1087	1092	16.00
17.00	1132	1092	1091	1105	1117	1122	1052	1051	1066	1077	1083	1050	1064	1076	1082	1079	1091	1096	1102	1108	1113	17.00
18.00	1153	1113	1112	1126	1138	1143	1073	1072	1086	1098	1104	1071	1085	1097	1102	1100	1111	1117	1123	1129	1134	18.00
19.00	1173	1133	1132	1147	1158	1164	1094	1093	1107	1119	1124	1091	1106	1118	1123	1120	1132	1138	1144	1149	1155	19.00
20.00	1194	1154	1153	1167	1179	1185	1114	1113	1128	1139	1145	1112	1127	1139	1144	1141	1153	1158	1164	1170	1175	20.00
21.00	1214	1175	1173	1188	1200	1205	1135	1134	1149	1160	1165	1132	1147	1159	1164	1162	1173	1179	1185	1190	1196	21.00
22.00	1235	1195	1194	1208	1220	1225	1155	1154	1169	1180	1186	1153	1167	1179	1184	1182	1194	1199	1205	1211	1216	22.00
23.00	1255	1215	1214	1229	1240	1245	1175	1174	1189	1200	1206	1173	1188	1199	1205	1202	1214	1219	1226	1231	1236	23.00
24.00	1275	1235	1234	1249	1260	1265	1196	1194	1209	1221	1226	1193	1208	1220	1225	1222	1234	1239	1246	1251	1257	24.00
25.00	1295	1255	1254	1269	1281	1286	1216	1215	1229	1241	1246	1213	1228	1240	1245	1242	1254	1259	1266	1271	1277	25.00
26.00	1315	1276	1274	1289	1301	1306	1236	1235	1249	1261	1266	1233	1248	1260	1265	1263	1274	1280	1286	1291	1297	26.00
27.00	1335	1296	1294	1309	1321	1326	1256	1255	1269	1281	1286	1253	1268	1280	1285	1283	1294	1300	1306	1311	1317	27.00
28.00	1355	1315	1314	1329	1341	1346	1276	1275	1290	1301	1306	1273	1288	1300	1305	1302	1314	1320	1326	1331	1337	28.00
29.00	1375	1335	1334	1349	1360	1366	1296	1294	1309	1321	1326	1293	1309	1319	1325	1322	1334	1339	1346	1351	1357	29.00
30.00	1395	1355	1354	1369	1380	1385	1315	1314	1329	1340	1346	1313	1328	1339	1345	1342	1354	1359	1366	1371	1376	30.00
31.00	1415	1375	1374	1388	1400	1405	1335	1334	1349	1360	1366	1333	1347	1359	1364	1362	1374	1379	1385	1391	1396	31.00
32.00	1434	1395	1393	1409	1420	1425	1355	1354	1368	1380	1385	1353	1367	1379	1384	1382	1393	1399	1405	1410	1416	32.00
33.00	1454	1414	1413	1428	1439	1445	1375	1373	1388	1400	1405	1372	1387	1398	1404	1401	1413	1418	1425	1430	1436	33.00
34.00	1474	1434	1433	1447	1459	1464	1394	1393	1408	1419	1425	1392	1406	1418	1424	1421	1433	1438	1444	1450	1455	34.00
35.00	1493	1454	1452	1467	1479	1484	1414	1413	1427	1439	1444	1411	1426	1438	1443	1441	1452	1458	1464	1469	1475	35.00
36.00	1513	1473	1472	1486	1498	1504	1433	1432	1447	1458	1464	1431	1446	1457	1463	1460	1472	1477	1484	1489	1494	36.00
37.00	1532	1493	1491	1506	1518	1523	1453	1452	1466	1478	1483	1451	1465	1477	1482	1480	1491	1497	1503	1509	1514	37.00
38.00	1552	1512	1511	1525	1537	1543	1472	1471	1486	1497	1503	1470	1485	1496	1502	1499	1511	1516	1523	1529	1533	38.00
39.00	1571	1532	1530	1545	1557	1562	1492	1491	1505	1517	1522	1490	1504	1516	1521	1519	1530	1536	1542	1547	1553	39.00
40.00	1591	1551	1550	1564	1576	1582	1511	1510	1525	1536	1542	1509	1524	1535	1541	1538	1550	1555	1561	1567	1572	40.00
41.00	1610	1570	1569	1584	1595	1601	1531	1530	1544	1556	1561	1528	1543	1555	1560	1557	1569	1575	1581	1586	1592	41.00
42.00	1630	1590	1589	1603	1615	1620	1550	1549	1563	1575	1581	1548	1562	1574	1579	1577	1589	1594	1600	1606	1611	42.00
43.00	1649	1609	1608	1622	1634	1640	1569	1568	1583	1595	1600	1567	1582	1593	1599	1596	1608	1613	1620	1625	1631	43.00
44.00	1668	1628	1627	1642	1654	1659	1588	1588	1602	1614	1619	1587	1601	1613	1618	1616	1627	1633	1639	1644	1650	44.00
45.00	1688	1648	1647	1661	1673	1678	1608	1607	1622	1633	1639	1606	1620	1632	1638	1635	1647	1652	1658	1664	1669	45.00
46.00	1707	1667	1666	1681	1692	1698	1627	1626	1641	1653	1658	1625	1640	1651	1657	1654	1666	1671	1678	1683	1689	46.00
47.00	1726	1686	1685	1700	1712	1717	1647	1646	1660	1672	1677	1644	1659	1671	1676	1674	1685	1691	1697	1702	1708	47.00
48.00	1746	1706	1705	1719	1731	1736	1666	1665	1679	1691	1697	1664	1678	1690	1695	1693	1704	1710	1716	1722	1727	48.00
49.00	1765	1725	1724	1739	1750	1756	1685	1684	1699	1710	1716	1683	1697	1709	1715	1712	1724	1729	1735	1741	1746	49.00
50.00	1784	1744	1743	1758	1769	1775	1704	1703	1718	1730	1735	1702	1717	1728	1734	1731	1743	1748	1755	1760	1765	50.00

TRANSMISSION-LOSS (DB RE 1 FT) FREQUENCY= 50.0 HZ WAVEHT= 0.0 FT CONVRT= 0.540000E+00 DB

HS (FT)	250	250	250	250	250	250	500	500	500	500	500	750	750	750	750	1000	1000	1000	1250	1250	1500	
HR (FT)	250	500	750	1000	1250	1500	500	750	1000	1250	1500	750	1000	1250	1500	1000	1250	1500	1250	1500	1500	
R (KM)																						
1.00	632	639	625	621	617	605	622	633	618	598	577	603	587	566	545	550	528	504	501	475	446	1.00
2.00	680	679	683	697	705	704	666	693	775	727	716	678	679	682	676	668	664	652	654	639	620	2.00
3.00	708	701	718	745	759	751	688	718	769	789	795	721	735	745	746	734	741	738	744	739	731	3.00
4.00	727	716	741	790	797	807	703	735	790	815	827	751	777	791	794	794	794	794	802	802	801	4.00
5.00	741	729	759	805	826	834	715	747	805	831	844	772	809	827	832	826	837	839	848	850	850	5.00
6.00	753	740	770	824	847	859	726	759	817	843	857	787	834	854	862	861	876	879	889	891	893	6.00
7.00	763	749	791	837	862	874	735	763	826	853	867	799	851	874	885	890	908	913	923	929	931	7.00
8.00	771	758	790	847	874	887	744	776	835	863	876	809	864	889	901	912	933	941	952	958	964	8.00
9.00	779	765	794	855	883	895	752	784	843	870	884	816	874	900	913	927	951	961	973	982	991	9.00
10.00	787	773	805	854	891	904	759	791	850	877	901	824	882	909	922	938	963	976	988	996	1010	10.00
11.00	793	779	812	871	898	912	765	799	857	884	898	831	889	916	930	947	973	986	999	1011	1024	11.00
12.00	800	786	818	877	904	918	772	805	864	891	905	837	896	923	937	954	981	994	1007	1021	1034	12.00
13.00	806	792	825	884	911	925	778	811	870	897	911	843	902	929	943	961	988	1001	1015	1029	1042	13.00
14.00	812	798	831	890	917	930	784	817	876	903	917	849	908	935	949	967	994	1008	1021	1035	1049	14.00
15.00	819	804	835	895	922	936	790	822	881	908	922	855	914	941	955	973	1000	1014	1027	1041	1054	15.00
16.00	823	809	842	901	928	942	795	828	887	914	928	860	919	946	960	978	1005	1019	1032	1046	1060	16.00
17.00	829	814	847	906	933	947	800	833	892	919	933	866	925	952	966	984	1011	1025	1038	1052	1065	17.00
18.00	833	820	852	911	938	952	806	838	897	924	938	871	930	957	971	989	1016	1030	1043	1057	1071	18.00
19.00	838	825	857	916	943	957	811	843	902	929	943	876	935	962	976	994	1021	1035	1048	1062	1076	19.00
20.00	843	829	862	921	948	962	816	849	907	934	948	881	940	967	981	999	1026	1040	1053	1067	1081	20.00
21.00	849	834	867	926	953	967	820	853	912	939	953	885	944	972	985	1004	1031	1044	1058	1071	1085	21.00
22.00	853	839	872	931	958	971	825	858	917	944	958	890	949	976	990	1008	1035	1049	1062	1076	1090	22.00
23.00	857	844	876	935	962	976	830	862	921	949	962	895	954	981	995	1013	1040	1054	1067	1081	1095	23.00
24.00	862	848	881	940	967	981	834	867	926	953	967	899	958	985	999	1017	1044	1058	1071	1085	1099	24.00
25.00	866	853	885	944	971	985	839	871	930	957	971	904	963	990	1004	1022	1049	1063	1076	1090	1104	25.00
26.00	871	857	889	949	975	989	843	876	935	962	975	908	967	994	1008	1026	1053	1067	1080	1094	1108	26.00
27.00	875	861	894	953	980	994	847	880	939	966	980	912	971	998	1012	1030	1057	1071	1085	1099	1112	27.00
28.00	879	865	898	957	984	998	852	884	943	970	984	917	976	1003	1017	1035	1062	1076	1089	1103	1117	28.00
29.00	884	870	902	961	988	1002	856	888	947	974	988	921	980	1007	1021	1039	1066	1080	1093	1107	1121	29.00
30.00	888	874	906	965	992	1006	860	892	951	978	992	925	984	1011	1025	1043	1070	1084	1097	1111	1125	30.00
31.00	892	879	910	969	996	1010	864	897	955	983	996	929	988	1015	1029	1047	1074	1088	1101	1115	1129	31.00
32.00	896	882	914	974	1001	1014	868	901	960	987	1001	933	992	1019	1033	1051	1078	1092	1105	1119	1133	32.00
33.00	900	886	919	978	1005	1019	872	905	964	991	1005	937	996	1023	1037	1055	1082	1096	1109	1123	1137	33.00
34.00	904	890	922	981	1009	1023	876	909	968	995	1009	941	1000	1027	1041	1059	1086	1100	1113	1127	1141	34.00
35.00	908	894	926	985	1012	1025	880	912	972	999	1012	945	1004	1031	1045	1063	1090	1104	1117	1131	1145	35.00
36.00	912	898	930	989	1016	1030	884	916	975	1002	1016	949	1008	1035	1049	1067	1094	1108	1121	1135	1149	36.00
37.00	916	902	934	993	1020	1033	888	920	979	1006	1020	953	1012	1039	1053	1071	1098	1112	1125	1139	1153	37.00
38.00	919	905	938	997	1024	1038	892	924	983	1010	1024	957	1016	1043	1057	1075	1102	1116	1129	1143	1157	38.00
39.00	923	909	942	1001	1028	1042	895	929	987	1014	1028	960	1019	1046	1060	1078	1105	1119	1133	1146	1160	39.00
40.00	927	913	946	1005	1032	1045	899	932	991	1018	1032	964	1023	1050	1064	1082	1109	1123	1136	1150	1164	40.00
41.00	931	917	949	1008	1035	1049	903	935	994	1021	1035	968	1027	1054	1068	1086	1113	1127	1140	1154	1168	41.00
42.00	934	920	953	1012	1039	1053	907	939	998	1025	1039	972	1031	1058	1072	1090	1117	1131	1144	1158	1172	42.00
43.00	938	924	957	1016	1043	1057	910	943	1002	1029	1043	975	1034	1061	1075	1093	1120	1134	1147	1161	1175	43.00
44.00	942	928	960	1019	1046	1060	914	946	1005	1032	1046	979	1038	1065	1079	1097	1124	1138	1151	1165	1179	44.00
45.00	945	931	964	1023	1050	1064	918	950	1009	1036	1050	983	1042	1069	1083	1101	1128	1142	1155	1169	1183	45.00
46.00	949	935	968	1027	1054	1068	921	953	1013	1040	1054	986	1045	1072	1086	1104	1131	1145	1158	1172	1186	46.00
47.00	953	939	971	1030	1057	1071	925	957	1016	1043	1057	990	1049	1076	1090	1108	1135	1149	1162	1176	1190	47.00
48.00	956	942	975	1034	1061	1075	928	961	1020	1047	1061	993	1053	1080	1093	1112	1139	1152	1166	1179	1193	48.00
49.00	960	946	978	1037	1064	1078	932	964	1024	1051	1064	997	1056	1083	1097	1115	1142	1156	1169	1183	1197	49.00
50.00	963	949	982	1041	1068	1082	935	968	1027	1054	1068	1001	1060	1087	1101	1119	1146	1160	1173	1187	1200	50.00

TRANSMISSION-LOSS (DB RE 1 FT) FREQUENCY= 100.0 HZ WAVEHT= 0.0 FT CONVRT= 9.540000E+00 DB

HS (FT)	250	250	250	250	250	250	500	500	500	500	500	750	750	750	750	1000	1000	1000	1250	1250	1500	
HR (FT)	250	500	750	1000	1250	1500	500	750	1000	1250	1500	750	1000	1250	1500	1000	1250	1500	1250	1500	1500	
R (KM)																						
1.00	619	642	659	649	674	627	636	625	597	592	591	600	563	558	549	516	511	502	505	494	493	1.00
2.00	650	640	713	712	734	727	685	687	682	686	681	670	652	653	647	607	605	599	603	595	586	2.00
3.00	664	699	739	749	803	802	711	719	739	752	751	707	716	724	721	678	680	675	680	674	668	3.00
4.00	681	713	756	824	849	854	729	739	778	799	802	730	763	779	779	739	744	741	747	743	739	4.00
5.00	691	723	764	815	877	887	741	754	806	831	839	745	795	818	824	788	759	798	806	804	801	5.00
6.00	699	732	774	861	905	909	751	765	827	856	865	757	818	846	855	827	843	846	856	857	856	6.00
7.00	706	739	787	872	909	923	760	777	844	875	887	768	835	867	878	859	879	884	896	900	902	7.00
8.00	712	746	795	882	915	933	767	785	859	891	903	777	849	882	895	883	907	914	928	934	939	8.00
9.00	717	751	802	891	929	944	774	793	870	904	917	784	860	895	908	904	930	939	954	962	969	9.00
10.00	722	756	809	898	937	952	780	800	880	915	929	792	870	905	919	922	950	960	976	985	994	10.00
11.00	726	761	815	904	944	959	785	807	888	925	939	799	878	914	929	938	967	978	996	1005	1015	11.00
12.00	730	765	821	911	950	966	790	813	896	933	947	805	885	922	937	951	983	994	1012	1023	1033	12.00
13.00	734	769	826	917	956	973	795	819	903	940	955	811	892	929	944	963	995	1008	1027	1039	1050	13.00
14.00	737	773	831	923	962	979	799	824	909	947	962	816	898	936	951	972	1006	1020	1039	1052	1064	14.00
15.00	740	776	836	928	967	987	803	830	915	953	969	822	904	941	957	981	1016	1030	1050	1063	1076	15.00
16.00	743	779	840	933	973	988	806	835	920	958	974	827	909	947	962	988	1024	1038	1059	1073	1087	16.00
17.00	746	782	844	938	978	993	810	839	925	963	979	832	915	952	968	995	1031	1046	1067	1081	1095	17.00
18.00	749	785	849	943	982	998	813	844	930	968	984	837	920	957	973	1000	1037	1052	1074	1089	1103	18.00
19.00	751	789	853	947	987	1003	816	849	935	973	989	842	924	962	978	1006	1043	1059	1080	1095	1110	19.00
20.00	753	790	856	952	991	1007	819	853	940	979	994	846	929	967	982	1011	1048	1064	1086	1101	1116	20.00
21.00	756	792	860	956	996	1012	822	857	944	983	998	851	934	971	987	1016	1053	1069	1091	1106	1121	21.00
22.00	758	795	864	960	1000	1016	825	861	948	987	1002	855	938	976	991	1020	1058	1073	1096	1111	1126	22.00
23.00	760	797	867	964	1004	1020	828	865	953	991	1007	859	942	980	996	1025	1063	1078	1100	1116	1131	23.00
24.00	762	799	871	968	1008	1024	830	869	957	995	1011	864	947	984	1000	1029	1067	1082	1105	1120	1135	24.00
25.00	764	801	874	972	1012	1028	833	873	961	1000	1015	868	951	989	1004	1033	1071	1087	1109	1124	1140	25.00
26.00	765	803	877	976	1015	1032	835	877	965	1004	1019	872	955	993	1008	1038	1075	1091	1113	1129	1144	26.00
27.00	767	805	880	979	1020	1035	837	880	969	1008	1023	876	959	997	1012	1042	1079	1095	1117	1133	1149	27.00
28.00	769	807	883	982	1024	1040	839	884	973	1011	1027	880	963	1001	1016	1046	1083	1099	1121	1137	1152	28.00
29.00	770	809	886	987	1027	1043	841	887	977	1015	1031	884	967	1005	1020	1049	1087	1103	1125	1141	1156	29.00
30.00	772	810	889	990	1031	1047	843	891	980	1019	1035	887	971	1008	1024	1053	1091	1107	1129	1145	1160	30.00
31.00	773	812	891	993	1034	1050	845	894	984	1023	1038	891	974	1012	1029	1057	1095	1111	1133	1148	1164	31.00
32.00	775	813	894	997	1038	1053	847	897	988	1026	1042	895	978	1016	1032	1061	1099	1114	1137	1152	1168	32.00
33.00	776	815	897	1000	1041	1057	849	901	991	1030	1046	899	982	1020	1035	1065	1103	1118	1140	1156	1171	33.00
34.00	778	816	899	1003	1045	1061	851	904	995	1034	1049	902	986	1023	1039	1068	1106	1122	1144	1160	1175	34.00
35.00	779	818	901	1006	1048	1064	852	907	998	1037	1053	906	989	1027	1043	1072	1110	1125	1148	1163	1179	35.00
36.00	780	819	904	1009	1051	1067	854	910	1002	1041	1056	909	993	1031	1046	1076	1113	1129	1151	1167	1182	36.00
37.00	781	820	906	1012	1055	1071	856	913	1005	1044	1060	913	996	1034	1050	1079	1117	1133	1155	1170	1186	37.00
38.00	783	822	909	1015	1058	1074	857	916	1008	1048	1063	916	1000	1038	1053	1083	1121	1136	1158	1174	1189	38.00
39.00	784	823	910	1018	1061	1077	859	919	1012	1051	1067	920	1003	1041	1057	1086	1124	1140	1162	1177	1193	39.00
40.00	785	824	913	1021	1064	1080	860	922	1015	1054	1070	923	1007	1045	1060	1090	1128	1143	1165	1181	1196	40.00
41.00	786	825	915	1024	1067	1083	861	924	1018	1058	1073	927	1010	1048	1064	1093	1131	1147	1169	1184	1200	41.00
42.00	787	826	917	1027	1070	1086	863	927	1021	1061	1077	930	1014	1052	1067	1097	1134	1150	1172	1188	1203	42.00
43.00	788	828	919	1030	1073	1089	864	930	1025	1064	1080	933	1017	1055	1071	1100	1138	1153	1176	1191	1207	43.00
44.00	789	829	920	1032	1076	1092	865	932	1028	1068	1083	937	1020	1059	1074	1103	1141	1157	1179	1195	1210	44.00
45.00	790	830	922	1035	1079	1095	867	935	1031	1071	1086	940	1024	1062	1077	1107	1145	1160	1182	1198	1214	45.00
46.00	791	831	924	1037	1082	1098	868	937	1034	1074	1090	943	1027	1065	1081	1110	1148	1163	1186	1201	1217	46.00
47.00	792	832	926	1040	1084	1101	869	940	1037	1077	1093	947	1030	1068	1084	1113	1151	1167	1189	1205	1220	47.00
48.00	793	833	927	1043	1087	1103	870	942	1040	1080	1096	950	1034	1072	1087	1117	1155	1170	1192	1208	1223	48.00
49.00	794	834	929	1045	1090	1106	871	945	1043	1083	1099	953	1037	1075	1091	1120	1158	1173	1196	1211	1227	49.00
50.00	795	835	931	1047	1092	1109	873	947	1046	1086	1102	956	1040	1078	1094	1123	1161	1177	1199	1215	1230	50.00

TRANSMISSION-LOSS (DB RE 1 FT) FREQUENCY= 250.0 HZ WAVEHT= 0.0 FT CONVRT= 9.5400000E+00 DB

ZS (FT)	250	250	250	250	250	250	500	500	500	500	500	750	750	750	750	1000	1000	1000	1250	1250	1500	
ZR (FT)	250	500	750	1000	1250	1500	500	750	1000	1250	1500	750	1000	1250	1500	1000	1250	1500	1250	1500	1500	
R (KM)																						
1.00	632	647	673	664	677	677	624	655	640	656	656	625	624	640	641	591	607	607	621	621	620	1.00
2.00	665	691	721	715	749	749	661	704	706	725	726	670	689	708	710	657	675	676	691	692	692	2.00
3.00	694	704	749	791	807	807	681	732	753	778	781	695	738	762	764	707	729	731	747	749	749	3.00
4.00	697	718	766	838	858	859	695	750	788	820	825	714	778	806	810	747	773	777	796	798	800	4.00
5.00	706	728	774	879	903	905	706	763	813	[illegible]	859	727	809	843	849	780	811	816	837	841	844	5.00
6.00	715	737	788	911	942	947	715	773	832	875	885	737	833	873	881	807	842	849	873	878	883	6.00
7.00	721	743	795	935	974	991	722	781	847	893	905	746	851	896	906	829	867	876	902	909	916	7.00
8.00	727	749	802	957	995	1009	728	789	860	908	920	753	864	914	925	848	888	898	926	935	943	8.00
9.00	732	755	807	964	1016	1027	734	795	872	920	933	759	875	927	940	864	906	917	946	956	966	9.00
10.00	737	759	812	973	1028	1042	738	801	882	931	944	765	884	937	951	879	922	933	964	974	985	10.00
11.00	741	764	817	981	1037	1057	743	807	892	941	955	770	891	946	960	893	936	948	979	990	1001	11.00
12.00	745	767	821	987	1045	1060	747	812	900	951	964	775	897	953	967	905	949	961	993	1005	1016	12.00
13.00	749	771	825	992	1051	1066	751	817	906	959	973	780	903	959	974	917	962	974	1006	1018	1029	13.00
14.00	752	774	829	997	1056	1071	754	821	916	967	981	784	908	964	979	928	973	986	1018	1030	1042	14.00
15.00	755	778	832	1001	1061	1075	758	825	923	975	989	788	912	969	984	939	985	997	1029	1042	1054	15.00
16.00	758	780	835	1006	1065	1081	761	830	930	982	996	792	917	974	989	950	995	1008	1040	1053	1065	16.00
17.00	761	783	838	1009	1069	1085	763	833	936	989	1003	795	921	978	993	960	1006	1018	1051	1063	1076	17.00
18.00	763	786	841	1013	1073	1089	766	837	942	995	1010	799	925	982	998	969	1016	1028	1061	1074	1086	18.00
19.00	766	788	844	1017	1077	1093	769	841	949	1001	1016	802	929	986	1002	978	1025	1038	1071	1084	1096	19.00
20.00	768	791	846	1020	1080	1095	771	844	953	1007	1022	806	933	990	1005	987	1034	1047	1081	1094	1106	20.00
21.00	770	793	849	1023	1084	1099	773	849	958	1012	1027	809	937	994	1009	996	1043	1056	1090	1103	1116	21.00
22.00	772	795	851	1027	1087	1102	776	851	963	1017	1032	812	940	997	1013	1004	1052	1065	1099	1112	1125	22.00
23.00	774	797	853	1030	1090	1105	778	854	967	1022	1037	815	944	1001	1016	1012	1061	1074	1108	1121	1134	23.00
24.00	776	799	856	1033	1093	1108	780	857	972	1027	1041	817	947	1004	1020	1020	1069	1082	1117	1130	1142	24.00
25.00	778	801	858	1035	1096	1112	782	860	976	1031	1046	820	950	1008	1023	1027	1076	1090	1125	1139	1151	25.00
26.00	780	803	860	1038	1099	1115	784	863	980	1035	1050	823	954	1011	1026	1034	1084	1097	1133	1146	1159	26.00
27.00	782	804	862	1041	1102	1119	786	866	983	1039	1054	825	957	1014	1029	1040	1091	1104	1140	1153	1167	27.00
28.00	783	806	864	1044	1105	1122	787	869	987	1043	1058	828	960	1017	1032	1047	1097	1111	1147	1161	1174	28.00
29.00	785	808	865	1046	1107	1127	789	871	990	1046	1061	830	963	1020	1036	1053	1104	1119	1154	1168	1181	29.00
30.00	787	809	867	1049	1110	1126	791	874	994	1050	1065	833	966	1023	1038	1058	1110	1124	1161	1174	1188	30.00
31.00	788	811	869	1051	1112	1128	792	876	997	1053	1068	835	969	1026	1041	1063	1116	1130	1167	1181	1195	31.00
32.00	790	812	871	1054	1115	1131	794	879	1000	1056	1072	837	971	1029	1044	1068	1121	1135	1173	1187	1201	32.00
33.00	791	814	872	1056	1117	1133	795	881	1003	1060	1075	839	974	1032	1047	1073	1126	1141	1179	1193	1207	33.00
34.00	792	815	874	1058	1120	1135	797	884	1006	1063	1078	841	977	1034	1050	1078	1131	1145	1184	1198	1212	34.00
35.00	794	816	875	1060	1122	1138	798	886	1009	1066	1081	844	980	1037	1053	1082	1136	1150	1189	1203	1218	35.00
36.00	795	818	877	1063	1124	1140	799	889	1012	1069	1084	846	982	1040	1055	1086	1140	1155	1194	1208	1223	36.00
37.00	796	819	878	1065	1127	1143	801	891	1015	1072	1087	848	985	1042	1058	1090	1144	1159	1198	1213	1227	37.00
38.00	798	820	880	1067	1129	1145	802	893	1018	1075	1090	850	987	1045	1061	1094	1148	1163	1202	1217	1232	38.00
39.00	799	822	881	1069	1131	1147	803	895	1021	1077	1093	851	990	1048	1063	1097	1152	1167	1206	1221	1236	39.00
40.00	800	823	882	1071	1133	1149	805	897	1023	1080	1095	853	992	1050	1066	1100	1156	1171	1210	1225	1240	40.00
41.00	801	824	884	1073	1135	1151	806	899	1026	1083	1099	855	995	1053	1068	1104	1159	1174	1214	1229	1244	41.00
42.00	802	825	885	1075	1137	1153	807	901	1029	1085	1101	857	997	1055	1071	1107	1162	1177	1218	1233	1248	42.00
43.00	803	826	886	1077	1139	1156	808	904	1031	1088	1103	859	1000	1058	1073	1110	1166	1181	1221	1236	1251	43.00
44.00	804	827	888	1079	1141	1158	809	906	1034	1091	1106	860	1002	1060	1076	1113	1169	1184	1224	1239	1254	44.00
45.00	805	828	889	1081	1143	1159	810	907	1037	1093	1109	862	1005	1063	1078	1116	1172	1187	1227	1243	1258	45.00
46.00	807	829	890	1082	1145	1161	812	909	1039	1096	1111	864	1007	1065	1080	1119	1175	1190	1230	1246	1261	46.00
47.00	808	830	891	1084	1147	1163	813	911	1042	1098	1114	865	1009	1067	1083	1121	1177	1193	1233	1249	1264	47.00
48.00	809	831	892	1086	1149	1165	814	913	1044	1101	1116	867	1012	1070	1085	1124	1180	1195	1236	1251	1267	48.00
49.00	810	832	893	1089	1151	1167	815	915	1047	1103	1119	868	1014	1072	1087	1127	1183	1198	1239	1254	1270	49.00
50.00	810	833	894	1090	1153	1169	816	917	1049	1106	1121	870	1016	1074	1090	1129	1185	1201	1242	1257	1272	50.00

TRANSMISSION-LOSS (DB RE 1 FT) FREQUENCY= 500.0 HZ WAVEHT= 0.0 FT CONVRT= 9.540000E+00 DB

ZS (FT)	250	250	250	250	250	250	500	500	500	500	500	750	750	750	750	1000	1000	1000	1250	1250	1500	
ZR (FT)	250	500	750	1000	1250	1500	500	750	1000	1250	1500	750	1000	1250	1500	1000	1250	1500	1250	1500	1500	
R (NM)																						
1.00	619	664	676	709	737	740	632	674	693	730	736	637	663	694	698	626	658	662	688	692	695	1.00
2.00	650	701	712	760	798	807	663	713	743	783	790	673	719	753	754	692	716	721	747	752	756	2.00
3.00	669	720	732	814	846	851	682	734	779	822	829	694	760	797	803	723	760	765	793	798	803	3.00
4.00	681	733	746	857	888	897	695	749	808	853	861	709	793	834	841	758	796	803	832	838	843	4.00
5.00	691	743	757	894	924	930	706	760	832	879	889	720	820	864	872	788	828	836	867	873	879	5.00
6.00	699	751	766	920	955	962	714	769	853	901	911	729	842	890	899	814	857	865	897	904	911	6.00
7.00	706	759	773	930	981	990	721	776	872	922	932	737	860	911	922	837	881	890	924	932	940	7.00
8.00	712	764	779	945	1002	1012	728	783	889	940	951	743	875	929	940	857	904	913	948	957	965	8.00
9.00	718	770	785	957	1018	1030	733	789	904	956	968	749	888	944	956	876	924	934	970	979	988	9.00
10.00	723	775	790	967	1031	1044	738	794	918	972	983	754	899	956	969	893	942	952	990	1000	1009	10.00
11.00	727	779	794	975	1041	1055	743	798	931	986	998	759	909	967	980	909	959	970	1008	1019	1028	11.00
12.00	731	783	799	983	1049	1063	747	803	944	999	1011	764	919	977	990	923	974	985	1024	1035	1045	12.00
13.00	735	787	803	989	1056	1071	751	807	955	1011	1024	768	927	986	1000	937	989	1000	1039	1050	1061	13.00
14.00	739	791	806	993	1062	1077	754	810	965	1023	1035	771	935	994	1009	949	1002	1014	1054	1065	1076	14.00
15.00	742	794	809	998	1067	1083	759	814	974	1033	1046	775	942	1002	1016	961	1015	1026	1067	1078	1090	15.00
16.00	745	797	813	1002	1072	1087	761	817	983	1043	1056	778	949	1009	1023	972	1026	1039	1079	1091	1103	16.00
17.00	748	800	816	1007	1077	1092	764	820	991	1052	1066	782	956	1016	1030	983	1037	1050	1091	1103	1115	17.00
18.00	751	803	819	1011	1081	1097	767	823	999	1061	1074	785	962	1023	1037	993	1048	1060	1102	1115	1127	18.00
19.00	753	805	821	1015	1085	1101	769	825	1004	1068	1082	787	968	1029	1044	1002	1058	1071	1113	1125	1138	19.00
20.00	756	808	824	1018	1089	1105	772	829	1010	1075	1089	790	974	1036	1050	1010	1067	1080	1123	1135	1148	20.00
21.00	759	811	826	1022	1093	1108	774	831	1016	1082	1096	793	980	1042	1056	1019	1076	1089	1132	1145	1159	21.00
22.00	761	813	829	1025	1097	1112	777	833	1021	1088	1102	795	985	1048	1062	1026	1084	1097	1141	1154	1167	22.00
23.00	763	815	831	1028	1100	1116	779	836	1026	1093	1108	798	991	1053	1067	1034	1092	1105	1149	1162	1176	23.00
24.00	765	818	833	1031	1104	1119	781	838	1030	1099	1113	800	996	1059	1073	1040	1099	1113	1157	1171	1184	24.00
25.00	767	820	835	1035	1107	1122	783	840	1034	1103	1118	802	1001	1064	1078	1047	1106	1120	1165	1178	1192	25.00
26.00	769	822	837	1037	1110	1125	785	842	1038	1108	1123	804	1006	1069	1084	1053	1113	1127	1172	1185	1199	26.00
27.00	771	824	839	1040	1113	1129	787	844	1042	1112	1127	806	1011	1075	1089	1059	1120	1133	1179	1192	1206	27.00
28.00	773	826	841	1043	1116	1131	789	846	1045	1116	1131	808	1016	1080	1094	1065	1126	1140	1185	1199	1213	28.00
29.00	775	827	843	1046	1119	1134	791	849	1049	1119	1135	810	1020	1085	1099	1070	1131	1145	1192	1205	1219	29.00
30.00	777	829	845	1048	1121	1137	793	850	1051	1123	1138	812	1025	1089	1104	1076	1137	1151	1198	1211	1225	30.00
31.00	778	831	847	1051	1124	1139	794	852	1054	1126	1141	814	1029	1094	1109	1081	1143	1157	1203	1217	1231	31.00
32.00	780	833	848	1053	1127	1142	796	853	1057	1129	1145	816	1033	1099	1113	1086	1148	1162	1209	1223	1237	32.00
33.00	782	834	850	1055	1129	1145	798	855	1060	1132	1148	817	1037	1103	1118	1090	1153	1167	1214	1228	1242	33.00
34.00	783	836	852	1058	1132	1147	799	857	1063	1135	1151	819	1041	1108	1122	1095	1158	1172	1219	1234	1248	34.00
35.00	785	838	853	1060	1134	1149	801	858	1065	1138	1154	821	1045	1112	1127	1100	1162	1177	1224	1239	1253	35.00
36.00	787	839	855	1063	1136	1152	802	860	1068	1141	1156	822	1049	1116	1131	1104	1167	1181	1229	1244	1258	36.00
37.00	788	841	856	1065	1139	1154	804	861	1070	1143	1159	824	1053	1120	1135	1108	1172	1186	1234	1249	1263	37.00
38.00	790	842	858	1067	1141	1156	805	863	1072	1146	1161	825	1056	1124	1139	1112	1176	1190	1239	1253	1267	38.00
39.00	791	844	859	1069	1143	1159	807	864	1074	1148	1164	827	1060	1128	1143	1116	1180	1195	1243	1258	1272	39.00
40.00	792	845	861	1071	1145	1161	808	866	1077	1151	1166	828	1063	1132	1147	1120	1184	1199	1248	1262	1276	40.00
41.00	794	846	862	1073	1147	1163	810	867	1079	1153	1169	830	1066	1136	1151	1124	1188	1203	1252	1266	1281	41.00
42.00	795	848	864	1075	1149	1165	811	869	1081	1155	1171	831	1069	1139	1154	1128	1192	1207	1256	1271	1285	42.00
43.00	796	849	865	1077	1151	1167	812	870	1083	1157	1173	833	1072	1143	1158	1131	1196	1211	1260	1275	1289	43.00
44.00	798	851	866	1079	1153	1169	814	871	1085	1160	1175	834	1075	1146	1161	1135	1200	1215	1264	1279	1293	44.00
45.00	799	852	868	1081	1155	1171	815	873	1087	1162	1177	835	1078	1150	1165	1138	1204	1219	1268	1283	1297	45.00
46.00	800	853	869	1083	1157	1173	816	874	1089	1164	1179	836	1081	1153	1168	1141	1207	1222	1272	1287	1301	46.00
47.00	802	854	870	1085	1159	1175	818	875	1091	1166	1181	838	1084	1156	1171	1145	1211	1226	1276	1291	1305	47.00
48.00	803	856	871	1086	1161	1177	819	877	1092	1168	1183	839	1086	1159	1174	1148	1214	1229	1280	1294	1309	48.00
49.00	804	857	873	1089	1163	1179	820	878	1094	1169	1185	840	1089	1162	1177	1151	1218	1233	1283	1298	1313	49.00
50.00	805	859	874	1090	1165	1180	821	879	1096	1171	1187	842	1091	1165	1180	1154	1221	1236	1287	1302	1316	50.00

TRANSMISSION-LOSS (DB RE 1 FT) FREQUENCY= 1000.0 HZ WAVEHT= 0.0 FT CONVRT= 9.540000E+00 DB

ZS (FT)	250	250	250	250	250	250	500	500	500	500	500	750	750	750	750	1000	1000	1000	1250	1250	1500	
ZR (FT)	250	500	750	1000	1250	1500	500	750	1000	1250	1500	750	1000	1250	1500	1000	1250	1500	1250	1500	1500	
R (NM)																						
1.00	612	656	709	760	805	817	628	692	720	772	791	642	710	764	774	655	705	713	754	762	770	1.00
2.00	644	698	742	810	861	869	660	714	766	821	830	675	756	811	820	704	755	764	805	913	821	2.00
3.00	663	717	761	845	899	907	679	734	799	855	865	695	788	844	854	740	792	800	842	851	859	3.00
4.00	677	731	775	873	930	939	693	749	825	883	893	709	814	871	882	769	822	831	874	883	891	4.00
5.00	689	742	787	896	956	965	704	760	847	907	918	721	836	895	906	795	849	859	902	911	920	5.00
6.00	697	752	795	915	978	989	713	769	866	927	939	730	855	916	927	817	873	883	927	937	946	6.00
7.00	705	760	804	931	997	1009	721	779	883	945	957	738	873	935	946	839	895	905	950	960	970	7.00
8.00	712	767	812	944	1013	1025	728	785	897	961	973	746	888	952	963	857	915	926	972	982	992	8.00
9.00	719	773	819	955	1027	1040	735	792	911	975	988	752	902	967	979	875	934	945	991	1002	1012	9.00
10.00	724	779	824	965	1035	1052	741	798	923	989	1002	759	915	981	994	891	951	962	1010	1020	1031	10.00
11.00	729	785	830	974	1049	1062	746	803	935	1002	1014	764	927	994	1007	906	967	979	1027	1039	1049	11.00
12.00	734	790	835	981	1057	1072	751	808	946	1013	1026	769	937	1006	1019	920	982	994	1043	1054	1065	12.00
13.00	739	794	840	989	1065	1080	756	813	956	1024	1037	774	947	1017	1030	934	996	1008	1059	1069	1081	13.00
14.00	741	799	845	995	1073	1087	761	819	966	1034	1049	778	956	1027	1040	947	1010	1022	1072	1084	1095	14.00
15.00	749	803	849	1001	1079	1094	765	822	976	1044	1058	783	965	1036	1049	959	1023	1035	1086	1097	1109	15.00
16.00	752	807	853	1007	1085	1100	769	827	985	1054	1067	787	973	1044	1059	970	1035	1047	1099	1110	1122	16.00
17.00	755	811	857	1012	1092	1105	773	831	993	1063	1076	791	980	1053	1066	982	1047	1059	1111	1123	1135	17.00
18.00	760	815	861	1018	1097	1112	777	834	1002	1072	1085	795	987	1060	1074	992	1058	1070	1122	1135	1147	18.00
19.00	763	818	865	1023	1103	1119	780	838	1010	1080	1094	799	994	1067	1081	1003	1068	1081	1134	1146	1158	19.00
20.00	767	822	868	1028	1108	1125	784	842	1017	1089	1102	802	1000	1074	1088	1012	1079	1092	1144	1157	1169	20.00
21.00	770	825	872	1032	1113	1129	787	845	1025	1096	1110	805	1007	1081	1095	1022	1089	1102	1155	1168	1180	21.00
22.00	773	829	875	1038	1118	1133	790	849	1032	1104	1117	809	1012	1087	1102	1031	1098	1111	1165	1178	1190	22.00
23.00	777	832	879	1042	1123	1137	794	852	1039	1111	1125	812	1018	1093	1109	1040	1108	1121	1175	1188	1200	23.00
24.00	780	835	882	1047	1127	1142	797	855	1046	1118	1132	815	1024	1099	1114	1049	1117	1130	1184	1197	1210	24.00
25.00	783	838	885	1051	1132	1147	800	859	1052	1125	1139	818	1029	1105	1119	1057	1125	1139	1193	1206	1219	25.00
26.00	786	841	888	1055	1136	1151	803	861	1059	1132	1146	821	1034	1111	1125	1065	1134	1147	1202	1215	1228	26.00
27.00	789	844	891	1059	1140	1155	806	864	1065	1138	1152	824	1040	1116	1131	1073	1142	1155	1211	1224	1237	27.00
28.00	791	847	894	1063	1144	1159	809	867	1071	1145	1159	827	1045	1121	1136	1080	1150	1163	1219	1232	1245	28.00
29.00	794	850	897	1067	1148	1163	811	870	1077	1151	1165	830	1049	1126	1141	1087	1158	1171	1227	1240	1254	29.00
30.00	797	853	899	1071	1152	1167	814	873	1082	1157	1171	832	1054	1131	1146	1094	1165	1179	1235	1249	1262	30.00
31.00	800	855	902	1075	1156	1171	817	875	1088	1163	1177	835	1059	1136	1151	1101	1172	1186	1242	1256	1269	31.00
32.00	802	858	905	1078	1160	1175	819	878	1093	1168	1183	838	1063	1141	1156	1107	1179	1193	1249	1263	1277	32.00
33.00	805	860	908	1082	1164	1179	822	881	1098	1174	1188	840	1068	1146	1160	1114	1186	1199	1257	1270	1284	33.00
34.00	807	863	910	1085	1167	1183	824	883	1103	1179	1194	843	1072	1150	1165	1120	1192	1206	1263	1277	1291	34.00
35.00	810	865	913	1089	1171	1186	827	886	1108	1185	1199	845	1076	1155	1169	1126	1198	1212	1270	1284	1298	35.00
36.00	812	868	915	1092	1174	1190	829	888	1113	1190	1204	848	1080	1159	1174	1131	1204	1218	1276	1290	1304	36.00
37.00	815	870	918	1096	1178	1193	832	891	1118	1195	1209	850	1084	1163	1178	1137	1210	1224	1283	1297	1311	37.00
38.00	817	873	920	1099	1181	1196	834	893	1122	1200	1214	853	1086	1168	1182	1142	1215	1230	1289	1303	1317	38.00
39.00	820	875	923	1102	1185	1200	837	896	1127	1205	1219	855	1092	1172	1187	1147	1222	1236	1295	1309	1323	39.00
40.00	822	877	925	1105	1188	1203	839	898	1131	1209	1224	858	1096	1176	1191	1152	1227	1241	1300	1315	1329	40.00
41.00	824	880	928	1108	1191	1205	841	900	1135	1214	1229	860	1100	1180	1195	1157	1232	1246	1306	1320	1334	41.00
42.00	827	882	930	1112	1194	1210	844	903	1140	1218	1233	862	1103	1184	1199	1162	1237	1252	1311	1326	1340	42.00
43.00	829	884	932	1115	1198	1213	846	905	1144	1223	1238	865	1107	1188	1202	1167	1242	1257	1317	1331	1345	43.00
44.00	831	886	935	1118	1201	1216	848	907	1148	1227	1242	867	1111	1191	1205	1172	1247	1262	1322	1336	1350	44.00
45.00	833	889	937	1121	1204	1219	850	909	1152	1231	1246	869	1114	1195	1210	1176	1252	1266	1327	1341	1356	45.00
46.00	836	891	939	1124	1207	1222	853	912	1155	1236	1250	871	1118	1199	1214	1180	1257	1271	1332	1346	1361	46.00
47.00	838	893	941	1127	1210	1225	855	914	1159	1240	1255	873	1121	1202	1217	1185	1261	1276	1337	1351	1365	47.00
48.00	840	895	944	1129	1213	1228	857	915	1163	1244	1259	876	1124	1206	1221	1189	1266	1280	1341	1356	1370	48.00
49.00	842	897	946	1132	1216	1231	859	918	1166	1248	1263	878	1128	1209	1224	1193	1270	1285	1346	1360	1375	49.00
50.00	844	900	948	1135	1219	1234	861	920	1170	1251	1266	880	1131	1213	1228	1197	1274	1289	1350	1365	1379	50.00

REFERENCES

1. C. W. Spofford, "The FACT Model, Volume 1," Maury Center Report 109, (November 1974).

2. C. S. Clay, "Sound Propagation in a Half Channel and Surface Duct," Technical Notes on Sound Propagation in the Sea II, Meteorology International, Inc., Monterey, CA, (1968).

3. F. M. Labianca, "Critique of C. S. Clay's 'Sound Transmission in a Half Channel and Surface Duct'," Reference 8 in Continuation of LRAPP, February 1972, Bell Laboratories.

4. H. E. Morris, "Comparison of Propagation Prediction Models with SUDS I Acoustic Data," NUC-TP-377, (April 1974).

5. H. W. Marsh and M. Schulkin, Report on the Status of Project AMOS, U. S. Naval Underwater Sound Laboratory, New London, Conn., May 1977.

6. H. P. Bucker, "Sound Propagation in a Channel with Lossy Boundaries," J. Acoust. Soc. Am., Vol. 48, 1187-1194, (1970). This is an extension, to include rough surface effects, of the normal mode code of Pedersen and Gordon, Reference 12.

7. F. M. Labianca, "Normal Modes, Virtual Modes, and Alternative Representations in the Theory of Surface-Duct Sound Propagation," J. Acoust. Soc. Am., Vol. 53, 1137-1147, (1973).

8. C. W. Spofford, "Improved Surface Duct Model for FACT," Science Applications, Inc. Report No. SAI-78-687-WA, (1977).

9. W. H. Furry, "The Bi-linear Modified-Index Profile," in Propagation of Short Radio Waves, edited by D. E. Kerr, McGraw-Hill, (1951).

10. H. W. Marsh, "Theory of Anomolous Propagation of Acoustic Waves in the Ocean," U. S. Naval Underwater Sound Laboratory Report No. 111, (1950).

11. M. Hall, "Mode Theory of Wave Propagation in a Bilinear Medium: the WKB Approximation," J. Acoust. Soc. Am., Vol. 60, 810-814, (1976).

12. M. A. Pedersen and D. F. Gordon, "Normal-Mode Theory Applied to Short-Range Propagation in an Underwater Acoustic Surface Duct," J. Acoust. Soc. Am., Vol. 37, 105-118, (1965); M. A. Pedersen and D. F. Gordon, "Theoretical Investigation of a Double Family of Normal Modes in an Underwater Acoustic Surface Duct," ibid, Vol. 47, 304-326, (1970).

13. The form of Eq. (2.6) is equivalent to Labianca's Eq. (23.a) (Ref. 7) if the following identifications between Labianca's notation and the present are made

$$k^2\lambda = \kappa^2 = (E + s^3D)/H^2$$

$$\phi(Z,\lambda) = \psi(x,E)$$

$$A(\lambda) \pm iB(\lambda) = T_{\pm}(E) \quad .$$

The factor of H arises from use of the dimension-less variable x instead of Z in the delta-function source term.

14. The intensity is normalized against a value of $4\pi r_0^2$ at unit distance r_0 (i.e., one foot or one meter).

15. M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions, Chapter 10, Dover Publications, (1965).

16. H. K. Brock, "The AESD PARABOLIC Equation Model," NORDA Tech Note 12, (1978).

17. R. E. Keenan and C. W. Spofford, "Rough Surface Loss for FACT," SAI Report No. SAI-78-688-WA, (November 1977).