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MECHANISM OF ARMOR PENETRATION

Second Partial Report

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the OBJECT is

To study the forces which act upon projectiles during armor penetration, and the effects thereof.

SUMMARY

Friction ~~has been~~ was found to have a negligible effect during armor penetration. The shearing stresses associated with friction may be calculated quite accurately. It has been found that these are less than 5,000 psi except during the first instant of contact.

An estimate has been made of the forces which arise from the inertial resistance of the plate material. These forces are proportional to the square of the projectile velocity, and are greater the blunter the ogive. These inertial forces may prevent projectiles from penetrating plate in a velocity range above the ballistic limit.

If the shape of the ogive is such as to force the plate to form a punch, the force acting upon the projectile decreases to a very small value after it has penetrated only a comparatively small distance. The energy a projectile needs in order to penetrate by punching is therefore much less than if it penetrates by pushing the

plate material aside. As an example, a flat-nosed projectile requires only one half the energy to penetrate a matching plate (e = d) that a projectile of conventional design does.

As long as the projectile penetrates in a ductile manner by pushing the plate material aside, the force which it encounters, and therefore the energy it needs for complete perforation, increases as the plate hardness increases. For a particular set of plates, [illegible] has been found to be proportional to (BHN + 200). The 200 points which must be added to the BHN in this relation finds a ready interpretation in the strain hardening of the plate material. The 200 points correspond to a hardening associated with a strain of nearly unity. It is pointed out that the higher the carbon content, the greater the strain hardening associated with a given strain, and therefore the higher the ballistic limit associated with a given BHN.

Consideration is given to the transverse forces which act upon the ogive at oblique incidence. A theory is thereby developed for armor penetration at obliquities by non-deforming projectiles in the case where no punches are formed. Upon striking the plate, the angle of obliquity is increased. This increase is given by a concise formula which takes account of the plate hardness, the velocity, and the original obliquity. The formula contains one unknown parameter, which must be determined experimentally for each type of projectile. This has been done for the standard cal. .30 AP projectile, and for two experimental types. The effect of obliquity upon the ballistic limit is adequately accounted for merely by considering the increase in length of projectile path through the plate.

It is demonstrated that when the ogive is not sufficiently hard, the transverse forces deform it in such a manner that the projectile then pursues a path which curves back to the face of the plate. This does not happen to the cores of standard cal. .50 AP ammunition. The transverse forces harm these cores in another way. They give rise to large bending moments, which in turn are associated with large surface tensile stresses. These tensile stresses result in fracture of cal. .50 AP cores when fired at obliquities of 20° or over against production plate.

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CONTENTS

LIST OF FIGURES

I INTRODUCTION

The first partial report[1] of this series was concerned primarily with the behavior of the plate during armor penetration. In this, the second partial report, emphasis is placed upon the behavior of the projectile during penetration. Two methods of approach are available in problems of this type. In one method, an exact solution is sought. In the other, the general nature of the solution is examined on the basis of dimensional arguments. The latter method is followed by the present authors. The results are therefore not precise, but it is believed that general principles are established which must be followed in any attempt to obtain more precise information. Wherever possible, these general principles are illustrated either by a comparison with past experiments, or, where the necessary empirical information was lacking, by new experiments.

No unique method presents itself for classifying the forces which act upon a projectile during armor penetration. In the present report, the forces are grouped as follows:

A. Forces acting parallel to surface of projectile (frictional forces).

B. Forces acting normal to surface of projectile.

   1. Forces due to inertial resistance of plate material to acceleration.

   2. Forces due to resistance of plate material to plastic deformation.

a. Forces along axis of projectile.
b. Forces transverse to axis of projectile.

The frictional forces (A) are found to have a negligible effect on the projectile. Also, for a projectile with a pointed ogive, the forces due to inertial resistance of plate material (B,1) have very little effect at normal velocities.

II FRICTIONAL FORCES

The plate reacts upon a projectile only with surface forces. Body forces, such as those associated with gravitation, are of course negligible. It is therefore appropriate to start the present investigation with a study of the nature of the surface forces acting upon projectiles.

Surface forces may be divided into two classes, those acting normal to the surface and those acting parallel to the surface. The forces of the first class may be called pressure forces, those of the second class, frictional forces. These two types of forces dissipate energy in radically different manners. The energy dissipated by the pressure forces is used in plastically deforming the plate material in the vicinity of the projectile. As a result of this deformation the plate material is heated to, at most, several hundred degrees.[1] The energy dissipated by the frictional forces, on the other hand, all goes into thermal energy along the surface of contact of the projectile and plate. The resultant intense concentration of thermal energy gives rise to a high local temperature. This high local temperature in turn reduces the friction force to a limiting value.

A. Temperature of Interface

Let ΔT be the rise in temperature at the interface due to friction after the two sides of the surface have moved a relative distance s with a constant relative velocity V. Further, let C be the thermal capacity per unit volume of the material, and σ the thermal conductivity of the material. It is shown in the appendix that if the frictional force per unit area, f, remains constant, then

$$\Delta T = \left(\frac{Vs}{\pi\sigma C}\right)^{1/2} f. \qquad (1)$$

In the derivation of this equation, a consistent set of units is assumed. For purposes of computation, it is convenient to express ΔT in $^{\circ}C$, V in ft/sec, s in inches, and f in psi. With these units, it is shown in Appendix A that Eq. (1) may be written as

$$\Delta T \cong \left(\frac{Vs}{2000}\right)^{1/2} f. \qquad (1a)$$

A plot of ΔT vs. s is given in Fig. 1 for various values of f. From this figure we see that if f is as great as 10,000 psi, the temperature of the interface reaches the melting temperature before the relative motion has become more than a small fraction of an inch.

B. <u>Frictional Stresses</u>

As the temperature of the surface approaches the melting temperature (of either the plate or projectile material), the frictional force per unit area will of course be reduced. The new value of the frictional force may be estimated from the necessity of a balance between the energy dissipated at the surface by friction and the energy conducted away. It is shown in Appendix A that if the temperature of the surface is to be raised by the amount ΔT and then maintained constant, the frictional force is given by the following equation:

$$f = 2\left(\frac{\sigma C}{\pi V s}\right)^{1/2} \Delta T. \qquad (2)$$

This equation is almost, but not quite, equivalent to Eq. (1). The difference arises from the circumstance that in one, <u>f</u> is maintained constant, while in the other, ΔT is maintained constant. If the same set of units is used as in Eq. (1a), Eq. (2) becomes

$$f \cong \frac{2}{3}\left(\frac{2000}{Vs}\right)^{1/2} \Delta T. \qquad (2a)$$

In view of the fact that the frictional force is initially of the order of magnitude of at least 10,000 psi, the temperature will rise rapidly during the first hundredth of an inch to some nearly constant value near the melting point, and <u>f</u> will become very small. The reduced value of <u>f</u> will be independent of its initial value, and will be given very nearly by Eq. (2a) in which ΔT is taken as 1500° C, corresponding to the melting temperature. Therefore,

$$f \cong 1000\left(\frac{2000}{Vs}\right)^{1/2}. \qquad (2b)$$

A plot of this equation is given as Fig. 2. From this figure it may be concluded that the frictional forces decrease rapidly as the projectile enters the plate — in fact they quickly become negligible as compared with the pressure forces.

It is of interest to note that Eqs. (1) and (2) may be derived, aside from the numerical coefficients, from dimensional arguments. The change in temperature ΔT of the surface can depend only upon the energy $\dot{Q}$ generated at the surface per unit time per unit area, upon the thermal capacity $\underline{C}$ of the material per unit volume, upon the thermal conductivity $\underline{\sigma}$, and upon the time $\underline{t}$ during which energy has been dissipated at the surface. The four quantities $\dot{Q}$, $\underline{C}$, $\underline{\sigma}$, $\underline{t}$, involve four dimensions: energy, length, time, temperature. Therefore, ΔT is related to these four quantities by a unique expression. This may readily be found to be:

$$\Delta T = A\,\dot{Q}\,\left[t/\sigma C\right]^{1/2}.$$

If a consistent set of units is used, general arguments show that the dimensionless constant $\underline{A}$ is of the order of magnitude of unity. If one now replaces $\dot{Q}$ by fV,

$\underline{t}$ by s/V, Eqs. (1) and (2) are obtained, aside from the numerical factors.

C. Width of Heated Zone

Dimensional arguments may also be used with profit to obtain the order of magnitude of the region which is heated to a temperature comparable with that of the interface. The width of this region, $\underline{w}$, can depend only upon the time interval $\underline{t}$ during which the surface has been at the elevated temperature, and the thermal diffusion coefficient of the material, $\underline{D}$. This coefficient has the dimensions $(\text{length})^2/\text{time}$. We therefore have the approximate equation

$$w \cong (Dt)^{1/2}.$$

Now for iron at room temperature $\underline{D}$ is 0.03 in^2/sec. Upon replacing $\underline{t}$ by s/V, we obtain

$$w \cong 0.001 \quad \frac{2000s}{V}^{\,1/2},$$

where $\underline{w}$ and $\underline{s}$ are in inches, and $\underline{V}$ is in ft/sec. In all cases of interest, the width of the zone heated to nearly the melting point is therefore of the order of magnitude of one thousandth of an inch.

III PLASTIC FORCES

In the previous section it has been shown that the frictional forces acting upon a projectile penetrating a plate are negligible compared with the normal pressure forces.

It is, therefore, only these normal pressure forces which do work upon the plate, and thereby dissipate the kinetic energy of the projectile. Part of this work goes into plastically deforming the plate material, part into imparting kinetic energy to it. If the velocity of the projectile is not too great, only a small fraction of the energy is dissipated as kinetic energy of the plate material. In the present section it will be assumed that the velocity is sufficiently low so that the normal pressure forces may be considered, to a good approximation, as arising primarily from the resistance of the plate material to plastic deformation. The inertial resistance of the plate will be considered only as providing an effective support for the plate during penetration. The velocity range over which this approximation is valid will be discussed in the following section, where the inertial forces are analyzed in some detail.

The plate acts primarily upon the ogive of the projectile. On any given element of the ogive, the force acts normal to the surface, and therefore has a component along the axis of the projectile, and a component transverse to this axis. In the case of normal incidence, the vector sum of all the transverse forces is equal to zero, and they have no effect of diverting the projectile from its original path. At oblique incidence these transverse forces do not entirely cancel, and the net transverse force determines, to a large extent, the behavior of the projectile. This case is discussed in a later section of the report, while the present section is devoted exclusively to the case of normal incidence.

The energy E necessary for complete perforation of a plate may be assumed proportional to some measure of the resistance of the plate material to plastic deformation. If there were no strain hardening, this measure could be taken to be the yield strength of the material. However, due to the presence of strain hardening, the effective plastic resistance will be greater than the yield strength. Since the rate of strain hardening is nearly linear, the measure of the effective plastic resistance R may be taken as

$$R = \text{Tensile strength} + (\text{slope of stress-strain curve})\,\varepsilon \qquad (3)$$

In this equation, ε may be defined as the effective strain. It will be of the order of magnitude of unity, but must, for each type of penetration, be determined empirically. The slope of the stress-strain curves is nearly independent of heat treatment, but increases with increasing carbon content.[2] From the firing records of a set of plates of various hardnesses, the second term in the right hand member of Eq. (3) may be determined. Thus for plates of the hardness range used in armor plate, Eq. (3) is nearly equivalent to the following equation:

$$R = 500\ (\text{BHN} + \Delta) \qquad (3a)$$

An example is shown in Fig. 3a, in which are represented all the data upon navy ballistic limits in Sullivan's extensive report.[3] For the plates investigated in Sullivan's report, it may be seen from Fig. 3a that

$$\Delta = 200.$$

Upon combining this equation with Eq. (3) and (3a), one obtains

$$R = \text{T.S.} + 100{,}000 \text{ psi.} \qquad (3b)$$

Dimensional considerations show that if the energy, $\underline{E}$, for complete perforation is proportional to a stress $\underline{R}$, then it must depend upon projectile caliber $\underline{d}$ and plate thickness $\underline{e}$ in the following way:

$$E = C\, d^n e^{3-n}\, R. \qquad (4)$$

The numerical constant $\underline{C}$ as well as the exponent $\underline{n}$ will depend upon the type of penetration. For any type of penetration, however, general arguments show that $\underline{C}$ will be of the order of magnitude of unity if a consistent set of units is used.

A. Ductile type of Penetration

One extreme type of penetration is that in which the plate material is pushed plastically aside. This is the type of penetration encountered when standard cal. .50 AP projectiles strike at normal incidence armor plate of BHN less than 380. For this type of penetration the exponent $\underline{n}$ in Eq. (4) is[1] 2. The equation therefore becomes

$$E_D = C_D\, d^2 e\, R. \tag{4a}$$

The verification of the dependence of E_D upon plate thickness $\underline{e}$, as well as upon $\underline{R}$, is given in Figure 3a. In this figure the heights of the horizontal lines are proportional to $\underline{e}$. These lines are seen to pass through the corresponding experimental points for plate BHN of less than 380. The numerical constant C_D, which corresponds to the horizontal lines in Fig. 3a, is given by the following equation:

$$C_D = 1.65. \tag{5a}$$

This numerical constant has very nearly the value $\pi/2$ given by Bethe's theory[4] for the penetration of thin plates.

The general features of the force arising from the plastic resistance of the plate material in the case of ductile type penetration may be inferred from Eq. (4a). In this equation, the factor d^2e is proportional to the volume Ω of the hole in the plate. Therefore, in the ductile type of penetration, one may expect that the resistance of the plate material will give rise to a force proportional to the derivative $d\Omega/dX$, where X is the coordinate specifying the position of the projectile. Thus

$$F_D = (4/\pi)\, C_D\, (d\Omega/dX)\, R. \tag{6}$$

The derivative $d\Omega/dX$ may be calculated graphically for any given plate thickness for any particular projectile.

Such a calculation for a standard cal. .50 projectile penetrating a 1/2" plate has been made, and the value for F_D is presented as Fig. 4. If the plate thickness were greater than the length of the projectile's ogive, the force would have a flat maximum.

B. Punching type of Penetration

The other extreme type of penetration is that in which a cylindrical punching is pushed out of the plate. This is the type of penetration obtained when flat-nosed projectiles strike at normal incidence a plate whose thickness is equal to or less than the projectile's caliber. Intermediate types of penetration are obtained with projectiles whose ogives are blunter than that of the cal. .50.

In this punching, or plugging, type of penetration, the exponent[1] _n_ is equal to unity, so that Eq. (4) becomes

$$E_P = C_P \, de^2 R \tag{4b}$$

Since experimental confirmation of this equation has not been presented heretofore, Table I* is presented as confirmation of the predicted linear dependence of the ballistic limit upon _e_. The plates were nearly all of the same hardness, and the projectiles were of Type D shown in Fig. 13. The near constancy of the values in the fourth column is a confirmation of the validity of the

*Page 34

proportionality of E_p with e^2 given by Eq. (4b).

In the hardness range in the vicinity of 300 BHN, E_p does not increase with plate hardness, contrary to Eq. (4b). Table II, the data for which were obtained with the same type projectiles, shows that E_p in fact decreases slightly with plate hardness in this range. This anomalous dependence upon hardness may be understood by an analysis of the force necessary to push out a punching. According to the ideas advanced in Ref. 1, this force may be represented as in Fig. 5. It starts to decrease linearly** with penetration distance until a critical distance is reached. At this critical distance the pertinent stress-strain curve reaches a maximum due to the adiabaticity of the deformation. After this critical distance all further deformation is confined to the immediate vicinity of a cylindrical surface.[5] The resultant high temperature reduces the shearing resistance to a negligible value. Now the harder the plate, the greater the rise in temperature associated with a given strain, and therefore the shorter will be the critical distance at which a punch is formed. An increase in plate hardness therefore introduces two counteracting effects, an increase in force, and a decrease in distance over which the force must act. This is illustrated in Fig. 5. From Table II* it is apparent that the second effect more than balances the first.

*Page 35
**Strain hardening will cause an initial slight rise

The above considerations may be verified by a measurement of the distance the flat-nosed projectiles must penetrate before a punching is started. This measurement has been made for the 3/16" plates, the ballistic limits of which are given in Table II. The critical distance X_c for the 205 BHN and the 388 BHN plates was 0.10" and 0.04", respectively.

For the plates with a BHN of about 300, the coefficient C_p in Eq. (4b) is given by the relation*

$$C_p = 0.74. \tag{5b}$$

A comparison of Eqs. (4a) and (5a) with Eqs. (4b) and (5b) gives the following approximate ratio:

$$E_p/E_D \cong (e/2d). \tag{7}$$

This equation shows that flat-nosed projectiles require only half the energy of pointed projectiles completely to perforate matching plate (e = d) in the usual hardness range. When $e < d$, the advantage of the flat-nosed projectile is

*After the first draft of the report was written, Report No. 7-43 of the U.S. Naval Proving Ground was received at this Arsenal. In this report, the ballistic limit of a 15 lb 3" flat-nosed projectile with respect to a 1.36" STS Plate is given as 576 f/s. Taking the BHN of this plate as 280, and upon substituting into Eq. (4b), one obtains $C_p = 0.70$. This is in satisfactory agreement with the value 0.74 obtained for the cal. .30 projectiles.

even greater.

When the ratio e/d becomes greater than 3, Eq. (7) implies that the energy required for a flat-nosed projectile completely to penetrate becomes greater than that required by a pointed projectile. This is not necessarily the case, since a flat-nosed projectile automatically acquires an ogive if the plate thickness is considerably larger than the projectile's calibre. This acquired ogive is formed out of a trapped plug, and the penetration thereafter is of the ductile type. Examples are shown in Fig. 3. Flat-nosed projectiles, however, are subjected to much greater inertial forces than projectiles with pointed ogives, as will be shown in the following section. These inertial forces are sufficiently great at a striking velocity of 1350 ft/sec to fracture the flat-nosed projectiles described in Appendix B. The use of A.P. flat-nosed projectiles in actual combat must wait until a steel can be found which will not fracture under the severe shock conditions which exist in the case of flat-nosed projectiles during impact.

IV INERTIAL FORCES

In the punching type of penetration, the inertial resistance of the plate material has no effect upon the energy required for perforation, provided of course this resistance does not give rise to fracture of the projectile. It is true that because of the inertia of the plug, more work is needed to start it moving out of the plate. But this additional work is stored as kinetic energy in the plug, and is available for helping the plug get completely out of the plate.

In the ductile type of penetration, the inertial resistance of the plate material may increase the energy needed for perforation. In this type of penetration the motion of the plate material is primarily radial, away from the axis of the projectile. At least part of the associated kinetic energy may also be useful in aiding the projectile to perforate the plate. For at least the major part of this kinetic energy of the plate material is used up as plastic deformation energy. If the final diameter of the hole in the plate is no larger than the projectile's calibre, then the inertial resistance of plate has not increased the energy required for perforation, aside from a small amount which goes into elastic vibration.[1] However, if the ogive is not tangent to the projectile body at the bourrelet, the plate material will still be flying radially away from the projectile at the bourrelet. The associated kinetic energy is a complete loss, since it is used up in expanding the hole to a diameter larger than that needed to let the projectile through. In appendix C it is shown that the relative increase in calibre of hole over the calibre of projectile is given by the approximate equation

$$\frac{\Delta d}{d} \approx \frac{\rho V^2}{\underline{R}} \tan^2 \theta$$

In this equation, θ is the angle the ogive makes at the bourrelet, ρ is the density of the plate, and $\underline{R}$, as before, is the resistance of the plate material to plastic deformation. The quantity ρV^2, which will be called inertial pressure, is given as a function of $\underline{V}$ in Fig. 6. From this figure it may be seen that if $\Delta d/d$ is to be kept much less than unity for velocities up to 2,000 ft/sec, and if $\underline{R}$ is less than 400,000 psi, $\tan^2 \theta$ must be much less than unity, i.e., θ must be considerably less than 45^o.

According to the above approximate equation, all detrimental effects of inertial pressure could be avoided merely by making the ogive tangent to the projectile body at the bourrelet. However, in the derivation of this equation, it was assumed that the plate material kept in contact with the ogive right up to the bourrelet. If the radius of curvature of the ogive is too small, this assumption is incorrect. This problem is analyzed in Appendix C. It is found that if the radius of curvature is greater than a critical value, the plate material will fly away from the ogive before reaching the bourrelet. This critical radius of curvature, R_c, is given by the following approximate equation:

$$R_c = (1/2)d\,\frac{\rho V^2}{R}.$$

As an example of the application of the above equation, suppose that a projectile is fired with a velocity of 3000 f/s against a plate of BHN 300. For such a plate the appropriate value of $\underline{R}$ is 250,000 psi. From the above equation for R_c, and from Figure 6 we then conclude that the projectile hole will be larger than the calibre of the projectile if the radius of curvature of the ogive is under two calibres.

The pressure which the projectile must withstand because of the inertia of the plate material may best be studied by neglecting the strength properties of the plate material. As long as the speed of the projectile is small compared with the speed of sound in the plate material, as is always the case in practice, the pressure which the projectile must sustain due only to the plate inertia is given approximately by the equation for the force per unit area due to air resistance at velocities considerably below the velocity of sound in air. This formula is

$$\text{Pressure} = \alpha\,\rho\,V^2, \qquad (8)$$

where α is a numerical constant, ρ the density of the medium, and $\underline{V}$ the velocity. If a consistent set of units is used, the inertial coefficient α is of the order of magnitude of unity. Values of α for various shaped projectiles are given in Table III, the data for which were taken from Cranz.[6]

The value of the inertial coefficient α is seen from Table III* to be about 0.8 for cylinders with flat faces, and about one tenth this value for projectiles with the usual ogives. Thus for flat-faced projectiles the inertial resistance of the plate is the limiting factor which determines how thick a homogeneous plate may be perforated without shattering of the projectile. At a velocity of 2,000 ft/sec such a projectile must withstand a pressure of at least 400,000 psi for its face. In order to demonstrate this effect, a slug was shot at 2000 ft/sec at a lead plate. Here the inertial pressure was 800,000 psi, and the plastic resistance was negligible. The slug fractured.

V TRANSVERSE FORCES

A. Effect upon Projectile Trajectory

The net force acting upon a projectile is directed along the projectile's axis only in the case where the impact has complete axial symmetry. In the general case of oblique impact, the net force has a transverse component. The magnitude as well as the direction of this transverse force depends upon a variety of factors.

*Page 36

As the ogive of a pointed projectile digs into the plate, the transverse force tends to tilt the projectile either away from the normal or towards the normal, according to whether the plate material is pushed plastically aside or is pushed towards the back of the plate as a plug. The two situations are illustrated schematically as Fig. 7. The direction of the transverse force may readily be determined by a consideration of the normal forces acting upon the surface of the ogive. If the projectile succeeds in penetrating to near the back of the plate in a ductile manner, the transverse force acting upon the ogive is always directed in such a manner as to turn the projectile towards the normal, irrespective as to whether or not a back plug is pushed out. The situation is illustrated schematically, and also with photographs, as Fig. 8.

In the case where a punching starts before the plate has deformed an appreciable amount, the initial transverse force tends to decrease the obliquity angle, and therefore to aid the projectile in passing through the plate. On the other hand, in the absence of a punching, the transverse force increases the obliquity angle, and therefore renders penetration more difficult. A quantitative estimate of this change in obliquity angle is presented below.

If a torque $\underline{N}$ acts upon a projectile with moment of inertia $\underline{I}$ for a time $\underline{T}$, the change in angle $\Delta\theta$ at the end of this time is given, in radians, by

$$\Delta\theta = N\,T^2/2I.$$

Now N is proportional to the transverse force acting upon the ogive. This transverse force, in turn, is proportional to some measure of the resistance of the plate material to plastic deformation, which will be taken as the quantity R, introduced in section III. The torque N must be an odd function of the obliquity angle, since it changes direction as the obliquity angle θ passes through zero. Hence, at least for small angles, N may also be taken as proportional to sin θ, so that

$$N \sim \sin\theta\, R$$

The duration of the torque N is inversely proportional to the velocity of the projectile,

$$T \sim 1/V,$$

and the moment of inertia is proportional to the density of projectile,

$$I \sim \rho\ .$$

Upon combining these relations, one obtains

$$\Delta\theta \sim \sin\theta\,(R/\rho V^2).$$

It is to be noted that both sides of this equation are dimensionless. Therefore if one writes

$$\Delta\theta = k \sin\theta\ (R/\rho V^2), \qquad (9)$$

and uses a consistent set of units, the dimensionless constant k will be of the order of magnitude of unity. This constant will of course depend upon the ogive shape. It will be larger the longer the ogive compared with the projectile's calibre, for the longer the ogive the longer will be the time during which the transverse force acts.

Some firings were made in order to illustrate Eq. (9), and to determine the constant $\underline{k}$ for several types of ogive. Three types of projectiles were used: standard cal. .30 M2 AP bullets were fired at 300 ft. range, and two types of experimental unjacketed projectiles were fired. The latter are illustrated in Fig. 13. In order to minimize yaw in these last two types, the plate was placed so close to the muzzle of the gun that the projectiles had only 1/8th inch free flight. The data are presented in tabular form as Appendix D, and are plotted in Fig. 10. To each type of projectile a $\underline{k}$ has been assigned which brings Eq. (9) into as close an agreement with the observation as possible. The $\underline{k}$ values for the standard cal. .30 bullets, for Type A and Type C projectiles were 1.7, 1.3 and 0.95, respectively. The shape of the ogive of the Type A unjacketed projectile was similar to that of the core of the standard cal. 30 bullet. It appears that the jacket of the standard bullet effectively lengthens the ogive, and thereby lengthens the time during which the transverse force acts. Type C projectile has a shorter ogive than Type A, and hence has a smaller $\underline{k}$ value.

Photographs no. 164, 165 and 166 in Figure 9 illustrate the effect of the back face of the plate in tilting the projectile towards the normal. In photographs no. 165 and 166, where the ogive has completely emerged from the back of the plate, this tilting towards the normal more than compensates for the tilting away from the normal at the plate's face. Thus in these two cases, the obliquity angle changes from 20° to 26° to 17°, and from 20° to 28° to 16°, respectively.

A special case exists if the ogive is insufficiently hard. Aside from the asymmetry associated with the front or back face, the projectile will follow a curved path, essentially the arc of a circle, if its ogive has been asymmetrically deformed. A consideration of the initial forces acting upon the ogive (see illustration A of Fig. 7) shows that the ogive is deformed in such a manner as to curve the path of the projectile out to the face of the plate. The softer the projectile, the more it will be deformed, and therefore, the more sharply curved will be the path of the projectile. In order to illustrate this effect, projectiles of various hardnesses were fired into a plate, and then sectioned. The results are shown as Fig. 9. A comparatively soft plate (155 BHN) was chosen so as to minimize the breaking up of the projectiles. The distortion of the ogives is seen to decrease rapidly as the projectile's hardness is raised, being detectable at a hardness Rc 57, but not at Rc 61. The distortion, and hence also the curvature of the path, is seen to increase as the obliquity of attack is increased. It will, presumably, also increase as the plate hardness is increased. The distortion is, however, apparently independent of incident velocity in the range used.

B. Effect of Obliquity upon Ballistic Limit

When no punches are formed, the increase in ballistic limit for complete perforation with obliquity may be regarded as due to the increase in length of path of the projectile in the plate. The factor by which the length of path has increased is $1/\cos(\theta + \Delta\theta)$. Eq. (4a) for the energy for perforation may therefore be generalized to

$$E_D = C_D \, d^2 e \, R \Big/ \cos(\theta + \Delta\theta).$$

Upon using Eq. (9), one obtains

$$E_D = C_D \, d^2 e \, R \Big/ \cos(\theta + q \sin\theta) \quad (10)$$

where

$$q = k \, R \Big/ \rho \, V^2. \quad (11)$$

The quantity q is implicitly a function of θ through V. It may be more conveniently written as

$$q = P \, (V_o/V)^2, \quad (11a)$$

where V_o is the ballistic limit at zero obliquity, and P is independent of both θ and V.

The obliquity function $f(\theta)$ may be defined as the ratio of the energy needed for perforation at the obliquity θ to that needed for perforation at zero obliquity. From Eq. (10) this function is determined by the equation

$$f(\theta) = 1/\cos(\theta + Pf \sin\theta). \qquad (12)$$

This is a transcendental equation for f. For such small values of θ that f differs only slightly from unity, f may be replaced by unity in the right hand member. This approximation gives

$$f(\theta) \cong 1/\cos(\theta + P \sin\theta). \qquad (12a)$$

For purposes of comparison with experiment, it is convenient to use an obliquity function of the form

$$f(\theta) = 1/\cos^s\theta . \qquad (12b)$$

From a comparison of the Taylor expansion of the right hand sides of Eqs. (12a) and (12b), it may be shown that Eqs. (12a) and (12b) are equivalent at small angles provided that

$$s = (1 + P)^2. \qquad (13)$$

The quantity P may be obtained by setting θ equal to zero in Eq. (10). One obtains

$$P = (\pi/8)\ (k/C_D)\ (L/e), \qquad (14)$$

where L is the length of a cylinder having the same calibre and volume as the projectile.

The obliquity function, as given by Eq. (12), is not a function of plate hardness, but is dependent upon plate thickness, increasing as the plate thickness decreases. A comparison with experiment of the approximate obliquity function of Eqs. (12b), (13) and (14), may readily be made. In Fig. 11 is given a plot of $\underline{s}$ vs. $1/P$ according to Eq. (13), and also $\underline{s}$ vs. (e/d) for standard cal. .50 bullets, obtained by averaging the data in Ref. 3 for 0 and 20° obliquities over all hardness ranges. Since the abscissa is on a logarithmic scale the experimental points should be upon the theoretical curve if the latter is shifted horizontally. The corresponding factor by which the theoretical master curve must be multiplied is equal to $(\pi/8)\ (k/C_D)\ (L/d)$. The experimental value of this factor is 1.4. Taking C_D as 1.65 from Section II, and upon using 3.5 as the value of L/d for standard cal. .50 cores, one obtains $k = 1.65$. This value of $\underline{k}$ is consistent with the value 1.7 obtained with cal. .30 bullets discussed above.

C. Fracture of Projectiles by Tensile Stresses

At normal incidence, the axial net force gives rise in the projectile to compressive stresses only. To the approximation that the velocity of elastic waves in the projectile may be treated as infinite compared with the velocity of the projectile itself, this compressive force is a maximum in the ogive and decreases linearly from the bourrelet to a zero value at the end of the projectile. At oblique incidence, on the other hand, the transverse forces give rise to tensile stresses which may be several times as large as the compressive stresses.

Before a punching has started the stress pattern in a projectile striking an armor plate at an oblique angle is very similar to that in a falling chimney, neglecting such irrelevancies as differences in shape, wind resistance, etc.. This is illustrated in Fig. 12. In each case a transverse force gives rise to a bending moment of such a sign that the tensile stress is positive on the side farthest away from the normal. A quantitative analysis of the stress pattern is given in Appendix F. It is there shown that if a transverse force f_{tr} acts at one end of a free cylinder of length $\underline{L}$, diameter $\underline{d}$, and cross-sectional area $\underline{A}$, the maximum tensile stress occurs at one third the length of cylinder away from this end, and has the magnitude:

$$\text{Max. tens. stress} = (32L/27d)f_{tr}/A.$$

In the case of a projectile, one may take $L \cong 3d$, and obtain

$$\text{Max. tens. stress} \cong 3.5\ (f_{tr}/A).$$

If the transverse and axial forces were equal, the maximum tensile stress due to the former would then be

3.5 times the maximum compressive stress due to the latter.

The transverse force, and therefore the maximum tensile stress, may be expected to be nearly proportional to the cross-sectional area of the ogive in a plane passing through the projectile axis. Therefore it is to be expected that the blunter the ogive the smaller will be the tensile stresses in the projectile during oblique impacts.

Standard cal. .50 A.P. projectiles do not, as a rule, fail as a result of compressive stresses when fired against homogeneous plate at zero obliquity. In a set of firings at normal incidence against production 1/2" plate, with incident velocities from 2000 to 3000 ft/sec, all cores were recovered intact. On the other hand, in a set of firings against the same plate at 20° and 30° obliquity over the same velocity range, no cores were recovered intact. A typical example is shown in illustration B of Fig. 8. Since in these oblique firings the compressive stresses are no greater than in the normal firings, the fractures can only arise from the tensile stresses due to the transverse stresses.

An example of a tensile crack in a 37 mm AP projectile fired at 20° obliquity is shown in Fig. 15.

TABLE I

Dependence of Ballistic Limit* for Punching Upon Plate Thickness
(Experimental Projectile, Type D, normal incidence)

Plate thickness (e)	BHN	V* (f/s)	V/e (f/sec in)
3/16"	275	680	3,600
1/4"	302	970	3,900
0.30"	321	1175	3,900

*Velocity which just pushes out plug.

TABLE II

Variation of Ballistic Limit* with Plate Hardness
(Experimental Projectile, Type D)
3/16" plate, Normal Incidence

BHN	V*
205	710
275	680
388	605

V*: Velocity which just completes a punching.

T A B L E III

Inertial Coefficients

Projectile	Authority	α
Cylinder with flat face	Poncelet and Didion	0.65
	F. le Dantei	.65
	P. C. Langley	.68
	Ch. Renard	.68
	Canovetti	.72
	J. Weisbach	.75
	F. V. Lossl	.85
	J. Smeaton	.98
	O. Lilienthal	1.01
	E. J. Marey	1.01
Sphere	N. Mayevski	.098
	Helie	.16
Ogive with 1/2 angle of:	Vallier	
31°		.074
33.6°		.080
37°		.086
48.2°		.122

APPENDIX A

Calculation of Frictional Stresses

In this appendix an analysis is given of the thermal effects produced by the rubbing together of two surfaces. The following notation is used

- f: Frictional force per unit area
- D: Thermal diffusion coefficient
- C: Thermal capacity per unit volume
- t: time
- X: coordinate normal to surface, with origin at surface
- s: relative displacement of two surfaces
- V: relative velocity of two surfaces
- $\dot{Q}$: rate of energy dissipated by friction per unit area
- T: temperature rise

The fundamental differential equation for thermal diffusion in a medium with no thermal sources or sinks is

$$\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial X^2}, \tag{a-1}$$

provided T does not depend upon the coordinates transverse to X. Since our surface, at X = 0, is to be a thermal source, this differential equation is to hold everywhere except along this surface. This equation is to be solved under two distinct sets of boundary conditions, corresponding to two problems in which the generation of heat at the surface is known, and in which the temperature at the surface is known.

The first problem may be solved by means of the well known particular solution of Eq. (a-1) of the type

$$T = \frac{\delta Q}{C} \frac{e^{-X^2/4Dt}}{(4\pi\ Dt)^{1/2}} \tag{a-2}$$

Such a solution satisfies Eq. (a-1) everywhere at all times except at t = 0 and X = 0. This solution gives in fact the temperature distribution due to the instantaneous application of the heat δQ per unit area along the surface X = 0 at t = 0. Hence the temperature distribution due to any rate $\dot{Q}(t)$ of heat generation may be obtained by superimposing solutions of this type. Thus

$$T(X,t) = \frac{1}{C}\int_0^t \frac{e^{-X^2/4D(t-t^1)}}{\{4\pi\ D(t-t^1)\}^{1/2}} \dot{Q}(t^1)dt^1 \ . \tag{a-3}$$

In the present application we are interested only in the temperature at X = 0 in the case where $\dot{Q}$ is a constant. For this particular case Eq. (a-3) reduces to

$$T = \frac{\dot{Q}\ t^{1/2}}{C\ (\pi\ D)^{1/2}} \tag{a-4}$$

Upon placing $\underline{D}$ by its expression in terms of $\underline{C}$ and $\underline{\sigma}$, namely

$$D = \sigma/C, \tag{a-5}$$

we obtain

$$T = \dot{Q}\ (t/\pi\sigma C)^{1/2} \tag{a-6}$$

In the second problem one considers that the temperature of the plane at $X = 0$ is suddenly raised by the amount ΔT_0 and then maintained at this new value. It is desired to find the rate at which energy must be supplied to the surface to maintain this temperature. The energy supply is twice the rate at which energy flows into the region $X > 0$. The rate at which energy flows into this region may be readily obtained from the solution of the problem in which the ΔT at $t = 0$ is $2\Delta T_0$ for $X < 0$, and 0 for $X > 0$. In this problem the plane at $X = 0$ is automatically elevated by the constant amount ΔT_0 for all subsequent times. The solution of Eq. (1-a) which satisfies this boundary condition may readily be seen to be

$$\Delta T = \frac{2\Delta T_0}{\pi} \int_0^\infty e^{-k^2 D\, t} \frac{\sin k X}{k} \, d\, k \qquad \text{(a-7)}$$

The rate of flow energy into the region $X > 0$ per unit area, which will be denoted by $(1/2)\dot{Q}$, is given by

$$(1/2)\dot{Q} = -\sigma \left. \frac{\partial \Delta T}{\partial X} \right|_{X = 0} .$$

Upon using Eq. (a-7), we find

$$(1/2)\dot{Q} = \Delta T_0 \left(\sigma C/\pi \; t\right)^{\frac{1}{2}} \qquad \text{(a-8)}$$

Upon substitution of $\dot{Q} = f\, V$ and $t = s/V$ into Eqs. (a-6) and (a-8), we obtain Eqs. (1) and (2) of the text, respectively.

In the text it is necessary to have the numerical value of σC. The values of σ and C do not vary markedly with temperature, so we shall be justified in using the room temperature values.

For steels

$$\sigma \cong 0.16 \text{ calories/cm sec}$$

$$C \cong 0.85 \text{ calories/cm}^3$$

Therefore

$$\sqrt{\sigma C} \cong 0.37 \text{ calories/cm}^2 \text{ sec}^{1/2}$$

Upon using the conversion factors

$$1 \text{ calorie} = 37.0 \text{ in-lbs}$$

$$1 \text{ cm}^2 = 0.155 \text{ in}^2,$$

we obtain

$$\sqrt{\sigma C} \cong 91 \text{ lb/in sec}^{1/2}.$$

Upon substitution of this relation into Eqs. (1) and (2), one obtains Eqs. (1a) and (2a), respectively.

APPENDIX B

Firing Record

PROJECTILE		PLATE		RESULT
TYPE	VEL.	e	BHN	
D(Flat nose)	670	3/16"	275	Punching - almost broken out of back of plate. (Hence, B.L. assumed to be at a slightly higher velocity: 680 f/s.)
"	980	1/4"	302	Punching - completed. Projectile stuck in plate. (Hence, B.L. limit assumed to be at a slightly lower velocity: 970 f/s.)
"	1165	.30 (matching plate)	321	Punching - almost broken out of back of plate. (Hence, B.L. assumed to be at a slightly higher velocity: 1175 f/s.)
"	675	3/16"	205	Punching - started
"	745	3/16"	205	Punching - completed, projectile stuck in plate. (Hence, B.L. is mean of 745 and 675: 710 f/s.)
"	615	3/16"	388	Punching - completed. Projectile and punching were stopped in sawdust very close to back of plate. (Hence, B.L. assumed to be at a slightly lower velocity: 605 f/s.)

APPENDIX C

Inertial Pressure

The pressure acting upon a projectile being slowly pushed through a plate is distributed fairly uniformly about its surface. This pressure distribution is modified when the projectile plows its way through at moderate velocities. The pressure is now greatest near the tip of the ogive, less along the sides, and of course zero in back. The situation is analogous to the pressure upon the sides of a moving submarine.

The change in pressure in each case becomes relatively important when the quantity ρV^2 approaches the magnitude of the pressure already existing. In the case of steel, when the pressure is from 200,000 to 400,000 psi, the change in pressure becomes important at 1,500 ft/sec, as may be seen from Fig. 6. In the case of a submarine, the term ρV^2 becomes relatively important when the velocity approaches $\sqrt{g\,h}$, e.g., 50 ft/sec at a depth of 100 ft.

When the velocity increases still further so that the quantity ρV^2 becomes greater than the quasi-static pressure, the pressure may even vanish over the latter part of the ogive. When this is the case, the medium being plowed through so to speak is carried away from the surface by inertial forces. In the case of the submarine, the water eventually again makes contact with the surface of the vessel further back. In the case of the projectile, which is plowing through a medium possessing some rigidity, the medium does not again make contact with the projectile. The hole in the plate has in such a case a diameter larger than the projectile. Since such an enlargement of the hole necessarily means a greater energy absorbed by the plate, it is important to establish rather precisely the condition under which the plate hole is so enlarged. Such an analysis is herein given.

The three dimensional problem of an actual penetration will be replaced by a simpler problem which can be handled precisely. The results of this analysis will then be taken as an approximate solution to the actual problem.

In the simplified problem, the projectile will be considered to pass through a ring of plate material as is illustrated in Fig. 14. This ring will be prevented by external forces from an axial motion, but is free to expand radially. We shall first inquire into the precise conditions which determine whether the inertia of the ring material will cause it to lose contact with the projectile ogive. If contact is to be lost, it is necessary that

$$\ddot{r}_{ring} > \ddot{r}_{proj.} \qquad \text{(c-1)}$$

where r_{proj} refers to the radius of the ogive in the plane of the ring.

An obvious calculation gives the right member of the above inequality as

$$\ddot{r}_{proj} = \frac{d^2r}{dx^2} V^2 .$$

In the interesting case near the back of the ogive, where its slope is small, this equation reduces to

$$\ddot{r}_{proj.} = - V^2/R_c \qquad \text{(c-2)}$$

where R_c is the radius of curvature of the ogive at the position adjacent to the ring.

The left hand member of (c-1) may be calculated using the condition that, where the ring is free, its increase in strain energy is equal to the loss of its kinetic energy. This condition gives

$$\rho \ddot{r}_{ring} = -R/ r,$$

where R is the tension in the ring. (c-3)

The inequality (c-1) may now be rewritten, after using (c-2) and (c-3) as

$$\rho V^2 > \frac{r}{R_c} R. \qquad \text{(c-4)}$$

The inequality (c-4) will always be satisfied at the bourrelet if the ogive is not there tangent to the body of the shell. In this case, the question arises as to how far the ring will expand. This expansion is given by the condition that the kinetic energy of the ring at the bourrelet will all be used in producing plastic deformation of the ring. This condition gives

$$\frac{\delta r}{r} = 1/2 \tan^2\theta \, \frac{\rho V^2}{R}$$

A P P E N D I X   D

Data upon Tipping Angle
(BHN of plate = 155)

θ : obliquity angle

Δ : increase of obliquity angle upon entering plate

V: striking velocity

I. Standard cal. .30 M2 AP

θ	V	Δ	(Δ)average
20°	2000	$17^\circ, 17^\circ, 11^\circ$	15°
	2500	$8^\circ, 10^\circ, 8^\circ$	9°
	3000	$9^\circ, 0^\circ, 2^\circ$	4°
30°	2000	$22^\circ, 24^\circ, 19^\circ$	22°
	2500	$18^\circ, 15^\circ, 14^\circ$	16°
	3000	$0^\circ, 14^\circ, 9^\circ$	8°

II. Experimental Projectiles, Type A

θ	V	Δ RC = 65	61	57	(Δ)ave
20°	2000	9°	11°	13°	11°
	2500	8°	6.5°	8.5°	7.6°
30°	2500		20.5°	15°	

III. Experimental Projectiles, Type C, RC = 65

θ	V	Δ
20°	1500	14°
	2000	7°
	2500	5°

APPENDIX E

Tensile Stresses in Projectiles

In this appendix will be calculated the tensile stresses inside a free bar which is subjected to a transverse force at one end. The general scheme of the calculation runs as follows. The acceleration of every element of the bar is obtained from the dynamic equations of motion. The problem is then reduced to a problem in statics by considering the reversed inertial forces as externally applied forces. From the static problem the moments in the bar may readily be computed and from the moments the tensile stresses are derived.

The acceleration $\underline{a}$ of an element at a distance $\underline{x}$ from the end at which the force F is applied is given by

$$a = (4F/Lm)\ (1-3x/2L),$$

where m is the mass per unit length, and the total length is $\underline{L}$. In the equivalent static problem, one therefore has the force

$$\underline{F} \text{ at } x = o$$

and a distributed force W per unit length given by

$$W = -ma = (4F/L)\ (3x/2L-1)$$

These static forces give rise to a moment M given by

$$M\ (z) = z(1-z)^2\ FL$$

where

$$z = x/L.$$

This moment has a maximum at $x = L/3$, and then has the value

$$M_{max} = (4/27)\ FL.$$

It may be shown that in a bar of circular cross-section subject to a bending moment M the maximum tensile stress is given by

$$T = 8M/Ad,$$

where <u>A</u> is the cross-sectional area, <u>d</u> the diameter of the bar. At the position $x = L/3$ the maximum tensile stress is therefore

$$T = (32/27)\ (L/d)\ F/A.$$

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1. C. Zener and J. H. Hollomon: "Mechanism of Armor Penetration, First Partial Report", Watertown Arsenal Report No. 710/454.

2. C. Zener and J. H. Hollomon: "Plastic Flow and Rupture of Metals, First Partial Report", Watertown Arsenal Report No. 732/11.

3. J. Sullivan: "Rolled Armor, Ballistic Properties of Rolled Face Hardened Armor and Rolled Homogeneous Armor at Various Hardnesses at Normal Incidence and at Various Obliquities", Watertown Arsenal Report No. 710/456.

4. H. A. Bethe: "Attempt of a Theory of Armor Penetration", Ordnance Laboratory, Frankford Arsenal, 1941.

5. T. von Karman: "On the Propagation of Plastic Deformation in Solids", NDRC Report A-29 (OSRD No. 365)

   C. Zener and J. H. Hollomon: "Addendum to von Karman's Theory of the Propagation of Plastic Deformation in Solids", NDRC Memo No. A-37M

6. C. Cranz: "Lehrbuch der Ballistic" 1st. vol., 5th Ed., pp. 54-66. (Berlin 1925)

FIGURE 1

TEMPERATURE RISE AT INTERFACE DUE TO A CONSTANT FRICTIONAL FORCE

(STEEL ON STEEL SURFACE, RELATIVE VELOCITY TAKEN AT 2,000 FT/SEC)

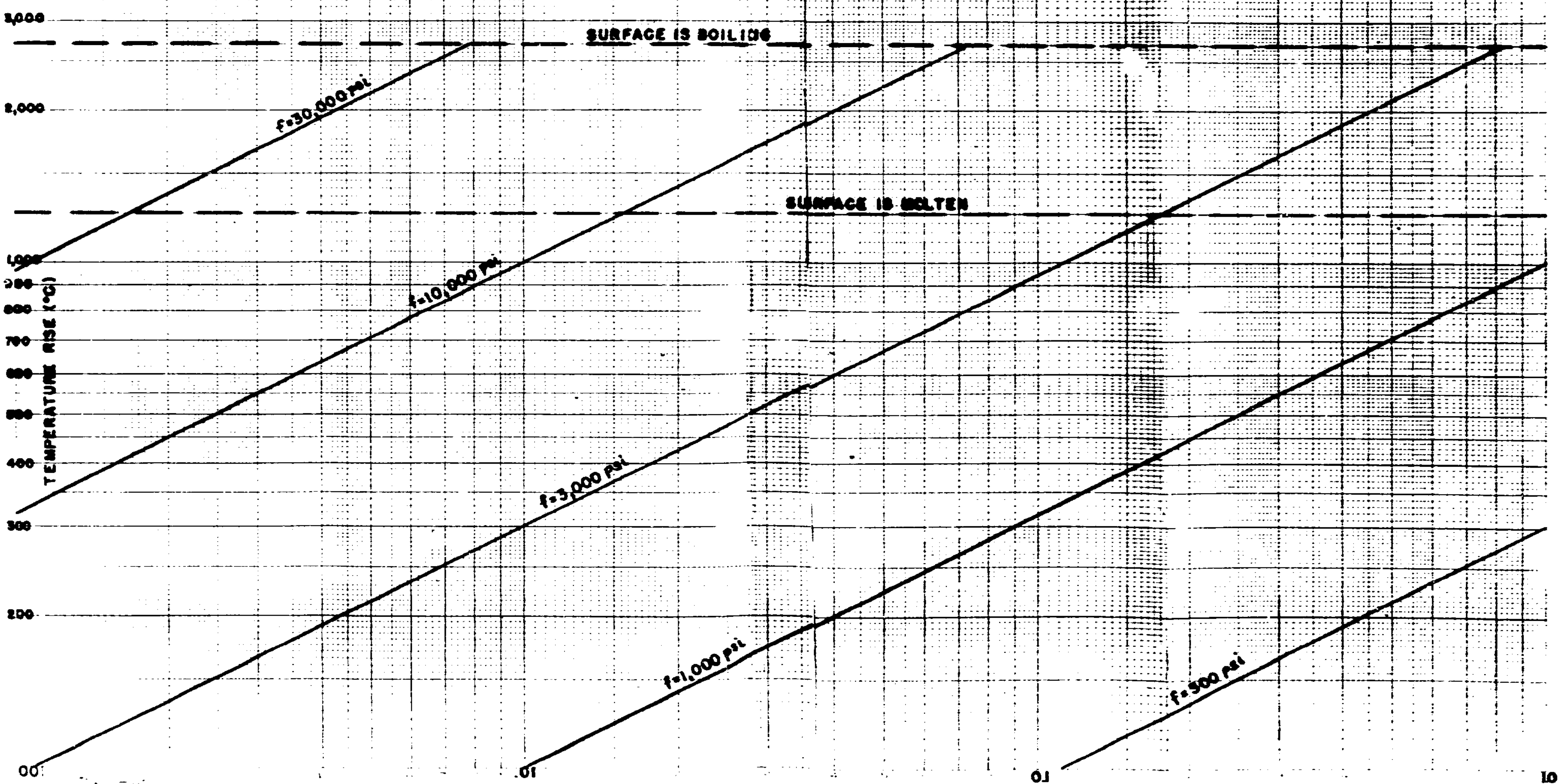
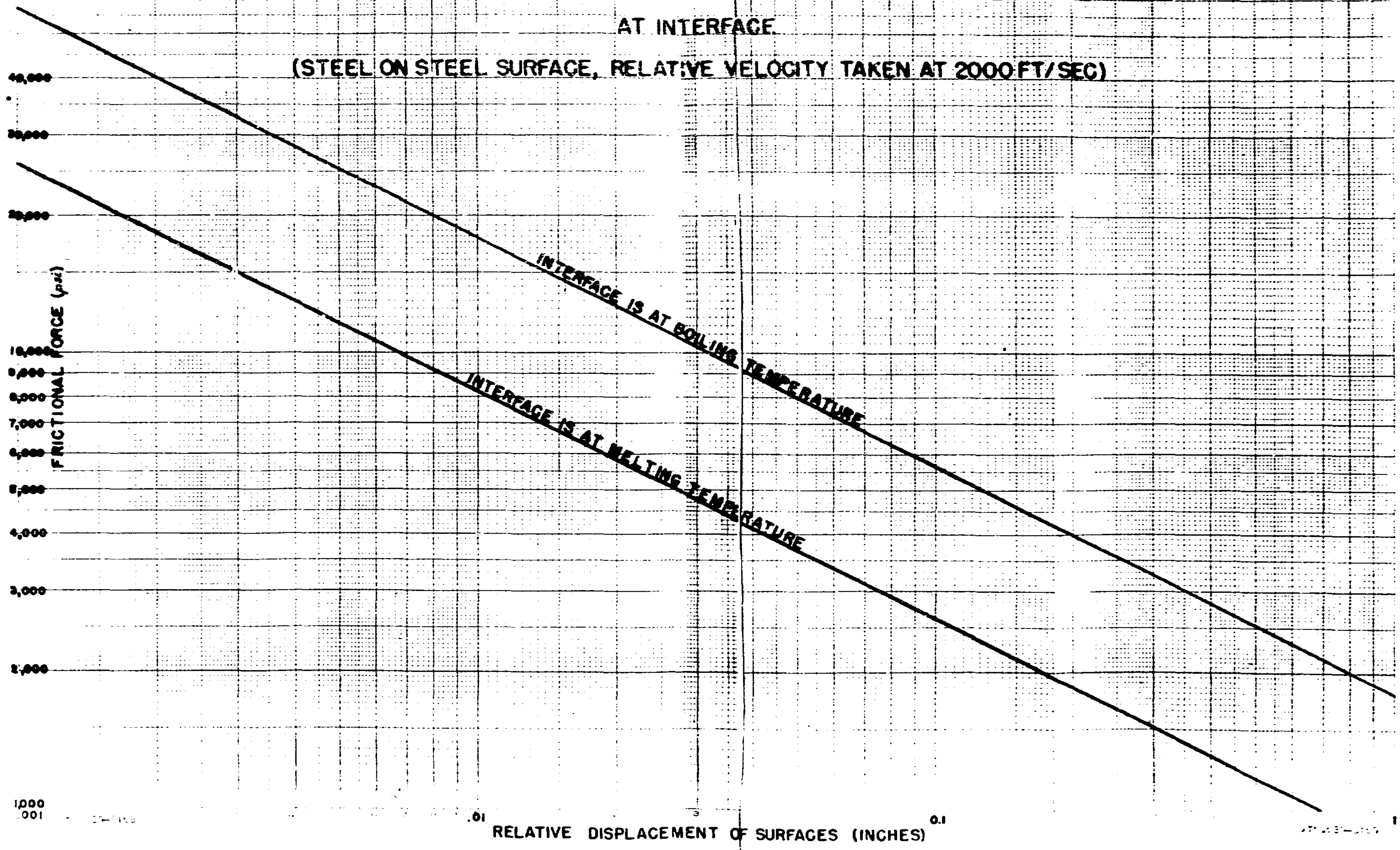


FIGURE 2

FRICTIONAL FORCE NECESSARY TO MAINTAIN A CONSTANT TEMPERATURE AT INTERFACE

(STEEL ON STEEL SURFACE, RELATIVE VELOCITY TAKEN AT 2000 FT/SEC)



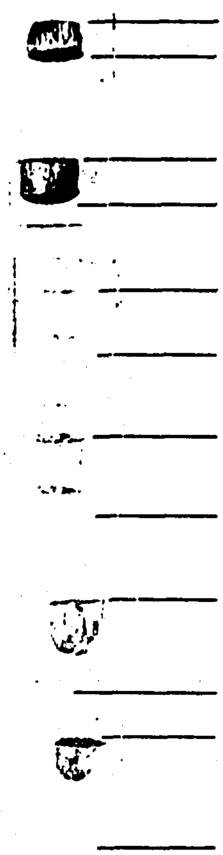


Plate thickness	Plate material
.188"	Steel
.25"	Steel
.35"	Dural
.440	Dural
.531	Dural
.625	Dural

FIGURE 3

Transition from free to trapped plug

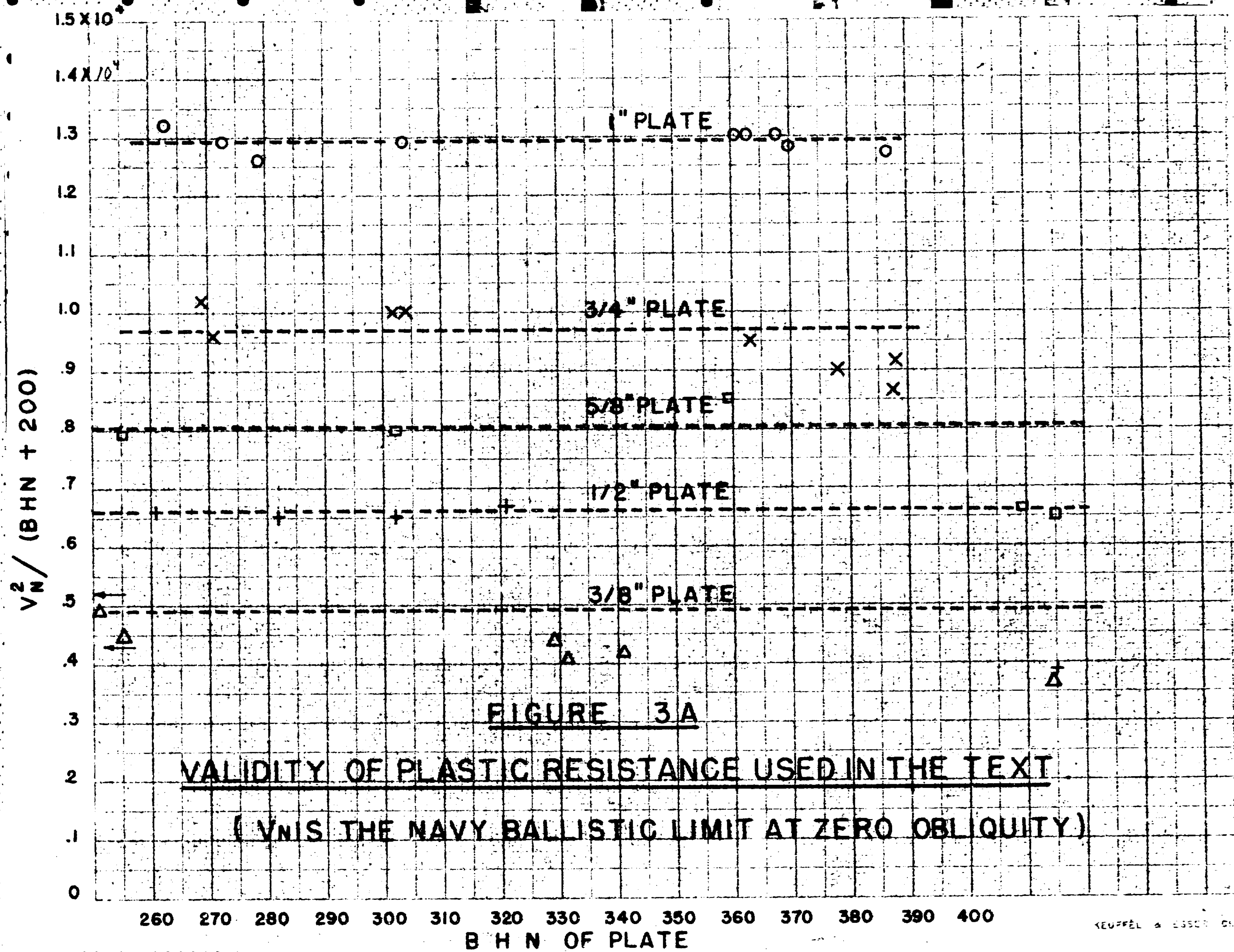


FIGURE 3A

VALIDITY OF PLASTIC RESISTANCE USED IN THE TEXT

(V_N IS THE NAVY BALLISTIC LIMIT AT ZERO OBLIQUITY)

FIGURE 4

PLASTIC RESISTANCE IN DART TYPE OF PENETRATION

(EXAMPLE OF STANDARD CAL. 50 CORE STRIKING ½" PLATE OF 300 BHN CALCULATED BY Eq 6)

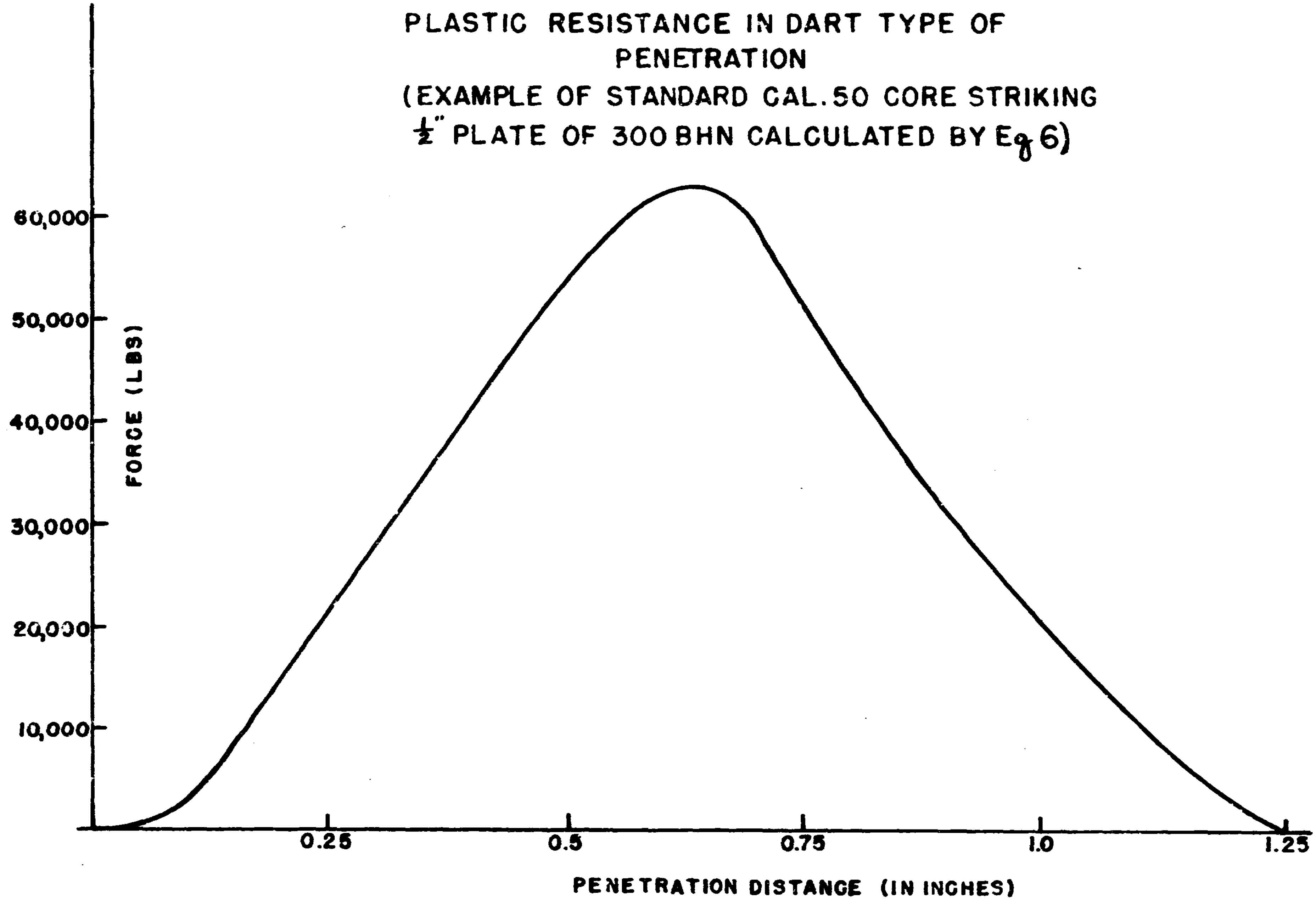
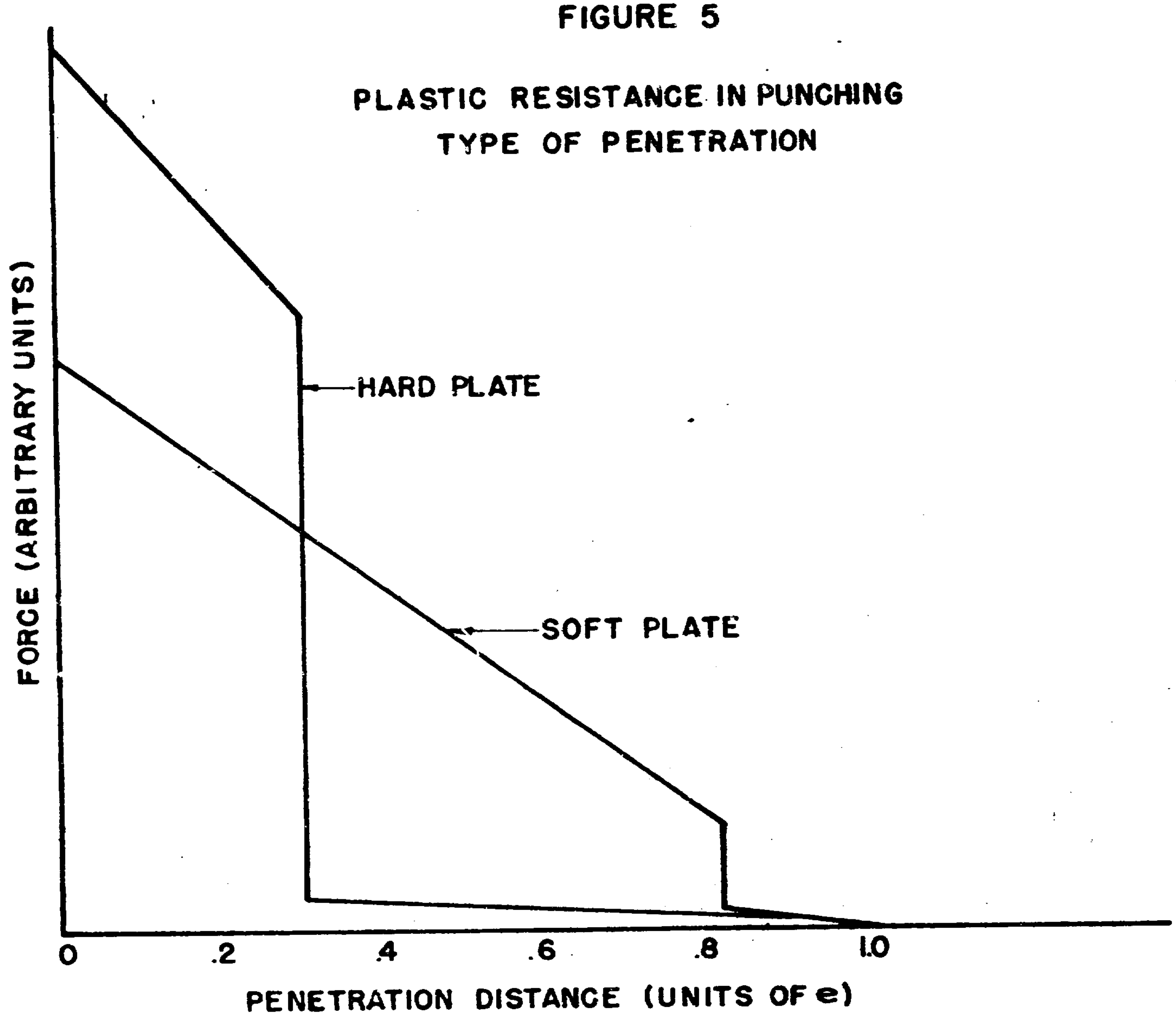


FIGURE 5

PLASTIC RESISTANCE IN PUNCHING
TYPE OF PENETRATION



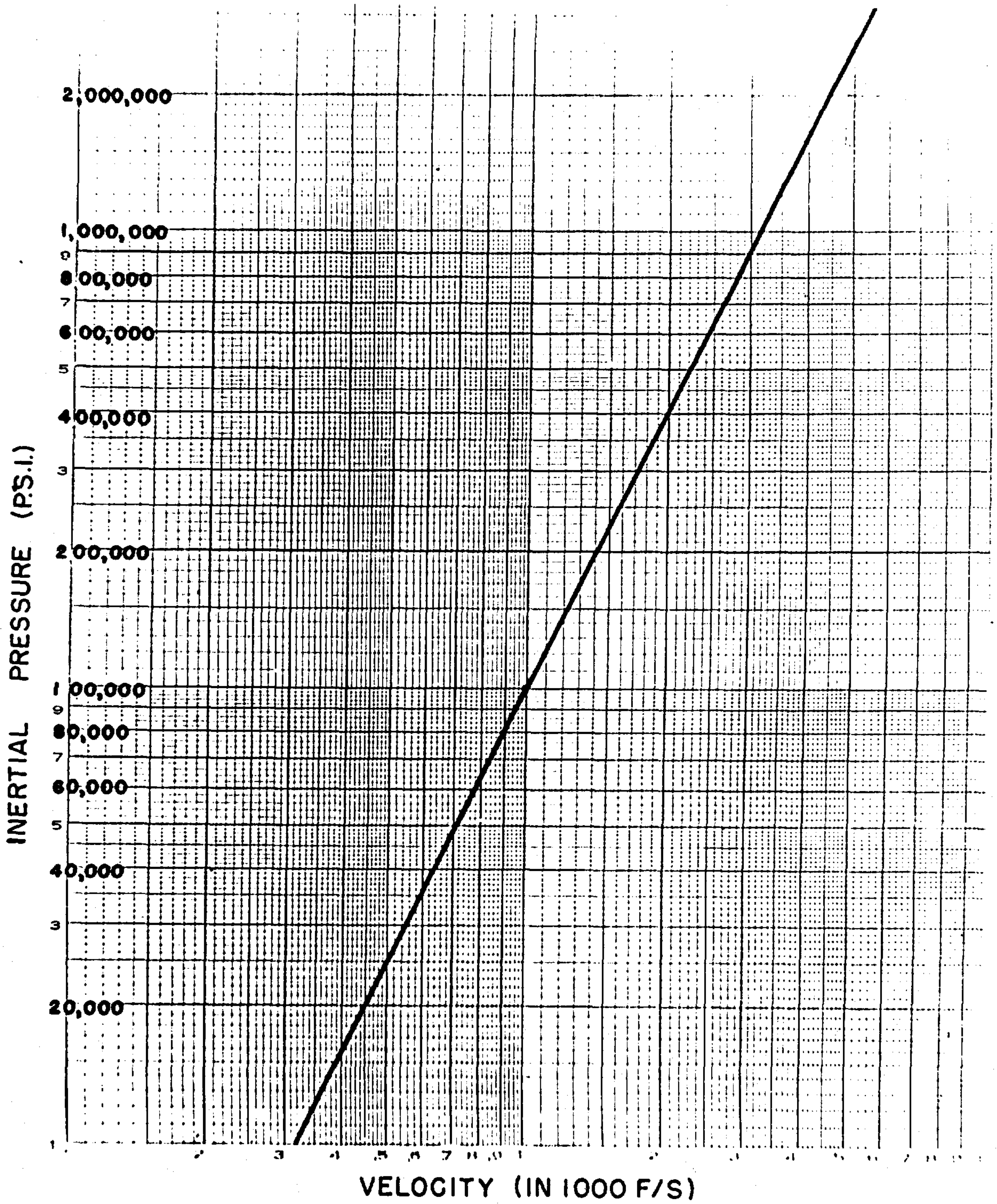
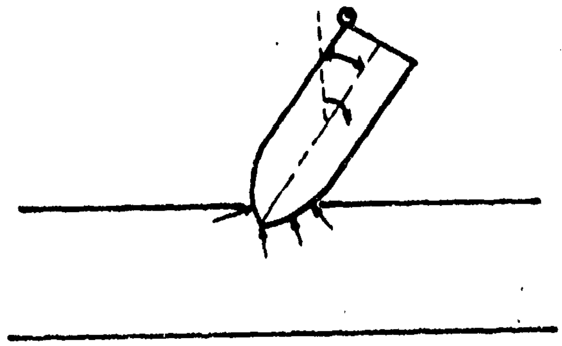
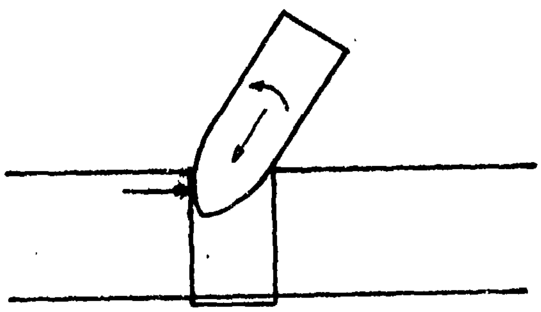


FIGURE 6

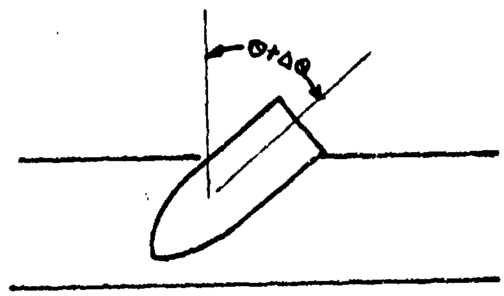
INERTIAL PRESSURE IN STEEL (ρv^2)



A. INITIAL FORCES ROTATE PROJECTILE CLOCKWISE.



B. IF PLUG IS FORMED AT ONCE, PROJECTILE ROTATES COUNTER-CLOCKWISE.



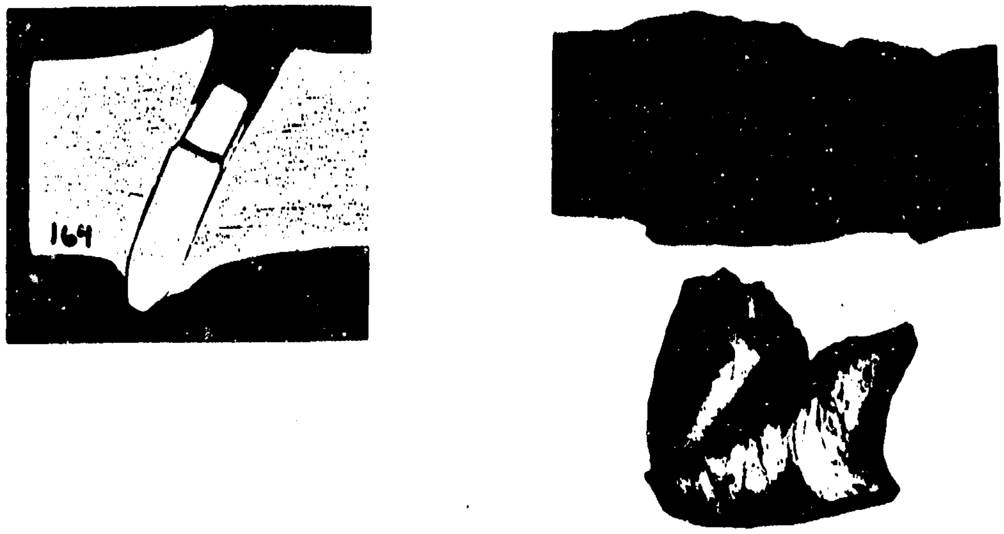
C. IF PLUG IS NOT FORMED AT ONCE, PROJECTILE PENETRATES IN DUCTILE FASHION AT AN INCREASED ANGLE OF OBLIQUITY.

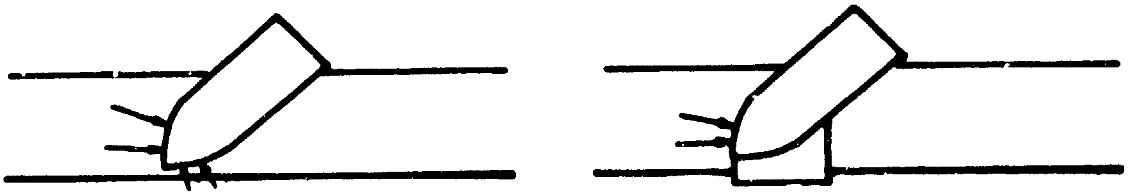
FIGURE 7

TYPES OF PLATE REACTION AT OBLIQUITIES (FACE OF PLATE)

CAL. .30 TYPE A
X 1

STANDARD CAL. .50 AP
X 2





A. CASE OF NO PLUG

B. CASE OF PLUG

FIGURE 8

PLATE REACTION AT OBLIQUITIES
(BACK OF PLATE)

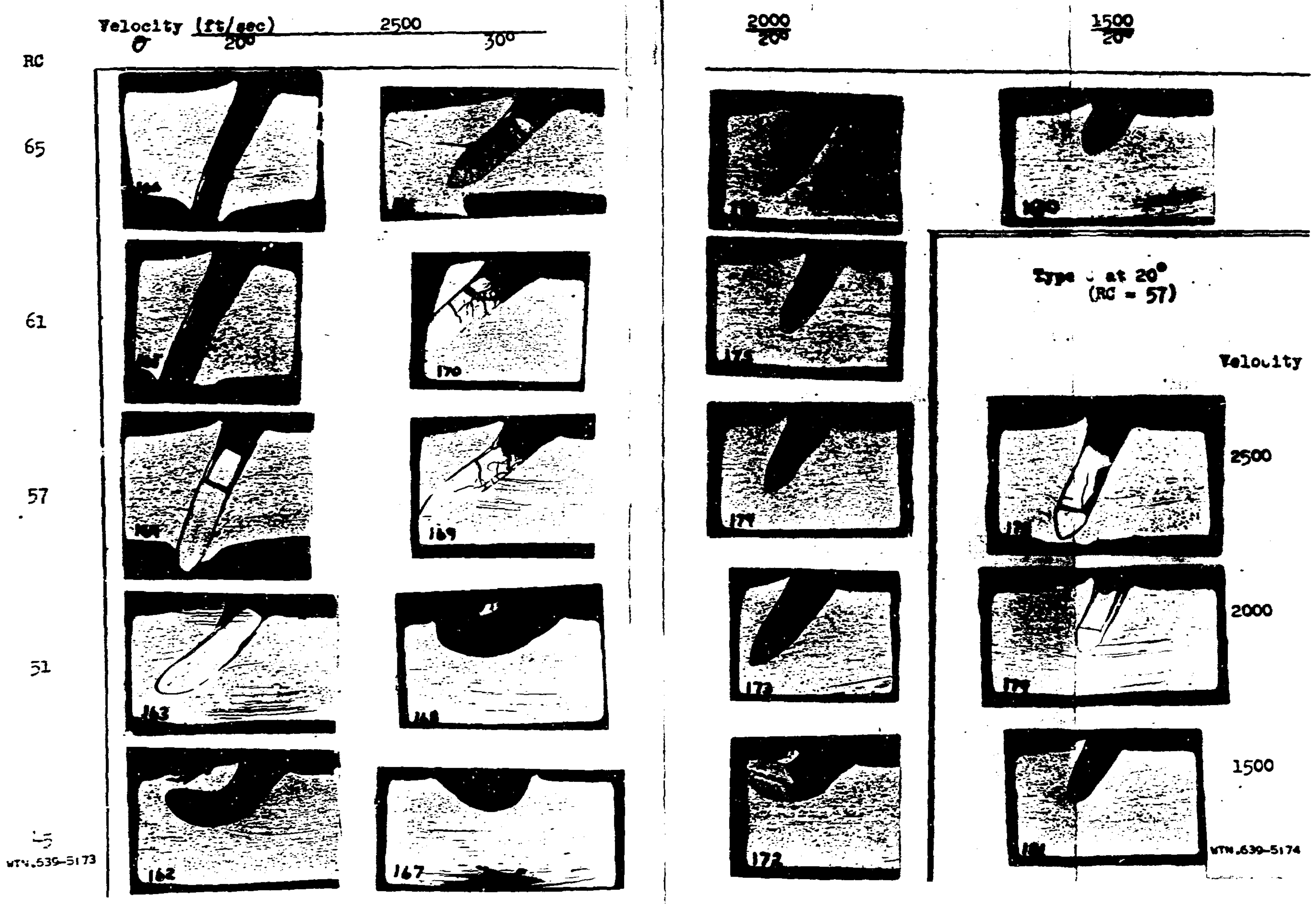
WTN.639-5172

FIGURE 9

SECTIONS FROM OBLIQUE IMPACTS

(TYPE A PROJECTILE)

PLATE HARDNESS: 155 BHN



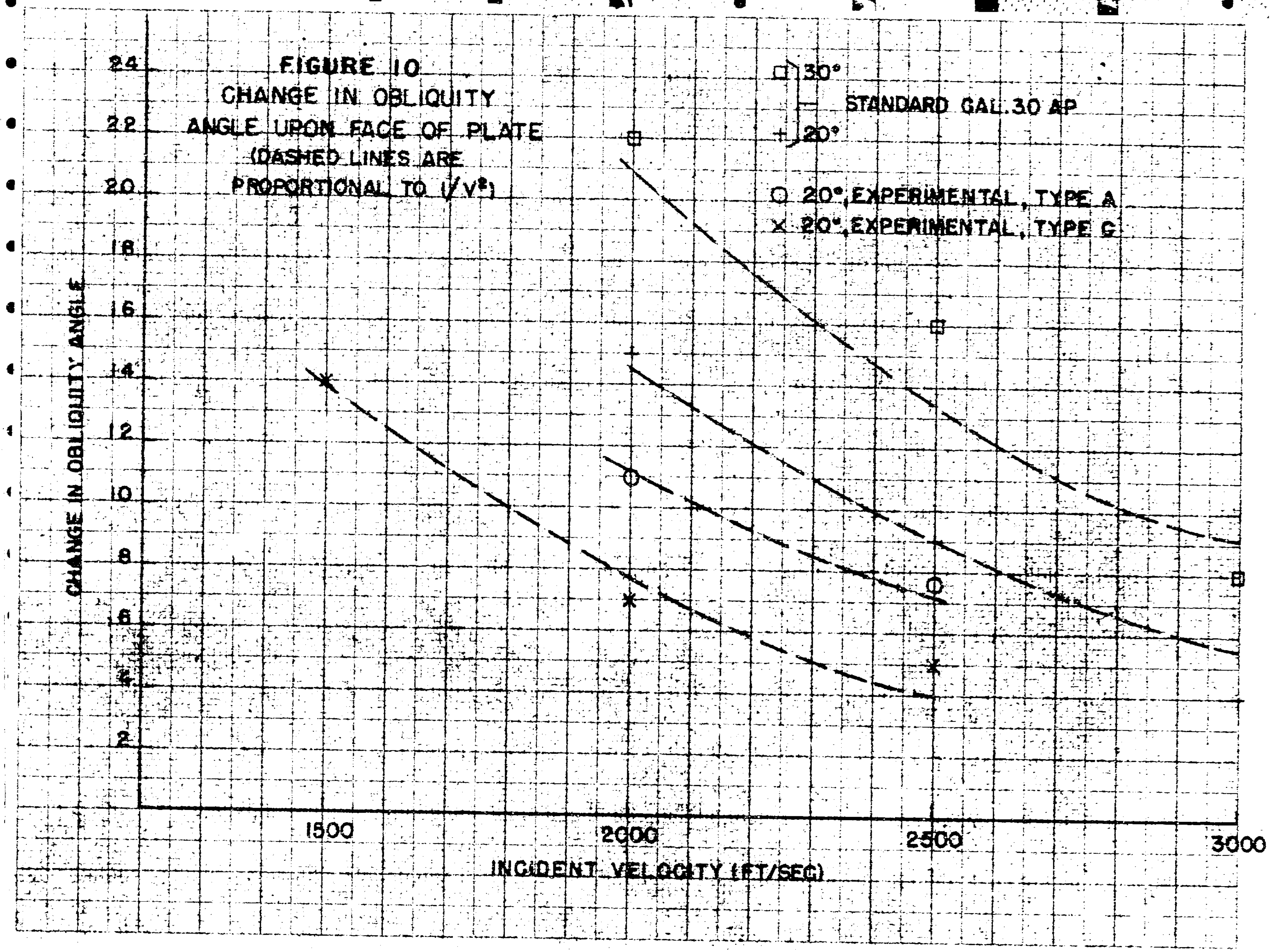


FIGURE 10
CHANGE IN OBLIQUITY
ANGLE UPON FACE OF PLATE
(DASHED LINES ARE
PROPORTIONAL TO 1/V²)
30°
STANDARD CAL.30 AP
20°
O 20°, EXPERIMENTAL, TYPE A
X 20°, EXPERIMENTAL, TYPE G
24
22
20
18
16
14
12
10
8
6
4
2
CHANGE IN OBLIQUITY ANGLE
1500
2000
2500
3000
INCIDENT VELOCITY (FT/SEC)

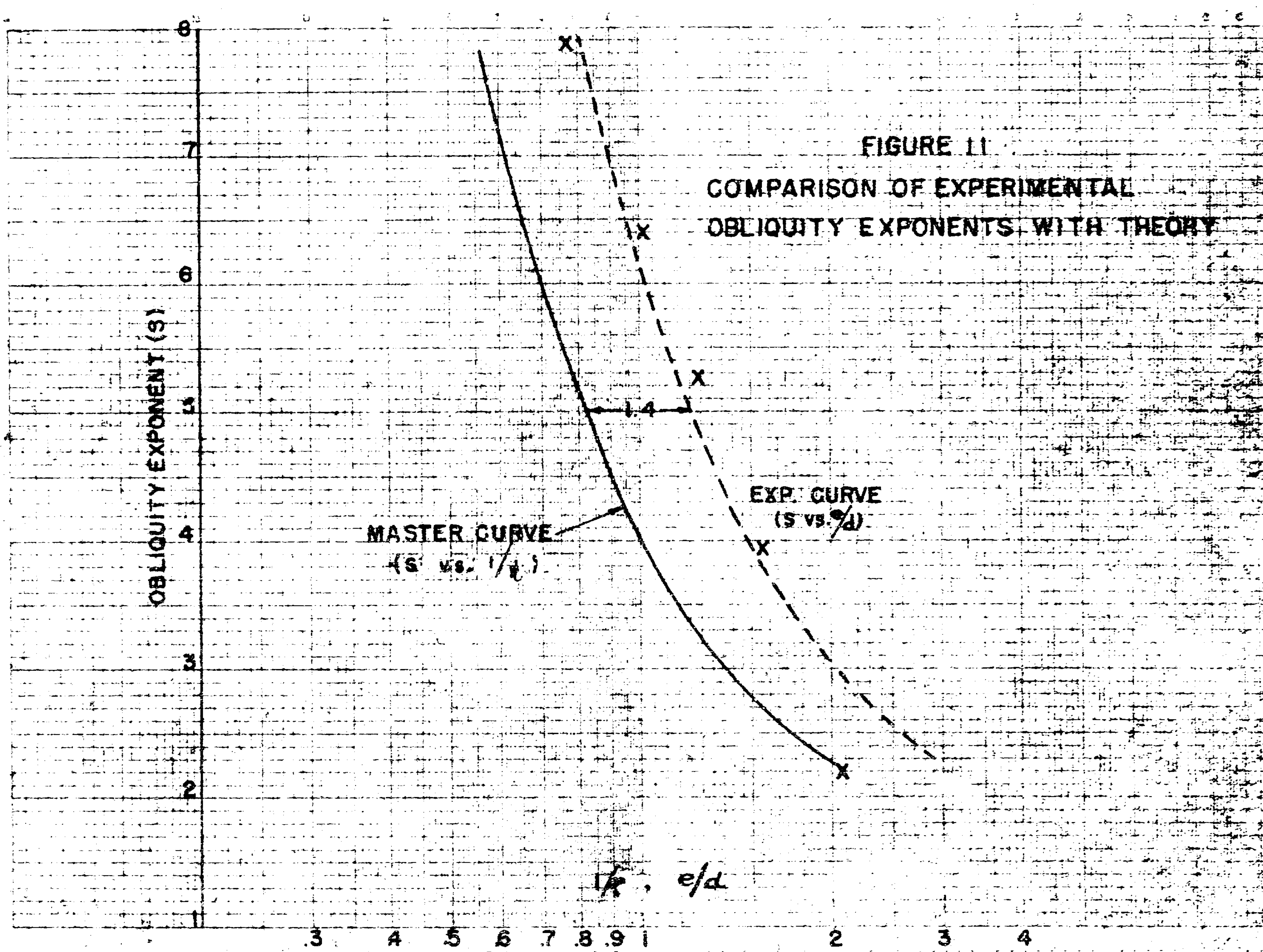


FIGURE 11
COMPARISON OF EXPERIMENTAL
OBLIQUITY EXPONENTS WITH THEORY
OBLIQUITY EXPONENT (S)
8
7
6
5
4
3
2
1
1.4
EXP. CURVE
(S vs. e/d)
MASTER CURVE
(S vs. 1/k)
1/k, e/d
.3
.4
.5
.6
.7
.8
.9
1
2
3
4

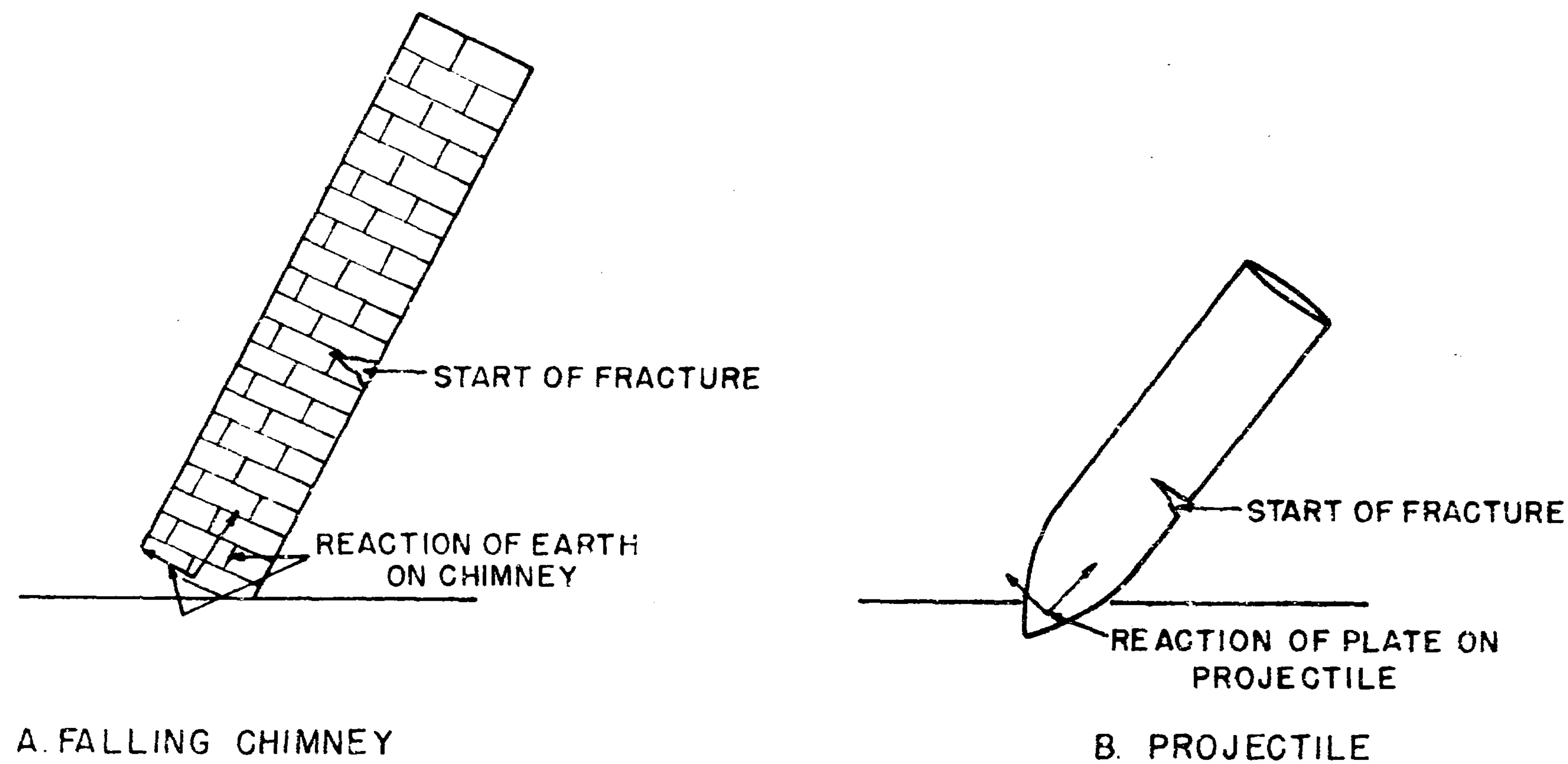
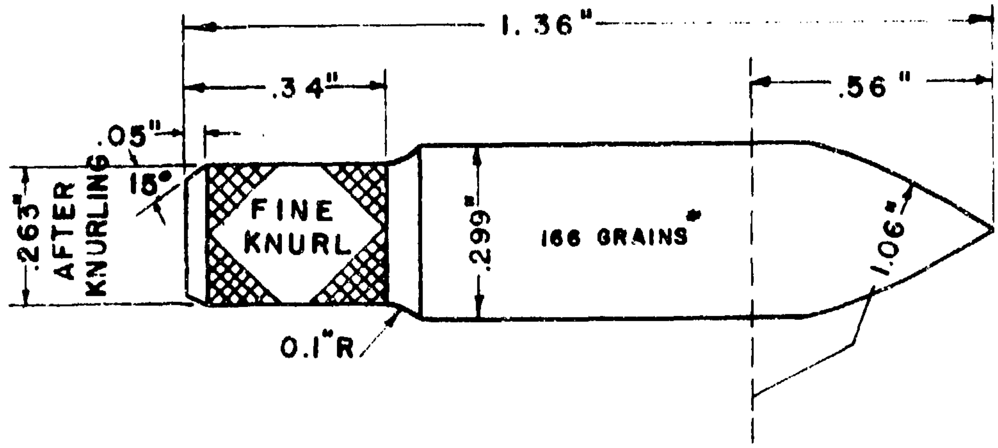


FIGURE 12

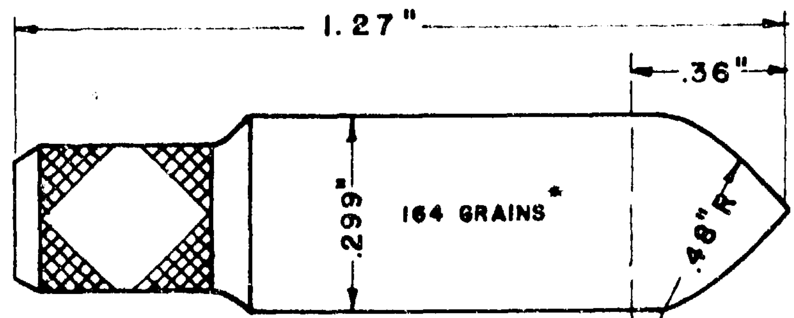
EXAMPLES OF FRACTURE BY TRANSVERSE FORCES.

FIG. 13 EXPERIMENTAL PROJECTILES

TYPE A — OGIVE SIMILAR TO THAT OF CAL.30 A.P. CORE.

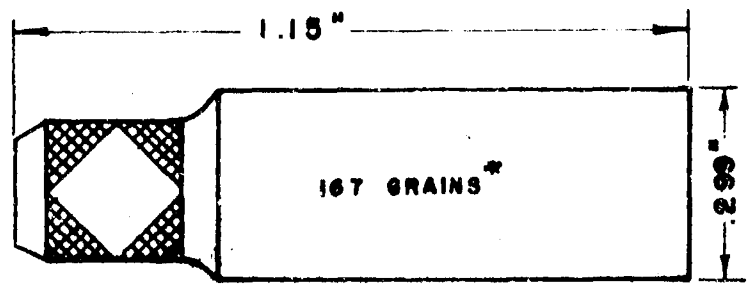


TYPE C — OGIVE = 1.6 CALIBRES



(TAIL DIMENSIONS AS ABOVE)

TYPE D — FLAT NOSED



(TAIL DIMENSIONS AS ABOVE)

SCALE: 3/1

*WEIGHT WITH ROTATING CAP

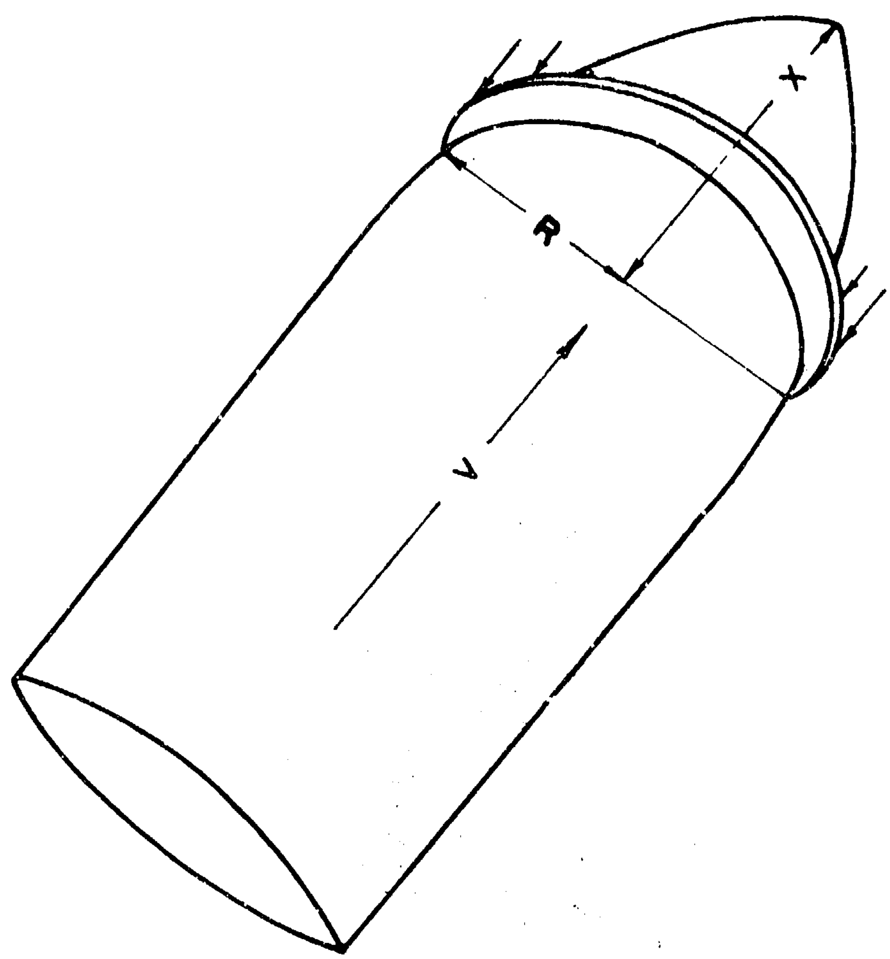


FIGURE 14

SIMPLIFIED PROBLEM USED IN CALCULATING INERTIA EFFECTS.

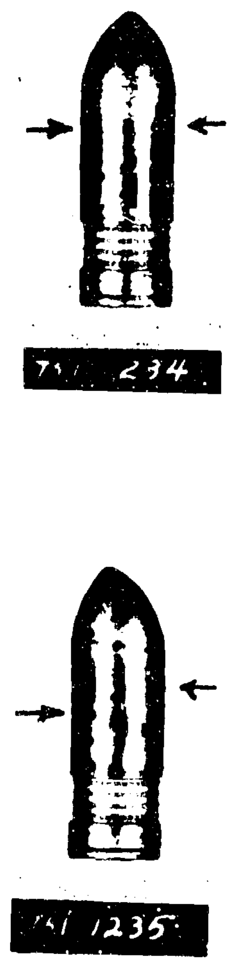


FIGURE 15

TENSILE CRACK IN 37mm AP PROJECTILE

WTN.639-5175