AD-A954 234

[redacted]

TM No.
TC-11-73

NAVAL UNDERWATER SYSTEMS CENTER
Newport, Rhode Island 02840

Technical Memorandum

PRE-FILTERING TO ENHANCE CLIPPER CORRELATOR PERFORMANCE

Date: 10 July 1973

Prepared by: Albert H. Nuttall
Albert H. Nuttall
Office of the Director of
Science and Technology

TM No.
TC-11-73

ABSTRACT

The use of appropriate pre-filtering in a clipper correlator yields performance within 1 dB of that attainable by means of a linear correlator which utilizes optimum pre-filters. A good choice of pre-filters are the Eckart filters for each of the individual system inputs. These conclusions hold for small signal-to-noise ratio and independent Gaussian noises at the system input.

ADMINISTRATIVE INFORMATION

This memorandum was prepared under Project No. A75205, Sub-Project No. ZF61112001, "Statistical Communication with Applications to Sonar Signal Processing," Principal Investigator Dr. A. H. Nuttall, Code TC. The sponsoring activity is Chief of Naval Material, Program Manager Dr. J. H. Huth.

The author of this memorandum is located at the New London Laboratory, Naval Underwater Systems Center, New London, Connecticut 06320.

INTRODUCTION

The use of a clipper correlator (CC) for signal detection and localization is a frequent occurrence. Here we wish to investigate the possibility of enhancing the performance of the CC by pre-filtering, prior to clipping. The linear correlator (LC) with its own optimum pre-filtering will be used as a comparison case.

A block diagram of the system of interest is given in Figure 1. Inputs

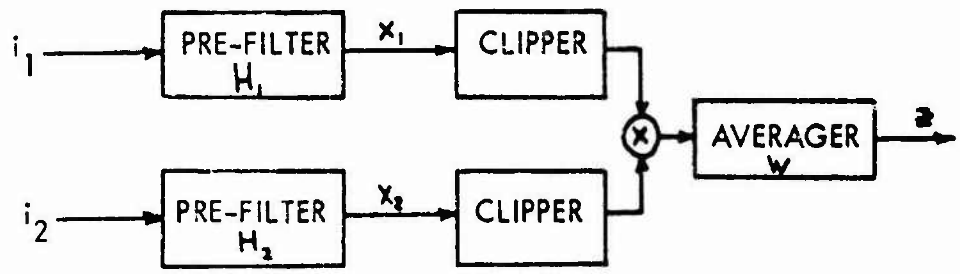

Figure 1. Clipper Correlator

i_1 and i_2 are given by

$$i_k = \left\{ \begin{array}{c} s_{i_k} + n_{i_k} \\ \text{OR} \\ n_{i_k} \end{array} \right\}, \quad k = 1,2 \tag{1}$$

where s_{i_k} is the input signal (if present) on channel k, and n_{i_k} is the accompanying input noise. The pre-filters are characterized by voltage-transfer functions H_k, and the averager is characterized by weighting w. The clippers yield outputs ± 1 depending on the polarity of their inputs. Thus system output

$$z = \int dt\, w(t)\, \mathrm{sgn}[x_1(t)]\, \mathrm{sgn}[x_2(t)]. \tag{2}$$

The time delay that is necessary in a correlator to line up the two received signals can be incorporated in the definition of transfer functions H_1 or H_2.

The waveforms x_k at the outputs of the pre-filters will be expressed as

$$x_k = \left\{ \begin{matrix} s_k + n_k \\ \text{OR} \\ n_k \end{matrix} \right\}, \quad k = 1, 2. \tag{3}$$

Since we assume that input noises n_{i_k} are independent, zero-mean, and stationary, filter output noises n_k are also. And if input signal s_{i_k} is zero-mean stationary, signal s_k possesses the same properties.

The LC is a special case of Figure 1, obtained by bypassing the clippers. The derivation of the optimum pre-filters in the LC is given in Ref. 1 for correlated Gaussian noises and arbitrary input signal-to-noise ratio (SNR). The optimum filters are extremely complicated, are not too informative, and require the detailed knowledge of the input signal and noise cross- and auto-spectra. Due to the non-linearity, this optimization is not possible for the CC, except via a numerical gradient-search approach. Accordingly, we limit consideration to uncorrelated Gaussian noises and small SNR at the input to the CC of Figure 1.

The measure of performance to be adopted for the system of Figure 1 is the deflection criterion. We define

$$d_z = \frac{\text{Change in Mean Output due to Signal}}{\text{Standard Deviation of Output for Noise-alone}}$$

$$= \frac{E\{z|S+N\} - E\{z|N\}}{\left[\mathrm{Var}\{z|N\}\right]^{1/2}} . \tag{4}$$

For the LC, when the clippers are absent, the output of Figure 1 will be denoted by the variable y, and the corresponding deflection by d_y.

LINEAR CORRELATOR PERFORMANCE

The derivation of the deflection for the LC is presented in Appendix A, for completeness. Under the assumption* that the averager effective duration L_w is much larger than the effective correlation extent of the noises, it is shown in (A-6) that

*This is in addition to the assumptions stated in the Introduction.

$$d_y^2 = \frac{R_{s_{12}}^2(0)}{N_1 N_2} \frac{L_w}{\int d\tau \, \rho_1(\tau) \rho_2(\tau)} , \tag{5}$$

where $R_{s_{12}}(0)$ is the cross-correlation (for zero delay) of signals s_k at the filter outputs, N_k is the noise power of the output noise n_k from filter H_k, ρ_k is the normalized correlation of n_k, and

$$L_w \equiv \frac{\left[\int dt \, w(t)\right]^2}{\int dt \, w^2(t)} . \tag{6}$$

Relation (5) holds for arbitrary signal and noise statistics and SNR.

In terms of the inputs to the system of Figure 1, (5) can be expressed as

$$d_y^2 = L_w \frac{\left[\int df \, H_1(f) H_2^*(f) G_{s_{12}}^{(i)}(f)\right]^2}{\int df \, |H_1(f)|^2 |H_2(f)|^2 G_{n_1}^{(i)}(f) G_{n_2}^{(i)}(f)} . \tag{7}$$

Equation (7) indicates that the only relevant quantity about the filters is the product $H_1(f) H_2^*(f)$. Also the detailed input signal and noise spectra, not merely their total powers, affect the deflection.

Application of Schwartz's inequality to (7) immediately yields

$$d_y^2 \le L_w \int df \frac{\left|G_{s_{12}}^{(i)}(f)\right|^2}{G_{n_1}^{(i)}(f) G_{n_2}^{(i)}(f)} \equiv d_{max}^2 . \tag{8}$$

This is the maximum attainable deflection, and can be realized only if the pre-filters satisfy

$$H_1(f) H_2^*(f) = \frac{G_{s_{12}}^{(i)*}(f)}{G_{n_1}^{(i)}(f) G_{n_2}^{(i)}(f)} . \tag{9}$$

For the special case of

$$s_{i_1}(t) = s_i(t), \quad s_{i_2}(t) = s_i(t - \tau_d), \tag{10}$$

then

$$G_{s_{12}}^{(i)}(f) = G_s^{(i)}(f)\exp(i2\pi f\tau_d), \tag{11}$$

and (8) yields

$$d_{max}^2 = L_w \int df \frac{G_s^{(i)^2}(f)}{G_{n_1}^{(i)}(f)\,G_{n_2}^{(i)}(f)}\,; \tag{12}$$

a particular solution of (9) is then

$$H_1(f) = \frac{\left[G_s^{(i)}(f)\right]^{1/2}}{G_{n_1}^{(i)}(f)}\exp(-i2\pi f\tau_d), \quad H_2(f) = \frac{\left[G_s^{(i)}(f)\right]^{1/2}}{G_{n_2}^{(i)}(f)}\,. \tag{13}$$

The filters in (13) are the Eckart filters (Ref. 2) for the individual inputs in Figure 1; the delay τ_d in H_1 is necessary to line up the particular signal forms in (10).

CLIPPER CORRELATOR PERFORMANCE

The derivation of the deflection for the CC is presented in Appendix B, under somewhat more general conditions than assumed above; they specialize to

$$d_z^2 = \frac{R_{s_{12}}^2(0)}{N_1 N_2}\;\frac{L_w}{\int d\tau \arcsin\{\rho_1(\tau)\}\arcsin\{\rho_2(\tau)\}} \tag{14}$$

for the case considered here (see (B-10)). An immediate observation to make is that d_z^2 is upper-bounded by the LC quantity d_y^2 in (5); this is proved in (B-11) - (B-15). The factor

$$Q_1 \equiv \frac{\int d\tau \arcsin\{\rho_1(\tau)\}\arcsin\{\rho_2(\tau)\}}{\int d\tau\,\rho_1(\tau)\,\rho_2(\tau)} \tag{15}$$

indicates how much the CC falls below the LC in performance as measured by quantity d_z^2. In Ref. 3, pages 5, 6, and 14, representative correlation functions are studied, and values of 10 log Q_1 in the range 1.20-2.06 dB were realized; furthermore, it is shown in Ref. 3, Appendix B, that the minimum value of 0 dB is possible, and that values of Q_1 significantly above 2.07 dB may not be possible.

If we require that the CC output deflection be identical to the LC output deflection, by increasing the input SNR for the CC, it is seen from (5) and (14) (since $R^2_{s_{12}}(0)$ is proportional to the square of signal power) that the CC requires

$$5 \log Q_1 \text{ dB} \tag{16}$$

more input SNR than the LC. Thus, the CC requires 0-1.04 dB more input SNR than the LC to realize the same output deflection, the exact amount depending on the shape of the correlations in (15), but not the bandwidths. (A common scale change in the numerator and denominator of (15) cancels out.)

The considerations above have compared the deflections of the LC and CC for arbitrary filters H_k. We now wish to specialize the filter choices to those that are optimum for the LC. (The problem set-up for maximization of d_z^2 is discussed in (B-17) - (B-22)). We adopt the signal model of (10) and let $\tau_d = 0$ for convenience. Then (B-24) yields

$$d_z^2 = \frac{d_{max}^2}{Q_2}, \tag{17}$$

where d_{max}^2 is given in (8), and

$$Q_2 = \frac{\int d\tau \arcsin\{q_1(\tau)\} \arcsin\{q_2(\tau)\}}{\int d\tau\, q_1(\tau) q_2(\tau)}; \tag{18}$$

the quantities q_k are the normalized Fourier transforms of the signal-to-noise spectral ratios, $G_s^{(i)}(f)/G_{n_k}^{(i)}(f)$, in the k-th channel input:

$$q_k(\tau) = \frac{\int df \exp(i2\pi f\tau)\, G_s^{(i)}(f)/G_{n_k}^{(i)}(f)}{\int df\, G_s^{(i)}(f)/G_{n_k}^{(i)}(f)}. \tag{19}$$

It therefore follows from (17) and (8) that if the optimum linear filters for the LC are used also for the CC, the CC requires an increased input SNR of

$$5 \log Q_2 \text{ dB} \tag{20}$$

in order to maintain an identical output deflection to that of the LC. Thus, as above, CC losses in the range 0 - 1.04 dB with respect to the optimum LC occur.

This degradation is slight enough to warrant use of the LC optimum filters in the CC as well.

COMMENTS

Although the pre-filters have not been specifically optimized for the CC, less than 1 dB degradation can be expected in most practical cases, with respect to the optimum that can be realized with a LC. The additional gain that could be achieved by an optimization of the filters for the CC itself is probably a small fraction of a dB and not worth the effort, considering that the optimization would have to be accomplished separately for each new set of signal and noise spectra.

For correlated input noises, the optimum pre-filters for the LC, which are derived in Ref. 1, could be used in the CC as well. (Their performance has not been investigated.) The difficulty of optimization for the CC probably precludes any other approach. If a bandwidth constraint on the filters is also imposed, the numerical optimization problem could be a formidable time-consuming task.

For larger input SNR, the analysis for the CC would have to be generalized, and the signal statistics would have to be known, such as Gaussian, for example. These effects on the deflection have not been investigated.

REFERENCES

1. D. W. Hyde and A. H. Nuttall, "Linear Pre-Filtering to Enhance Correlator Performance," USL Tech. Memo. No. 2020-34-69, 27 February 1969.

2. C. Eckart, "Optimal Rectifier Systems for the Detection of Steady Signals," Rpt. SIO 12692, Univ. of California, Scripps Institute of Oceanography, Marine Phys. Laboratory, SIO Ref. 52-11, 1952.

3. A. H. Nuttall, "Comparisons of Energy Detectors and Correlators with AGC or Clipping at the Element Outputs or at the Half-Beam Outputs," 26 February 1973.

APPENDIX A

DERIVATION OF LINEAR CORRELATOR PERFORMANCE

The LC is available from Figure 1 upon by-passing the clippers. The system output is then

$$y = \int dt\, w(t)\, x_1(t)\, x_2(t). \tag{A-1}$$

There follows, using (3) and the assumptions listed in the Introduction,

$$E\{y|S+N\} = \int dt\, w(t)\, E\{[s_1(t)+n_1(t)][s_2(t)+n_2(t)]\}$$

$$= \int dt\, w(t)\, R_{s_{12}}(0). \tag{A-2}$$

The signal need not be Gaussian.

It follows immediately from (A-2) that $E\{y|N\} = 0$. The variance of y is therefore

$$Var\{y|N\} = E\{y^2|N\} = \iint du\, dv\, w(u)\, w(v)\, E\{n_1(u)\, n_1(v)\, n_2(u)\, n_2(v)\}$$

$$= \iint du\, dv\, w(u) w(v) E\{n_1(u)\, n_1(v)\}\, E\{n_2(u) n_2(v)\}$$

$$= N_1 N_2 \iint du\, dv\;\; w(u)\, w(v)\; \rho_1(u-v)\rho_2(u-v)$$

$$= N_1 N_2 \int d\tau\;\; \phi_w(\tau)\, \rho_1(\tau)\rho_2(\tau), \tag{A-3}$$

where

$$\phi_w(\tau) \equiv \int dt\, w(t) w(t-\tau). \tag{A-4}$$

Now if the averager effective duration is much larger than the effective correlation extent of the noises, (A-3) becomes

$$Var\{y|N\} \cong N_1 N_2\, \phi_w(0) \int d\tau\; \rho_1(\tau)\; \rho_2(\tau)$$

$$= N_1 N_2 \int dt\, w^2(t) \int d\tau\, \rho_1(\tau)\, \rho_2(\tau). \tag{A-5}$$

Therefore

$$d_y^2 = \frac{\left[\text{Change in Mean Output due to Signal}\right]^2}{\text{Variance of Output for Noise-alone}}$$

$$= \frac{R_{s_{12}}^2(0)}{N_1 N_2} \frac{L_W}{\int d\tau \, \rho_1(\tau) \rho_2(\tau)}, \qquad \text{(A-6)}$$

where L_W is defined in (6).

APPENDIX B

DERIVATION OF CLIPPER CORRELATOR PERFORMANCE

The CC output is given by (2). Therefore

$$E\{z|S+N\} = \int dt\, w(t)\, E\{\mathrm{sgn}[x_1(t)]\,\mathrm{sgn}[x_2(t)]\}. \tag{B-1}$$

Now the average in (B-1) is, from (3),

$$E\{\mathrm{sgn}[s_1(t)+n_1(t)]\,\mathrm{sgn}[s_2(t)+n_2(t)]\}$$

$$= \iiiint ds_1\, ds_2\, dn_1\, dn_2\, \mathrm{sgn}[s_1+n_1]\,\mathrm{sgn}[s_2+n_2]\, p(s_1,s_2)\, p_1(n_1)\, p_2(n_2)$$

$$= \iint ds_1\, ds_2\, p(s_1,s_2)\left[1-2P_1(-s_1)\right]\left[1-2P_2(-s_2)\right], \tag{B-2}$$

where P_k is the cumulative probability distribution of noise n_k. For small SNR, the signal joint probability density function $p(s_1,s_2)$ peaks near the origin; thus $1-2P_k(-s_k)$ does not vary significantly where $p(s_1,s_2)$ is non-zero. Accordingly, we approximate

$$1-2P_k(-s_k) \cong 1-2P_k(0)+2p_k(0)s_k \quad \text{for } s_k \text{ near } 0. \tag{B-3}$$

Now the noises have zero-mean; if they also have symmetric density functions around zero, then $P_k(0)=\frac{1}{2}$, and (B-2) becomes

$$4\, p_1(0)\, p_2(0) \iint ds_1\, ds_2\, s_1\, s_2\, p(s_1,s_2)$$

$$= 4\, p_1(0)\, p_2(0)\, R_{s_{12}}(0). \tag{B-4}$$

The general result of (B-4)(for small SNR) is now specialized to Gaussian noise. Then

$$p_k(n) = (2\pi N_k)^{-1/2} \exp\left(-\frac{n^2}{2N_k}\right), \tag{B-5}$$

and (B-4) becomes

$$\frac{2}{\pi}\,\frac{R_{s_{12}}(0)}{\sqrt{N_1 N_2}}. \tag{B-6}$$

Therefore (B-1) is given by

$$E\{z|S+N\} = \int dt\, w(t)\, \frac{2}{\pi}\, \frac{R_{s_{12}}(0)}{\sqrt{N_1 N_2}} . \tag{B-7}$$

It follows immediately from (B-7) that $E\{z|N\} = 0$. Therefore

$$Var\{z|N\} = E\{z^2|N\} = \iint du\, dv\, w(u)\, w(v) \cdot$$

$$E\{sgn[n_1(u)]\, sgn[n_2(u)]\, sgn[n_1(v)]\, sgn[n_2(v)]\}$$

$$= \left(\frac{2}{\pi}\right)^2 \iint du\, dv\, w(u)\, w(v)\, \arcsin\{\rho_1(u-v)\}\, \arcsin\{\rho_2(u-v)\}$$

$$= \left(\frac{2}{\pi}\right)^2 \int d\tau\, \phi_w(\tau)\, \arcsin\{\rho_1(\tau)\}\, \arcsin\{\rho_2(\tau)\}, \tag{B-8}$$

using the properties of the independent Gaussian noises and (A-4). And if L_w in (6) is much larger than the effective correlation extent of the noises,

$$Var\{z|N\} \cong \left(\frac{2}{\pi}\right)^2 \int dt\, w^2(t) \int d\tau\, \arcsin\{\rho_1(\tau)\}\, \arcsin\{\rho_2(\tau)\}. \tag{B-9}$$

The deflection follows upon use of (B-7), (B-9) and (6) in (4):

$$d_z^2 = \frac{R_{s_{12}}^2(0)}{N_1 N_2}\, \frac{L_w}{\int d\tau\, \arcsin\{\rho_1(\tau)\}\, \arcsin\{\rho_2(\tau)\}} . \tag{B-10}$$

We now wish to show that

$$Q_1 \equiv \frac{\int d\tau\, \arcsin\{\rho_1(\tau)\}\, \arcsin\{\rho_2(\tau)\}}{\int d\tau\, \rho_1(\tau)\, \rho_2(\tau)} \geq 1 \tag{B-11}$$

for all ρ_1 and ρ_2. To do this, we expand

$$\arcsin\{\rho_k(\tau)\} = \rho_k(\tau) + \sum_{\ell=1}^{\infty} a_\ell\, \rho_k^{2\ell+1}(\tau), \tag{B-12}$$

where $a_\ell > 0$, all ℓ. Therefore, the numerator of Q_1 equals

$$\int d\tau\, \rho_1(\tau)\, \rho_2(\tau) + R \tag{B-13}$$

where R is composed of terms of the form

$$\int d\tau\, \rho_1^m(\tau)\, \rho_2^n(\tau) \equiv \int d\tau\, b_m(\tau)\, c_n(\tau) \tag{B-14}$$

with positive coefficients. Now (B-14) is equal to

$$\int df\, B_m(f) C_n(f) = \int df \left[\underbrace{B_1(f) \circledast \cdots \circledast B_1(f)}_{\text{(n times)}}\right]\left[\underbrace{C_1(f) \circledast \cdots \circledast C_1(f)}_{\text{(n times)}}\right], \qquad \text{(B-15)}$$

where $B_m(f)$ and $C_n(f)$ are the Fourier transforms of $b_m(\tau)$ and $c_n(\tau)$ respectively. But since $B_1(f)$ and $C_1(f)$ are non-negotive, being the Fourier tronsforms of $\rho_1(\tau)$ and $\rho_2(\tau)$ respectively, (B-15) is obviously non-negative. Therefore $R \geq 0$ in (B-13), and (B-11) is proved true.

A bound on the worst possible degradotion of the CC relctive to the LC hos not been attained for generol ρ_1 and ρ_2. However for the case of $\rho_1(\tau) = \rho_2(\tau)$, we can show that $Q_1 \leq (\pi/2)^2$:

$$\arcsin^2\{x\} \leq \left(\frac{\pi}{2}x\right)^2 \quad \text{for } |x| \leq 1. \qquad \text{(B-16)}$$

Substitution of (B-16) in (B-11) yields the desired relotion. The moximum difference in input SNR for the CC is therefore 1.96 dB relative to the LC.

The deflection in (B-10) is in terms of quontities at the filter outputs. To relate the deflection to the input statistics, and to indicate the problem of optimizing the filters in the CC for maximum deflection, we let

$$A_k(f) = H_k(f)\left[G^{(1)}_{n_k}(f)/N_k\right]^{1/2}, \quad K = 1,2. \qquad \text{(B-17)}$$

Then

$$\int df\, |A_k(f)|^2 = 1, \quad K = 1,2 \qquad \text{(B-18)}$$

$$\rho_k(\tau) = \int df \exp(i2\pi f\tau)\, |A_k(f)|^2, \quad K = 1,2 \qquad \text{(B-19)}$$

$$R_{s_{1,2}}(0) = \sqrt{N_1 N_2} \int df\, C(f) A_1(f) A_2^*(f), \qquad \text{(B-20)}$$

where

$$C(f) \equiv \frac{G^{(1)}_{s_{12}}(f)}{\left[G^{(1)}_{n_1}(f)\, G^{(1)}_{n_2}(f)\right]^{1/2}}. \qquad \text{(B-21)}$$

Then (B-10) can be expressed as

$$d_z^2 = L_w \frac{\left[\int df\, C(f) A_1(f) A_2^*(f)\right]^2}{\int d\tau \arcsin\left\{\int df \exp(i2\pi f\tau)|A_1(f)|^2\right\} \arcsin\left\{\int df \exp(i2\pi f\tau)|A_2(f)|^2\right\}} . \qquad \text{(B-22)}$$

The dependence of the deflection on general filters can be investigated by means of (B-22) in conjunction with (B-17) and (B-21).

The optimization problem for the CC consists of choosing complex functions A_1 and A_2 such that (B-22) is maximized, subject to constraints (B-18). (It is interesting to note that the individual signal and noise spectra are irrelevant except insofar as they enter via the single quantity C(f) in (B-21).) The maximization of (B-22) is very difficult, even with computer aid, and must be accomplished numerically for each C(f) of interest. Since a simpler approach, which comes very close to the performance of the optimum LC, can be found, this tack is not pursued further.

For the LC, and for the signal model of (10) with $\tau_d = 0$, the optimum filters were given in (13). Substitution in (B-17) yields

$$A_k(f) = \left[\frac{G_s^{(i)}(f)/G_{n_k}^{(i)}(f)}{\int df\, G_s^{(i)}(f)/G_{n_k}^{(i)}(f)}\right]^{1/2} . \qquad \text{(B-23)}$$

And substitution of (B-23) in (B-22) yields, with use of (B-21), the deflection for the CC as

$$d_z^2 = L_w \int df \frac{\left[G_s^{(i)}(f)\right]^2}{G_{n_1}^{(i)}(f) G_{n_2}^{(i)}(f)} \; \frac{\int d\tau\, q_1(\tau)\, q_2(\tau)}{\int d\tau \arcsin\{q_1(\tau)\} \arcsin\{q_2(\tau)\}} \equiv \frac{d_{max}^2}{Q_2} , \qquad \text{(B-24)}$$

where

$$q_k(\tau) = \frac{\int df \exp(i2\pi f\tau) G_s^{(i)}(f)/G_{n_k}^{(i)}(f)}{\int df\, G_s^{(i)}(f)/G_{n_k}^{(i)}(f)} . \qquad \text{(B-25)}$$

Thus the CC requires an increased input SNR of 5 log Q_2 dB in order to maintain the identical output deflection as the LC, when both use the optimum filters appropriate to the LC.

Pre-Filtering to Enhance Clipper Correlator Performance
Albert H. Nuttall
Office of the Director of Science and Technology
TM No. TC-11-73
10 July 1973
Project No. A75205
UNCLASSIFIED

DISTRIBUTION LIST

External

DLP/MAT 03L4 (Dr. J. H. Huth)

Internal

Codes
LA151 (2)
LA152 (10)
LA182 (R. Bernier)
LA12 (L. Hull)
T (W. Von Winkle)
TC (A. H. Nuttall)(3)
TC1 (D. W. Hyde)
TC1 (G. C. Carter)
TC (P. G. Cable)
TC1 (A. H. Quazi)
TD111 (H. Newman)
TD111 (R. Kneipfer)
SA1 (H. Fridge)
TD111 (A. F. Magaraci)
TA111 (W. Roderick)
SA111 (J. Pratt)
TD11 (J. Kingsbury)
C2 (W. Marsh)
PA4 (L. Einstein)
TD11i (J. Ianniello)
TD111 (N. Owsley)
SA21 (J. Snow)
SA12 (R. Garber)
PA11 (M. Pastore)