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A Method for Calculating the Pressure Distribution of a Body of Revolution Moving in a Circular Path through a Perfect Incompressible Fluid

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Summary.—A method is developed for calculating the pressure distribution of a body of revolution moving along a circular path in an infinite inviscid fluid otherwise still.

The method is based on the use of source distributions on the surface of the body. Their strength is determined by solving three integral equations corresponding to the elementary cases of a longitudinal translation, a lateral translation and a rotation of the body about a lateral axis. The distributions of the relative velocity on the surface are then obtained by integrations. The complete solution for any particular case is obtained by a linear combination of the three elementary cases.

An arrangement of the numerical work is given by which it can be carried out fairly automatically by a computer of average experience; several computers may share the work between them. Tables are given facilitating the calculation of the kernels of the integrals which forms the major part of the numerical work.

Introduction.—Unlike the problems of calculating the pressure distribution along a given body of revolution in motion along a straight line through a perfect fluid with or without an angle of incidence for which numerous solutions have been given, the case of a body of revolution moving along a circular path appears to have been treated so far only for a few special cases. The exact solution of this problem is known for an ellipsoid and its various degenerations from the classical investigations of the nineteenth century which give also the lines of approach to the general problem. An approximate solution for very elongated bodies was given by M. M. Munk in connection with his work on the theory of airships. Very few experimental results are available owing to the difficulty of measuring sufficiently accurately the pressure distribution of a model attached to a whirling arm.

It is easy to see that the motion of a body along a circle can be obtained by a superposition of a longitudinal translation, a lateral translation and a rotation of the body about its centre (= centre of its axis). These elementary motions can be treated separately and, methods being available for calculating the effect of the translations, there remains only the question of calculating the relative flow caused by a rotation of the body about its centre.

This problem is closely related to the problem of calculating the transverse flow past the body, and any of the methods developed for this purpose (*e.g.* by Th. v. Karman, C. Kaplan or Mrs. Flügge-Lotz) can readily be extended to the case of a rotating body.

The present investigation was undertaken with a view to developing a method for calculating the pressure distribution for a torpedo. This means in mathematical language that the shape of the body may be composed of arcs of different curves (often straight lines and arcs of circles) and that its curvature may have isolated discontinuities.

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The methods of v. Karman and Kaplan which use either explicitly or implicitly a generation of the body by a distribution of singularities on its axis, will then fail in principle, because they require the possibility of an analytic continuation of the flow into the interior of the body which is free from singularities except on its axis. The method of Mrs. Flügge-Lotz which uses a generation of the body by a distribution of sources on its surface, is free from this restriction, but has the practical disadvantage of requiring an excessive amount of numerical work.

The present report is an attempt to reduce the work involved in the method of Mrs. Flügge-Lotz. The integrations which have to be carried out for this method are performed with the aid of a mechanical integration formula which takes account of the singularities of the integrands. Tables are given facilitating the numerical calculation of the kernels of the integrals and the whole numerical work is reduced to the filling of certain forms which can be done fairly automatically by a computer of average experience. The numerical calculation is arranged in such a way that several computers can be engaged in the work simultaneously. If at all possible, the work should be divided up among several persons, as it is still so laborious that a single computer is likely to get tired of it. It may be worth investigating if the present method can serve as a basis for the application of high-speed electronic calculating equipment to the problem in question.

With a view to the application of the method to the case of a torpedo the arrangement of the numerical work was made under the assumption that the shape of the body is 'normal', *i.e.*, that it can be given in cylindrical co-ordinates by an equation $r = r(x)$, where $r(x)$ is a single-valued function (not necessarily the same analytical expression in different parts of the body) with a continuous derivative $r'(x)$ and a piecewise continuous second derivative $r''(x)$. The shape is supposed to have a finite radius of curvature at the ends of the body. The case of pointed ends can be treated in the same way after rounding off the ends with a small radius of curvature; this has only a local effect on the pressure distribution which remains practically unaltered elsewhere. The case of an abnormal shape of the body, *e.g.*, of a body with a flat nose can be treated in a similar manner, using the same integration formula but integrating along the arc of the shape instead of along the axis of the body. It was further assumed in setting up the arrangement for the numerical work that the equations defining the shape have a comparatively simple structure so that it is not too difficult and not too laborious to calculate r, r' and r'' for a great number of points.

Throughout the present report, the maximum radius of the body is used as unit of length. This is convenient as the singularities generating the body under the various flow conditions have the form of source rings with either constant (longitudinal translation) or sinusoidal (latitudinal translation or rotation) distribution of their strength per unit length of arc of the ring. When dealing with such source rings, the radius of the ring is a natural unit of length and, the body being supposed to be elongated, its maximum radius will be of the order of the local radius almost everywhere on the axis of the body except very near to its ends.

The author regrets that, due to circumstances beyond his control, he has to present his results in a comparatively condensed form.

PART A.—GENERAL THEORY

1. *Systems of Co-ordinates and their Transformation.*—Consider a body of revolution revolving with a constant angular velocity Ω about a fixed axis in an inviscid fluid otherwise still. All its points will then move with constant velocity ΩR along circles of radius R about points of the axis of rotation. Let C be the circle of radius R_0 and centre O_0 along which the centre O_b of the body (= centre of its axis) is moving with the velocity ΩR_0.

The complete motion can then be described by using a system of co-ordinates (X, Y, Z; unit vectors $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$) with the origin O_0, the axis of X coincident with the line O_0O_b at the time $t = 0$,

the axis of Y perpendicular to the axis of X, coincident with the position of O_aO_b at the time $t = \pi/2\Omega$, the axis of Z perpendicular to both the axes of X and Y and having the direction of the vector $\mathbf{\Omega}$. These co-ordinates will be referred to as absolute co-ordinates.

The use of the absolute co-ordinates has the practical disadvantage that the position of the body and the pattern of the flow are time-dependent. It is easier to use a system of relative co-ordinates (x, y, z) with the unit vectors $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ attached to the body in which the flow pattern is independent of the time t. On the other hand, the flow is a time-dependent potential flow in (X, Y, Z) for which the pressure is readily obtained from Kelvin's formula. In (x, y, z), the flow is not a potential flow but has a constant vorticity. It is, however, possible to transform Kelvin's formula into the system (x, y, z) and to obtain in this way a convenient formula for the calculation of the pressure distribution.

In order to perform the transformation $(X, Y, Z) \rightleftarrows (x, y, z)$, consider first an auxiliary system of co-ordinates (T, N, B) with the origin at the centre O_b of the body. Let the axis of T be tangent to the circle C, pointing into the direction in which the body is moving. Let further the axis of N coincide with the line O_aO_b pointing into the interior of C and let finally the axis of B be perpendicular to the plane of C and point into the direction of $\mathbf{\Omega}$ (Fig. 1).

The absolute co-ordinates of O_b at the time t are obviously given by

$$\mathbf{O_aO_b} = R_0 \cos \Omega t \mathbf{X} + R_0 \sin \Omega t \mathbf{Y} + O\mathbf{Z} \quad \ldots \quad (1)$$

The angles between the axes (X, Y, Z) and (T, N, B) are easily seen to be

$$\left.\begin{array}{lll} (X, T) = \frac{\pi}{2} + \Omega t & (X, N) = \pi + \Omega t & (X, B) = \frac{\pi}{2} \\ (Y, T) = \Omega t & (Y, N) = \frac{\pi}{2} + \Omega t & (Y, B) = \frac{\pi}{2} \\ (Z, T) = \frac{\pi}{2} & (Z, N) = \frac{\pi}{2} & (Z, B) = 0 \end{array}\right\} \quad \ldots \quad (2)$$

This gives the transformation

$$\begin{bmatrix} X - R_0 \cos \Omega t \\ Y - R_0 \sin \Omega t \\ Z \end{bmatrix} = \begin{bmatrix} -\sin \Omega t & -\cos \Omega t & 0 \\ \cos \Omega t & -\sin \Omega t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}, \quad \ldots \quad (3a)$$

and the inverse transformation

$$\begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} -\sin \Omega t & \cos \Omega t & 0 \\ -\cos \Omega t & -\sin \Omega t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X - R_0 \cos \Omega t \\ Y - R_0 \sin \Omega t \\ Z \end{bmatrix}. \quad \ldots \quad (3b)$$

Consider now the system of co-ordinates (x, y, z) with the unit vectors $(\mathbf{x}, \mathbf{y}, \mathbf{z})$. Its origin is again the centre O_b of the body. The axis of x is the axis of symmetry of the body, pointing into the direction in which the body is moving and forming an angle α (= angle of pitch) with the plane of C. The axis of y is perpendicular to the axis of x, in the plane of C, pointing into its interior and forming an angle β (= angle of yaw) with the axis of N. The axis of z is perpendicular to both the axes of x and y, pointing into the direction of the vector product $\mathbf{x} \times \mathbf{y}$.

The angles between the axes (T, N, B) and (x, y, z) are then easily seen to be given by

$$\left.\begin{array}{lll} \cos (T, x) = \cos \alpha \cos \beta & \cos (T, y) = \sin \beta & \cos (T, z) = -\cos \beta \sin \alpha \\ \cos (N, x) = -\cos \alpha \sin \beta & \cos (N, y) = \cos \beta & \cos (N, z) = \sin \alpha \sin \beta \\ \cos (B, x) = \sin \alpha & \cos (B, y) = 0 & \cos (B, z) = \cos \alpha \end{array}\right\} \quad \ldots \quad (4)$$

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This gives the transformation

$$\begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} \cos\alpha\ \cos\beta & \sin\beta & -\cos\beta\ \sin\alpha \\ -\cos\alpha\ \sin\beta & \cos\beta & \sin\alpha\ \sin\beta \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \ldots \quad \ldots \quad \ldots \quad (5a)$$

and inversely

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\alpha\ \cos\beta & -\cos\alpha\ \sin\beta & \sin\alpha \\ \sin\beta & \cos\beta & 0 \\ -\sin\alpha\ \cos\beta & \sin\alpha\ \sin\beta & \cos\alpha \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}. \quad \ldots \quad \ldots \quad \ldots \quad (5b)$$

The transformation $(X, Y, Z) \rightleftarrows (x, y, z)$ is then obtained by combining (3) and (5)

$$\begin{bmatrix} X - R_0 \cos \Omega t \\ Y - R_0 \sin \Omega t \\ Z \end{bmatrix} = \begin{bmatrix} -\cos\alpha\ \sin(\Omega t - \beta) & -\cos(\Omega t - \beta) & \sin(\Omega t - \beta)\sin\alpha \\ \cos\alpha\ \cos(\Omega t - \beta) & -\sin(\Omega t - \beta) & -\cos(\Omega t - \beta)\sin\alpha \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad (6a)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\cos\alpha\ \sin(\Omega t - \beta) & \cos\alpha\ \cos(\Omega t - \beta) & \sin\alpha \\ -\cos(\Omega t - \beta) & -\sin(\Omega t - \beta) & 0 \\ \sin\alpha\ \sin(\Omega t - \beta) & -\sin\alpha\ \cos(\Omega t - \beta) & \cos\alpha \end{bmatrix} \begin{bmatrix} X - R_0 \cos \Omega t \\ Y - R_0 \sin \Omega t \\ Z \end{bmatrix}. \quad \ldots \quad (6b)$$

Instead of using the relative cartesian co-ordinates (x, y, z), it is more convenient to use relative cylindrical co-ordinates (x, r, θ) for the practical calculation :

$$x = x \qquad y = r\cos\theta \qquad z = r\sin\theta\,, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7a)$$

$$x = x \qquad r = \sqrt{(y^2 + z^2)} \qquad \theta = \tan^{-1}\frac{z}{y}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7b)$$

For the calculation of the relative velocity at a point P of the surface of the body (co-ordinates x_0, r_0, θ_0), it is necessary to introduce a further system of co-ordinates (n, t, b ; unit vectors $\mathbf{n}, \mathbf{t}, \mathbf{b}$) attached to this point. The axis of n is normal to the surface of the body, pointing into its exterior ; the axis of t is tangential to the meridian curve of the body passing through P and is pointing from its head ($x = +l/2$) to its tail ($x = -l/2$). The axis of b is perpendicular to the axes of n and t, pointing into the direction of increasing values of the angular co-ordinate θ (= direction of the vector product $\mathbf{n} \times \mathbf{t}$) (Fig. 2).

Only the angles between the axes (x, y, z) and (n, t, b) will be needed, as the transformation $(x, y, z) \rightleftarrows (n, t, b)$ will only be used for velocity components, *i.e.*, for vectors.

If the shape of the body is given by an equation

$$r = r(x) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

in the cylindrical co-ordinates, the cosines of these angles (= scalar products of the corresponding unit vectors) are easily seen to be

$$\begin{aligned} \mathbf{n}\,.\,\mathbf{x} &= -\frac{r'}{\sqrt{(1 + r'^2)}} & \mathbf{n}\,.\,\mathbf{y} &= \frac{\cos\theta}{\sqrt{(1 + r'^2)}} & \mathbf{n}\,.\,\mathbf{z} &= \frac{\sin\theta}{\sqrt{(1 + r'^2)}} \\ \mathbf{t}\,.\,\mathbf{x} &= -\frac{1}{\sqrt{(1 + r'^2)}} & \mathbf{t}\,.\,\mathbf{y} &= \frac{-r'\cos\theta}{\sqrt{(1 + r'^2)}} & \mathbf{t}\,.\,\mathbf{z} &= \frac{-r'\sin\theta}{\sqrt{(1 + r'^2)}} \\ \mathbf{b}\,.\,\mathbf{x} &= 0 & \mathbf{b}\,.\,\mathbf{y} &= -\sin\theta & \mathbf{b}\,.\,\mathbf{z} &= \cos\theta \end{aligned} \quad \ldots \quad \ldots \quad (9)$$

2. *Absolute Velocity of a Point of the Body* [*Co-ordinates* (X, Y, Z); (x, y, z) *and* (x, r, θ)].—The absolute velocity of a point P of the body, *i.e.*, of a point with fixed co-ordinates (x, y, z), is

$$\mathbf{V}_{abs} = \mathbf{\Omega} \times \mathbf{O_aP}\,, \qquad (10)$$

where

$$\mathbf{\Omega} = \Omega\mathbf{Z} \qquad (11)$$

is the vector of the angular velocity and O_a the origin of the absolute co-ordinates. If, at the time t, P has the absolute co-ordinates X, Y, Z, the vector $\mathbf{O_aP}$ is

$$\mathbf{O_aP} = X\mathbf{X} + Y\mathbf{Y} + Z\mathbf{Z}\,. \qquad (12)$$

The absolute velocity is then

$$\mathbf{V}_{abs} = \begin{vmatrix} \mathbf{X} & \mathbf{Y} & \mathbf{Z} \\ 0 & 0 & \Omega \\ X & Y & Z \end{vmatrix} = \Omega(-Y\mathbf{X} + X\mathbf{Y})\,. \qquad (13)$$

Introducing the relative co-ordinates (x, y, z), $\mathbf{V}_{abs}$ becomes with respect to (6)

$$\mathbf{V}_{abs} = \Omega[(R_0 \cos\beta - y)\cos\alpha\mathbf{x} + (R_0 \sin\beta + x\cos\alpha - z\sin\alpha)\mathbf{y} - (R_0\cos\beta - y)\sin\alpha\mathbf{z}]\,. \qquad (14)$$

This gives

$$V_{abs}^2 = \Omega^2[(R_0\cos\beta - y)^2 + (R_0\sin\beta + x\cos\alpha - z\sin\alpha)^2] \qquad (15)$$

$$= \Omega^2[(R_0\cos\beta - r\cos\theta)^2 + (R_0\sin\beta + x\cos\alpha - r\sin\theta\ \sin\alpha)^2]\,. \qquad (15a)$$

Equation (14) can also be written in the form

$$\mathbf{V}_{abs} = \Omega[R_0\cos\alpha\ \cos\beta\mathbf{x} + R_0\sin\beta\mathbf{y} - R_0\sin\alpha\ \cos\beta\mathbf{z} + \cos\alpha(x\mathbf{y} - y\mathbf{x}) + \sin\alpha(y\mathbf{z} - z\mathbf{y})]\,. \qquad (14a)$$

This means that the complete movement of the body can be considered as the result of a superposition of the following elementary movements:

$$\left.\begin{array}{l} \text{(i) Translation in the direction of } x \text{ with the velocity } V_x = \Omega R_0\cos\alpha\ \cos\beta \\ \text{(ii) Translation in the direction of } y \text{ with the velocity } V_y = \Omega R_0\sin\beta \\ \text{(iii) Translation in the direction of } z \text{ with the velocity } V_z = -\Omega R_0\sin\alpha\ \cos\beta \\ \text{(iv) Rotation about the axis of } x \text{ with the angular velocity } \Omega_x = \Omega\sin\alpha \\ \text{(v) Rotation about the axis of } z \text{ with the angular velocity } \Omega_z = \Omega\cos\alpha \end{array}\right\}\,. \qquad (16)$$

No rotation occurs about the axis of y, this axis being chosen in a plane perpendicular to the vector $\mathbf{\Omega}$.

3. *Relative Particle Velocity of the Fluid at a Point of the Body* [*Co-ordinates* (X, Y, Z) *and* (x, y, z)].—If P is a point on the surface of the body or outside the body in a fixed relative position (constant co-ordinates x, y, z) and if $\Phi(X, Y, Z, t)$ is the time-dependant potential of the flow, the relative particle velocity of the fluid at P will obviously be

$$\mathbf{V}_{rel} = (\Phi_x - \mathbf{V}_{abs}\cdot\mathbf{x})\mathbf{x} + (\Phi_y - \mathbf{V}_{abs}\cdot\mathbf{y})\mathbf{y} + (\Phi_z - \mathbf{V}_{abs}\cdot\mathbf{z})\mathbf{z}\,. \qquad (17)$$

The derivatives Φ_x, Φ_y, Φ_z can be obtained from $\Phi(X, Y, Z, t)$ by the transformation equation (6); these expressions are, however, of no particular interest for the present purpose. Introducing the components of $\mathbf{V}_{abs}$ from (14), the relative velocity becomes

$$\begin{aligned}\mathbf{V}_{rel} = {} & [\Phi_x - \Omega \cos\alpha(R_0 \cos\beta - y)]\mathbf{x} \\ & + [\Phi_y - \Omega(R_0 \sin\beta + x\cos\alpha - z\sin\alpha)]\mathbf{y} \\ & + [\Phi_z - \Omega \sin\alpha(R_0\cos\beta - y)]\mathbf{z} \qquad \ldots \qquad (18)\end{aligned}$$

and consequently with respect to equation (15)

$$\begin{aligned}V_{rel}^2 = {} & V_{abs}^2 - 2\Omega[(\Phi_x \cos\alpha - \Phi_z \sin\alpha)(R_0\cos\beta - y) \\ & + \Phi_y(R_0 \sin\beta + x\cos\alpha - z\sin\alpha)] + [\Phi_x^2 + \Phi_y^2 + \Phi_z^2]\,. \qquad \ldots \qquad (19)\end{aligned}$$

4. *Calculation of the Pressure.*—The absolute flow caused by the revolving body is a time-dependant potential flow; the pressure is therefore to be calculated from Kelvin's formula

$$p - p_0 = -\rho\Phi_t - \tfrac{1}{2}\rho(\Phi_X^2 + \Phi_Y^2 + \Phi_Z^2)\,. \qquad \ldots \qquad (20)$$

p_0 will in general depend on t. In the present case p_0 can, however, be assumed as constant. This equation can be given a more convenient form by introducing the relative co-ordinates, as the relative flow is independent of the time t.

In the system (x, y, z) the derivatives of Φ with respect to X, Y, Z and t are

$$\Phi_X = \Phi_x \frac{\partial x}{\partial X} + \Phi_y \frac{\partial y}{\partial X} + \Phi_z \frac{\partial z}{\partial X} \qquad \ldots \qquad (21)$$

and correspondingly for Φ_Y, Φ_Z and Φ_t. The derivatives of x, y, z with respect to X, Y, Z are obviously the elements of the matrix in equation (6*b*)

$$\frac{\partial x}{\partial X} = -\cos\alpha \sin(\Omega t - \beta), \text{ etc.}; \qquad \ldots \qquad (22)$$

the derivatives of x, y, z with respect to t are the components of $-\mathbf{V}_{abs}$

$$\frac{\partial x}{\partial t} = -\Omega(R_0\cos\beta - y)\cos\alpha, \text{ etc.} \qquad \ldots \qquad (23)$$

This gives

$$\begin{aligned}p - p_0 = {} & +\rho\Omega[(R_0\cos\beta - y)(\Phi_x \cos\alpha - \Phi_z \sin\alpha) \\ & + (R_0 \sin\beta + x\cos\alpha - z\sin\alpha)\Phi_y] - \tfrac{1}{2}\rho(\Phi_x^2 + \Phi_y^2 + \Phi_z^2)\,, \qquad \ldots \qquad (24)\end{aligned}$$

or with respect to (19)

$$p - p_0 = \tfrac{1}{2}\rho(V_{abs}^2 - V_{rel}^2)\,. \qquad \ldots \qquad (25)$$

This equation is formally identical with Bernoulli's equation for a stationary translation of the body. It is, however, to be noted that V_{abs} is not constant but varies with the distance of the point P from the axis of Z.

5. *Absolute Velocity on the Surface of the Body [Co-ordinates (n, t, b)].*—For the calculation of the flow conditions, the absolute velocity on the surface of the body has to be determined in the systems (n, t, b) of the surface co-ordinates attached to the points P of the surface. This is easily performed with the aid of the scalar products of the unit vectors $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ and $(\mathbf{n}, \mathbf{t}, \mathbf{b})$ which are given in equation (9). Introducing

$$\mathbf{x} = (\mathbf{n}\,.\,\mathbf{x})\mathbf{n} + (\mathbf{t}\,.\,\mathbf{x})\mathbf{t} + (\mathbf{b}\,.\,\mathbf{x})\mathbf{b} \qquad \ldots \qquad (26)$$

and the corresponding expressions for $\mathbf{y}$ and $\mathbf{z}$ into (14a), the absolute velocity becomes

$$\begin{aligned}\mathbf{V}_{abs} = {} & V_x\left[-\frac{r'}{\sqrt{(1+r'^2)}}\,\mathbf{n} - \frac{1}{\sqrt{(1+r'^2)}}\,\mathbf{t} + 0\,\mathbf{b}\right] \\ & + V_y\left[\frac{\cos\theta}{\sqrt{(1+r'^2)}}\,\mathbf{n} - \frac{r'\cos\theta}{\sqrt{(1+r'^2)}}\,\mathbf{t} - \sin\theta\,\mathbf{b}\right] \\ & + V_z\left[\frac{\sin\theta}{\sqrt{(1+r'^2)}}\,\mathbf{n} - \frac{r'\sin\theta}{\sqrt{(1+r'^2)}}\,\mathbf{t} + \cos\theta\,\mathbf{b}\right] \\ & + \Omega_x(0\,\mathbf{n} + 0\,\mathbf{t} + r\,\mathbf{b}) \\ & + \Omega_z\left[\frac{(x+rr')\cos\theta}{\sqrt{(1+r'^2)}}\,\mathbf{n} - \frac{(xr'-r)\cos\theta}{\sqrt{(1+r'^2)}}\,\mathbf{t} - x\sin\theta\,\mathbf{b}\right], \qquad \ldots \quad (27)\end{aligned}$$

with V_x, V_y, V_z, Ω_x, Ω_z from (16).

6. *Relative Velocity on the Surface of the Body* [*Co-ordinates* (n, t, b)].—6.1. *General Remarks.*—It was shown in equations (14a) and (16) that the complete movement of the body can be considered as the result of the superposition of a longitudinal translation, two lateral translations and two rotations. The potential equation being linear, these elementary movements can be considered separately.

The following investigation starts from the general equations for the calculation of the relative velocity on the surface of a body moving arbitrarily in an inviscid fluid otherwise still. These equations will then be specialised so as to give the solutions for the elementary translations and rotations (16). At last the solution of the complete problem will be obtained as a linear combination of the elementary solutions.

6.2. *General Equations for* $\mathbf{V}_{rel}$ *in Vector Notation.*—Consider a solid body moving in an inviscid fluid otherwise still, so that a point $P = (x, r, \theta)$ of its surface S has the absolute velocity $\mathbf{V}_{abs}$ at a certain time t. In general $\mathbf{V}_{abs}$ will be different for different points of S, although subject to certain restrictions expressing the condition that the form of the body remains unaltered. Only in the case of a simple translation of the body, $\mathbf{V}_{abs}$ is constant for all points of S.

The surface of the body being impervious for the fluid, the relative particle velocity on S must be in the tangential plane of the body. The moving body will therefore generate a velocity field in the fluid with a normal component on S equal to the normal component of $\mathbf{V}_{abs}$. This velocity field can be produced by a suitable source distribution D on S.

A single elementary source of strength dg at the point H of S induces the velocity

$$d\mathbf{V}_D = \frac{\mathbf{R}}{R^3}\,dg \quad \ldots \quad (28)$$

at the point P, where $\mathbf{R}$ is the vector $\mathbf{HP}$. The total velocity $\mathbf{V}_D$ induced at P by a distribution of the strength

$$\frac{dg}{d\sigma} = f(x, r, \theta) \quad \ldots \quad (29)$$

per unit area of S is obtained by an integration, taking into account that the source at the point P gives a finite contribution to the normal component of $\mathbf{V}_D$ which equals half of its total output per unit of area.

$$\mathbf{V}_D(x, r, \theta) = 2\pi f(x, r, \theta)\,\mathbf{n} + \int_S f(\xi, \rho, \Theta)\frac{\mathbf{R}}{R^3}\,d\sigma\,. \quad \ldots \quad (30)$$

The condition of equal normal components of the two vectors $\mathbf{V}_D$ and $\mathbf{V}_{abs}$ is then

$$\mathbf{V}_{abs} \cdot \mathbf{n} = \mathbf{V}_D \cdot \mathbf{n}, \qquad (31)$$

or

$$\mathbf{V}_{abs} \cdot \mathbf{n} = 2\pi f(x, r, \theta) + \int_S f(\xi, \rho, \Theta) \frac{\mathbf{R} \cdot \mathbf{n}}{R^3} d\sigma. \qquad (32)$$

For a given distribution of $\mathbf{V}_{abs}$ on S, this is a linear integral equation of the second kind for the strength f of the distribution D.

If f has been determined from (32), the relative velocity on the surface is easily obtained as the difference of $\mathbf{V}_D$ and $\mathbf{V}_{abs}$,

$$\mathbf{V}_{rel} = -\mathbf{V}_{abs} + \mathbf{V}_D, \qquad (33)$$

or, the normal components of $\mathbf{V}_{abs}$ and $\mathbf{V}_D$ being equal,

$$\mathbf{V}_{rel} = \left[-\mathbf{V}_{abs} \cdot \mathbf{t} + \int_S f(\xi, \rho, \Theta) \frac{\mathbf{R} \cdot \mathbf{t}}{R^3} d\sigma \right] \mathbf{t}$$
$$+ \left[-\mathbf{V}_{abs} \cdot \mathbf{b} + \int_S f(\xi, \rho, \Theta) \frac{\mathbf{R} \cdot \mathbf{b}}{R^3} d\sigma \right] \mathbf{b}. \qquad (34)$$

6.3. *Introduction of Co-ordinates into the Kernels of the General Equations.*—For the practical calculation, the expressions

$$\frac{\mathbf{R} \cdot \mathbf{n}}{R^3} d\sigma, \quad \frac{\mathbf{R} \cdot \mathbf{t}}{R^3} d\sigma \quad \text{and} \quad \frac{\mathbf{R} \cdot \mathbf{b}}{R^3} d\sigma \qquad (35)$$

in the kernels of equations (32) and (34) have to be expressed in terms of the co-ordinates.

Equations (32) and (34) are quite general and hold for any form of the moving body. With a view to the problem under consideration, the body will now be assumed to be a body of revolution.

With $P = (x, y, z)$ and $\Pi = (\xi, \eta, \zeta)$, the vector $\mathbf{R}$ is obviously

$$\mathbf{R} = (x - \xi)\,\mathbf{x} + (y - \eta)\,\mathbf{y} + (z - \zeta)\,\mathbf{z}. \qquad (36)$$

Its absolute value is

$$R = \sqrt{\{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2\}}. \qquad (37)$$

Introducing cylindrical co-ordinates, $\mathbf{R}$ and R become

$$\mathbf{R} = (x - \xi)\mathbf{x} + (r \cos \theta - \rho \cos \Theta)\mathbf{y} + (r \sin \theta - \rho \sin \Theta)\mathbf{z}, \qquad (38)$$

$$R = \sqrt{\{(x - \xi)^2 + r^2 + \rho^2 - 2r\rho \cos(\theta - \Theta)\}}. \qquad (39)$$

The scalar products of $\mathbf{R}$ and the unit vectors $\mathbf{n}$, $\mathbf{t}$, $\mathbf{b}$ are then with respect to (9)

$$\mathbf{R} \cdot \mathbf{n} = (x - \xi)(\mathbf{n} \cdot \mathbf{x}) + (r \cos \theta - \rho \cos \Theta)(\mathbf{n} \cdot \mathbf{y}) + (r \sin \theta - \rho \sin \Theta)(\mathbf{n} \cdot \mathbf{z})$$
$$= \frac{r - r'(x - \xi) - \rho \cos(\theta - \Theta)}{\sqrt{(1 + r'^2)}}, \qquad (40)$$

and correspondingly

$$\mathbf{R} \cdot \mathbf{t} = -\frac{(x - \xi) + r'[r - \rho \cos(\theta - \Theta)]}{\sqrt{(1 + r'^2)}}, \qquad (41)$$

$$\mathbf{R} \cdot \mathbf{b} = \rho \sin(\theta - \Theta). \qquad (42)$$

The element of area $d\sigma$ being

$$d\sigma = \rho\, d\Theta \sqrt{(1 + \rho'^2)}\, d\xi \qquad (43)$$

in cylindrical co-ordinates, equations (32) and (34) can now be written in the forms

$$\mathbf{V}_{abs} \cdot \mathbf{n} = 2\pi f(x, r, 0) + \int_{-l/2}^{+l/2} \int_0^{2\pi} f(\xi, \rho, \Theta) \frac{[r - r'(x - \xi) - \rho \cos(\theta - \Theta)]\rho}{[\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho \cos(\theta - \Theta)\}}]^3} \sqrt{\left(\frac{1 + \rho'^2}{1 + r'^2}\right)} \, d\Theta \, d\xi , \quad \ldots \quad \ldots \quad (44)$$

or

$$g(x,r,0) = r\sqrt{(1 + r'^2)}\mathbf{V}_{abs} \cdot \mathbf{n} - \int_{-l/2}^{+l/2} \int_0^{2\pi} g(\xi, \rho, \Theta) \frac{r[r - r'(x - \xi) - \rho \cos(\theta - \Theta)]}{2\pi[\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho \cos(\theta - \Theta)\}}]^3} \, d\Theta \, d\xi , \quad \ldots \quad (45)$$

with

$$g(x,r,0) = 2\pi r \sqrt{(1 + r'^2)} f(x, r, 0) , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (46)$$

and correspondingly

$$\mathbf{V}_{rel} = \mathbf{t}\left[-\mathbf{V}_{abs} \cdot \mathbf{t} - \frac{1}{\sqrt{(1 + r'^2)}} \int_{-l/2}^{+l/2} \int_0^{2\pi} g(\xi, \rho, \Theta) \frac{(x - \xi) + r'[r - \rho \cos(\theta - \Theta)]}{2\pi[\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho \cos(\theta - \Theta)\}}]^3} \, d\Theta \, d\xi\right]$$
$$+ \mathbf{b}\left[-\mathbf{V}_{abs} \cdot \mathbf{b} + \int_{-l/2}^{+l/2} \int_0^{2\pi} g(\xi, \rho, \Theta) \frac{\rho \sin(\theta - \Theta)}{2\pi[\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho \cos(\theta - \Theta)\}}]^3} \, d\Theta \, d\xi\right] . \quad \ldots \quad \ldots \quad (47)$$

Equations (45) and (47) will now be specialised in order to obtain the solutions for the elementary movements (16).

6.4(*a*). *Calculation of* $\mathbf{V}_{rel}$ *for a longitudinal translation* ($\mathbf{V}_{abs} = V_x \mathbf{x}$).—For reasons of symmetry of the flow, the strength g of the source distribution is a function of the only variable x and $\mathbf{V}_{rel}$ has the direction of $\mathbf{t}$ on the surface.

Introducing into the general equation (45)

$$\mathbf{V}_{abs} = V_x \mathbf{x}, \; \mathbf{x} \cdot \mathbf{n} = -\frac{r'}{\sqrt{(1 + r'^2)}}, \quad \theta - \Theta = \chi \text{ and } g(x) = g_1(x) V_x , \quad \ldots \quad \ldots \quad (48)$$

the integral equation becomes

$$g_1(x) = -rr' - \int_{-l/2}^{+l/2} g_1(\xi) \left[\int_0^{2\pi} \frac{r[r - r'(x - \xi) - \rho \cos \chi] \, d\chi}{2\pi[\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho \cos \chi\}}]^3}\right] d\xi . \quad \ldots \quad (49)$$

The integration with respect to χ can be carried out with the aid of complete elliptic integrals. The result is

$$g_1(x) = -rr' - \int_{-l/2}^{+l/2} g_1(\xi) \left[\frac{r[r - \rho - r'(x - \xi)]}{(x - \xi)^2 + (r - \rho)^2} G_1(k^2) + G_2(k^2)\right] \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r + \rho)^2\}}} \quad (50)$$

$$= -rr' - \int_{-l/2}^{+l/2} g_1(\xi) K_{10}(x, \xi) \, d\xi , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (50a)$$

with

$$k^2 = \frac{4r\rho}{(x - \xi)^2 + (r + \rho)^2} , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (51)$$

$$G_1(k^2) = \frac{2}{\pi} E(k^2) , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

$$G_2(k^2) = \frac{1}{\pi} (K(k^2) - E(k^2)) . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (53)$$

The relative velocity is then obtained from (47)

$$\frac{\mathbf{V}_{\text{rel long}}}{V_x} = \frac{\mathbf{t}}{\sqrt{(1+r'^2)}}\left[1 - \int_{-l/2}^{+l/2} g_1(\xi)\left\{\int_0^{2\pi} \frac{[x-\xi+r'(r-\rho\cos\chi)]\,d\chi}{2\pi[\sqrt{\{(x-\xi)^2+r^2+\rho^2-2r\rho\cos\chi\}}]^3}\right\}d\xi\right] \quad (54)$$

or, after integrating with respect to χ

$$\frac{\mathbf{V}_{\text{rel long}}}{V_x} = \frac{\mathbf{t}}{\sqrt{(1+r'^2)}}\left[1 - \int_{-l/2}^{+l/2} g_1(\xi)\left\{\frac{(x-\xi)+r'(r-\rho)}{(x-\xi)^2+(r-\rho)^2}\,G_1(k^2) + \frac{r'}{r}\,G_2(k^2)\right\}\frac{d\xi}{\sqrt{\{(x-\xi)^2+(r+\rho)^2\}}}\right] \quad \ldots \quad (55)$$

$$= \frac{\mathbf{t}}{\sqrt{(1+r'^2)}}\left[1 - \int_{-l/2}^{+l/2} g_1(\xi)\left\{K_{11}(x,\xi) + \frac{r'}{r}\,K_{12}(x,\xi)\right\}d\xi\right]. \quad \ldots \quad (55a)$$

Appendix to Section 6.4(a).—Potential of the Source Distribution for the Longitudinal Translation

The potential of a single elementary source of strength dq at the point $H = (\xi, \rho, \Theta)$ of the surface is given by

$$d\Phi = -\frac{dq}{R}. \quad \ldots \quad (56)$$

The source distribution on the surface will then have the potential

$$\Phi = -\int_S f(\xi, \rho, \Theta)\,\frac{1}{R}\,d\sigma. \quad \ldots \quad (57)$$

This gives

$$\Phi_{\text{long}} = -V_x \int_{-l/2}^{+l/2} g_1(\xi)\left[\int_0^{2\pi} \frac{d\chi}{2\pi\sqrt{\{(x-\xi)^2+r^2+\rho^2-2r\rho\cos\chi\}}}\right]d\xi \quad (58)$$

or, after integrating with respect to χ

$$\Phi_{\text{long}} = -V_x \int_{-l/2}^{+l/2} g_1(\xi)\,G_2(k^2)\,\frac{d\xi}{\sqrt{\{(x-\xi)^2+(r+\rho)^2\}}} \quad \ldots \quad (59)$$

$$= -V_x \int_{-l/2}^{+l/2} g_1(\xi)\,K_{13}(x,\xi)\,d\xi, \quad \ldots \quad (59a)$$

with

$$G_2(k^2) = \frac{c}{\pi}\,K(k^2). \quad \ldots \quad (60)$$

6.4(*b*). *Calculation of* $\mathbf{V}_{\text{rel}}$ *for a Lateral Translation.*

$$\left[\mathbf{V}_{\text{abs}} = \begin{matrix} V_y\,\mathbf{y} \\ V_z\,\mathbf{z} \end{matrix}\right].$$

In the case of a lateral translation of the body in the directions of y or z, it is easily seen that the solution of the integral equation (45) must be the product of a function of x and $\cos\theta$ or $\sin\theta$, respectively. In fact, let

$$\mathbf{V}_{\text{abs}} = \begin{matrix} V_y\,\mathbf{y} \\ V_z\,\mathbf{z} \end{matrix}, \quad \begin{matrix} \mathbf{y} \\ \mathbf{z} \end{matrix}\cdot\mathbf{n} = \frac{\begin{matrix}\cos\theta \\ \sin\theta\end{matrix}}{\sqrt{(1+r'^2)}} \quad \text{and} \quad g(x, r, \theta) = \begin{matrix} V_y \\ V_z \end{matrix}\,g_3(x)\,\begin{matrix}\cos\theta \\ \sin\theta\end{matrix}. \quad \ldots \quad (61)$$

The integral equation can then be written in the form

$$g_2(x) = r - \int_{-l/2}^{+l/2} g_2(\xi) \left[\int_0^{2\pi} \frac{\begin{matrix}\cos\\ \sin\end{matrix}\,\Theta\; r[r - r'(x-\xi) - \rho\cos(\theta - \Theta)]}{\begin{matrix}\cos\\ \sin\end{matrix}\,\theta\; 2\pi[\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho\cos(\theta-\Theta)\}}]^3}\, d\Theta \right] d\xi\,. \quad (62)$$

Taking account of the relations

$$\begin{matrix}\cos\\ \sin\end{matrix}\,\Theta \quad \begin{matrix}\cos\\ \sin\end{matrix}\,[\theta - (\theta - \Theta)] = \begin{matrix}\cos\\ \sin\end{matrix}\,\theta\,\cos(\theta - \Theta) \pm \begin{matrix}\sin\\ \cos\end{matrix}\,\theta\,\sin(\theta - \Theta) \quad \ldots \quad (63)$$

and

$$\int_0^{2\pi} F(\cos\chi)\sin\chi\, d\chi = 0\,, \quad \chi = \theta - \Theta\,, \quad \ldots \quad (64)$$

the kernel of (62) is reduced to

$$\int_0^{2\pi} \frac{r[r - r'(x-\xi) - \rho\cos\chi]\cos\chi}{2\pi[\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho\cos\chi\}}]^3}\, d\chi\,. \quad \ldots \quad (65)$$

The integration with respect to χ can again be carried out with the aid of complete elliptic integrals. The result is

$$g_2(x) = r - \int_{-l/2}^{+l/2} g_2(\xi) \left[\frac{r[r - \rho - r'(x-\xi)]}{(x-\xi)^2 + (r-\rho)^2} H_1(k^2) + H_2(k^2) \right] \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \quad (66)$$

$$= r - \int_{-l/2}^{+l/2} g_2(\xi)\, K_{20}(x, \xi)\, d\xi\,, \quad \ldots \quad (66a)$$

with k^2 as given by (51) and

$$H_1(k^2) = \frac{2}{\pi}\,\frac{(1 + k'^2)E(k^2) - 2k'^2K(k^2)}{k^2} \quad \ldots \quad (67)$$

$$H_2(k^2) = \frac{1}{\pi}\,\frac{(1 + 3k'^2)K(k^2) - (3 + k'^2)E(k^2)}{k^2} \quad \ldots \quad (68)$$

$$k'^2 = 1 - k^2\,. \quad \ldots \quad (69)$$

The relative velocity is then obtained from (47). This gives with respect to (9), (61), (63) and (64)

$$\mathbf{V}_{\text{rel lat}} = \begin{matrix}V_y\\ V_z\end{matrix} \left[\frac{\begin{matrix}\cos\\ \mathbf{t}\sin\theta\end{matrix}}{\sqrt{(1 + r'^2)}} \left\{ r' - \int_{-l/2}^{+l/2} g_2(\xi) \int_0^{2\pi} \frac{[(x-\xi) + r'(r - \rho\cos\chi)]\cos\chi}{2\pi[\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho\cos\chi\}}]^3}\, d\chi\, d\xi \right\} \right.$$
$$\left. \pm \begin{matrix}\sin\\ \cos\end{matrix}\,\theta\,\mathbf{b} \left\{ 1 + \int_{-l/2}^{+l/2} g_2(\xi) \int_0^{2\pi} \frac{\rho\sin^2\chi}{2\pi[\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho\cos\chi\}}]^3}\, d\chi\, d\xi \right\} \right] \quad \ldots \quad (70)$$

and finally by integration with respect to χ

$$\mathbf{V}_{\text{rel lat}} = \begin{matrix}V_y\\ V_z\end{matrix} \left\{ \frac{\begin{matrix}\cos\\ \mathbf{t}\sin\theta\end{matrix}}{\sqrt{(1 + r'^2)}} \left[r' - \int_{-l/2}^{+l/2} g_2(\xi) \left\{ \frac{(x-\xi) + r'(r-\rho)}{(x-\xi)^2 + (r-\rho)^2} H_1(k^2) \right.\right.\right.$$
$$\left.\left. + \frac{r'}{r} H_2(k^2) \right\} \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \right]$$
$$\left. \pm \begin{matrix}\sin\\ \cos\end{matrix}\,\theta\,\mathbf{b} \left[1 + \frac{1}{r} \int_{-l/2}^{+l/2} g_2(\xi)\, H_3(k^2) \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \right] \right\}, \quad \ldots \quad (71)$$

with

$$H_3(k^2) = \frac{2}{\pi}\,\frac{(1 + k'^2)K(k^2) - 2E(k^2)}{k^2}\,, \quad \ldots \quad (72)$$

or with the use of symbols for the kernels

$$\mathbf{V}_{\text{rel lat}} = \begin{matrix} V_y \\ V_z \end{matrix} \left\{ \begin{matrix} \cos \\ \mathbf{t} \sin\theta \\ \hline \sqrt{(1+r'^2)} \end{matrix} \left[r' - \int_{-l/2}^{+l/2} g_2(\xi) K_{21}(x, \xi)\, d\xi - \frac{r'}{r} \int_{-l/2}^{+l/2} g_2(\xi) K_{22}(x, \xi)\, d\xi \right] \right.$$
$$\left. + \begin{matrix} \sin \\ -\cos \end{matrix} \theta\, \mathbf{b} \left[1 + \frac{1}{r} \int_{-l/2}^{+l/2} g_2(\xi) K_{23}(x, \xi)\, d\xi \right] \right\}. \qquad \ldots \quad (71a)$$

Appendix to Section 6.4(b).—Potential of the Source Distribution for a Lateral Translation

Introducing into the general formula (57) for the potential of a source distribution its strength in the case of a lateral translation

$$f(x, r, \theta) = \begin{matrix} V_y \\ V_z \end{matrix} \left(\frac{g_2(x)}{2\pi r \sqrt{(1+r'^2)}} \right) \begin{matrix} \cos\theta \\ \sin\theta \end{matrix}, \qquad \ldots \quad (73)$$

the potential becomes

$$\Phi_{\text{lat}} = -\begin{matrix} V_y \\ V_z \end{matrix} \int_{-l/2}^{+l/2} g_2(\xi) \left[\int_0^{2\pi} \frac{\begin{matrix} \cos\Theta \\ \sin\Theta \end{matrix}}{2\pi\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho\cos(\theta - \Theta)\}}}\, d\Theta \right] d\xi \qquad (74)$$

and after integrating with respect to Θ

$$\Phi_{\text{lat}} = -\begin{matrix} V_y \cos \\ V_z \sin \end{matrix} \theta \int_{-l/2}^{+l/2} g_2(\xi)\, H_2(k^2) \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \qquad \ldots \quad (75)$$

$$= -\begin{matrix} V_y \cos \\ V_z \sin \end{matrix} \theta \int_{-l/2}^{+l/2} g_2(\xi)\, K_{21}(x, \xi)\, d\xi\,. \qquad \ldots \quad (75a)$$

6.4(c). *Calculation of* $\mathbf{V}_{\text{rel}}$ *for a Rotation about the Axes of* y *or* z.

$$\mathbf{V}_{\text{abs}} = \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \begin{bmatrix} z\,.\,\mathbf{x} & - & x\,.\,\mathbf{z} \\ x\,.\,\mathbf{y} & & y\,.\,\mathbf{x} \end{bmatrix}.$$

As in the case of a lateral translation, the solution of the integral equation (45) is easily seen to be the product of a function of x and either $\sin\theta$ or $\cos\theta$. With

$$\mathbf{V}_{\text{abs}} = \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \begin{bmatrix} z\,.\,\mathbf{x} & - & x\,.\,\mathbf{z} \\ x\,.\,\mathbf{y} & & y\,.\,\mathbf{x} \end{bmatrix} \qquad \ldots \quad (76)$$

$$\mathbf{V}_{\text{abs}}\,.\,\mathbf{n} = \mp \frac{x + rr'}{\sqrt{(1+r'^2)}} \begin{matrix} \Omega_y \sin \\ \Omega_z \cos \end{matrix} \theta \qquad \ldots \quad (77)$$

$$g(x, r, \theta) = \mp \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} g_2(x) \begin{matrix} \sin \\ \cos \end{matrix} \theta\,, \qquad \ldots \quad (78)$$

equation (45) becomes

$$g_2(x) = \frac{r(x + rr')}{l/2} - \int_{-l/2}^{+l/2} g_2(\xi) \int_0^{2\pi} \frac{r[r - r'(x-\xi) - \rho\cos\chi]\cos\chi}{2\pi[\sqrt{\{(x-\xi)^2 + r^2 + \rho^2 - 2r\rho\cos\chi\}}]^3}\, d\chi\,. \qquad \ldots \quad (79)$$

The kernel of this equation is the same as in the case of a lateral translation (*cf.* (65)). The final form of the integral equation is therefore

$$g_3(x) = \frac{r(x + rr')}{l/2} - \int_{-l/2}^{+l/2} g_3(\xi) \left[\frac{r[r - \rho - r'(x - \xi)]}{(x - \xi)^2 + (r - \rho)^2} H_1(k^2) + H_2(k^2)\right] \frac{d\xi}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}, \quad \ldots \quad (80)$$

$$= \frac{r(x + rr')}{l/2} - \int_{-l/2}^{+l/2} g_3(\xi) K_{30}(x, \xi)\, d\xi\,. \quad \ldots \quad (80a)$$

The relative velocity is then obtained from (47) with

$$\mathbf{V}_{\text{abs}} \cdot \mathbf{t} = \begin{matrix} + \\ - \end{matrix} \begin{matrix} \Omega_y \sin\theta \\ \Omega_z \cos\theta \end{matrix} \left(\frac{r'x - r}{\sqrt{(1 + r'^2)}}\right) \quad \ldots \quad (81)$$

$$\mathbf{V}_{\text{abs}} \cdot \mathbf{b} = \begin{matrix} - \\ - \end{matrix} \begin{matrix} \Omega_y \cos\theta \\ \Omega_z \sin\theta \end{matrix} \left(x\right), \quad \ldots \quad (82)$$

which gives in the same way as in equations (70) to (72) above the final result:

$$\mathbf{V}_{\text{rel rot } y,z} = \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \left\{ \frac{\begin{matrix} - \sin \\ + \cos \end{matrix}\theta\, \mathbf{t}}{\sqrt{(1 + r'^2)}} \left[\frac{r'x - r}{l/2} - \int_{-l/2}^{+l/2} g_3(\xi) \left\{\frac{x - \xi + r'(r - \rho)}{(x - \xi)^2 + (r - \rho)^2} H_1(k^2) + \frac{r'}{r} H_2(k^2)\right\} \frac{d\xi}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}\right] + \begin{matrix} \cos \\ \sin \end{matrix}\theta\, \mathbf{b} \left[\frac{x}{l/2} + \frac{1}{r} \int_{-l/2}^{+l/2} g_3(\xi) H_2(k^2) \frac{d\xi}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}\right]\right\} \quad \ldots \quad (83)$$

$$= \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \left\{ \frac{\begin{matrix} - \sin \\ + \cos \end{matrix}\theta\, \mathbf{t}}{\sqrt{(1 + r'^2)}} \left[\frac{r'x - r}{l/2} - \int_{-l/2}^{+l/2} g_3(\xi) K_{31}(x, \xi)\, d\xi - \frac{r'}{r} \int_{-l/2}^{+l/2} g_3(\xi) K_{32}(x, \xi)\, d\xi\right] + \begin{matrix} \cos \\ \sin \end{matrix}\theta\, \mathbf{b} \left[\frac{x}{l/2} + \frac{1}{r} \int_{-l/2}^{+l/2} g_3(\xi) K_{33}(x, \xi)\, d\xi\right]\right\}. \quad (83a)$$

Appendix to Section 6.4(*c*).—*Potential of the Source Distribution for a Rotation about the Axes of y or z*

The potential is obtained in the same way as for the lateral translation (equations (73) to (75)):

$$f(x, r, \theta) = \mp \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \frac{g_3(x)}{2\pi r \sqrt{(1 + r'^2)}} \begin{matrix} \sin \\ \cos \end{matrix}\theta \quad \ldots \quad (84)$$

$$\Phi_{\text{rot } y,z} = \pm \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \int_{-l/2}^{+l/2} g_3(\xi) \left[\int_0^{2\pi} \frac{\begin{matrix} \sin \\ \cos \end{matrix}\Theta\, d\Theta}{2\pi\sqrt{\{(x - \xi)^2 + r^2 + \rho^2 - 2r\rho\cos(\theta - \Theta)\}}}\right] d\xi \quad \ldots \quad (85)$$

$$\Phi_{\text{rot } y,z} = \pm \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \begin{matrix} \sin \\ \cos \end{matrix}\theta \int_{-l/2}^{+l/2} g_3(\xi) H_2(k^2) \frac{d\xi}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}} \quad \ldots \quad (86)$$

$$= \pm \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \begin{matrix} \sin \\ \cos \end{matrix}\theta \int_{-l/2}^{+l/2} g_3(\xi) K_{32}(x, \xi)\, d\xi\,. \quad \ldots \quad (86a)$$

6.4(*d*). $\mathbf{V}_{rel}$ *for a rotation about the axis of* x

$$[\mathbf{V}_{abs} = \Omega_x \cdot (y \cdot \mathbf{z} - z \cdot \mathbf{y})] .$$

It is obvious that a rotation of the body about the axis of x cannot create a velocity field in a perfect fluid. $\mathbf{V}_{rel}$ is therefore simply $-\mathbf{V}_{abs}$.

$$\mathbf{V}_{rel\,rot\,x} = -\Omega_x(y \cdot \mathbf{z} - z \cdot \mathbf{y}) \quad \ldots \quad (87)$$

$$= -\frac{l}{2}\Omega_x \cdot \frac{r}{l/2} \cdot \mathbf{b} . \quad \ldots \quad (88)$$

7. *Summary of Sections* 1 *to* 6.—The preceding results can be summarised as follows :

(1) If a body of revolution is revolving with a constant angular velocity Ω about a fixed axis in an inviscid fluid otherwise still, the pressure at any point P of its surface is given by

$$p - p_0 = \tfrac{1}{2}\rho(V_{abs}^2 - V_{rel}^2) . \quad \ldots \quad (89)$$

(2) $\mathbf{V}_{abs}$ is given by

$$V_{abs}^2 = \Omega^2[(R_0 \cos\beta - r\cos\theta)^2 + (R_0 \sin\beta + x\cos\alpha - r\sin\theta\sin\alpha)^2] . \quad \ldots \quad (90)$$

(3) In order to calculate V_{rel}^2, the motion of the body is considered as the result of a superposition of the following elementary motions :

(*a*) Translation in the direction of x with the velocity $V_x = \Omega R_0 \cos\alpha \cos\beta$

(*b*) Translation in the direction of y with the velocity $V_y = \Omega R_0 \sin\beta$

(*c*) Translation in the direction of z with the velocity $V_z = -\Omega R_0 \sin\alpha \cos\beta$ .. (91)

(*d*) Rotation about the axis of x with the angular velocity $\Omega_x = \Omega \sin\alpha$

(*e*) Rotation about the axis of z with the angular velocity $\Omega_z = \Omega \cos\alpha$.

These elementary motions can be treated separately.

(4) $\mathbf{V}_{rel}$ for the elementary motions. The condition of zero normal velocity at the surface S is satisfied by a source distribution D on S. Its strength is determined from an integral equation. $\mathbf{V}_{rel}$ is then obtained as the difference $\mathbf{V}_D - \mathbf{V}_{abs}$ where $\mathbf{V}_D$ is the velocity induced by the distribution at the point P.

(*a*) Longitudinal translation.

Integral equation :

$$g_1(x) = -rr' - \int_{-l/2}^{+l/2} g_1(\xi)\left[\frac{r[r - \rho - r'(x-\xi)]}{(x-\xi)^2 + (r-\rho)^2} G_1(k^2) + G_2(k^2)\right]\frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} . \quad \ldots \quad (92)$$

Relative velocity :

$$\mathbf{V}_{rel\,long} = V_x \frac{\mathbf{t}}{\sqrt{(1+r'^2)}}\left[1 - \int_{-l/2}^{+l/2} g_1(\xi)\left\{\frac{x - \xi + r'(r-\rho)}{(x-\xi)^2 + (r-\rho)^2} G_1(k^2) + \frac{r'}{r} G_2(k^2)\right\}\frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}}\right] . \quad \ldots \quad (93)$$

Potential of the source distribution:

$$\Phi_{\text{long}} = -V_x \int_{-l/2}^{+l/2} g_1(\xi)\, G_3(k^2) \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \quad \ldots \quad (94)$$

(*b*), (*c*) Lateral translations.

$$g_2(x) = r - \int_{-l/2}^{+l/2} g_2(\xi) \left[\frac{r[r-\rho-r'(x-\xi)]}{(x-\xi)^2 + (r-\rho)^2} H_1(k^2) + H_2(k^2)\right] \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \quad (95)$$

$$\mathbf{V}_{\text{rel lat}} = \begin{matrix} V_y \\ V_z \end{matrix} \left\{ \frac{\mathbf{t} \begin{matrix}\cos \\ \sin\end{matrix} \theta}{\sqrt{(1+r'^2)}} \left[r' - \int_{-l/2}^{+l/2} g_2(\xi) \left\{ \frac{x-\xi+r'(r-\rho)}{(x-\xi)^2 + (r-\rho)^2} H_1(k^2) + \frac{r'}{r} H_2(k^2) \right\} \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \right] \right.$$

$$\left. \pm \begin{matrix}\sin \\ \cos\end{matrix} \theta\, \mathbf{b} \left[1 + \frac{1}{r} \int_{-l/2}^{+l/2} g_2(\xi)\, H_2(k^2) \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \right] \right\} \quad \ldots \quad (96)$$

$$\Phi_{\text{lat}} = \begin{matrix} -V_y \cos\theta \\ -V_z \sin\theta \end{matrix} \int_{-l/2}^{+l/2} g_2(\xi)\, H_2(k^2) \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \quad \ldots \quad (97)$$

(*d*) Rotation about the longitudinal axis.

$$\mathbf{V}_{\text{rel rot } x} = -\frac{l}{2} \Omega_x \frac{r}{l/2} \mathbf{b} \quad \ldots \quad (98)$$

$$\Phi_{\text{rot } x} = 0 \quad \ldots \quad (99)$$

(*e*) Rotation about the lateral axes.

$$g_3(x) = \frac{r(x+rr')}{l/2} - \int_{-l/2}^{+l/2} g_3(\xi) \left[\frac{r[r-\rho-r'(x-\xi)]}{(x-\xi)^2 + (r-\rho)^2} H_1(k^2) + H_2(k^2)\right] \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \quad \ldots \quad (100)$$

$$\mathbf{V}_{\text{rel rot } y,z} = \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \left\{ \frac{\begin{matrix}-\sin\theta \\ +\cos\theta\end{matrix}\, \mathbf{t}}{\sqrt{(1+r'^2)}} \left[\frac{r'x - r}{l/2} - \int_{-l/2}^{+l/2} g_3(\xi) \left\{ \frac{x-\xi+r'(r-\rho)}{(x-\xi)^2 + (r-\rho)^2} H_1(k^2) + \frac{r'}{r} H_2(k^2) \right\} \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \right] \right.$$

$$\left. + \begin{matrix}\cos \\ \sin\end{matrix} \theta\, \mathbf{b} \left[\frac{x}{l/2} + \frac{1}{r} \int_{-l/2}^{+l/2} g_3(\xi)\, H_2(k^2) \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \right] \right\} \quad \ldots \quad (101)$$

$$\Phi_{\text{rot } y,z} = \pm \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \begin{matrix}\sin \\ \cos\end{matrix} \theta \int_{-l/2}^{+l/2} g_3(\xi)\, H_2(k^2) \frac{d\xi}{\sqrt{\{(x-\xi)^2 + (r+\rho)^2\}}} \quad \ldots \quad (102)$$

PART B.—ADAPTATION OF THE THEORY FOR THE NUMERICAL WORK

1. *Discussion of the Integral Equations and the Formulae for* $\mathbf{V}_{rel}$ *and* Φ.—1.1. *Structure of the Kernels.*—The kernels of the integrals for the lateral translations are identical with the kernels for the rotations about the axes of y and z. The kernels for the longitudinal translation have a very similar structure. The only differences are that the elliptic integrals H_r are replaced by the corresponding G_r and that the b-component of $V_{rel\,long}$ is zero.

The kernels are built up from the following expressions

$$r_1 = \sqrt{\{(x-\xi)^2 + (\gamma-\rho)^2\}} \quad \ldots \quad (103)$$

$$r_2 = \sqrt{\{(x-\xi)^2 + (\gamma+\rho)^2\}} \quad \ldots \quad (104)$$

$$b = (\gamma-\rho) - \gamma'(x-\xi) \quad \ldots \quad (105)$$

$$c = (x-\xi) + \gamma'(\gamma-\rho)\,. \quad \ldots \quad (106)$$

A geometric interpretation of these quantities is given in Fig. 3. Introducing the abbreviations

$$A(x,\xi) = \frac{1}{r_2} \quad \ldots \quad (107)$$

$$B(x,\xi) = \frac{\gamma b}{r_1^2} \quad \ldots \quad (108)$$

$$C(x,\xi) = \frac{c}{r_1^2}\,, \quad \ldots \quad (109)$$

the kernels become

$$\left.\begin{aligned} K_{10} &= (BG_1 + G_2)A & K_{20} &= (BH_1 + H_2)A \\ K_{11} &= CG_1A & K_{21} &= CH_1A \\ K_{12} &= G_2A & K_{22} &= H_2A \\ K_{13} &= G_3A & K_{23} &= H_3A \end{aligned}\right\} \quad \ldots \quad (110)$$

The modulus k^2 (equation (51)) of the functions G_r and H_r can also be expressed in terms of r_1 and r_2

$$k^2 = \frac{r_2^2 - r_1^2}{r_2^2}\,. \quad \ldots \quad (111)$$

For the numerical work it has some advantages to use in general the complementary modulus

$$k'^2 = \frac{r_1^2}{r_2^2}\,. \quad \ldots \quad (112)$$

This will be done in the following :

The equations for the g_r, $\mathbf{V}_{rel}$ and Φ are then

$$g_1(x) = -\gamma\gamma' - \int_{-l/2}^{+l/2} g_1(\xi)\, K_{10}(x,\xi)\, d\xi \quad \ldots \quad (113)$$

$$\mathbf{V}_{rel\,long} = V_x \cdot \frac{\mathbf{t}}{\sqrt{(1+\gamma'^2)}} \left[1 - \int_{-l/2}^{+l/2} g_1(\xi)\, K_{11}(x,\xi)\, d\xi - \frac{\gamma'}{\gamma}\int_{-l/2}^{+l/2} g_1(\xi)\, K_{12}(x,\xi)\, d\xi\right] \quad \ldots \quad (114)$$

$$\Phi_{long} = -V_x \int_{-l/2}^{+l/2} g_1(\xi)\, K_{20}(x,\xi)\, d\xi \quad \ldots \quad (115)$$

$$g_2(x) = r - \int_{-l/2}^{+l/2} g_2(\xi)\, K_{20}(x, \xi)\, d\xi \quad \ldots \quad (116)$$

$$\mathbf{V}_{\text{rel lat}} = \begin{matrix} V_y \\ V_z \end{matrix} \left\{ \frac{\mathbf{t} \begin{matrix} \cos \\ \sin \end{matrix} \theta}{\sqrt{(1 + r'^2)}} \left[r' - \int_{-l/2}^{+l/2} g_2(\xi)\, K_{21}(x, \xi)\, d\xi - \frac{r'}{r} \int_{-l/2}^{+l/2} g_2(\xi)\, K_{22}(x, \xi)\, d\xi \right] \right.$$
$$\left. \pm \begin{matrix} \sin \\ \cos \end{matrix} \theta\, \mathbf{b} \left[1 + \frac{1}{r} \int_{-l/2}^{+l/2} g_2(\xi)\, K_{23}(x, \xi)\, d\xi \right] \right\} \quad \ldots \quad (117)$$

$$\Phi_{\text{lat}} = \begin{matrix} - V_y \cos \theta \\ - V_z \sin \theta \end{matrix} \int_{-l/2}^{+l/2} g_2(\xi)\, K_{23}(x, \xi)\, d\xi \quad \ldots \quad (118)$$

$$g_3(x) = \frac{r(x + rr')}{l/2} - \int_{-l/2}^{+l/2} g_3(\xi)\, K_{30}(x, \xi)\, d\xi \quad \ldots \quad (119)$$

$$\mathbf{V}_{\text{rel rot } y, z} = \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \left\{ \frac{\begin{matrix} - \sin \\ + \cos \end{matrix} \theta \,.\, \mathbf{t}}{\sqrt{(1 + r'^2)}} \left[\frac{r'x - r}{l/2} - \int_{-l/2}^{+l/2} g_3(\xi)\, K_{21}(x, \xi)\, d\xi \right. \right.$$
$$\left. \left. - \frac{r'}{r} \int_{-l/2}^{+l/2} g_3(\xi)\, K_{22}(x, \xi)\, d\xi \right] + \begin{matrix} \cos \\ \sin \end{matrix} \theta \,.\, \mathbf{b} \left[\frac{x}{l/2} + \frac{1}{r} \int_{-l/2}^{+l/2} g_3(\xi)\, K_{23}(x, \xi)\, d\xi \right] \right\} \quad \ldots \quad (120)$$

$$\Phi_{\text{rel rot } y, z} = \pm \frac{l}{2} \begin{matrix} \Omega_y \sin \\ \Omega_z \cos \end{matrix} \theta \int_{-l/2}^{+l/2} g_3(\xi)\, K_{23}(x, \xi)\, d\xi \quad \ldots \quad (121)$$

$$\mathbf{V}_{\text{rel rot } x} = - \frac{l}{2} \Omega_x \frac{r}{l/2} \mathbf{b} \quad \ldots \quad (122)$$

$$\Phi_{\text{rot } x} = 0 \,. \quad \ldots \quad (123)$$

1.2. *The Functions G_ν and H_ν.*—The functions G_ν and H_ν were defined in equations (52, 53, 60, 67, 68 and 72) in terms of the standard integrals

$$E(k^2) = \int_0^{\pi/2} \sqrt{\{1 - k^2 \sin^2 \psi\}}\, d\psi \quad \text{and} \quad K(k^2) = \int_0^{\pi/2} \frac{d\psi}{\sqrt{(1 - k^2 \sin^2 \psi)}} \,. \quad (124)$$

Their developments for $k^2 \ll 1$ and $k'^2 \ll 1$ are easily obtained from the well-known developments E and K

$$\left. \begin{aligned} & k^2 \ll 1 : \\ & G_1 = 1 - \frac{1}{4} k^2 - \frac{3}{64} k^4 - \frac{5}{256} k^6 \ldots \\ & G_2 = \frac{1}{4} k^2 + \frac{3}{32} k^4 + \frac{15}{256} k^6 \ldots \\ & G_3 = 1 + \frac{1}{4} k^2 + \frac{9}{64} k^4 + \frac{25}{256} k^6 \ldots \\ & H_1 = \frac{3}{8} k^2 + \frac{3}{32} k^4 + \frac{45}{1024} k^6 \ldots \\ & H_2 = - \frac{1}{8} k^2 + \frac{15}{1024} k^6 \ldots \\ & H_3 = \frac{1}{8} k^2 + \frac{3}{32} k^4 + \frac{75}{1024} k^6 \ldots \end{aligned} \right\} \quad \ldots \quad (125)$$

$k'^2 \ll 1$ Let $\Lambda = \ln \frac{4}{k'}$

$$\left.\begin{aligned}
G_1 &= \frac{2}{\pi}\left[1 + \tfrac{1}{2}(\Lambda - \tfrac{1}{2})k'^2 + \frac{3}{16}\left\{\Lambda - \frac{13}{12}\right\}k'^4 \ldots\right] \\
G_2 &= \frac{1}{\pi}\left[(\Lambda - 1) - \tfrac{1}{4}\Lambda k'^2 - \frac{3}{64}\left\{\Lambda - \frac{5}{6}\right\}k'^4 \ldots\right] \\
G_3 &= \frac{2}{\pi}\left[\Lambda + \tfrac{1}{4}(\Lambda - 1)k'^2 + \frac{9}{64}\left\{\Lambda - \frac{7}{6}\right\}k'^4 \ldots\right] \\
H_1 &= \frac{2}{\pi}\left[1 - \frac{3}{2}\left\{\Lambda - \frac{7}{6}\right\}k'^2 - \frac{21}{16}\left\{\Lambda - \frac{115}{84}\right\}k'^4 \ldots\right] \\
H_2 &= \frac{1}{\pi}\left[(\Lambda - 3) + \frac{11}{4}\left\{\Lambda - \frac{14}{11}\right\}k'^2 + \frac{165}{64}\left\{\Lambda - \frac{91}{66}\right\}k'^4 \ldots\right] \\
H_3 &= \frac{2}{\pi}\left[(\Lambda - 2) + \frac{5}{4}\left\{\Lambda - \frac{7}{5}\right\}k'^2 + \frac{81}{64}\left\{\Lambda - \frac{25}{18}\right\}k'^4 \ldots\right]
\end{aligned}\right\}. \quad (126)$$

For $k'^2 = 0$, the functions G_1 and H_1 are finite, whereas G_2, G_3, H_2 and H_3 have a logarithmic singularity.

If tables of the complete elliptic integrals

$$\left.\begin{aligned}
B(k^2) &= \int_0^{\pi/2} \frac{\cos^2\psi\, d\psi}{\sqrt{(1 - k^2\sin^2\psi)}}, \qquad C(k^2) = \int_0^{\pi/2} \frac{\sin^2\psi\cos^2\psi\, d\psi}{\{\sqrt{(1 - k^2\sin^2\psi)}\}^3} \\
D(k^2) &= \int_0^{\pi/2} \frac{\sin^2\psi\, d\psi}{\sqrt{(1 - k^2\sin^2\psi)}}
\end{aligned}\right\} \quad (127)$$

are available (*e.g.*, Jahnke-Emde, Tafeln höherer Funktionen, 4 Aufe., Leipzig, 1948), the following alternative formulae for G_2, H_1, H_2, H_3 will be found useful for the numerical calculation of these functions:

$$G_2 = \frac{1}{\pi}k^2 D(k^2) \quad \ldots \quad (128)$$

$$\left.\begin{aligned}
H_1 &= \frac{2}{\pi}[2B(k^2) - E(k^2)] \\
&= \frac{2}{\pi}k^2[D(k^2) - C(k^2)]
\end{aligned}\right\} \quad \ldots \quad (129)$$

$$\left.\begin{aligned}
H_2 &= \frac{1}{\pi}[D(k^2) + E(k^2) - 3B(k^2)] \\
&= \frac{1}{\pi}k^2[2C(k^2) - D(k^2)]
\end{aligned}\right\} \quad \ldots \quad (130)$$

$$\left.\begin{aligned}
H_3 &= \frac{2}{\pi}k^2 C(k^2) \\
&= \frac{2}{\pi}[2D(k^2) - K(k^2)]
\end{aligned}\right\} \quad \ldots \quad (131)$$

A three-figure table of the G_r and H_r is given at the end of this report; its accuracy will in general be sufficient for the present purpose.

1.3. *Singularities of the Kernels and Limiting Cases.*—In order to develop a suitable method for the evaluation of the integrals occurring in the numerical work, the singularities of the kernels have to be considered. A special treatment is necessary for the ends of the body where the elliptic integrals degenerate.

1.3(*a*). *Singularities of the kernels for* $x \neq \pm l/2$.—Only the case $\xi \to x$ has to be considered. Introducing the development

$$\rho = r - r'(x - \xi) + \tfrac{1}{2}r''(x - \xi)^2 - \ldots, \qquad (132)$$

the expressions A, B, C and k'^2 become

$$A = 1/\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}} \to 1/2r \qquad (133)$$

$$B = \frac{r[r - \rho - r'(x - \xi)]}{(x - \xi)^2 + (r - \rho)^2} = \frac{r}{1 + \left(\frac{r - \rho}{x - \xi}\right)^2} \frac{\frac{r - \rho}{x - \xi} - r'}{x - \xi} \to -\frac{rr''}{2(1 + r'^2)} \qquad (134)$$

$$C = \frac{(x - \xi) + r'(r - \rho)}{(x - \xi)^2 + (r - \rho)^2} = \frac{1 + r'\left(\frac{r - \rho}{x - \xi}\right)}{1 + \left(\frac{r - \rho}{x - \xi}\right)^2} \frac{1}{x - \xi} \to \frac{1}{x - \xi} \qquad (135)$$

$$k'^2 = \frac{(x - \xi)^2 + (r - \rho)^2}{(x - \xi)^2 + (r + \rho)^2} = \frac{1 + \left(\frac{r - \rho}{x - \xi}\right)^2}{(x - \xi)^2 + (r + \rho)^2}(x - \xi)^2 \to \frac{1 + r'^2}{4r^2}(x - \xi)^2. \qquad (136)$$

The behaviour of the functions G_r and H_r is obtained from the first terms of their developments (126) for $k'^2 \ll 1$ with

$$A = \ln\frac{4}{k'} \to \ln\frac{8r}{\sqrt{(1 + r'^2)}\,|x - \xi|}. \qquad (137)$$

The kernels become then

$$\begin{matrix} K_{10} \\ K_{20} \end{matrix} = \left(B\begin{matrix} G_1 \\ H_1 \end{matrix} + \begin{matrix} G_2 \\ H_2 \end{matrix}\right)A \to \frac{1}{2r\pi}\left(-\frac{rr''}{1 + r'^2} + \ln\frac{8r}{\sqrt{(1 + r'^2)}\,|x - \xi|} - \frac{1}{3}\right) \qquad (138)$$

$$\begin{matrix} K_{11} \\ K_{21} \end{matrix} = C\begin{matrix} G_1 \\ H_1 \end{matrix}A \to \frac{1}{\pi r(x - \xi)} \qquad (139)$$

$$\begin{matrix} K_{12} \\ K_{22} \end{matrix} = \begin{matrix} G_2 \\ H_2 \end{matrix}A \to \frac{1}{2\pi r}\left(\ln\frac{8r}{\sqrt{(1 + r'^2)}\,|x - \xi|} - \frac{1}{3}\right) \qquad (140)$$

$$\begin{matrix} K_{13} \\ K_{23} \end{matrix} = \begin{matrix} G_3 \\ H_3 \end{matrix}A \to \frac{1}{\pi r}\left(\ln\frac{8r}{\sqrt{(1 + r'^2)}\,|x - \xi|} - \frac{0}{2}\right). \qquad (141)$$

K_{11} and K_{21} have a singularity of the type $(x - \xi)^{-1}$; the other kernels have a logarithmic singularity for $\xi \to x$.

1.3(*b*). *Ends of the body.*—The ends of the body require a special treatment, as the elliptic integrals G_r and H_r degenerate on the axis. It is sufficient to consider the head ($x = +l/2$), as the corresponding formulae for the stern are easily obtained by a change of the direction of the axis of x. Practically, the calculation for the stern has but little importance, as the flow conditions there differ too much from the behaviour of a perfect fluid.

Let

$$r\left(+\frac{l}{2}\right) = 0\,,\quad r'\left(+\frac{l}{2}\right) = -\infty \text{ in such a way that } \lim_{x\to l/2} rr' \text{ is finite.} \qquad \ldots \quad \ldots \quad (142)$$

Near $\eta = 0$, ρ is then given by

$$\rho(\xi) \doteq \sqrt{\left\{2 \lim_{x\to l/2} (-rr')\left(\frac{l}{2} - \xi\right)\right\}}\,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (143)$$

The integral equation for $g_1(x)$ becomes then

$$g_1\left(\frac{l}{2}\right) = \lim_{x\to l/2} (-rr') - \int_{-l/2}^{+l/2} g_1(\xi) \lim_{x\to l/2} \left\{\frac{r[r - \rho - r'(x-\xi)]}{\sqrt{\{(x-\xi)^2 + (r-\rho)^2\}}} G_1 + G_2\right\} d\xi\,, \qquad \ldots \quad (144)$$

or

$$g_1\left(\frac{l}{2}\right) = \lim_{x\to l/2} (-rr')\left(1 - \int_{-l/2}^{+l/2} g_1(\xi) \frac{l/2 - \xi}{\left[\sqrt{\left\{\left(\frac{l}{2} - \xi\right)^2 + \rho^2\right\}}\right]^3} d\xi\right) \qquad \ldots \quad \ldots \quad (145)$$

with respect to

$$G_1 \doteq 1\,,\quad G_2 \doteq \frac{k^2}{4} \text{ for } k^2 = \frac{4r\rho}{(x-\xi)^2 + (r+\rho)^2} \ll 1\,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (146)$$

The kernel of this equation has a singularity of the type

$$\frac{l/2 - \xi}{\left[\sqrt{\left\{\left(\frac{l}{2} - \xi\right)^2 + \rho^2\right\}}\right]^3} \to \frac{1}{\left[\sqrt{\left\{2 \lim_{x\to l/2} (-rr')\right\}}\right]^3} \left(\frac{l}{2} - \xi\right)^{-1/2} \text{ for } \xi \to \frac{l}{2}\,. \qquad \ldots \quad (147)$$

The values of $g_2(l/2)$ and $g_3(l/2)$ are easily seen to be zero. It is, however, advisable to determine the limits of $g_2(x)/r(x)$ and $g_3(x)/r(x)$ which can be obtained in a similar manner as $g_1(l/2)$. The results are

$$\lim_{x\to l/2} \frac{g_2(x)/r(x)}{g_3(x)/r(x)} = \frac{1}{1 + \dfrac{\lim rr'}{l/2}} - \int_{-l/2}^{+l/2} \frac{g_2(\xi)/\rho}{g_3(\xi)/\rho} \frac{\rho^2}{2\left[\sqrt{\left\{\left(\frac{l}{2} - \xi\right)^2 + \rho^2\right\}}\right]^3} \left[\frac{3\left(\frac{l}{2} - \xi\right) \lim (-rr')}{\left(\frac{l}{2} - \xi\right)^2 + \rho^2} - 1\right] d\xi \qquad (148)$$

with the singularity of the kernel

$$\frac{\rho^2}{2\left[\sqrt{\left\{\left(\frac{l}{2} - \xi\right)^2 + \rho^2\right\}}\right]^3} \left[\frac{3\left(\frac{l}{2} - \xi\right) \lim (-rr')}{\left(\frac{l}{2} - \xi\right)^2 + \rho^2} - 1\right] \to \frac{1}{4\sqrt{\{2 \lim (-rr')\}}} \left(\frac{l}{2} - \xi\right)^{-1/2}$$

$$\text{for } \xi \to \frac{l}{2}\,. \qquad \ldots \quad \ldots \quad (149)$$

Consider now the equations for $\mathbf{V}_{rel}$. In longitudinal translation, $\mathbf{V}_{rel}$ will obviously be zero at the ends of the body where the relative flow past the body has stagnation points

$$\lim_{x\to l/2} \mathbf{V}_{rel\,long}(x) = 0\,. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (150)$$

In the case of a lateral translation, $\mathbf{V}_{rel}$ has a finite limit at the ends

$$\lim_{x\to l/2} \mathbf{V}_{rel\,lat}(x) = \frac{V_y}{V_z}\left(-\frac{\cos}{\sin}\theta\,\mathbf{t} \pm \frac{\sin}{\cos}\theta\,\mathbf{b}\right)\left(1 + \int_{-l/2}^{+l/2} \frac{g_2(\xi)}{\rho} \frac{\rho^2\,d\xi}{2\left[\sqrt{\left\{\left(\frac{l}{2} - \xi\right)^2 + \rho^2\right\}}\right]^3}\right). \qquad (151)$$

The singularity of the kernel is

$$\frac{\rho^2}{2\left[\sqrt{\left\{\left(\frac{l}{2}-\xi\right)^2+\rho^2\right\}}\right]^3} \to \frac{1}{2\sqrt{\{2\lim(-rr')\}}}\left(\frac{l}{2}-\xi\right)^{-1/2} \text{ for } \xi \to \frac{l}{2}. \quad \ldots \quad (152)$$

For a rotation of the body about its longitudinal axis, the relative velocity is obviously zero at the ends

$$\lim_{x\to l/2} \mathbf{V}_{\text{rel rot } x}(x) = 0. \quad \ldots \quad (153)$$

Finally, the relative velocity for a rotation about the axes of y or z is obtained as

$$\lim_{x\to l/2} \mathbf{V}_{\text{rel rot } y,z}(x) = \frac{l}{2}\frac{\Omega_y}{\Omega_z}\begin{pmatrix}+\sin\\-\cos\end{pmatrix}\theta\,\mathbf{t} + \begin{pmatrix}\cos\\\sin\end{pmatrix}\theta\,\mathbf{b}\Bigg)\left(\frac{x}{l/2} + \int_{-l/2}^{+l/2}\frac{g_2(\xi)}{\rho}\frac{\rho^2\,d\xi}{2\left[\sqrt{\left\{\left(\frac{l}{2}-\xi\right)^2+\rho^2\right\}}\right]^3}\right) \quad (154)$$

with the same kernel as (151) and its singularity (152). Consider at last the equations for the potential. The limit of Φ_{long} is

$$\lim_{x\to l/2} \Phi_{\text{long}} = -V_x\int_{-l/2}^{+l/2} g_1(\xi)\frac{d\xi}{\sqrt{\left\{\left(\frac{l}{2}-\xi\right)^2+\rho^2\right\}}}, \quad \ldots \quad (155)$$

with

$$\frac{1}{\sqrt{\left\{\left(\frac{l}{2}-\xi\right)^2+\rho^2\right\}}} \to \frac{1}{\sqrt{\{2\lim(-rr')\}}}\left(\frac{l}{2}-\xi\right)^{-1/2} \text{ for } \xi \to \frac{l}{2}. \quad \ldots \quad (156)$$

The limits of Φ_{lat} and Φ_{rot} are zero for reasons of symmetry.

2. *An Integration Formula for the Numerical Evaluation of the Integrals.*—The numerical evaluation of the integrals occurring in the integral equations and in the equations for $\mathbf{V}_{\text{rel}}$ and Φ is most conveniently carried out by using an integration formula of the type

$$I(x) = \int_{-l/2}^{+l/2} g(\xi)\,K(x,\xi)\,d\xi = \sum g(\xi_\nu)\,p_\nu K(x,\xi_\nu). \quad \ldots \quad (157)$$

As the kernels of the integrals have singularities, the points ξ_ν and the corresponding weights p_ν have to be determined in such a way that the singularities are automatically taken account of. This can be achieved by a suitable transformation of the variable of integration ξ.

The singularities can be of the types

$$\left.\begin{array}{ll}(a)\ K(x,\xi) \sim \log|x-\xi| & \text{for } \xi \to x\\ (b)\ K(x,\xi) \sim (x-\xi)^{-1} & \text{for } \xi \to x\\ (c)\ K\left(\pm\frac{l}{2},\xi\right) \sim \left|\pm\frac{l}{2}-\xi\right|^{-1/2} & \text{for } \xi \to \pm\frac{l}{2}\end{array}\right\}. \quad \ldots \quad (158)$$

Consider

$$h(\xi)\,; \qquad d\xi = \frac{1}{h'(\xi)}\,dh(\xi). \quad \ldots \quad (159)$$

Then

$$I(x) = \int_{h(-l/2)}^{h(+l/2)} g(\xi)\,K(x,\xi)\frac{1}{h'(\xi)}\,dh(\xi). \quad \ldots \quad (160)$$

In order to remove the singularities (158) as far as possible, $h(\xi)$ has to satisfy the following conditions :

$$\left.\begin{array}{l}(a)\ h(x-\xi) = -h(\xi-x), \qquad h(x) = 0\\ (b)\ \lim\limits_{\xi\to x} K(x,\xi)\dfrac{1}{h'(\xi)} = \text{finite or zero in the cases 158 } (a) \text{ and } (c)\end{array}\right\}. \quad \ldots \quad (161)$$

The first condition is necessary with respect to the singularities of the type 158 (b) of the kernels K_{11} and K_{21}. The corresponding integrals have to be calculated in the form of their so-called principal values ; this requires that the points ξ_ν have to be distributed symmetrically with respect to x, at least in the neighbourhood of x. This gives

$$\tfrac{1}{2}(\xi_\nu + \xi_{-\nu}) = x \quad \ldots \quad (162)$$

and together with 161 (a)

$$p_{-\nu} = p_\nu \quad \ldots \quad (163)$$

The second condition is required in order to remove the singularities 158 (a) and (c).

A transformation satisfying these conditions can obviously be chosen in many different ways. A comparatively simple transformation of this kind is

$$h(\xi) = \{\mathrm{sgn}\,(\xi - x)\} \sqrt{\left(\frac{|\xi - x|}{|\xi - x| + 1}\right)} \quad \ldots \quad (164)$$

$$\frac{1}{h'(\xi)} = 2\sqrt{\{|\xi - x|\,(|\xi - x| + 1)^2\}} \quad \ldots \quad (165)$$

The condition 161 (a) is obviously satisfied. 161 (b) is also satisfied since

$$\lim_{\xi \to x} K(x, \xi) \frac{1}{h'(\xi)} = 2 \lim_{\xi \to x} \{K(x, \xi) \sqrt{(|\xi - x|)}\}, \quad \ldots \quad (166)$$

which is zero in the case 158 (a) and finite in the case 158 (c). The transformation has further the properties

$$\lim_{\xi \to \infty} h(\xi) = 1 \quad \ldots \quad (167)$$

and

$$\lim_{\xi \to \infty} \frac{1}{\xi^2} \frac{1}{h'(\xi)} = 2, \quad \ldots \quad (168)$$

which are convenient when dealing with the practically most important case of an elongated body.

Introducing (164) and (165) into (160), the integral becomes

$$I(x) = \int_{h(-l/2)}^{h(+l/2)} g(\xi)\, K(x, \xi)\, 2[\sqrt{\{|\xi - x|\,(|\xi - x| + 1)^2\}}]$$
$$d\left[\{\mathrm{sgn}\,(\xi - x)\} \sqrt{\left(\frac{|\xi - x|}{|\xi - x| + 1}\right)}\right] \quad \ldots \quad (169)$$

This integral can be evaluated approximately by using the trapezoidal rule.

Let first $x \neq \pm\, l/2$, and let

$$-\frac{l}{2} = \xi_{-n-1} = \xi_{-n} < \xi_{-n+1} < \ldots < \xi_{-1} < \xi_0$$
$$= x < \xi_1 < \ldots < \xi_{m-1} < \xi_m = \xi_{m+1} = +\frac{l}{2} \quad \ldots \quad (170)$$

be a set of points on the axis of the body satisfying the condition

$$x - \xi_{-\nu} = \xi_\nu - x \text{ for } \nu < Min\,(n, m) \quad \ldots \quad (171)$$

[*cf.* equation (162)].

The general trapezoidal rule for non-equidistant points gives then

$$I(x) \simeq \sum_{\nu=-n}^{+m}{}' g(\xi_\nu)\, K(x, \xi_\nu)\, p_\nu \quad \ldots \quad (172)$$

with the weights

$$p_\nu = \frac{h(\xi_{\nu+1}) - h(\xi_{\nu-1})}{2h'(\xi_\nu)} \quad \ldots \quad (173)$$

or, with respect to (164) and (165)

$$p_\nu = \sqrt{\{|\xi_\nu - x|(|\xi_\nu - x| + 1)^3\}}\left[\{\operatorname{sgn}(\xi_{\nu+1} - x)\}\sqrt{\left(\frac{|\xi_{\nu+1} - x|}{|\xi_{\nu+1} - x| + 1}\right)}\right.$$
$$\left. - \{\operatorname{sgn}(\xi_{\nu-1} - x)\}\sqrt{\left(\frac{|\xi_{\nu-1} - x|}{|\xi_{\nu-1} - x| + 1}\right)}\right]. \quad \ldots \quad (174)$$

Σ' in (172) indicates that the term corresponding to $\nu = 0$ is zero, as the singularity of K is of the types 158 (a) or 158 (b).

Let now $x = + l/2$ and

$$-\frac{l}{2} = \xi_{-n-1} = \xi_{-n} < \xi_{-n+1} < \ldots < \xi_{-1} < \xi_0 = \xi_1 = +\frac{l}{2}. \quad \ldots \quad (175)$$

In this case, K has a singularity of the type 158 (c). The trapezoidal rule gives then

$$I\left(\frac{l}{2}\right) = \sum_{\nu=-n}^{-1} g(\xi_\nu)\, K\left(\frac{l}{2}, \xi_\nu\right) p_\nu + g\left(\frac{l}{2}\right) \lim_{\xi \to l/2}\left[K\left(\frac{l}{2}, \xi\right)\sqrt{\left(\frac{l}{2} - \xi\right)}\right]\sqrt{\left(\frac{\frac{l}{2} - \xi_{-1}}{\frac{l}{2} - \xi_{-1} + 1}\right)}, \quad (176)$$

with the same values of the p as in equation (174). The limits of $K\sqrt{(l/2 - \xi)}$ in this formula are given in equations (147), (149), (152) and (156) for the various cases.

For the practical application of the integration formulae (172) and (176), it is convenient to use a standard set of values $|x - \xi_\nu|$ which correspond to approximately equal values of Δh and are numerically convenient. In the subsequent numerical examples, the standard set

$\nu - 0$	± 1	± 2	± 3	± 4	± 5	± 6	± 7	± 8	± 9	± 10	± 11	± 12	± 13	± 14
$x - \xi_\nu = 0$	$\mp 0{\cdot}01$	$\mp 0{\cdot}04$	$\mp 0{\cdot}1$	$\mp 0{\cdot}2$	$\mp 0{\cdot}4$	$\mp 0{\cdot}7$	∓ 1	$\mp 1{\cdot}5$	$\mp 2{\cdot}5$	∓ 4	∓ 6	∓ 9	∓ 15	$\mp \infty$
p_ν	0·0199	0·0428	0·0774	0·134	0·245	0·320	0·376	0·658	1·24	1·85	2·46	4·02	12·72	—

(177)

$$p_0 = 0{\cdot}0995 \text{ in (176)} \quad \ldots \quad (177a)$$

was used. In general, the two ends of the body $\xi = \pm l/2$ will not coincide with one of the points ξ_ν for any particular point x. The first two points $\xi_{-n-1} < \xi_{-n} < -l/2$ and the first two points $\xi_{m+1} > \xi_m > +l/2$ in (177) were then replaced by $-l/2$ or $+l/2$, respectively, and the new values of the weights $p_{-n}, p_{-n+1}, p_{m-1}, p_m$ were calculated from (174).

Example: $l = 10$, $x = +3$

$$\xi_{-13} = \xi_{-12} = -5, \underbrace{\xi_{-11} = -3, \ldots, \xi_0 = +3, \ldots, \xi_8 = +4{\cdot}5,}_{\text{from (177).}} \xi_9 = \xi_{10} = +5$$

The weights are then

$$p_{-12} = \sqrt{(8{\cdot}9^3)}\left(\sqrt{\frac{8}{9}} - \sqrt{\frac{6}{7}}\right) = 0{\cdot}451 \quad \ldots \quad (178)$$

$$p_{-11} = \sqrt{(6{\cdot}7^3)}\left(\sqrt{\frac{8}{9}} - \sqrt{\frac{4}{5}}\right) = 0{\cdot}839$$

p_{-10} to p_7 as in (177)

$$p_8 = \sqrt{(1{\cdot}5 \times 2{\cdot}5^3)}\left(\sqrt{\frac{2}{3}} - \sqrt{\frac{1}{2}}\right) = 0{\cdot}530$$

$$p_9 = \sqrt{(2{\cdot}3^3)}\left(\sqrt{\frac{2}{3}} - \sqrt{\frac{1{\cdot}5}{2{\cdot}5}}\right) = 0{\cdot}146.$$

The results of the subsequent numerical examples suggest that the set (177) gives a sufficient accuracy of the numerical integrations in most cases of practical interest, especially for the lateral translation and for the rotation. In the case of the longitudinal translation where the requirements on the accuracy are considerably greater, the set (177) gave quite satisfactory results for an ellipsoid. The results for a longitudinal translation of a torpedo with a hemispherical head were still fair but the velocity near the head of the body was found up to about 2 per cent less than the velocity which was obtained in a previous calculation with the use of the vortex-ring method. The latter results which were obtained with about twice the number of points ξ_ν were in excellent agreement with American experimental results. This suggests that it may be advisable to use more points ξ_ν for the calculation of the longitudinal flow which unfortunately will lead to a corresponding increase of the already considerable numerical work.

3. *Procedure for the Numerical Calculation.*—In the actual numerical work, the functions g_ν, the relative velocity and the potential will be calculated for a certain number of points x (in the following referred to as pivotal points) along the axis of the body. The selection of these points is therefore the first problem for the computer. Generally speaking, the pivotal points should give sufficient guidance to draw the curves $g_\nu(x)$, $\mathbf{V}_{rel}(x) \cdot \mathbf{t}$, $\mathbf{V}_{rel}(x) \cdot \mathbf{b}$ and $\Phi(x)$. They should be spaced rather closely together in those parts of the body where irregularities (suction peaks, influence of discontinuities of the curvature, etc.), may be expected. The front end of the body ($x = +l/2$) should be a pivotal point, but it is in general not necessary to use $x = -l/2$ as one, because the flow conditions near the stern of the body will differ too much from the behaviour of a perfect fluid. It is advisable to start the work with about 12 to 15 pivotal points; more points can be added later in the course of the work wherever they may be required.

The subsequent procedure of the numerical work is given under the assumption that the shape of the body, although it may have isolated discontinuities of the curvature, is composed of arcs of relatively simple analytical curves (*e.g.*, straight lines, circles, ellipses, parabolas, etc.), so that it is not difficult and not too laborious to obtain sufficiently accurate values of r, r' and r'' for every point x and of ρ and ρ' for every point ξ_ν.

Consider first the case $x \neq l/2$.

By far the greatest part of the numerical work is required for the calculation of the eight kernels (110) for the points ξ_ν. The calculation follows closely the pattern given in equations (103) to (112). The first step is the calculation of the expressions b, c, r_1^2 and r_2^2 (equations (103) to (106)) on Form I. The calculation of b requires some attention near $\xi = x$, as b is there of the order ε^2 and has to be calculated from r and ρ which are of the order 1. In order to obtain b with an accuracy of 3 significant decimals, r and ρ have to be calculated with about 7 significant decimals, r' with 5 to 6 significant decimals. Form II gives then the calculation of pA, B†, C, k'^2 and the elliptic integrals $G_{1,2,3}$ and $H_{1,2,3}$, and Form III the final assembly of the kernels.

The calculation is then split up into three parts corresponding to the longitudinal translation (Forms IV and V), the lateral translation (Forms VI and VII) and the rotation of the body (Forms VIII and IX). All necessary instructions for the numerical work are given on these forms.

The solution of the three integral equations for the $g_\nu(x)$ is carried out by iteration. In the case of $g_1(x)$, the direct iteration according to the scheme

$$\left.\begin{aligned} h(x) &= f(x) + \int h(\xi)\, K(x, \xi)\, d\xi \\ h_0(x) &= f(x) \\ h_n(x) &= f(x) + \int h_{n-1}(\xi)\, K(x, \xi)\, d\xi \end{aligned}\right\} \quad \ldots \quad (179)$$

† In some cases, the calculation of B can be simplified near $\xi = x$, especially if both x and ξ are on a part of the body where the shape is an arc of circle.

was found to oscillate and to converge only very slowly. A much quicker convergence was obtained by the scheme

$$h_0(x) = \tfrac{1}{2} f(x)$$

$$h_n^*(x) = f(x) + \int h_{n-1}(\xi)\, K(x, \xi)\, d\xi\,, \quad h_n(x) = \tfrac{1}{2}(h_{n-1}(x) + h_n^*(x)) \quad \ldots \quad (180)$$

$g_2(x)$ and $g_3(x)$ could be obtained from (179). Two or three steps of the iteration procedure were in general sufficient and a fourth step had only to be carried out for the pivotal points near to the ends of the body. After each step of the iteration, the results $g_{\nu\mu}(x)$ were represented by a curve from which the values $g_{\nu\mu}(\xi)$ were read for the subsequent step of the procedure.

Special forms are needed for the case $x = l/2$ where the elliptic integrals in the kernels degenerate (equations 142 to 156). These forms I (a) to IX (a) contain also all the necessary instructions for the numerical work.

After carrying out the calculation of $\mathbf{V}_{|\nu|}$ and Φ for the elementary translations and rotations of the body, the result for the complete motion can easily be obtained by a superposition (equation (16)).

4. *Formulae and Forms for the Numerical Work.—Body of Revolution in Translation or Rotation. Calculation of the Kernels for $x \neq l/2$.*

FORMULAE :

1. *Longitudinal translation:*

$$K_{10} = \left\{ \frac{r[r - \rho - r'(x - \xi)]}{(x - \xi)^2 + (r - \rho)^2}\, G_1(k^2) + G_2(k^2) \right\} \frac{1}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}$$

$$K_{11} = \frac{(x - \xi) + r'(r - \rho)}{(x - \xi)^2 + (r - \rho)^2}\, G_1(k^2) \frac{1}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}$$

$$K_{12} = G_2(k^2) \frac{1}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}$$

$$K_{13} = G_3(k^2) \frac{1}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}$$

2. *Lateral translation and rotation:*

$$K_{20} = \left\{ \frac{r[r - \rho - r'(x - \xi)]}{(x - \xi)^2 + (r - \rho)^2}\, H_1(k^2) + H_2(k^2) \right\} \frac{1}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}$$

$$K_{21} = \frac{(x - \xi) + r'(r - \rho)}{(x - \xi)^2 + (r - \rho)^2}\, H_1(k^2) \frac{1}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}$$

$$K_{22} = H_2(k^2) \frac{1}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}$$

$$K_{23} = H_3(k^2) \frac{1}{\sqrt{\{(x - \xi)^2 + (r + \rho)^2\}}}$$

EXAMPLE 1.—*Ellipsoid* $b : a = 1 : 4$

In order to check the accuracy of the method, the calculation was first carried out for the case of an ellipsoid of fineness ratio $b : a = 1 : 4$. Figs. 4 to 6 give the distributions of the relative velocity in longitudinal translation, lateral translation and rotation about a lateral axis, respectively, in comparison with the known exact solutions. In general, the agreement is very good. Close to the ends of the body, the error of the present method tends to become greater, but is still small enough for practical purposes. The calculations were carried out with the standard set (177) of points ξ_ν. If a greater accuracy near the ends of the body should be desirable, this is likely to be achieved by using more points ξ_ν for the points x near the ends.

EXAMPLE 2.—*Torpedo of Fineness Ratio* 1 : 8

As a second numerical example, the calculation was carried out for a torpedo of fineness ratio 1 : 8. Its shape, composed of arcs of circles and straight lines, was given by the equations :

1. $$\frac{r}{r_{max}} = \sqrt{\left\{1 - \left(\frac{x}{r_{max}} - 7\right)^2\right\}} \text{ for } +7 \leqslant \frac{x}{r_{max}} \leqslant +8$$
   (Hemispherical head).

2. $$\frac{r}{r_{max}} = 1 \text{ for } -2{\cdot}003 \leqslant \frac{x}{r_{max}} \leqslant +7$$
   (Straight cylindrical central part).

3. $$\frac{r}{r_{max}} = \sqrt{\left\{13{\cdot}715^2 - \left(\frac{x}{r_{max}} + 2{\cdot}003\right)^2\right\}} - 12{\cdot}715 \text{ for } -4{\cdot}385 \leqslant \frac{x}{r_{max}} \leqslant -2{\cdot}003$$
   (Transition to the tail).

4. $$\frac{r}{r_{max}} = 0{\cdot}1763\,\frac{x}{r_{max}} + 1{\cdot}5647 \text{ for } -8 \leqslant \frac{x}{r_{max}} \leqslant -4{\cdot}385$$
   (Conical tail).

At the rear end of the body, $x/r_{max} = -8$, r/r_{max} is not zero as required by the definition of a 'normal' shape. It was assumed for the calculation that the end was suitably rounded off. The rear end of the body was, however, not included in the number of the pivotal points as the result of the calculation was likely to have little practical value beyond $x = -6$ or -7.

Body of Revolution in Translation or Rotation.—I. b, c, r_1^2, r_2^2

									b	c	r_1^2	r_2^2
[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]
ξ*	$x - \xi$	$r'(x - \xi)$	ρ	$4r\rho$	$r - \rho$	$r'(r - \rho)$	$(r - \rho)^2$	$(x - \xi)^2$	[6] − [3]	[2] + [7]	[8] + [9]	[12] + [5]
	15							225				
	9							81				
	6							36				
	4							16				
	2·5							6·25				
	1·5							2·25				
	1							1				
	0·7		(4)					0·49				
	0·4		(5)					0·16				
	0·2		(6)					0·04				
	0·1		(6)					0·01				
	0·04		(7)					0·0016				
	0·01		(7)					0·0001				
(x)	0	0	(r)	($4r^2$)	0	0	0	0	0	0	0	($4r^2$)
	− 0·01		(7)					0·0001				
	− 0·04		(7)					0·0016				
	− 0·1		(6)					0·01				
	− 0·2		(6)					0·04				
	− 0·4		(5)					0·16				
	− 0·7		(4)					0·49				
	− 1							1				
	− 1·5							2·25				
	− 2·5							6·25				
	− 4							16				
	− 6							36				
	− 8							81				
	−15							225				

* Replace the last $\xi < -l/2$ by $-l/2$ and the first $\xi > +l/2$ by $l/2$ and correct [2] and [9] correspondingly.

Constants of the pivotal point :

$x =$ ($\neq l/2$)

$r =$ (7 dec)

$r' =$ (5)

$r'' =$ (3)

The numbers in brackets give the number of significant decimals where more than 3 are required.

Body of Revolution in Translation or Rotation.—II. pA, B, C, $G_{1,2,3}$ *and* $H_{1,2,3}$

		pA	B	C	k'^2						
[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
ξ	p*	$p/\sqrt{r_2^2}$	$\frac{rb}{r_1^2}$	$\frac{c}{r_1^2}$	$\frac{r_1^2}{r_2^2}$†	G_1	G_2	G_3	H_1	H_2	H_3
	12·72										
	4·02										
	2·46										
	1·85										
	1·24										
	0·658										
	0·376										
	0·320										
	0·245										
	0·134										
	0·0774										
	0·0428										
	0·0199										
(x)	—	—	$\left(\frac{-\pi r^2}{2(1+r^2)}\right)$	$\pm\infty$	0	0·637	∞	∞	0·637	∞	∞
	0·0199										
	0·0428										
	0·0774										
	0·134										
	0·245										
	0·320										
	0·376										
	0·658										
	1·24										
	1·85										
	2·46										
	4·02										
	12·72										

Pivotal point
$x =$ $(\neq l/2)$

* Correct according to (178).

† If $k'^2 > 0{\cdot}9$, use $k'^2 = 1 - 4r\rho/r_2^2$ instead.

Body of Revolution in Translation or Rotation.—III. *Kernels*

							pK_{10}	pK_{11}	pK_{12}	pK_{13}	pK_{20}	pK_{21}	pK_{22}	pK_{23}	
[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]	
t	BG_1	BH_1	CG_1	CH_1	$[2] + G_2$	$[3] + H_2$	$[6] - pA$	$[4]\,pA$	$G_2\,pA$	$G_3\,pA$	$[7]\,pA$	$[5]\,pA$	$H_2\,pA$	$H_3\,pA$	$x =$ $(\neq l/2)$
(x)	$\left(\frac{-\pi r^2}{x(1+r^2)}\right)$	$\left(\frac{-\pi r^2}{x(1+r^2)}\right)$	$\pm\infty$	$\pm\infty$	∞	∞	—	—	—	—	—	—	—	—	
							Longitudinal translation				Lateral translation and rotation				

Body of Revolution in Longitudinal Translation.—Calculation of g_1, $\mathbf{V}_{\text{rel long}}$ and Φ_{long} for $x \neq l/2$

FORMULAE :

1. *Integral Equation:*

$$g_1(x) = -rr' - \int_{-l/2}^{+l/2} g_1(\xi)\, K_{10}(x, \xi)\, d\xi$$

Solution:

$$g = -\tfrac{1}{2}rr'$$

$$g_{11}^* = -rr' - \int_{-l/2}^{+l/2} g_{10}(\xi)\, K_{10}(x, \xi)\, d\xi, \quad g_{11} = \tfrac{1}{2}(g_{10} + g_{11}^*)$$

$$g_{12}^* = -rr' - \int_{-l/2}^{+l/2} g_{11}(\xi)\, K_{10}(x, \xi)\, d\xi, \quad g_{12} = \tfrac{1}{2}(g_{11} + g_{12}^*)$$

---

2. *Relative Velocity:*

$$\mathbf{V}_{\text{rel long}} = V_x \frac{\mathbf{t}}{\sqrt{(1 + r'^2)}} \left[1 - \int_{-l/2}^{+l/2} g_1(\xi)\, K_{11}(x, \xi)\, d\xi - \frac{r'}{r} \int_{-l/2}^{+l/2} g_1(\xi)\, K_{12}(x, \xi)\, d\xi\right]$$

3. *Potential:*

$$\Phi_{\text{long}} = -V_x \int_{-l/2}^{+l/2} g_1(\xi)\, K_{13}(x, \xi)\, d\xi$$

Body of Revolution in Translation or Rotation.—IV. Solution of the Integral Equation for Longitudinal Translation

$g_{10}(\xi)$

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
ξ	$-\frac{1}{2}\rho\rho'$	ρK_{10}	[2] . [3]	$g_{11}(\xi)$	[3] . [5]	$g_{12}(\xi)$	[3] . [7]	$g_{13}(\xi)$	[3] . [9]	$g_{14}(\xi)$	[3] . [11]
		Σ [4] =		Σ [6] =		Σ [8] =		Σ [10] =		Σ [12] =	
		$g_{11}^* = -\pi' - \Sigma$ [4] =		$g_{12}^* = -\pi' - \Sigma$ [6] =		$g_{13}^* = -\pi' - \Sigma$ [8] =		$g_{14}^* = -\pi' - \Sigma$ [10] =		$g_{15}^* = -\pi' - \Sigma$ [12] =	
$g_{10} = -\frac{1}{2}\pi'$ =		$g_{11} = \frac{g_{10} + g_{11}^*}{2}$ =		$g_{12} = \frac{g_{11} + g_{12}^*}{2}$ =		$g_{13} = \frac{g_{12} + g_{13}^*}{2}$ =		$g_{14} = \frac{g_{13} + g_{14}^*}{2}$ =		$g_{15} = \frac{g_{14} + g_{15}^*}{2}$ =	

$x =$ $(\neq l/2)$

$-\pi' =$

Body of Revolution in Translation or Rotation.—V. Relative Velocity and Potential for Longitudinal Translation

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
ξ	$g_1(\xi)$	pK_{11}	[2] . [3]	pK_{13}	[2] . [5]	pK_{12}	[2] . [7]
		Σ [4] =		Σ [6] =		Σ [8] =	

$x =$ $(\neq l/2)$

$\frac{r'}{r} =$

$\frac{1}{\sqrt{(1 + r'^2)}} =$

$$\frac{\mathbf{V}_{\text{rel long}}}{V_x} = \frac{1 - \Sigma[4] - \frac{r'}{r}\Sigma[6]}{\sqrt{(1 + r'^2)}}\,\mathbf{t} = \ldots\ldots \mathbf{t}$$

$$\Phi_{\text{long}} = -V_x \Sigma[8] = \ldots\ldots V_x$$

Body of Revolution in Lateral Translation. Calculation of g_2, $\mathbf{V}_{\text{relat}}$ and Φ_{lat} for $x \neq l/2$

FORMULAE :

1. *Integral Equation:*

$$g_2(x) \quad r - \int_{-l/2}^{+l/2} g_2(\xi)\, K_{20}(x, \xi)\, d\xi$$

Solution:

$$g_{20} \quad r$$

$$g_{21} \quad r - \int_{l/2}^{+l/2} g_{20}(\xi)\, K_{20}(x, \xi)\, d\xi$$

$$g_{22} \quad r - \int_{-l/2}^{+l/2} g_{21}(\xi)\, K_{20}(x, \xi)\, d\xi$$

---

2. *Relative Velocity:*

$$\mathbf{V}_{\text{relat}} \quad \begin{matrix} V_y \\ V_z \end{matrix} \left\{ \begin{matrix} \mathbf{t} \begin{matrix} \cos \\ \sin \end{matrix} \theta \\ \sqrt{(1 + r'^2)} \left[r' - \int_{-l/2}^{+l/2} g_2(\xi)\, K_{21}(x, \xi)\, d\xi - \frac{r'}{r} \int_{-l/2}^{+l/2} g_2(\xi)\, K_{22}(x, \xi)\, d\xi \right] \end{matrix} \right.$$

$$\left. \pm \begin{matrix} \sin \\ \cos \end{matrix} \theta\, \mathbf{b} \left[1 + \frac{1}{r} \int_{-l/2}^{+l/2} g_2(\xi)\, K_{22}(x, \xi)\, d\xi \right] \right\}$$

3. *Potential:*

$$\Phi_{\text{lat}} \quad - \begin{matrix} V_y \cos \\ V_z \sin \end{matrix} \theta \int_{-l/2}^{+l/2} g_2(\xi)\, K_{22}(x, \xi)\, d\xi\,.$$

(78396)

Body of Revolution in Translation or Rotation.—VI. Solution of the Integral Equation for Lateral Translation

$g_{20}(\xi)$

1	2	3	4	5	6	7	8	9	10	11	12
ξ	ρ	ρK_{20}	2 . 3	$g_{21}(\xi)$	3 . 5	$g_{22}(\xi)$	3 . 7	$g_{23}(\xi)$	3 . 9	$g_{24}(\xi)$	3 . 11
		Σ 4 =		Σ 6 =		Σ 8 =		Σ 10 =		Σ 12 =	
$g_{20} = r$		$g_{21} = r - \Sigma$ 4		$g_{22} = r - \Sigma$ 6		$g_{23} = r - \Sigma$ 8		$g_{24} = r - \Sigma$ 10		$g_{25} = r - \Sigma$ 12	
=		=		=		=		=		=	

$x =$ $(= l/2)$

$r =$

Body of Revolution in Translation or Rotation.—VII. *Relative Velocity and Potential for Lateral Translation*

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
ξ	$g_2(\xi)$	pK_{21}	[2] . [3]	pK_{22}	[2] . [5]	pK_{23}	[2] . [7]
		$\Sigma[4] =$		$\Sigma[6] =$		$\Sigma[8] =$	

$x =$ $(\neq l/2)$

$r =$

$r' =$

$\frac{r'}{r} =$

$\frac{1}{\sqrt{(1 + r'^2)}} =$

$$\mathbf{V}_{\text{rel lat}} = \begin{matrix} V_y \\ V_z \end{matrix} \left[\frac{r' - \Sigma[4] - \frac{r'}{r}\Sigma[6]}{\sqrt{(1 + r'^2)}} \begin{matrix} \cos \\ \sin \end{matrix} \theta \, \mathbf{t} \pm \left(1 + \frac{\Sigma[8]}{r}\right) \begin{matrix} \sin \\ \cos \end{matrix} \theta \, \mathbf{b} \right]$$

$$= \begin{matrix} V_y \\ V_z \end{matrix} \left(\ldots\ldots \begin{matrix} \cos \\ \sin \end{matrix} \theta \, \mathbf{t} \pm \ldots\ldots \begin{matrix} \sin \\ \cos \end{matrix} \theta \, \mathbf{b} \right)$$

$$\Phi_{\text{lat}} = -\Sigma[8] \begin{matrix} V_y \cos \\ V_z \sin \end{matrix} \theta = \ldots\ldots \begin{matrix} V_y \cos \\ V_z \sin \end{matrix} \theta \, .$$

Body of Revolution in Rotation.—Calculation of g_3, $\mathbf{V}_{\text{rel rot}}$ and Φ_{rot} for $x \neq l/2$

FORMULAE :

1. *Integral Equation:*

$$g_3(x) = \frac{r(x + rr')}{l/2} - \int_{-l/2}^{+l/2} g_3(\xi)\, K_{30}(x, \xi)\, d\xi$$

Solution:

$$g_{30} = \frac{r(x + rr')}{l/2}$$

$$g_{31} = \frac{r(x + rr')}{l/2} - \int_{-l/2}^{+l/2} g_{30}(\xi)\, K_{40}(x, \xi)\, d\xi$$

$$g_{32} = \frac{r(x + rr')}{l/2} - \int_{-l/2}^{+l/2} g_{31}(\xi)\, K_{30}(x, \xi)\, d\xi\,.$$

---

2. *Relative Velocity:*

$$\mathbf{V}_{\text{rel rot } y,z} = \frac{l}{2}\,\Omega_y \left\{ \begin{matrix} -\sin \\ +\cos \end{matrix}\, \theta\, \mathbf{t} \left[\frac{r'x - r}{l/2} - \int_{-l/2}^{+l/2} g_3(\xi)\, K_{31}(x, \xi)\, d\xi \right.\right.$$

$$\left. - \frac{r'}{r} \int_{-l/2}^{+l/2} g_3(\xi)\, K_{32}(x, \xi)\, d\xi \right]$$

$$\left. + \begin{matrix} \cos \\ \sin \end{matrix}\, \theta\, \mathbf{b} \left[\frac{x}{l/2} + \frac{1}{r} \int_{-l/2}^{+l/2} g_3(\xi)\, K_{33}(x, \xi)\, d\xi \right] \right\}$$

$$\mathbf{V}_{\text{rel rot } x} = -\frac{l}{2}\,\Omega_x \frac{r}{l/2}\,\mathbf{b}\,.$$

3. *Potential:*

$$\Phi_{\text{rot } y,z} = \pm \frac{l}{2}\,\Omega_y \begin{matrix} \sin \\ \cos \end{matrix}\, \theta \int_{-l/2}^{+l/2} g_3(\xi)\, K_{33}(x, \xi)\, d\xi$$

$$\Phi_{\text{rot } x} = 0\,.$$

Body of Revolution in Translation or Rotation.—VIII. *Solution of the Integral Equation for Rotation*

$g_{33}(\xi)$

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
ξ	$\frac{p(\xi+\rho\rho')}{l/2}$	pK_{33}	[2] . [3]	$g_{31}(\xi)$	[3] . [5]	$g_{32}(\xi)$	[3] . [7]	$g_{33}(\xi)$	[3] . [9]	$g_{34}(\xi)$	[3] . [11]
		$\Sigma[4] =$		$\Sigma[6] =$		$\Sigma[8] =$		$\Sigma[10] =$		$\Sigma[12] =$	
$g_{33} = \frac{r(x+rr')}{l/2}$ $=$		$g_{31} = \frac{r(x+rr')}{l/2}$ $-\Sigma[4] =$		$g_{32} = \frac{r(x+rr')}{l/2}$ $-\Sigma[6] =$		$g_{33} = \frac{r(x+rr')}{l/2}$ $-\Sigma[8] =$		$g_{34} = \frac{r(x+rr')}{l/2}$ $-\Sigma[10] =$		$g_{35} = \frac{r(x+rr')}{l/2}$ $-\Sigma[12] =$	

$x =$ $(\neq l/2)$

$\frac{r(x+rr')}{l/2} =$

Body of Revolution in Translation or Rotation.—IX. *Relative Velocity and Potential for Rotation*

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
ξ	$g_3(\xi)$	pK_{31}	[2] . [3]	pK_{32}	[2] . [5]	pK_{33}	[2] . [7]
		$\Sigma[4] =$		$\Sigma[6] =$		$\Sigma[8] =$	

$x =$ $(\neq l/2)$

$r =$

$\dfrac{1}{\sqrt{(1 + r'^2)}} =$

$\dfrac{r'}{r} =$

$\dfrac{x}{l/2} =$

$\dfrac{r'x - r}{l/2} =$

$$\mathbf{V}_{\text{rel rot } y,z} = \frac{l}{2}\,\frac{\Omega_y}{\Omega_z}\left(\frac{\dfrac{r'x - r}{l/2} - \Sigma[4] - \dfrac{r'}{r}\Sigma[6]}{\sqrt{(1 + r'^2)}}\begin{pmatrix} -\sin \\ +\cos \end{pmatrix}\theta\,\mathbf{t} + \left(\frac{x}{l/2} + \frac{\Sigma[8]}{r}\right)\begin{matrix}\cos \\ \sin\end{matrix}\,\theta\,\mathbf{b}\right)$$

$$= \frac{l}{2}\,\frac{\Omega_y}{\Omega_z}\left[\ldots\ldots\begin{pmatrix} -\sin \\ +\cos \end{pmatrix}\theta\,\mathbf{t} + \ldots\ldots\begin{matrix}\cos \\ \sin\end{matrix}\,\theta\,\mathbf{b}\right]$$

$$\mathbf{V}_{\text{rel rot } x} = -\frac{l}{2}\,\Omega_x\,\frac{r}{l/2}\,\mathbf{b} = \underline{-\frac{l}{2}\,\Omega_x \ldots\ldots\,\mathbf{b}}$$

$$\Phi_{\text{rot } y,z} = \pm\frac{l}{2}\,\frac{\Omega_y}{\Omega_z}\begin{matrix}\sin \\ \cos\end{matrix}\,\theta\,\Sigma[8] = \underline{\pm\frac{l}{2}\,\frac{\Omega_y}{\Omega_z}\begin{matrix}\sin \\ \cos\end{matrix}\,\theta \ldots\ldots}$$

$$\Phi_{\text{rot } x} = 0\,.$$

Body of Revolution in Translation or Rotation.—Calculation of the Kernels for $x = l/2$

FORMULAE : Let $\lim_{x \to l/2} (-rr') = a$.

1. *Longitudinal Translation:*

$$K_{10}\left(\frac{l}{2}, \xi\right) = \frac{l/2 - \xi}{\left[\sqrt{\left\{\left(\frac{l}{2} - \xi\right)^2 + \rho^2\right\}}\right]^3}$$

with
$$\lim_{\xi \to l/2} \left[K_{10}\left(\frac{l}{2}, \xi\right) \sqrt{\left(\frac{l}{2} - \xi\right)}\right] = \frac{1}{[\sqrt{(2a)}]^3}$$

$$K_{12}\left(\frac{l}{2}, \xi\right) = \frac{1}{\sqrt{\left\{\left(\frac{l}{2} - \xi\right)^2 + \rho^2\right\}}}$$

with
$$\lim_{\xi \to l/2} \left[K_{12}\left(\frac{l}{2}, \xi\right) \sqrt{\left(\frac{l}{2} - \xi\right)}\right] = \frac{1}{\sqrt{(2a)}} .$$

2. *Lateral Translation and Rotation:*

$$K_{30}\left(\frac{l}{2}, \xi\right) = \frac{\rho^2}{2\left[\sqrt{\left\{\left(\frac{l}{2} - \xi\right)^2 + \rho^2\right\}}\right]^3} \left[\frac{3a\left(\frac{l}{2} - \xi\right)}{\left(\frac{l}{2} - \xi\right)^2 + \rho^2} - 1\right]$$

with
$$\lim_{\xi \to l/2} \left[K_{30}\left(\frac{l}{2}, \xi\right) \sqrt{\left(\frac{l}{2} - \xi\right)}\right] = \frac{1}{4\sqrt{(2a)}}$$

$$K_{31}\left(\frac{l}{2}, \xi\right) = \frac{\rho^2}{2\left[\sqrt{\left\{\left(\frac{l}{2} - \xi\right)^2 + \rho^2\right\}}\right]^3}$$

with
$$\lim_{\xi \to l/2} \left[K_{31}\left(\frac{l}{2}, \xi\right) \sqrt{\left(\frac{l}{2} - \xi\right)}\right] = \frac{1}{2\sqrt{(2a)}}$$

Body of Revolution in Translation or Rotation.—I (*a*) *to* III (*a*). *Kernels for* $x = l/2$

							pK_{13}		pK_{19}	pK_{21}			pK_{20}
[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
ξ*	$\frac{l}{2} - \xi$	ρ	ρ^2	$\left(\frac{l}{2} - \xi\right)^2$	[4] + [5]	p**	[7]/$\sqrt{[6]}$	[8]/[6]	[2] . [9]	$\frac{1}{2}$. [4] . [9]	$3a \cdot \frac{[2]}{[6]}$	[12] − 1	[11] . [13]
	15			225		12·72							
	9			81		4·02							
	6			36		2·46							
	4			16		1·85							
	2·5			6·25		1·24							
	1·5			2·25		0·658							
	1			1		0·376							
	0·7			0·49		0·320							
	0·4			0·16		0·245							
	0·2			0·04		0·134							
	0·1			0·01		0·0774							
	0·04			0·0016		0·0428							
	0·01			0·0001		0·0199							
(l/2)							$\frac{0·0704}{\sqrt{a}}$		$\frac{0·0352}{\sqrt{a^3}}$	$\frac{0·0352}{\sqrt{a}}$			$\frac{0·0176}{\sqrt{a}}$
	0	0	0	0	0	—	=	—	=	=	1·500	0·500	=

$x =$

$a = \lim_{x \to l/2} (-rr') =$

* Replace the last $\xi < -l/2$ by $-l/2$ and correct [2] and [5] correspondingly.

** Correct according to (178).

Body of Revolution in Longitudinal Translation.—g_1, $\mathbf{V}_{\text{rel long}}$ *and* Φ_{long} *for* $x = l/2$

FORMULAE :

1. *Integral equation:*

$$g_1(l/2) = \lim_{x \to l/2} (-rr') \left[1 - \int_{-l/2}^{+l/2} g_1(\xi)\, K_{10}\left(\frac{l}{2}, \xi\right) d\xi \right]$$

Solution:

$$g_{10}(l/2) = \tfrac{1}{2} \lim_{x \to l/2} (-rr')$$

$$g_{11}^*(l/2) = \lim_{x \to l/2} (-rr') \left[1 - \int_{-l/2}^{+l/2} g_{10}(\xi)\, K_{10}\left(\frac{l}{2}, \xi\right) d\xi \right], \quad g_{11} = \tfrac{1}{2}(g_{10} + g_{11}^*)$$

---

2. *Relative Velocity:*

$$\mathbf{V}_{\text{rel long}}(l/2) = 0$$

3. *Potential:*

$$\Phi_{\text{long}}(l/2) = -V_x \int_{-l/2}^{+l/2} g_1(\xi)\, K_{13}\left(\frac{l}{2}, \xi\right) d\xi\,.$$

Body of Revolution in Translation or Rotation.—IV (*a*). *Solution of the Integral Equation for Longitudinal Translation*

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
ξ	$-\frac{1}{2}\rho\rho'$	ξK_{10}	[3] . [2]	$g_{11}(\xi)$	[3] . [5]	$g_{12}(\xi)$	[3] . [7]	$g_{13}(\xi)$	[3] . [9]	$g_{14}(\xi)$	[3] . [11]
$(l/2)$	$(\frac{1}{2}a)$	$\frac{0\cdot0352}{\sqrt{\pi^2}}$ =									
		$\Sigma[4] =$		$\Sigma[6] =$		$\Sigma[8] =$		$\Sigma[10] =$		$\Sigma[12] =$	
		$1 - \Sigma[4] =$		$1 - \Sigma[6] =$		$1 - \Sigma[8] =$		$1 - \Sigma[10] =$		$1 - \Sigma[12] =$	
		$g_{11}^* = a(1 - \Sigma[4])$ =		$g_{12}^* = a(1 - \Sigma[6])$ =		$g_{13}^* = a(1 - \Sigma[8])$ =		$g_{14}^* = a(1 - \Sigma[10])$ =		$g_{15}^* = a(1 - \Sigma[12])$ =	
$g_{10} = \frac{1}{2}a$ =		$g_{11} = \frac{g_{10} + g_{11}^*}{2}$ =		$g_{12} = \frac{g_{11} + g_{12}^*}{2}$ =		$g_{13} = \frac{g_{12} + g_{13}^*}{2}$ =		$g_{14} = \frac{g_{13} + g_{14}^*}{2}$ =		$g_{15} = \frac{g_{14} + g_{15}^*}{2}$ =	

$x =$ $= l/2$

$a = \lim_{x \to l/2} (-rr') =$

Body of Revolution in Translation or Rotation.—V (*a*). *Relative Velocity and Potential for Longitudinal Translation*

[1]	[2]	[3]	[4]
ξ	$g_1(\xi)$	ϕK_{13}	[2] . [3]
$(l/2)$		$\frac{0 \cdot 0704}{\sqrt{a}}$ =	
		$\Sigma[4]$ =	

$x =$ $= l/2$

$a = \lim_{x \to l/2} (-rr') =$

$\mathbf{V}_{\text{rel long}}(l/2) = 0$

$\Phi_{\text{long}}(l/2) = -V_x \Sigma[4] = \ldots\ldots V_x$

Body of Revolution in Lateral Translation.—$\lim_{x \to l/2} g_z(x)/r(x)$, $\mathbf{V}_{\text{rel lat}}$ *and* Φ_{lat} *for* $x = l/2$

FORMULAE :

1. *Integral Equation:*

$$\lim_{x \to l/2} \frac{g_z(x)}{r(x)} = 1 - \int_{-l/2}^{+l/2} \frac{g_z(\xi)}{\rho} K_{zz}(l/2, \xi)\, d\xi$$

Solution:

$$\lim_{x \to l/2} \frac{g_{z0}(x)}{r(x)} = 1$$

$$\lim_{x \to l/2} \frac{g_{zn}(x)}{r(x)} = 1 - \int_{-l/2}^{+l/2} \frac{g_{z(n-1)}(\xi)}{\rho} K_{zz}(l/2, \xi)\, d\xi$$

2. *Relative Velocity:*

$$\mathbf{V}_{\text{rel lat}}(l/2) = \begin{matrix} V_y \\ V_z \end{matrix} \left[\begin{matrix} -\cos \\ \sin \end{matrix} \theta\, \mathbf{t} \begin{matrix} +\sin \\ -\cos \end{matrix} \theta\, \mathbf{b} \right] \left[1 + \int_{-l/2}^{+l/2} \frac{g_z(\xi)}{\rho} K_{zl}(l/2, \xi)\, d\xi \right]$$

3. *Potential:*

$$\Phi_{\text{lat}}(l/2) = 0\,.$$

Body of Revolution in Translation or Rotation.—VI (*a*). *Solution of the Integral Equation for Lateral Translation*

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
ξ	ϕK_m	$g_m(\xi)/\rho$	[2] . [3]	$g_m(\xi)/\rho$	[2] . [5]	$g_m(\xi)/\rho$	[2] . [7]	$g_m(\xi)/\rho$	[2] . [9]
$(l/2)$	$\frac{0{\cdot}0176}{\sqrt{\pi}}$ =								
Σ[2] =		Σ[4] =		Σ[6] =		Σ[8] =		Σ[10] =	
$\frac{g_m(l/2)}{r(l/2)} = 1 - \Sigma$[2] =		$\frac{g_m(l/2)}{r(l/2)} = 1 - \Sigma$[4] =		$\frac{g_m(l/2)}{r(l/2)} = 1 - \Sigma$[6] =		$\frac{g_m(l/2)}{r(l/2)} = 1 - \Sigma$[8] =		$\frac{g_m(l/2)}{r(l/2)} = 1 - \Sigma$[10] =	

$x = \qquad\qquad = l/2$

$a = \lim_{x \to l/2} (-rr') =$

Body of Revolution in Translation or Rotation.—VII (*a*). *Relative Velocity and Potential for Lateral Translation*

[1]	[2]	[3]	[4]
ξ	$g_2(\xi)/\rho$	ρK_{11}	[2] . [3]
$(l/2)$		$\frac{0 \cdot 0352}{\sqrt{a}}$ =	
		$\Sigma[4]$ =	

$x =$ $= l/2$

$a = \lim_{x \to l/2} (-rr') =$

$$V_{\text{rel lat}}(l/2) = \begin{matrix} V_y \\ V_z \end{matrix} \left[- \begin{matrix} \cos \\ \sin \end{matrix} \theta \, \mathbf{t} \begin{matrix} + \sin \\ - \cos \end{matrix} \theta \, \mathbf{b} \right] \left[1 + \Sigma[4] \right]$$

$$= \begin{matrix} V_y \\ V_z \end{matrix} \left[- \begin{matrix} \cos \\ \sin \end{matrix} \theta \, \mathbf{t} \begin{matrix} + \sin \\ - \cos \end{matrix} \theta \, \mathbf{b} \right] \cdot \ldots\ldots$$

$$\Phi_{\text{lat}}(l/2) = 0 \, .$$

Body of Revolution in Rotation.— $\lim_{x \to l/2} g_3(x)/r(x)$, $\mathbf{V}_{\text{rel rot}}$ *and* Φ_{rot} *for* $x = l/2$

FORMULAE :

1. *Integral Equation:*

$$\lim_{x \to l/2} \frac{g_3(x)}{r(x)} = 1 + \frac{\lim rr'}{l/2} - \int_{-l/2}^{+l/2} \frac{g_3(\xi)}{\rho} K_{33}(l/2, \xi)\, d\xi \,.$$

Solution:

$$\lim_{x \to l/2} \frac{g_{30}(x)}{r(x)} = 1 + \frac{\lim rr'}{l/2}$$

$$\lim_{x \to l/2} \frac{g_{3n}(x)}{r(x)} = 1 + \frac{\lim rr'}{l/2} - \int_{-l/2}^{+l/2} \frac{g_{3,n-1}(\xi)}{\rho} K_{33}(l/2, \xi)\, d\xi \,.$$

2. *Relative velocity:*

$$\mathbf{V}_{\text{rel rot}\,x}\,(l/2) = 0$$

$$\mathbf{V}_{\text{rel rot}\,y,z}\,(l/2) = \frac{l}{2} \begin{matrix} \Omega_y \\ \Omega_z \end{matrix} \left[\begin{matrix} +\sin \\ -\cos \end{matrix} \theta\, \mathbf{t} + \begin{matrix} \cos \\ \sin \end{matrix} \theta\, \mathbf{b} \right] \left[1 + \int_{-l/2}^{+l/2} \frac{g_3(\xi)}{\rho} K_{33}(l/2, \xi)\, d\xi \right].$$

3. *Potential:*

$$\Phi_{\text{rot}\,x}\,(l/2) = \Phi_{\text{rot}\,y,z}\,(l/2) = 0 \,.$$

Body of Revolution in Translation or Rotation.—VIII (*a*). *Solution of the Integral Equation for Rotation*

$$\frac{\xi + \rho\rho'}{l/2}$$

[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
ξ	ρK_{20}	$g_{20}(\xi)/\rho$	[2] · [3]	$g_{21}(\xi)/\rho$							
$(l/2)$	$\frac{0 \cdot 0176}{\sqrt{a}}$ =										
		Σ [4] =		Σ [5] =		Σ [8] =		Σ [10] =		Σ [12] =	
		$\frac{g_{21}(l/2)}{r(l/2)}$ =		$\frac{g_{20}(l/2)}{r(l/2)}$ =		$\frac{g_{20}(l/2)}{r(l/2)}$ =		$\frac{g_{20}(l/2)}{r(l/2)}$ =		$\frac{g_{21}(l/2)}{r(l/2)}$ =	

$$\frac{g_{20}(l/2)}{r(l/2)} = 1 - \frac{a}{l/2} - \Sigma\,[2n + 2]$$

$x =$ $= l/2$

$a = \lim_{x \to l/2} (-rr') =$

$1 - \frac{a}{l/2} =$

Body of Revolution in Translation or Rotation.—IX (*a*). *Relative Velocity and Potential for Rotation*

[1]	[2]	[3]	[4]
ξ	$g_a(\xi)/p$	pK_{11}	[2] . [3]
$(l/2)$		$\frac{0 \cdot 0352}{\sqrt{a}}$ =	
		$\Sigma[4]$ =	

$x =$ $= l/2$

$a = \lim_{x \to l/2} (-rr') =$

$$\mathbf{V}_{\text{rel rot } x}(l/2) = 0$$

$$\mathbf{V}_{\text{rel rot } y,z}(l/2) = \frac{l}{2}\frac{\Omega_y}{\Omega_z}\begin{bmatrix} +\sin \\ -\cos \end{bmatrix}\theta\,\mathbf{t} + \begin{matrix}\cos \\ \sin\end{matrix}\theta\,\mathbf{b}\Big](1 + \Sigma[4])$$

$$= \frac{l}{2}\frac{\Omega_y}{\Omega_z}\begin{bmatrix} +\sin \\ -\cos \end{bmatrix}\theta\,\mathbf{t} + \begin{matrix}\cos \\ \sin\end{matrix}\theta\,\mathbf{b}\Big] \cdot \ldots\ldots$$

$$\Phi_{\text{rot } x}(l/2) = \Phi_{\text{rot } y,z}(l/2) = 0\,.$$

Tables of the Functions G_s and H_s

For $k'^2 < 0 \cdot 0^4 10$, the functions G_s and H_s are

$$G_1(0 \cdot 0^{4+n}x) = 0 \cdot 637$$

$$G_2(0 \cdot 0^{4+n}x) = G_2(0 \cdot 0^4 x) + 0 \cdot 366n$$

$$G_3(0 \cdot 0^{4+n}x) = G_3(0 \cdot 0^4 x) + 0 \cdot 732n$$

$$H_1(0 \cdot 0^{4+n}x) = 0 \cdot 637$$

$$H_2(0 \cdot 0^{4+n}x) = H_2(0 \cdot 0^4 x) + 0 \cdot 366n$$

$$H_3(0 \cdot 0^{4+n}x) = H_3(0 \cdot 0^4 x) + 0 \cdot 732n$$

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
$0{\cdot}0^{4}10$	0·637	$1{\cdot}955_{15}$	$4{\cdot}547_{30}$	0·637	$1{\cdot}319_{15}$	$3{\cdot}274_{30}$	$0{\cdot}9^{4}90$
11		$1{\cdot}940_{14}$	$4{\cdot}517_{28}$		$1{\cdot}304_{14}$	$3{\cdot}244_{28}$	89
12		$1{\cdot}926_{12}$	$4{\cdot}489_{24}$		$1{\cdot}290_{12}$	$3{\cdot}216_{25}$	88
13		$1{\cdot}914_{12}$	$4{\cdot}465_{24}$		$1{\cdot}278_{12}$	$3{\cdot}191_{24}$	87
14		$1{\cdot}902_{11}$	$4{\cdot}441_{22}$		$1{\cdot}266_{11}$	$3{\cdot}167_{22}$	86
15		$1{\cdot}891_{10}$	$4{\cdot}419_{20}$		$1{\cdot}255_{10}$	$3{\cdot}145_{20}$	85
16		$1{\cdot}881_{10}$	$4{\cdot}399_{20}$		$1{\cdot}245_{10}$	$3{\cdot}125_{20}$	84
17		$1{\cdot}871_{9}$	$4{\cdot}3[illegible]_{18}$		$1{\cdot}235_{9}$	$3{\cdot}105_{18}$	83
18		$1{\cdot}862_{9}$	$4{\cdot}361_{18}$		$1{\cdot}226_{9}$	$3{\cdot}087_{17}$	82
19		$1{\cdot}853_{8}$	$4{\cdot}343_{16}$		$1{\cdot}217_{8}$	$3{\cdot}070_{17}$	81
$0{\cdot}0^{4}20$	0·637	$1{\cdot}845_{8}$	$4{\cdot}327_{16}$	0·637	$1{\cdot}209_{8}$	$3{\cdot}053_{15}$	$0{\cdot}9^{4}80$
21		$1{\cdot}837_{7}$	$4{\cdot}311_{14}$		$1{\cdot}201_{7}$	$3{\cdot}038_{15}$	79
22		$1{\cdot}830_{7}$	$4{\cdot}297_{14}$		$1{\cdot}194_{7}$	$3{\cdot}023_{14}$	78
23		$1{\cdot}823_{7}$	$4{\cdot}283_{14}$		$1{\cdot}187_{7}$	$3{\cdot}009_{14}$	77
24		$1{\cdot}816_{6}$	$4{\cdot}269_{12}$		$1{\cdot}180_{7}$	$2{\cdot}995_{13}$	76
25		$1{\cdot}810_{7}$	$4{\cdot}257_{14}$		$1{\cdot}173_{6}$	$2{\cdot}982_{12}$	75
26		$1{\cdot}803_{7}$	$4{\cdot}243_{12}$		$1{\cdot}167_{6}$	$2{\cdot}970_{12}$	74
27		$1{\cdot}797_{6}$	$4{\cdot}231_{12}$		$1{\cdot}161_{6}$	$2{\cdot}958_{12}$	73
28		$1{\cdot}791_{5}$	$4{\cdot}219_{10}$		$1{\cdot}155_{5}$	$2{\cdot}946_{11}$	72
29		$1{\cdot}786_{5}$	$4{\cdot}209_{10}$		$1{\cdot}150_{6}$	$2{\cdot}935_{11}$	71
$0{\cdot}0^{4}30$	0·637	$1{\cdot}781_{4}$	$4{\cdot}199_{12}$	0·637	$1{\cdot}144_{5}$	$2{\cdot}924_{11}$	$0{\cdot}9^{4}70$
31		$1{\cdot}775_{5}$	$4{\cdot}187_{10}$		$1{\cdot}139_{5}$	$2{\cdot}913_{10}$	69
32		$1{\cdot}770_{5}$	$4{\cdot}177_{10}$		$1{\cdot}134_{5}$	$2{\cdot}903_{10}$	68
33		$1{\cdot}765_{4}$	$4{\cdot}167_{8}$		$1{\cdot}129_{5}$	$2{\cdot}893_{9}$	67
34		$1{\cdot}761_{5}$	$4{\cdot}159_{10}$		$1{\cdot}124_{5}$	$2{\cdot}884_{9}$	66
35		$1{\cdot}756_{5}$	$4{\cdot}149_{10}$		$1{\cdot}119_{4}$	$2{\cdot}875_{9}$	65
36		$1{\cdot}751_{4}$	$4{\cdot}139_{8}$		$1{\cdot}115_{4}$	$2{\cdot}866_{9}$	64
37		$1{\cdot}747_{4}$	$4{\cdot}131_{8}$		$1{\cdot}111_{4}$	$2{\cdot}857_{8}$	63
38		$1{\cdot}743_{4}$	$4{\cdot}123_{8}$		$1{\cdot}107_{5}$	$2{\cdot}849_{8}$	62
39		$1{\cdot}739_{4}$	$4{\cdot}115_{8}$		$1{\cdot}102_{3}$	$2{\cdot}841_{8}$	61
$0{\cdot}0^{4}40$	0·637	$1{\cdot}735_{4}$	$4{\cdot}107_{8}$	0·637	$1{\cdot}099_{4}$	$2{\cdot}833_{8}$	$0{\cdot}9^{4}60$

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
$0{\cdot}0^{4}40$	0·637	1·735 $_{4}$	4·107 $_{8}$	0·637	1·099 $_{4}$	2·833 $_{8}$	$0{\cdot}9^{4}60$
41		1·731 $_{4}$	4·099 $_{8}$		1·095 $_{4}$	2·825 $_{7}$	59
42		1·727 $_{4}$	4·091 $_{8}$		1·091 $_{4}$	2·818 $_{8}$	58
43		1·723 $_{3}$	4·083 $_{6}$		1·087 $_{3}$	2·810 $_{7}$	57
44		1·720 $_{4}$	4·077 $_{8}$		1·084 $_{4}$	2·803 $_{8}$	56
45		1·716 $_{4}$	4·069 $_{8}$		1·080 $_{4}$	2·795 $_{7}$	55
46		1·712 $_{3}$	4·061 $_{6}$		1·076 $_{3}$	2·788 $_{6}$	54
47		1·709 $_{3}$	4·055 $_{6}$		1·073 $_{3}$	2·782 $_{7}$	53
48		1·706 $_{4}$	4·049 $_{8}$		1·070 $_{3}$	2·775 $_{6}$	52
49		1·702 $_{3}$	4·041 $_{6}$		1·067 $_{4}$	2·769 $_{7}$	51
$0{\cdot}0^{4}50$	0·637	1·699 $_{3}$	4·035 $_{6}$	0·637	1·063 $_{3}$	2·762 $_{6}$	$0{\cdot}9^{4}50$
51		1·696 $_{3}$	4·029 $_{6}$		1·060 $_{3}$	2·756 $_{6}$	49
52		1·693 $_{3}$	4·023 $_{6}$		1·057 $_{3}$	2·750 $_{7}$	48
53		1·690 $_{3}$	4·017 $_{6}$		1·054 $_{3}$	2·743 $_{6}$	47
54		1·687 $_{3}$	4·011 $_{6}$		1·051 $_{3}$	2·737 $_{6}$	46
55		1·684 $_{3}$	4·005 $_{6}$		1·048 $_{2}$	2·731 $_{5}$	45
56		1·681 $_{3}$	3·999 $_{6}$		1·046 $_{3}$	2·726 $_{6}$	44
57		1·678 $_{2}$	3·993 $_{4}$		1·043 $_{3}$	2·720 $_{5}$	43
58		1·676 $_{3}$	3·989 $_{6}$		1·040 $_{3}$	2·715 $_{6}$	42
59		1·673 $_{3}$	3·983 $_{6}$		1·037 $_{3}$	2·709 $_{5}$	41
$0{\cdot}0^{4}60$	0·637	1·670 $_{2}$	3·977 $_{4}$	0·637	1·034 $_{2}$	2·704 $_{5}$	$0{\cdot}9^{4}40$
61		1·668 $_{3}$	3·973 $_{6}$		1·032 $_{3}$	2·699 $_{5}$	39
62		1·665 $_{3}$	3·967 $_{6}$		1·029 $_{2}$	2·694 $_{6}$	38
63		1·662 $_{2}$	3·961 $_{4}$		1·027 $_{3}$	2·688 $_{5}$	37
64		1·660 $_{3}$	3·957 $_{6}$		1·024 $_{2}$	2·683 $_{5}$	36
65		1·657 $_{2}$	3·951 $_{4}$		1·022 $_{2}$	2·678 $_{5}$	35
66		1·655 $_{2}$	3·947 $_{4}$		1·020 $_{3}$	2·673 $_{4}$	34
67		1·653 $_{3}$	3·943 $_{6}$		1·017 $_{2}$	2·669 $_{5}$	33
68		1·650 $_{2}$	3·937 $_{4}$		1·015 $_{3}$	2·664 $_{4}$	32
69		1·648 $_{2}$	3·933 $_{4}$		1·012 $_{2}$	2·660 $_{5}$	31
$0{\cdot}0^{4}70$	0·637	1·646 $_{3}$	3·929 $_{6}$	0·637	1·010 $_{2}$	2·655 $_{4}$	$0{\cdot}9^{4}30$

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
$0{\cdot}0^{4}70$	0·637	$1{\cdot}646_{3}$	$3{\cdot}929_{6}$	0·637	$1{\cdot}010_{2}$	$2{\cdot}655_{4}$	$0{\cdot}9^{4}30$
71		$1{\cdot}643_{2}$	$3{\cdot}923_{4}$		$1{\cdot}008_{2}$	$2{\cdot}651_{4}$	29
72		$1{\cdot}641_{2}$	$3{\cdot}919_{4}$		$1{\cdot}006_{2}$	$2{\cdot}647_{5}$	28
73		$1{\cdot}639_{2}$	$3{\cdot}915_{4}$		$1{\cdot}004_{2}$	$2{\cdot}642_{4}$	27
74		$1{\cdot}637_{2}$	$3{\cdot}911_{4}$		$1{\cdot}002_{3}$	$2{\cdot}638_{4}$	26
75		$1{\cdot}635_{2}$	$3{\cdot}907_{4}$		$0{\cdot}999_{2}$	$2{\cdot}634_{4}$	25
76		$1{\cdot}633_{3}$	$3{\cdot}903_{6}$		$0{\cdot}997_{2}$	$2{\cdot}630_{4}$	24
77		$1{\cdot}630_{2}$	$3{\cdot}897_{4}$		$0{\cdot}995_{2}$	$2{\cdot}626_{5}$	23
78		$1{\cdot}628_{2}$	$3{\cdot}893_{4}$		$0{\cdot}993_{2}$	$2{\cdot}621_{4}$	22
79		$1{\cdot}626_{2}$	$3{\cdot}889_{4}$		$0{\cdot}991_{2}$	$2{\cdot}617_{4}$	21
$0{\cdot}0^{4}80$	0·637	$1{\cdot}624_{2}$	$3{\cdot}885_{4}$	0·637	$0{\cdot}989_{2}$	$2{\cdot}613_{4}$	$0{\cdot}9^{4}20$
81		$1{\cdot}622_{2}$	$3{\cdot}881_{4}$		$0{\cdot}987_{2}$	$2{\cdot}609_{4}$	19
82		$1{\cdot}620_{1}$	$3{\cdot}877_{2}$		$0{\cdot}985_{2}$	$2{\cdot}605_{3}$	18
83		$1{\cdot}619_{2}$	$3{\cdot}875_{4}$		$0{\cdot}983_{2}$	$2{\cdot}602_{4}$	17
84		$1{\cdot}617_{2}$	$3{\cdot}871_{4}$		$0{\cdot}981_{2}$	$2{\cdot}598_{4}$	16
85		$1{\cdot}615_{2}$	$3{\cdot}867_{4}$		$0{\cdot}979_{2}$	$2{\cdot}594_{4}$	15
86		$1{\cdot}613_{2}$	$3{\cdot}863_{4}$		$0{\cdot}977_{2}$	$2{\cdot}590_{4}$	14
87		$1{\cdot}611_{2}$	$3{\cdot}859_{4}$		$0{\cdot}975_{2}$	$2{\cdot}586_{3}$	13
88		$1{\cdot}609_{2}$	$3{\cdot}855_{4}$		$0{\cdot}973_{1}$	$2{\cdot}583_{4}$	12
89		$1{\cdot}607_{1}$	$3{\cdot}851_{2}$		$0{\cdot}972_{2}$	$2{\cdot}579_{4}$	11
$0{\cdot}0^{4}90$	0·637	$1{\cdot}606_{2}$	$3{\cdot}849_{4}$	0·637	$0{\cdot}970_{2}$	$2{\cdot}575_{3}$	$0{\cdot}9^{4}10$
91		$1{\cdot}604_{2}$	$3{\cdot}845_{4}$		$0{\cdot}968_{1}$	$2{\cdot}572_{4}$	09
92		$1{\cdot}602_{2}$	$3{\cdot}841_{4}$		$0{\cdot}967_{2}$	$2{\cdot}568_{3}$	08
93		$1{\cdot}600_{1}$	$3{\cdot}837_{2}$		$0{\cdot}965_{2}$	$2{\cdot}565_{4}$	07
94		$1{\cdot}599_{2}$	$3{\cdot}835_{4}$		$0{\cdot}963_{1}$	$2{\cdot}561_{3}$	06
95		$1{\cdot}597_{2}$	$3{\cdot}831_{4}$		$0{\cdot}962_{2}$	$2{\cdot}558_{3}$	05
96		$1{\cdot}595_{1}$	$3{\cdot}827_{2}$		$0{\cdot}960_{2}$	$2{\cdot}555_{4}$	04
97		$1{\cdot}594_{2}$	$3{\cdot}825_{4}$		$0{\cdot}958_{2}$	$2{\cdot}551_{3}$	03
98		$1{\cdot}592_{2}$	$3{\cdot}821_{4}$		$0{\cdot}956_{1}$	$2{\cdot}548_{4}$	02
99		$1{\cdot}590_{1}$	$3{\cdot}817_{2}$		$0{\cdot}955_{2}$	$2{\cdot}544_{3}$	01
$0{\cdot}0^{4}100$	0·637	$1{\cdot}589_{15}$	$3{\cdot}815_{30}$	0·637	0·953	$2{\cdot}541_{30}$	$0{\cdot}9^{4}00$

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
$0{\cdot}0^{2}10$	0·637	$1{\cdot}589_{15}$	$3{\cdot}815_{30}$	0·637	0·953	$2{\cdot}541_{30}$	$0{\cdot}9^{2}90$
11		$1{\cdot}574_{14}$	$3{\cdot}785_{28}$	0·636	0·938	$2{\cdot}511_{28}$	89
12		$1{\cdot}560_{13}$	$3{\cdot}757_{26}$	0·636	0·924	$2{\cdot}483_{25}$	88
13		$1{\cdot}547_{12}$	$3{\cdot}731_{24}$	0·636	0·911	$2{\cdot}458_{24}$	87
14		$1{\cdot}535_{11}$	$3{\cdot}707_{22}$	0·636	0·899	$2{\cdot}434_{22}$	86
15		$1{\cdot}524_{10}$	$3{\cdot}685_{20}$	0·636	0·888	$2{\cdot}412_{20}$	85
16		$1{\cdot}514_{10}$	$3{\cdot}665_{20}$	0·636	0·878	$2{\cdot}392_{19}$	84
17		$1{\cdot}504_{9}$	$3{\cdot}645_{18}$	0·636	0·869	$2{\cdot}373_{18}$	83
18		$1{\cdot}495_{8}$	$3{\cdot}627_{16}$	0·636	0·860	$2{\cdot}355_{17}$	82
19		$1{\cdot}487_{8}$	$3{\cdot}611_{16}$	0·636	0·851	$2{\cdot}338_{17}$	81
$0{\cdot}0^{2}20$	0·637	$1{\cdot}479_{8}$	$3{\cdot}595_{16}$	0·636	0·843	$2{\cdot}321_{15}$	$0{\cdot}9^{2}80$
21		$1{\cdot}471_{8}$	$3{\cdot}579_{16}$	0·636	0·835	$2{\cdot}306_{15}$	79
22		$1{\cdot}463_{7}$	$3{\cdot}563_{14}$	0·636	0·828	$2{\cdot}291_{14}$	78
23		$1{\cdot}456_{6}$	$3{\cdot}549_{12}$	0·636	0·821	$2{\cdot}277_{14}$	77
24		$1{\cdot}450_{7}$	$3{\cdot}537_{14}$	0·636	0·814	$2{\cdot}263_{13}$	76
25		$1{\cdot}443_{6}$	$3{\cdot}523_{12}$	0·636	0·807	$2{\cdot}250_{12}$	75
26		$1{\cdot}437_{6}$	$3{\cdot}511_{12}$	0·636	0·801	$2{\cdot}238_{12}$	74
27		$1{\cdot}431_{6}$	$3{\cdot}499_{12}$	0·636	0·795	$2{\cdot}226_{12}$	73
28		$1{\cdot}425_{6}$	$3{\cdot}487_{12}$	0·636	0·789	$2{\cdot}214_{11}$	72
29		$1{\cdot}419_{5}$	$3{\cdot}475_{10}$	0·636	0·784	$2{\cdot}203_{11}$	71
$0{\cdot}0^{2}30$	0·637	$1{\cdot}414_{5}$	$3{\cdot}465_{10}$	0·636	0·778	$2{\cdot}192_{11}$	$0{\cdot}9^{2}70$
31		$1{\cdot}409_{5}$	$3{\cdot}455_{10}$	0·636	0·773	$2{\cdot}181_{10}$	69
32		$1{\cdot}404_{5}$	$3{\cdot}445_{10}$	0·636	0·768	$2{\cdot}171_{10}$	68
33		$1{\cdot}399_{5}$	$3{\cdot}435_{10}$	0·636	0·763	$2{\cdot}161_{9}$	67
34		$1{\cdot}394_{5}$	$3{\cdot}425_{10}$	0·635	0·759	$2{\cdot}152_{9}$	66
35		$1{\cdot}389_{4}$	$3{\cdot}415_{8}$	0·635	0·754	$2{\cdot}143_{9}$	65
36		$1{\cdot}385_{4}$	$3{\cdot}407_{8}$	0·635	0·750	$2{\cdot}134_{9}$	64
37		$1{\cdot}381_{5}$	$3{\cdot}399_{10}$	0·635	0·745	$2{\cdot}125_{8}$	63
38		$1{\cdot}376_{4}$	$3{\cdot}389_{8}$	0·635	0·741	$2{\cdot}117_{8}$	62
39		$1{\cdot}372_{4}$	$3{\cdot}381_{8}$	0·635	0·737	$2{\cdot}109_{8}$	61
$0{\cdot}0^{2}40$	0·637	$1{\cdot}368_{4}$	$3{\cdot}373_{7}$	0·635	0·733	$2{\cdot}101_{8}$	$0{\cdot}9^{2}60$

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
$0{\cdot}0^{3}40$	0·637	1·368 $_{4}$	3·373 $_{7}$	0·635	0·733	2·101 $_{8}$	$0{\cdot}9^{3}60$
41		1·364 $_{4}$	3·366 $_{7}$	0·635	0·729	2·093 $_{7}$	59
42		1·360 $_{3}$	3·359 $_{7}$	0·635	0·726	2·086 $_{7}$	58
43		1·357 $_{4}$	3·352 $_{7}$	0·635	0·722	2·079 $_{8}$	57
44		1·353 $_{3}$	3·345 $_{7}$	0·635	0·718	2·071 $_{8}$	56
45		1·350 $_{4}$	3·338 $_{7}$	0·635	0·714	2·063 $_{7}$	55
46		1·346 $_{3}$	3·331 $_{7}$	0·635	0·711	2·056 $_{6}$	54
47		1·343 $_{4}$	3·324 $_{7}$	0·635	0·708	2·050 $_{7}$	53
48		1·339 $_{3}$	3·317 $_{7}$	0·635	0·704	2·043 $_{6}$	52
49		1·336 $_{3}$	3·310 $_{7}$	0·635	0·701	2·037 $_{6}$	51
$0{\cdot}0^{3}50$	0·637	1·333 $_{3}$	3·303 $_{6}$	0·635	0·698	2·031 $_{6}$	$0{\cdot}9^{3}50$
51	0·638	1·330 $_{4}$	3·297 $_{5}$	0·635	0·695	2·025 $_{6}$	49
52		1·326 $_{3}$	3·292 $_{6}$	0·635	0·692	2·019 $_{7}$	48
53		1·323 $_{3}$	3·286 $_{6}$	0·635	0·689	2·012 $_{6}$	47
54		1·320 $_{2}$	3·280 $_{6}$	0·635	0·686	2·006 $_{6}$	46
55		1·318 $_{3}$	3·274 $_{5}$	0·635	0·683	2·000 $_{5}$	45
56		1·315 $_{3}$	3·269 $_{6}$	0·635	0·681	1·995 $_{6}$	44
57		1·312 $_{3}$	3·263 $_{6}$	0·635	0·678	1·989 $_{5}$	43
58		1·309 $_{3}$	3·257 $_{5}$	0·634	0·675	1·984 $_{6}$	42
59		1·306 $_{2}$	3·252 $_{6}$	0·634	0·673	1·978 $_{5}$	41
$0{\cdot}0^{3}60$	0·638	1·304 $_{3}$	3·246 $_{5}$	0·634	0·670	1·973 $_{5}$	$0{\cdot}9^{3}40$
61		1·301 $_{3}$	3·241 $_{5}$	0·634	0·668	1·968 $_{5}$	39
62		1·298 $_{2}$	3·236 $_{5}$	0·634	0·665	1·963 $_{6}$	38
63		1·296 $_{3}$	3·231 $_{5}$	0·634	0·663	1·957 $_{5}$	37
64		1·293 $_{2}$	3·226 $_{5}$	0·634	0·660	1·952 $_{5}$	36
65		1·291 $_{2}$	3·221 $_{5}$	0·634	0·658	1·947 $_{5}$	35
66		1·289 $_{3}$	3·216 $_{5}$	0·634	0·655	1·942 $_{4}$	34
67		1·286 $_{2}$	3·211 $_{5}$	0·634	0·653	1·938 $_{5}$	33
68		1·284 $_{3}$	3·206 $_{5}$	0·634	0·650	1·933 $_{4}$	32
69		1·281 $_{2}$	3·201 $_{5}$	0·634	0·648	1·929 $_{5}$	31
$0{\cdot}0^{3}70$	0·638	1·279 $_{2}$	3·196 $_{4}$	0·634	0·645	1·924 $_{4}$	$0{\cdot}9^{3}30$

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
$0{\cdot}0^{3}70$	0·638	$1{\cdot}279_{2}$	$3{\cdot}196_{4}$	0·634	0·645	$1{\cdot}924_{4}$	$0{\cdot}9^{3}30$
71		$1{\cdot}277_{2}$	$3{\cdot}192_{4}$	0·634	0·643	$1{\cdot}920_{4}$	29
72		$1{\cdot}275_{2}$	$3{\cdot}188_{5}$	0·634	0·641	$1{\cdot}916_{5}$	28
73		$1{\cdot}273_{3}$	$3{\cdot}183_{4}$	0·634	0·639	$1{\cdot}911_{4}$	27
74		$1{\cdot}270_{2}$	$3{\cdot}179_{4}$	0·634	0·637	$1{\cdot}907_{4}$	26
75		$1{\cdot}268_{2}$	$3{\cdot}175_{4}$	0·634	0·634	$1{\cdot}903_{4}$	25
76		$1{\cdot}266_{2}$	$3{\cdot}171_{4}$	0·634	0·632	$1{\cdot}899_{4}$	24
77		$1{\cdot}264_{2}$	$3{\cdot}167_{5}$	0·634	0·630	$1{\cdot}895_{5}$	23
78		$1{\cdot}262_{2}$	$3{\cdot}162_{4}$	0·634	0·628	$1{\cdot}890_{4}$	22
79		$1{\cdot}260_{2}$	$3{\cdot}158_{4}$	0·634	0·626	$1{\cdot}886_{4}$	21
$0{\cdot}0^{3}80$	0·638	$1{\cdot}258_{2}$	$3{\cdot}154_{4}$	0·634	0·624	$1{\cdot}882_{4}$	$0{\cdot}9^{3}20$
81		$1{\cdot}256_{2}$	$3{\cdot}150_{4}$	0·634	0·622	$1{\cdot}878_{4}$	19
82		$1{\cdot}254_{2}$	$3{\cdot}146_{3}$	0·634	0·620	$1{\cdot}874_{3}$	18
83		$1{\cdot}252_{2}$	$3{\cdot}143_{4}$	0·634	0·618	$1{\cdot}871_{4}$	17
84		$1{\cdot}250_{2}$	$3{\cdot}139_{4}$	0·634	0·617	$1{\cdot}867_{4}$	16
85		$1{\cdot}248_{2}$	$3{\cdot}135_{4}$	0·634	0·615	$1{\cdot}863_{4}$	15
86		$1{\cdot}246_{1}$	$3{\cdot}131_{4}$	0·634	0·613	$1{\cdot}859_{4}$	14
87		$1{\cdot}245_{2}$	$3{\cdot}127_{3}$	0·634	0·612	$1{\cdot}855_{3}$	13
88		$1{\cdot}243_{2}$	$3{\cdot}124_{4}$	0·634	0·610	$1{\cdot}852_{4}$	12
89		$1{\cdot}241_{2}$	$3{\cdot}120_{4}$	0·634	0·608	$1{\cdot}848_{3}$	11
$0{\cdot}0^{3}90$	0·638	$1{\cdot}239_{2}$	$3{\cdot}116_{3}$	0·633	0·606	$1{\cdot}845_{3}$	$0{\cdot}9^{3}10$
91		$1{\cdot}237_{1}$	$3{\cdot}113_{4}$	0·633	0·604	$1{\cdot}842_{4}$	09
92		$1{\cdot}236_{2}$	$3{\cdot}109_{3}$	0·633	0·602	$1{\cdot}838_{3}$	08
93		$1{\cdot}234_{2}$	$3{\cdot}106_{4}$	0·633	0·601	$1{\cdot}835_{4}$	07
94		$1{\cdot}232_{1}$	$3{\cdot}102_{3}$	0·633	0·599	$1{\cdot}831_{3}$	06
95		$1{\cdot}231_{2}$	$3{\cdot}099_{3}$	0·633	0·597	$1{\cdot}828_{3}$	05
96		$1{\cdot}229_{2}$	$3{\cdot}096_{4}$	0·633	0·595	$1{\cdot}825_{4}$	04
97		$1{\cdot}227_{1}$	$3{\cdot}092_{3}$	0·633	0·594	$1{\cdot}821_{3}$	03
98		$1{\cdot}226_{2}$	$3{\cdot}089_{4}$	0·633	0·592	$1{\cdot}818_{4}$	02
99		$1{\cdot}224_{2}$	$3{\cdot}085_{3}$	0·633	0·590	$1{\cdot}814_{3}$	01
$0{\cdot}0^{3}100$	0·638	$1{\cdot}222_{15}$	$3{\cdot}082_{30}$	0·633	0·589	$1{\cdot}811_{30}$	$0{\cdot}9^{3}00$

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
$0{\cdot}0^{2}10$	0·638	$1{\cdot}222_{15}$	$3{\cdot}082_{30}$	0·633	0·589	$1{\cdot}811_{30}$	$0{\cdot}9^{2}90$
11	0·638	$1{\cdot}207_{14}$	$3{\cdot}052_{28}$	0·633	0·574	$1{\cdot}781_{28}$	89
12	0·638	$1{\cdot}193_{13}$	$3{\cdot}024_{25}$	0·633	0·560	$1{\cdot}753_{25}$	88
13	0·638	$1{\cdot}180_{12}$	$2{\cdot}999_{24}$	0·632	0·548	$1{\cdot}728_{23}$	87
14	0·639	$1{\cdot}168_{11}$	$2{\cdot}975_{22}$	0·632	0·537	$1{\cdot}705_{22}$	86
15	0·639	$1{\cdot}157_{10}$	$2{\cdot}953_{20}$	0·632	0·526	$1{\cdot}683_{20}$	85
16	0·639	$1{\cdot}147_{10}$	$2{\cdot}933_{20}$	0·631	0·516	$1{\cdot}663_{20}$	84
17	0·639	$1{\cdot}137_{9}$	$2{\cdot}913_{18}$	0·631	0·507	$1{\cdot}643_{17}$	83
18	0·639	$1{\cdot}128_{8}$	$2{\cdot}895_{17}$	0·631	0·498	$1{\cdot}626_{17}$	82
19	0·639	$1{\cdot}120_{9}$	$2{\cdot}873_{16}$	0·630	0·490	$1{\cdot}609_{16}$	81
$0{\cdot}0^{2}20$	0·639	$1{\cdot}111_{7}$	$2{\cdot}862_{16}$	0·630	0·482	$1{\cdot}593_{16}$	$0{\cdot}9^{2}80$
21	0·639	$1{\cdot}104_{8}$	$2{\cdot}846_{14}$	0·630	0·474	$1{\cdot}577_{15}$	79
22	0·639	$1{\cdot}096_{7}$	$2{\cdot}832_{14}$	0·629	0·467	$1{\cdot}562_{13}$	78
23	0·640	$1{\cdot}089_{7}$	$2{\cdot}818_{14}$	0·629	0·460	$1{\cdot}549_{14}$	77
24	0·640	$1{\cdot}082_{6}$	$2{\cdot}804_{13}$	0·629	0·453	$1{\cdot}535_{13}$	76
25	0·640	$1{\cdot}076_{7}$	$2{\cdot}791_{12}$	0·629	0·447	$1{\cdot}522_{12}$	75
26	0·640	$1{\cdot}069_{6}$	$2{\cdot}779_{12}$	0·628	0·441	$1{\cdot}510_{11}$	74
27	0·640	$1{\cdot}063_{5}$	$2{\cdot}767_{12}$	0·628	0·436	$1{\cdot}499_{12}$	73
28	0·640	$1{\cdot}058_{6}$	$2{\cdot}755_{11}$	0·628	0·430	$1{\cdot}487_{11}$	72
29	0·640	$1{\cdot}052_{5}$	$2{\cdot}744_{11}$	0·628	0·424	$1{\cdot}476_{11}$	71
$0{\cdot}0^{2}30$	0·640	$1{\cdot}047_{6}$	$2{\cdot}733_{10}$	0·627	0·419	$1{\cdot}465_{10}$	$0{\cdot}9^{2}70$
31	0·640	$1{\cdot}041_{5}$	$2{\cdot}723_{10}$	0·627	0·414	$1{\cdot}455_{10}$	69
32	0·641	$1{\cdot}036_{5}$	$2{\cdot}713_{10}$	0·627	0·409	$1{\cdot}445_{9}$	68
33	0·641	$1{\cdot}031_{4}$	$2{\cdot}703_{9}$	0·627	0·405	$1{\cdot}436_{9}$	67
34	0·641	$1{\cdot}027_{5}$	$2{\cdot}694_{9}$	0·626	0·401	$1{\cdot}427_{10}$	66
35	0·641	$1{\cdot}022_{5}$	$2{\cdot}685_{10}$	0·626	0·396	$1{\cdot}417_{9}$	65
36	0·641	$1{\cdot}017_{4}$	$2{\cdot}675_{8}$	0·626	0·391	$1{\cdot}408_{8}$	64
37	0·641	$1{\cdot}013_{4}$	$2{\cdot}667_{9}$	0·626	0·387	$1{\cdot}400_{8}$	63
38	0·641	$1{\cdot}009_{4}$	$2{\cdot}658_{8}$	0·625	0·383	$1{\cdot}392_{8}$	62
39	0·641	$1{\cdot}005_{5}$	$2{\cdot}650_{8}$	0·625	0·379	$1{\cdot}384_{8}$	61
$0{\cdot}0^{2}40$	0·641	$1{\cdot}000_{4}$	$2{\cdot}642_{8}$	0·625	0·376	$1{\cdot}376_{8}$	$0{\cdot}9^{2}60$

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
$0{\cdot}0^{2}40$	0·641	1·000 $_{4}$	2·642 $_{8}$	0·625	0·376	1·376 $_{8}$	$0{\cdot}9^{2}60$
41	0·641	0·996 $_{3}$	2·634 $_{7}$	0·625	0·372	1·368 $_{8}$	59
42	0·642	0·993 $_{4}$	2·627 $_{8}$	0·624	0·368	1·360 $_{6}$	58
43	0·642	0·989 $_{4}$	2·619 $_{7}$	0·624	0·365	1·354 $_{7}$	57
44	0·642	0·985 $_{3}$	2·612 $_{7}$	0·624	0·362	1·347 $_{8}$	56
45	0·642	0·982 $_{4}$	2·605 $_{7}$	0·624	0·358	1·339 $_{7}$	55
46	0·642	0·978 $_{3}$	2·598 $_{7}$	0·623	0·355	1·332 $_{6}$	54
47	0·642	0·975 $_{4}$	2·591 $_{7}$	0·623	0·352	1·326 $_{7}$	53
48	0·642	0·971 $_{3}$	2·584 $_{6}$	0·623	0·348	1·319 $_{6}$	52
49	0·642	0·968 $_{3}$	2·578 $_{7}$	0·623	0·345	1·313 $_{6}$	51
$0{\cdot}0^{2}50$	0·642	0·965 $_{4}$	2·571 $_{6}$	0·622	0·343	1·307 $_{6}$	$0{\cdot}9^{2}50$
51	0·642	0·961 $_{3}$	2·565 $_{6}$	0·622	0·340	1·301 $_{7}$	49
52	0·643	0·958 $_{3}$	2·559 $_{6}$	0·622	0·337	1·294 $_{6}$	48
53	0·643	0·955 $_{3}$	2·553 $_{6}$	0·622	0·334	1·288 $_{6}$	47
54	0·643	0·952 $_{3}$	2·547 $_{6}$	0·621	0·331	1·282 $_{5}$	46
55	0·643	0·949 $_{3}$	2·541 $_{5}$	0·621	0·328	1·277 $_{5}$	45
56	0·643	0·946 $_{2}$	2·536 $_{6}$	0·621	0·325	1·272 $_{6}$	44
57	0·643	0·944 $_{3}$	2·530 $_{5}$	0·621	0·323	1·266 $_{5}$	43
58	0·643	0·941 $_{3}$	2·525 $_{6}$	0·620	0·320	1·261 $_{6}$	42
59	0·643	0·938 $_{3}$	2·519 $_{5}$	0·620	0·318	1·255 $_{5}$	41
$0{\cdot}0^{2}60$	0·643	0·935 $_{2}$	2·514 $_{5}$	0·620	0·315	1·250 $_{5}$	$0{\cdot}9^{2}40$
61	0·643	0·933 $_{3}$	2·509 $_{6}$	0·620	0·313	1·245 $_{6}$	39
62	0·643	0·930 $_{3}$	2·503 $_{5}$	0·620	0·310	1·239 $_{4}$	38
63	0·644	0·927 $_{2}$	2·498 $_{5}$	0·619	0·308	1·235 $_{5}$	37
64	0·644	0·925 $_{3}$	2·493 $_{4}$	0·619	0·305	1·230 $_{5}$	36
65	0·644	0·922 $_{2}$	2·489 $_{5}$	0·619	0·303	1·225 $_{5}$	35
66	0·644	0·920 $_{2}$	2·484 $_{5}$	0·619	0·301	1·220 $_{4}$	34
67	0·644	0·918 $_{3}$	2·479 $_{4}$	0·618	0·299	1·216 $_{5}$	33
68	0·644	0·915 $_{2}$	2·475 $_{5}$	0·618	0·297	1·211 $_{4}$	32
69	0·644	0·913 $_{2}$	2·470 $_{5}$	0·618	0·295	1·207 $_{4}$	31
$0{\cdot}0^{2}70$	0·644	0·911 $_{3}$	2·465 $_{4}$	0·618	0·293	1·203 $_{5}$	$0{\cdot}9^{2}30$

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
0·0^{2}70	0·644	0·911 $_{3}$	2·465 $_{4}$	0·618	0·293	1·203 $_{5}$	0·9^{2}30
71	0·644	0·908 $_{2}$	2·461 $_{4}$	0·618	0·291	1·198 $_{4}$	29
72	0·644	0·906 $_{2}$	2·457 $_{5}$	0·617	0·289	1·194 $_{5}$	28
73	0·644	0·904 $_{2}$	2·452 $_{4}$	0·617	0·287	1·189 $_{4}$	27
74	0·645	0·902 $_{2}$	2·448 $_{4}$	0·617	0·285	1·185 $_{4}$	26
75	0·645	0·900 $_{3}$	2·444 $_{5}$	0·617	0·283	1·181 $_{3}$	25
76	0·645	0·897 $_{2}$	2·439 $_{4}$	0·616	0·281	1·178 $_{5}$	24
77	0·645	0·895 $_{2}$	2·435 $_{4}$	0·616	0·279	1·173 $_{4}$	23
78	0·645	0·893 $_{2}$	2·431 $_{4}$	0·616	0·277	1·169 $_{4}$	22
79	0·645	0·891 $_{2}$	2·427 $_{4}$	0·616	0·275	1·165 $_{4}$	21
0·0^{2}80	0·645	0·889 $_{2}$	2·423 $_{4}$	0·616	0·273	1·161 $_{4}$	0·9^{2}20
81	0·645	0·887 $_{2}$	2·419 $_{4}$	0·615	0·271	1·157 $_{4}$	19
82	0·645	0·885 $_{2}$	2·415 $_{3}$	0·615	0·269	1·153 $_{2}$	18
83	0·645	0·883 $_{2}$	2·412 $_{4}$	0·615	0·268	1·151 $_{4}$	17
84	0·645	0·881 $_{2}$	2·408 $_{4}$	0·615	0·266	1·147 $_{4}$	16
85	0·646	0·879 $_{2}$	2·404 $_{4}$	0·615	0·264	1·143 $_{4}$	15
86	0·646	0·877 $_{1}$	2·400 $_{3}$	0·614	0·262	1·139 $_{3}$	14
87	0·646	0·876 $_{2}$	2·397 $_{4}$	0·614	0·261	1·136 $_{4}$	13
88	0·646	0·874 $_{2}$	2·393 $_{4}$	0·614	0·259	1·132 $_{4}$	12
89	0·646	0·872 $_{2}$	2·389 $_{3}$	0·614	0·258	1·128 $_{2}$	11
0·0^{2}90	0·646	0·870 $_{2}$	2·386 $_{4}$	0·614	0·256	1·126 $_{4}$	0·9^{2}10
91	0·646	0·868 $_{1}$	2·382 $_{3}$	0·613	0·254	1·122 $_{3}$	09
92	0·646	0·867 $_{2}$	2·379 $_{3}$	0·613	0·253	1·119 $_{4}$	08
93	0·646	0·865 $_{2}$	2·376 $_{4}$	0·613	0·251	1·115 $_{3}$	07
94	0·646	0·863 $_{2}$	2·372 $_{3}$	0·613	0·250	1·112 $_{3}$	06
95	0·646	0·861 $_{1}$	2·369 $_{4}$	0·613	0·248	1·109 $_{4}$	05
96	0·646	0·860 $_{2}$	2·365 $_{3}$	0·612	0·247	1·105 $_{2}$	04
97	0·647	0·858 $_{2}$	2·362 $_{3}$	0·612	0·245	1·103 $_{3}$	03
98	0·647	0·856 $_{1}$	2·359 $_{3}$	0·612	0·244	1·100 $_{4}$	02
99	0·647	0·855 $_{2}$	2·356 $_{3}$	0·612	0·242	1·096 $_{3}$	01
0·0100	0·647	0·853 $_{15}$	2·353 $_{30}$	0·612	0·241	1·093 $_{28}$	0·9^{2}00

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
0·010	0·647	$0·853_{15}$	$2·353_{30}$	0·612	0·241	$1·093_{28}$	0·990
11	0·648	$0·838_{14}$	$2·323_{28}$	0·610	0·228	$1·065_{26}$	89
12	0·648	$0·824_{13}$	$2·295_{25}$	0·608	0·216	$1·039_{24}$	88
13	0·649	$0·811_{13}$	$2·270_{23}$	0·606	0·205	$1·015_{23}$	87
14	0·650	$0·798_{12}$	$2·247_{22}$	0·604	0·194	$0·992_{21}$	86
15	0·651	$0·787_{10}$	$2·225_{20}$	0·602	0·185	$0·971_{19}$	85
16	0·652	$0·777_{10}$	$2·205_{19}$	0·600	0·176	$0·952_{18}$	84
17	0·653	$0·767_{9}$	$2·186_{18}$	0·599	0·168	$0·934_{17}$	83
18	0·653	$0·758_{9}$	$2·168_{17}$	0·597	0·160	$0·917_{16}$	82
19	0·654	$0·749_{9}$	$2·151_{16}$	0·596	0·153	$0·901_{15}$	81
0·020	0·655	$0·740_{8}$	$2·135_{15}$	0·594	0·146	$0·886_{14}$	0·980
21	0·656	$0·732_{7}$	$2·120_{14}$	0·592	0·140	$0·872_{14}$	79
22	0·656	$0·725_{8}$	$2·106_{14}$	0·591	0·134	$0·858_{13}$	78
23	0·657	$0·717_{7}$	$2·092_{14}$	0·589	0·128	$0·845_{12}$	77
24	0·658	$0·710_{6}$	$2·078_{12}$	0·588	0·123	$0·833_{12}$	76
25	0·659	$0·704_{7}$	$2·066_{12}$	0·586	0·118	$0·821_{11}$	75
26	0·659	$0·697_{6}$	$2·054_{12}$	0·585	0·113	$0·810_{11}$	74
27	0·660	$0·691_{6}$	$2·042_{12}$	0·583	0·108	$0·799_{10}$	73
28	0·661	$0·685_{6}$	$2·030_{10}$	0·582	0·1038	$0·789_{11}$	72
29	0·661	$0·679_{5}$	$2·020_{11}$	0·580	0·0994	$0·778_{10}$	71
0·030	0·662	$0·674_{6}$	$2·009_{10}$	0·579	0·0954	$0·768_{9}$	0·970
31	0·663	$0·668_{5}$	$1·999_{10}$	0·578	0·0916	$0·759_{9}$	69
32	0·663	$0·663_{5}$	$1·989_{9}$	0·576	0·0878	$0·750_{9}$	68
33	0·664	$0·658_{5}$	$1·980_{10}$	0·574	0·0842	$0·741_{8}$	67
34	0·665	$0·653_{5}$	$1·970_{9}$	0·572	0·0806	$0·733_{8}$	66
35	0·665	$0·648_{5}$	$1·961_{8}$	0·571	0·0771	$0·725_{8}$	65
36	0·666	$0·643_{4}$	$1·953_{9}$	0·570	0·0737	$0·717_{8}$	64
37	0·667	$0·639_{5}$	$1·944_{8}$	0·569	0·0705	$0·709_{8}$	63
38	0·667	$0·634_{4}$	$1·936_{8}$	0·568	0·0673	$0·701_{7}$	62
39	0·668	$0·630_{4}$	$1·928_{8}$	0·566	0·0643	$0·694_{7}$	61
0·040	0·669	$0·626_{4}$	$1·920_{7}$	0·565	0·0614	$0·687_{7}$	0·980

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
0·040	0·669	0·626 $_4$	1·920 $_7$	0·565	0·0614	0·687 $_7$	0·960
41	0·669	0·622 $_4$	1·913 $_8$	0·564	0·0586	0·680 $_7$	59
42	0·670	0·618 $_4$	1·905 $_8$	0·562	0·0560	0·673 $_6$	58
43	0·671	0·614 $_4$	1·897 $_6$	0·561	0·0534	0·667 $_7$	57
44	0·671	0·610 $_4$	1·891 $_7$	0·559	0·0508	0·660 $_6$	56
45	0·672	0·606 $_4$	1·884 $_7$	0·558	0·0483	0·654 $_6$	55
46	0·673	0·602 $_3$	1·877 $_7$	0·557	0·0458	0·648 $_6$	54
47	0·673	0·599 $_4$	1·870 $_6$	0·555	0·0434	0·642 $_6$	53
48	0·674	0·595 $_3$	1·864 $_6$	0·554	0·0412	0·636 $_6$	52
49	0·675	0·592 $_4$	1·858 $_6$	0·552	0·0390	0·630 $_5$	51
0·050	0·675	0·588 $_3$	1·852 $_6$	0·551	0·0369	0·625 $_6$	0·950
51	0·676	0·585 $_3$	1·846 $_6$	0·550	0·0349	0·619 $_5$	49
52	0·676	0·582 $_4$	1·840 $_6$	0·549	0·0330	0·614 $_5$	48
53	0·677	0·578 $_3$	1·834 $_6$	0·547	0·0311	0·609 $_5$	47
54	0·678	0·575 $_3$	1·828 $_6$	0·546	0·0292	0·604 $_5$	46
55	0·678	0·572 $_3$	1·822 $_5$	0·545	0·0274	0·599 $_5$	45
56	0·679	0·569 $_3$	1·817 $_5$	0·544	0·0255	0·594 $_5$	44
57	0·679	0·566 $_3$	1·812 $_6$	0·543	0·0236	0·589 $_4$	43
58	0·680	0·563 $_3$	1·806 $_5$	0·541	0·0218	0·585 $_5$	42
59	0·681	0·560 $_3$	1·801 $_5$	0·540	0·0201	0·580 $_4$	41
0·060	0·681	0·557 $_2$	1·796 $_5$	0·539	0·0184	0·576 $_4$	0·940
61	0·682	0·555 $_3$	1·791 $_5$	0·538	0·0168	0·572 $_4$	39
62	0·682	0·552 $_3$	1·786 $_5$	0·537	0·0152	0·568 $_5$	38
63	0·683	0·549 $_3$	1·781 $_5$	0·535	0·0136	0·563 $_4$	37
64	0·684	0·546 $_2$	1·776 $_4$	0·534	0·0121	0·559 $_4$	36
65	0·684	0·544 $_3$	1·772 $_5$	0·533	0·0107	0·555 $_4$	35
66	0·685	0·541 $_2$	1·767 $_4$	0·532	0·0093	0·551 $_4$	34
67	0·685	0·539 $_3$	1·763 $_5$	0·531	0·0079	0·547 $_4$	33
68	0·686	0·536 $_2$	1·758 $_5$	0·529	0·0066	0·543 $_4$	32
69	0·686	0·534 $_3$	1·753 $_4$	0·528	0·0052	0·539 $_4$	31
0·070	0·687	0·531 $_2$	1·749 $_4$	0·527	0·0038	0·535 $_4$	0·930

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
0·070	0·687	0·531 $_2$	1·749 $_4$	0·527	0·0038	0·535 $_4$	0·930
71	0·688	0·529 $_3$	1·745 $_4$	0·526	0·0025	0·531 $_3$	29
72	0·688	0·526 $_2$	1·741 $_4$	0·525	0·0012	0·528 $_4$	28
73	0·689	0·524 $_2$	1·737 $_5$	0·524	0·0000	0·524 $_3$	27
74	0·689	0·522 $_3$	1·732 $_4$	0·523	−0·0012	0·521 $_4$	26
75	0·690	0·519 $_2$	1·728 $_4$	0·522	−0·0024	0·517 $_3$	25
76	0·690	0·517 $_2$	1·724 $_4$	0·520	−0·0035	0·514 $_4$	24
77	0·691	0·515 $_3$	1·720 $_4$	0·519	−0·0047	0·510 $_3$	23
78	0·692	0·512 $_2$	1·716 $_4$	0·518	−0·0058	0·507 $_4$	22
79	0·692	0·510 $_2$	1·712 $_4$	0·517	−0·0069	0·503 $_3$	21
0·080	0·693	0·508 $_2$	1·708 $_3$	0·516	−0·0080	0·500 $_3$	0·920
81	0·693	0·506 $_2$	1·705 $_4$	0·515	−0·0091	0·4[illegible]	19
82	0·694	0·504 $_2$	1·701 $_4$	0·514	−0·0102	0·494 $_4$	18
83	0·694	0·502 $_2$	1·697 $_3$	0·513	−0·0112	0·490 $_3$	17
84	0·695	0·500 $_2$	1·694 $_4$	0·512	−0·0122	0·487 $_3$	16
85	0·695	0·498 $_2$	1·690 $_3$	0·511	−0·0132	0·484 $_3$	15
86	0·696	0·496 $_2$	1·687 $_4$	0·509	−0·0141	0·481 $_3$	14
87	0·696	0·494 $_2$	1·683 $_3$	0·508	−0·0151	0·478 $_3$	13
88	0·697	0·492 $_2$	1·680 $_3$	0·507	−0·0160	0·475 $_3$	12
89	0·698	0·490 $_2$	1·677 $_4$	0·506	−0·0170	0·472 $_3$	11
0·090	0·698	0·488 $_2$	1·673 $_3$	0·505	−0·0179	0·469 $_3$	0·910
91	0·699	0·486 $_2$	1·670 $_3$	0·504	−0·0188	0·466 $_2$	09
92	0·699	0·484 $_2$	1·667 $_4$	0·503	−0·0197	0·464 $_3$	08
93	0·700	0·482 $_2$	1·663 $_3$	0·502	−0·0205	0·461 $_2$	07
94	0·700	0·480 $_2$	1·660 $_3$	0·501	−0·0213	0·459 $_3$	06
95	0·701	0·478 $_2$	1·657 $_3$	0·500	−0·0221	0·456 $_3$	05
96	0·701	0·476 $_1$	1·654 $_3$	0·499	−0·0229	0·453 $_2$	04
97	0·702	0·475 $_2$	1·651 $_3$	0·498	−0·0237	0·451 $_3$	03
98	0·702	0·473 $_2$	1·648 $_3$	0·497	−0·0245	0·448 $_2$	02
99	0·703	0·471 $_2$	1·645 $_3$	0·496	−0·0252	0·446 $_3$	01
0·100	0·703	0·469	1·642	0·495	−0·0259	0·443	0·900

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
0·10	$0{\cdot}703_{6}$	$0{\cdot}469_{17}$	$1{\cdot}642_{29}$	$0{\cdot}495_{10}$	$-0{\cdot}0259_{69}$	$0{\cdot}443_{24}$	0·90
11	$0{\cdot}709_{5}$	$0{\cdot}452_{15}$	$1{\cdot}613_{26}$	$0{\cdot}485_{10}$	$-0{\cdot}0328_{58}$	$0{\cdot}419_{23}$	89
12	$0{\cdot}714_{5}$	$0{\cdot}437_{15}$	$1{\cdot}587_{24}$	$0{\cdot}475_{9}$	$-0{\cdot}0386_{50}$	$0{\cdot}398_{19}$	88
13	$0{\cdot}719_{4}$	$0{\cdot}422_{13}$	$1{\cdot}563_{21}$	$0{\cdot}466_{9}$	$-0{\cdot}0436_{43}$	$0{\cdot}379_{18}$	87
14	$0{\cdot}723_{5}$	$0{\cdot}409_{12}$	$1{\cdot}542_{20}$	$0{\cdot}457_{9}$	$-0{\cdot}0479_{36}$	$0{\cdot}361_{16}$	86
15	$0{\cdot}728_{5}$	$0{\cdot}397_{12}$	$1{\cdot}522_{20}$	$0{\cdot}448_{9}$	$-0{\cdot}0515_{32}$	$0{\cdot}345_{15}$	85
16	$0{\cdot}733_{4}$	$0{\cdot}385_{11}$	$1{\cdot}502_{17}$	$0{\cdot}439_{8}$	$-0{\cdot}0547_{28}$	$0{\cdot}330_{14}$	84
17	$0{\cdot}737_{5}$	$0{\cdot}374_{11}$	$1{\cdot}485_{17}$	$0{\cdot}431_{8}$	$-0{\cdot}0575_{23}$	$0{\cdot}316_{13}$	83
18	$0{\cdot}742_{4}$	$0{\cdot}363_{10}$	$1{\cdot}468_{16}$	$0{\cdot}423_{8}$	$-0{\cdot}0598_{20}$	$0{\cdot}303_{12}$	82
19	$0{\cdot}746_{4}$	$0{\cdot}353_{10}$	$1{\cdot}452_{15}$	$0{\cdot}415_{8}$	$-0{\cdot}0618_{17}$	$0{\cdot}291_{11}$	81
0·20	$0{\cdot}750_{5}$	$0{\cdot}343_{9}$	$1{\cdot}437_{14}$	$0{\cdot}407_{8}$	$-0{\cdot}0635_{15}$	$0{\cdot}280_{11}$	0·80
21	$0{\cdot}755_{4}$	$0{\cdot}334_{9}$	$1{\cdot}423_{13}$	$0{\cdot}399_{7}$	$-0{\cdot}0650_{12}$	$0{\cdot}269_{10}$	79
22	$0{\cdot}759_{4}$	$0{\cdot}325_{8}$	$1{\cdot}410_{13}$	$0{\cdot}392_{8}$	$-0{\cdot}0662_{10}$	$0{\cdot}259_{9}$	78
23	$0{\cdot}763_{4}$	$0{\cdot}317_{8}$	$1{\cdot}397_{13}$	$0{\cdot}384_{7}$	$-0{\cdot}0672_{9}$	$0{\cdot}250_{9}$	77
24	$0{\cdot}767_{4}$	$0{\cdot}309_{8}$	$1{\cdot}384_{11}$	$0{\cdot}377_{7}$	$-0{\cdot}0681_{7}$	$0{\cdot}241_{9}$	76
25	$0{\cdot}771_{4}$	$0{\cdot}301_{8}$	$1{\cdot}373_{11}$	$0{\cdot}370_{7}$	$-0{\cdot}0688_{5}$	$0{\cdot}232_{8}$	75
26	$0{\cdot}775_{4}$	$0{\cdot}293_{7}$	$1{\cdot}362_{11}$	$0{\cdot}363_{7}$	$-0{\cdot}0693_{4}$	$0{\cdot}224_{8}$	74
27	$0{\cdot}779_{4}$	$0{\cdot}286_{7}$	$1{\cdot}351_{10}$	$0{\cdot}356_{7}$	$-0{\cdot}0697_{3}$	$0{\cdot}216_{7}$	73
28	$0{\cdot}783_{4}$	$0{\cdot}279_{7}$	$1{\cdot}341_{10}$	$0{\cdot}349_{7}$	$-0{\cdot}0700_{1}$	$0{\cdot}209_{7}$	72
29	$0{\cdot}787_{4}$	$0{\cdot}272_{7}$	$1{\cdot}331_{10}$	$0{\cdot}342_{6}$	$-0{\cdot}0701_{1}$	$0{\cdot}202_{7}$	71
0·30	$0{\cdot}791_{3}$	$0{\cdot}265_{6}$	$1{\cdot}321_{9}$	$0{\cdot}336_{7}$	$-0{\cdot}0702_{0}$	$0{\cdot}195_{6}$	0·70
31	$0{\cdot}794_{4}$	$0{\cdot}259_{6}$	$1{\cdot}312_{9}$	$0{\cdot}329_{6}$	$-0{\cdot}0702_{2}$	$0{\cdot}189_{6}$	69
32	$0{\cdot}798_{4}$	$0{\cdot}253_{6}$	$1{\cdot}303_{8}$	$0{\cdot}323_{7}$	$-0{\cdot}0700_{2}$	$0{\cdot}183_{6}$	68
33	$0{\cdot}802_{3}$	$0{\cdot}247_{6}$	$1{\cdot}295_{9}$	$0{\cdot}316_{6}$	$-0{\cdot}0698_{3}$	$0{\cdot}177_{6}$	67
34	$0{\cdot}805_{4}$	$0{\cdot}241_{6}$	$1{\cdot}286_{8}$	$0{\cdot}310_{6}$	$-0{\cdot}0695_{3}$	$0{\cdot}171_{6}$	66
35	$0{\cdot}809_{4}$	$0{\cdot}235_{6}$	$1{\cdot}278_{8}$	$0{\cdot}304_{6}$	$-0{\cdot}0692_{4}$	$0{\cdot}165_{5}$	65
36	$0{\cdot}813_{3}$	$0{\cdot}229_{6}$	$1{\cdot}270_{7}$	$0{\cdot}298_{6}$	$-0{\cdot}0688_{5}$	$0{\cdot}160_{5}$	64
37	$0{\cdot}816_{4}$	$0{\cdot}223_{5}$	$1{\cdot}263_{7}$	$0{\cdot}292_{6}$	$-0{\cdot}0683_{5}$	$0{\cdot}155_{5}$	63
38	$0{\cdot}820_{3}$	$0{\cdot}218_{5}$	$1{\cdot}256_{8}$	$0{\cdot}286_{6}$	$-0{\cdot}0678_{6}$	$0{\cdot}150_{4}$	62
39	$0{\cdot}823_{4}$	$0{\cdot}213_{6}$	$1{\cdot}248_{7}$	$0{\cdot}280_{6}$	$-0{\cdot}0672_{6}$	$0{\cdot}146_{5}$	61
0·40	$0{\cdot}827_{2}$	$0{\cdot}207_{5}$	$1{\cdot}241_{7}$	$0{\cdot}274_{6}$	$-0{\cdot}0666_{6}$	$0{\cdot}141_{4}$	0·60

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
0·40	0·827 $_{3}$	0·207 $_{5}$	1·241 $_{7}$	0·274 $_{6}$	−0·0666 $_{6}$	0·141 $_{4}$	0·60
41	0·830 $_{3}$	0·202 $_{5}$	1·234 $_{6}$	0·268 $_{6}$	−0·0660 $_{7}$	0·136 $_{4}$	59
42	0·833 $_{4}$	0·197 $_{5}$	1·228 $_{7}$	0·262 $_{5}$	−0·0653 $_{7}$	0·132 $_{4}$	58
43	0·837 $_{3}$	0·192 $_{5}$	1·221 $_{6}$	0·257 $_{6}$	−0·0646 $_{8}$	0·128 $_{4}$	57
44	0·840 $_{4}$	0·187 $_{4}$	1·215 $_{6}$	0·251 $_{5}$	−0·0638 $_{8}$	0·124 $_{4}$	56
45	0·844 $_{3}$	0·183 $_{5}$	1·209 $_{6}$	0·246 $_{6}$	−0·0630 $_{8}$	0·120 $_{4}$	55
46	0·847 $_{3}$	0·178 $_{4}$	1·203 $_{6}$	0·240 $_{5}$	−0·0622 $_{8}$	0·116 $_{4}$	54
47	0·850 $_{3}$	0·174 $_{5}$	1·197 $_{6}$	0·235 $_{6}$	−0·0614 $_{9}$	0·112 $_{3}$	53
48	0·853 $_{3}$	0·169 $_{4}$	1·191 $_{5}$	0·229 $_{5}$	−0·0605 $_{9}$	0·109 $_{4}$	52
49	0·857 $_{3}$	0·165 $_{5}$	1·186 $_{6}$	0·224 $_{5}$	−0·0596 $_{10}$	0·105 $_{3}$	51
0·50	0·860	0·160	1·180	0·219	−0·0586	0·102	0·50
0·50	0·860 $_{3}$	0·160 $_{4}$	1·180 $_{5}$	0·219 $_{5}$	−0·0586 $_{10}$	0·1017 $_{34}$	0·50
51	0·863 $_{3}$	0·156 $_{4}$	1·175 $_{5}$	0·214 $_{6}$	−0·0576 $_{9}$	0·0983 $_{32}$	49
52	0·866 $_{3}$	0·152 $_{4}$	1·170 $_{5}$	0·208 $_{5}$	−0·0567 $_{10}$	0·0951 $_{31}$	48
53	0·869 $_{4}$	0·148 $_{4}$	1·165 $_{5}$	0·203 $_{5}$	−0·0557 $_{10}$	0·0920 $_{31}$	47
54	0·873 $_{3}$	0·144 $_{4}$	1·160 $_{5}$	0·198 $_{5}$	−0·0547 $_{11}$	0·0889 $_{30}$	46
55	0·876 $_{3}$	0·140 $_{4}$	1·155 $_{5}$	0·193 $_{5}$	−0·0536 $_{10}$	0·0859 $_{29}$	45
56	0·879 $_{3}$	0·136 $_{4}$	1·150 $_{5}$	0·188 $_{5}$	−0·0526 $_{10}$	0·0830 $_{28}$	44
57	0·882 $_{3}$	0·132 $_{4}$	1·145 $_{4}$	0·183 $_{5}$	−0·0516 $_{11}$	0·0802 $_{28}$	43
58	0·885 $_{3}$	0·128 $_{4}$	1·141 $_{5}$	0·178 $_{4}$	−0·0505 $_{10}$	0·0774 $_{27}$	42
59	0·888 $_{3}$	0·124 $_{4}$	1·136 $_{4}$	0·174 $_{5}$	−0·0495 $_{11}$	0·0747 $_{27}$	41
0·60	0·891 $_{3}$	0·120 $_{3}$	1·132 $_{5}$	0·169 $_{5}$	−0·0484 $_{11}$	0·0720 $_{26}$	0·40
61	0·894 $_{3}$	0·117 $_{4}$	1·127 $_{4}$	0·164 $_{5}$	−0·0473 $_{12}$	0·0694 $_{25}$	39
62	0·897 $_{3}$	0·113 $_{3}$	1·123 $_{4}$	0·159 $_{5}$	−0·0461 $_{11}$	0·0669 $_{25}$	38
63	0·900 $_{3}$	0·110 $_{4}$	1·119 $_{4}$	0·154 $_{4}$	−0·0450 $_{11}$	0·0644 $_{24}$	37
64	0·903 $_{3}$	0·106 $_{4}$	1·115 $_{4}$	0·150 $_{5}$	−0·0439 $_{11}$	0·0620 $_{23}$	36
65	0·906 $_{3}$	0·1023 $_{34}$	1·111 $_{4}$	0·145 $_{5}$	−0·0428 $_{12}$	0·0597 $_{24}$	35
66	0·909 $_{3}$	0·0989 $_{33}$	1·107 $_{4}$	0·140 $_{5}$	−0·0416 $_{11}$	0·0573 $_{22}$	34
67	0·912 $_{2}$	0·0956 $_{34}$	1·103 $_{4}$	0·135 $_{4}$	−0·0405 $_{12}$	0·0551 $_{22}$	33
68	0·914 $_{3}$	0·0922 $_{34}$	1·099 $_{4}$	0·131 $_{4}$	−0·0393 $_{12}$	0·0529 $_{22}$	32
69	0·917 $_{3}$	0·0888 $_{33}$	1·095 $_{4}$	0·127 $_{5}$	−0·0381 $_{12}$	0·0507 $_{21}$	31
0·70	0·920 $_{3}$	0·0855 $_{32}$	1·091 $_{3}$	0·122 $_{4}$	−0·0369 $_{12}$	0·0486 $_{21}$	0·30

k'^2	G_1	G_2	G_3	H_1	H_2	H_3	k^2
0·70	$0·920_{3}$	$0·0855_{32}$	$1·091_{3}$	$0·122_{4}$	$-0·0369_{12}$	$0·0486_{21}$	0·30
71	$0·923_{3}$	$0·0823_{33}$	$1·088_{4}$	$0·118_{4}$	$-0·0357_{12}$	$0·0465_{21}$	29
72	$0·926_{3}$	$0·0790_{32}$	$1·084_{4}$	$0·114_{5}$	$-0·0345_{12}$	$0·0444_{20}$	28
73	$0·929_{2}$	$0·0758_{32}$	$1·080_{3}$	$0·109_{4}$	$-0·0333_{12}$	$0·0424_{20}$	27
74	$0·931_{3}$	$0·0726_{31}$	$1·077_{4}$	$0·105_{5}$	$-0·0321_{12}$	$0·0404_{19}$	26
75	$0·934_{3}$	$0·0695_{30}$	$1·073_{3}$	$0·1004_{42}$	$-0·0309_{12}$	$0·0385_{18}$	25
76	$0·937_{3}$	$0·0665_{31}$	$1·070_{3}$	$0·0962_{43}$	$-0·0297_{12}$	$0·0367_{19}$	24
77	$0·940_{3}$	$0·0634_{30}$	$1·067_{4}$	$0·0919_{43}$	$-0·0285_{12}$	$0·0348_{18}$	23
78	$0·943_{2}$	$0·0604_{30}$	$1·063_{3}$	$0·0876_{42}$	$-0·0273_{12}$	$0·0330_{18}$	22
79	$0·945_{3}$	$0·0574_{30}$	$1·060_{3}$	$0·0834_{42}$	$-0·0261_{12}$	$0·0312_{17}$	21
0·80	$0·948_{3}$	$0·0544_{30}$	$1·057_{3}$	$0·0792_{42}$	$-0·0249_{12}$	$0·0295_{18}$	0·20
81	$0·951_{2}$	$0·0514_{29}$	$1·054_{4}$	$0·0750_{41}$	$-0·0237_{13}$	$0·0277_{17}$	19
82	$0·953_{3}$	$0·0485_{29}$	$1·050_{3}$	$0·0709_{41}$	$-0·0224_{12}$	$0·0260_{16}$	18
83	$0·956_{3}$	$0·0456_{30}$	$1·047_{3}$	$0·0668_{42}$	$-0·0212_{13}$	$0·0244_{17}$	17
84	$0·959_{2}$	$0·0426_{28}$	$1·044_{3}$	$0·0626_{41}$	$-0·0199_{12}$	$0·0227_{16}$	16
85	$0·961_{3}$	$0·0398_{28}$	$1·041_{3}$	$0·0585_{40}$	$-0·0187_{12}$	$0·0211_{15}$	15
86	$0·964_{3}$	$0·0370_{28}$	$1·038_{3}$	$0·0545_{41}$	$-0·0175_{13}$	$0·0196_{16}$	14
87	$0·967_{2}$	$0·0342_{27}$	$1·035_{3}$	$0·0504_{40}$	$-0·0162_{12}$	$0·0180_{15}$	13
88	$0·969_{3}$	$0·0315_{28}$	$1·032_{3}$	$0·0464_{39}$	$-0·0150_{13}$	$0·0165_{15}$	12
89	$0·972_{3}$	$0·0287_{27}$	$1·029_{2}$	$0·0425_{40}$	$-0·0137_{12}$	$0·0150_{15}$	11
0·90	$0·975_{2}$	$0·0260_{27}$	$1·027_{3}$	$0·0385_{39}$	$-0·0125_{13}$	$0·0135_{14}$	0·10
91	$0·977_{3}$	$0·0233_{27}$	$1·024_{3}$	$0·0346_{40}$	$-0·01124_{125}$	$0·0121_{14}$	09
92	$0·980_{2}$	$0·0206_{27}$	$1·021_{3}$	$0·0306_{39}$	$-0·00999_{124}$	$0·0106_{14}$	08
93	$0·982_{3}$	$0·0179_{26}$	$1·018_{3}$	$0·0267_{38}$	$-0·00875_{125}$	$0·0092_{13}$	07
94	$0·985_{2}$	$0·0153_{25}$	$1·015_{2}$	$0·0229_{39}$	$-0·00750_{125}$	$0·0079_{14}$	06
95	$0·987_{3}$	$0·0128_{26}$	$1·013_{3}$	$0·0190_{38}$	$-0·00625_{125}$	$0·0065_{13}$	05
96	$0·990_{2}$	$0·0102_{26}$	$1·010_{2}$	$0·0152_{38}$	$-0·00500_{125}$	$0·0052_{14}$	04
97	$0·992_{3}$	$0·0076_{25}$	$1·008_{3}$	$0·0114_{39}$	$-0·00375_{125}$	$0·0038_{13}$	03
98	$0·995_{2}$	$0·0051_{26}$	$1·005_{2}$	$0·0075_{37}$	$-0·00250_{125}$	$0·0025_{12}$	02
99	$0·997_{3}$	$0·0025_{25}$	$1·003_{3}$	$0·0038_{38}$	$-0·00125_{125}$	$0·0013_{13}$	·01
1·00	1·000	0·0000	1·000	0	0	0·0000	0·00

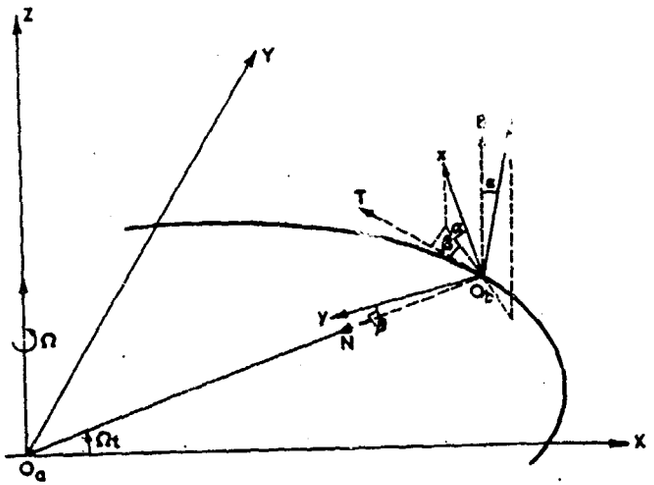


FIG. 1. Systems of co-ordinates 1.—Absolute co-ordinates (X, Y, Z) and relative co-ordinates (T, N, B) and (x, y, z).

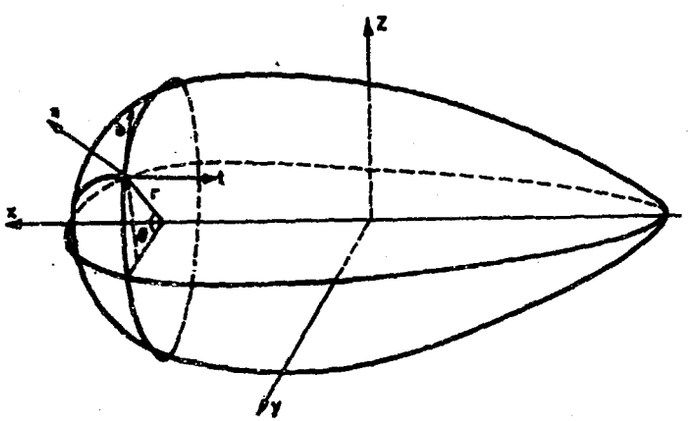


FIG. 2. Systems of co-ordinates 2.—Relative co-ordinates (x, y, z), (x, r, θ) and (n, t, b).

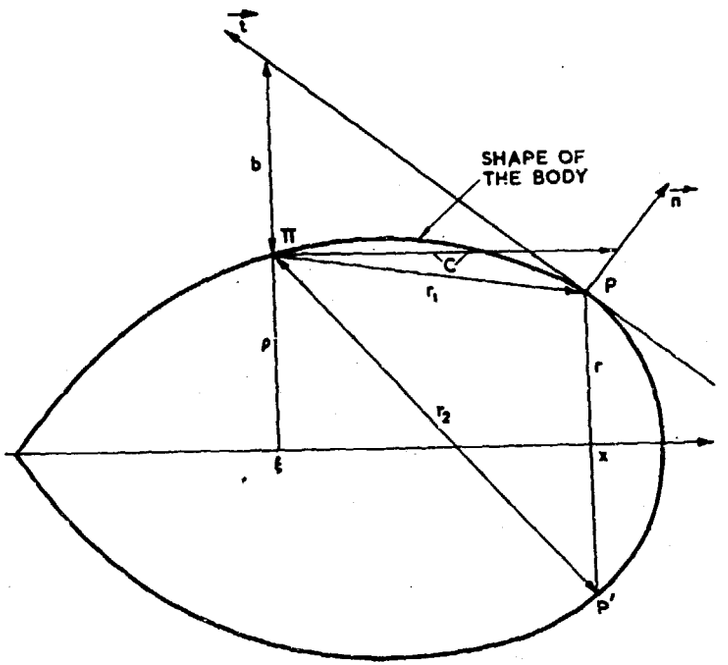


FIG. 3. Geometric interpretation of the expressions r_1, r_2, b and c.

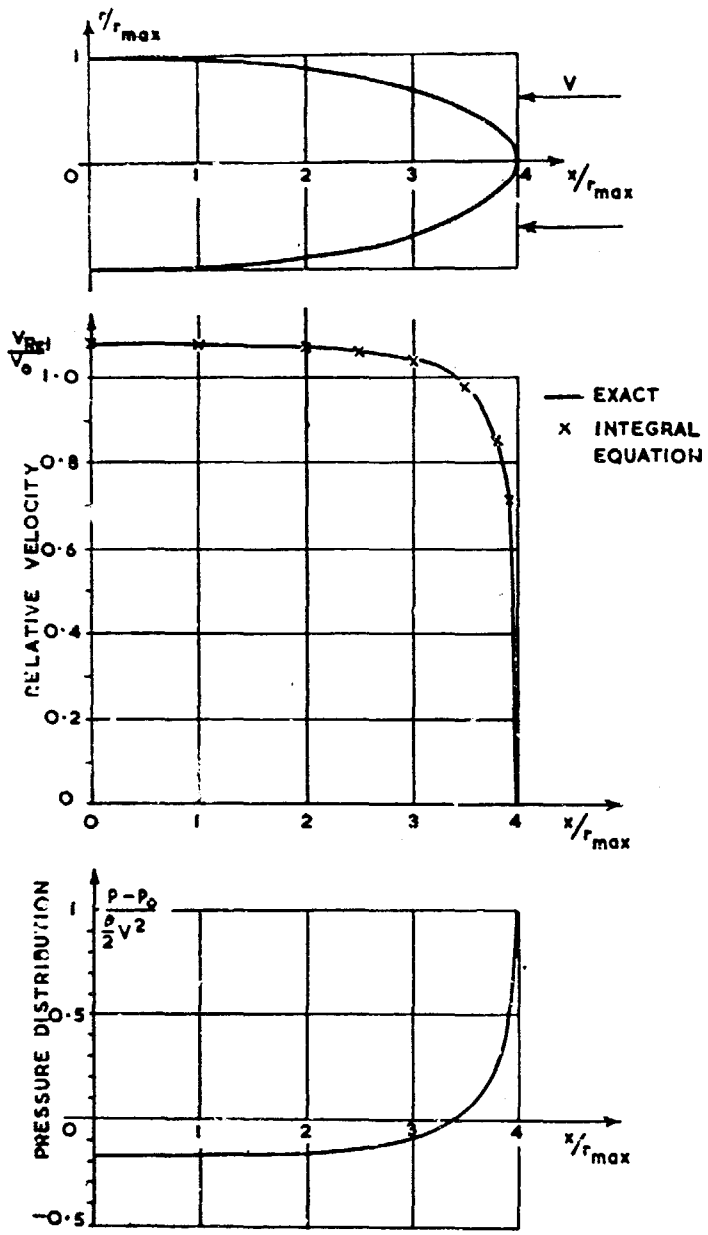


FIG. 4. Ellipsoid 1 : 4.—Longitudinal translation.

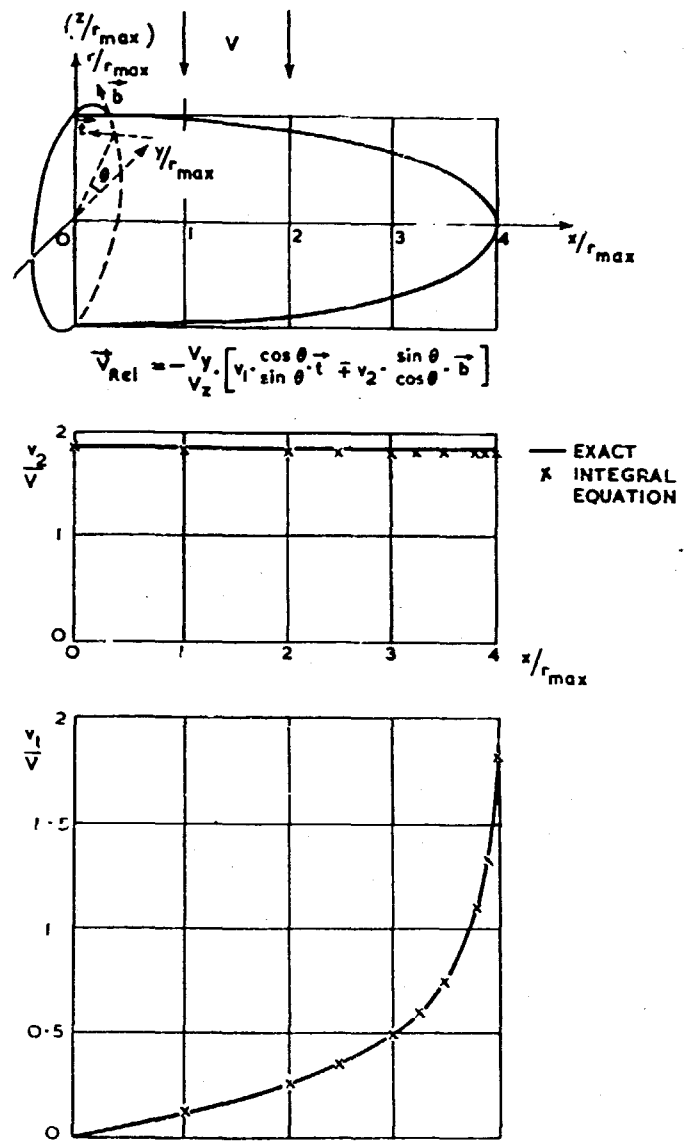
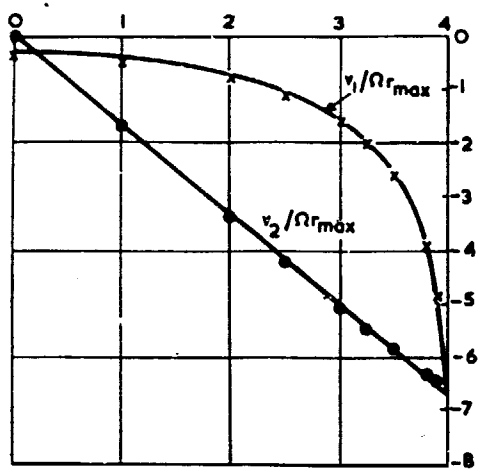


FIG. 5. Ellipsoid 1 : 4.—Lateral translation.



$\frac{x}{r_{max}}$	$v_1/\Omega r_{max}$	EXACT	$v_2/\Omega r_{max}$	EXACT
4		(−6·75)		(−6·75)
3·9	−4·83	(−4·92)	−6·46	(−6·58)
3·8	−3·94	(−3·97)	−6·35	(−6·42)
3·5	−2·56	(−2·57)	−5·90	(−5·91)
3·25	−1·97	(−1·98)	−5·45	(−5·49)
3	−1·57	(−1·57)	−5·02	(−5·07)
2·5	−1·07	(−1·07)	−4·20	(−4·22)
2	−0·76	(−0·76)	−3·36	(−3·38)
1	−0·43	(−0·41)	−1·66	(−1·69)
0	−0·34	(−0·31)	0	(0)

FIG. 6. Ellipsoid 1 : 4.—Relative velocity in the case of rotation about the axis of y or z.

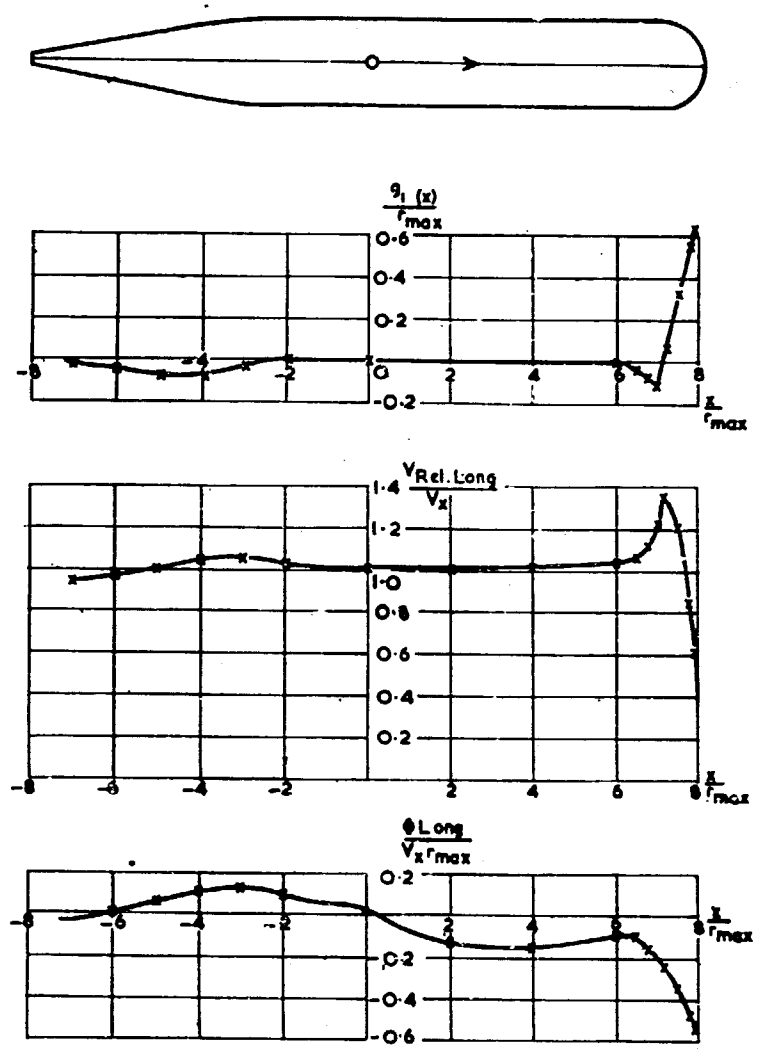


FIG. 7. Torpedo in longitudinal translation.—Source distribution, relative velocity, and potential.

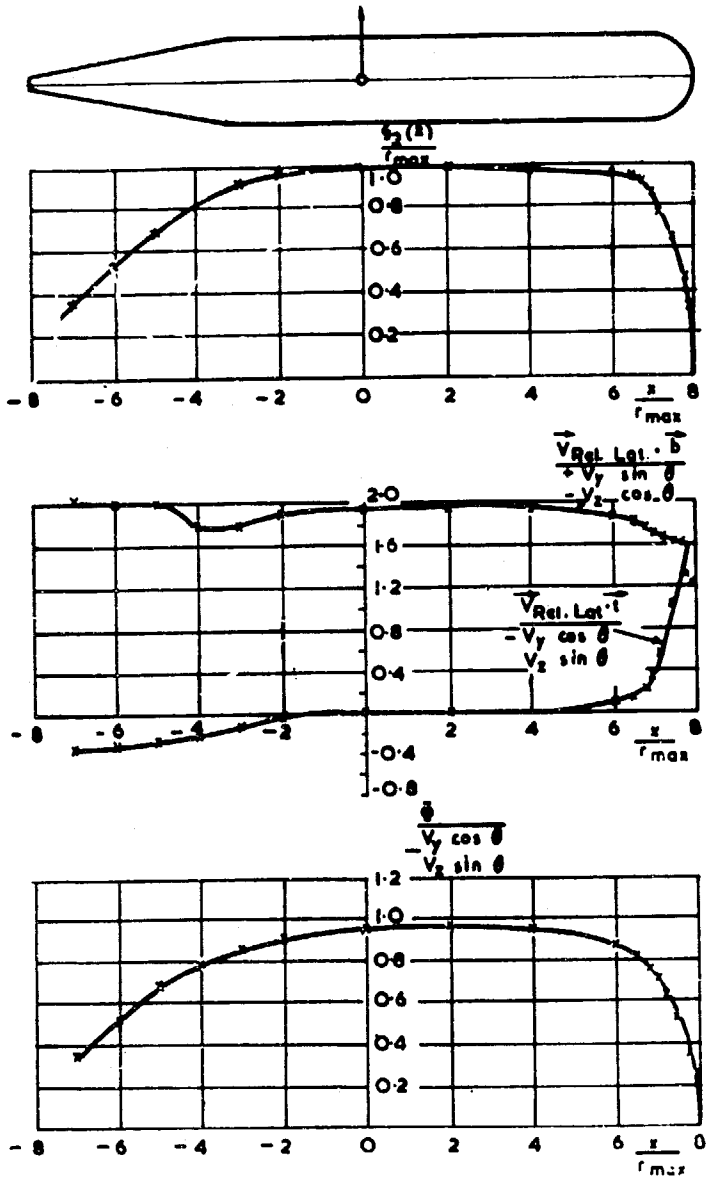


FIG. 8. Torpedo in lateral translation.—Source distribution, relative velocity, and potential.

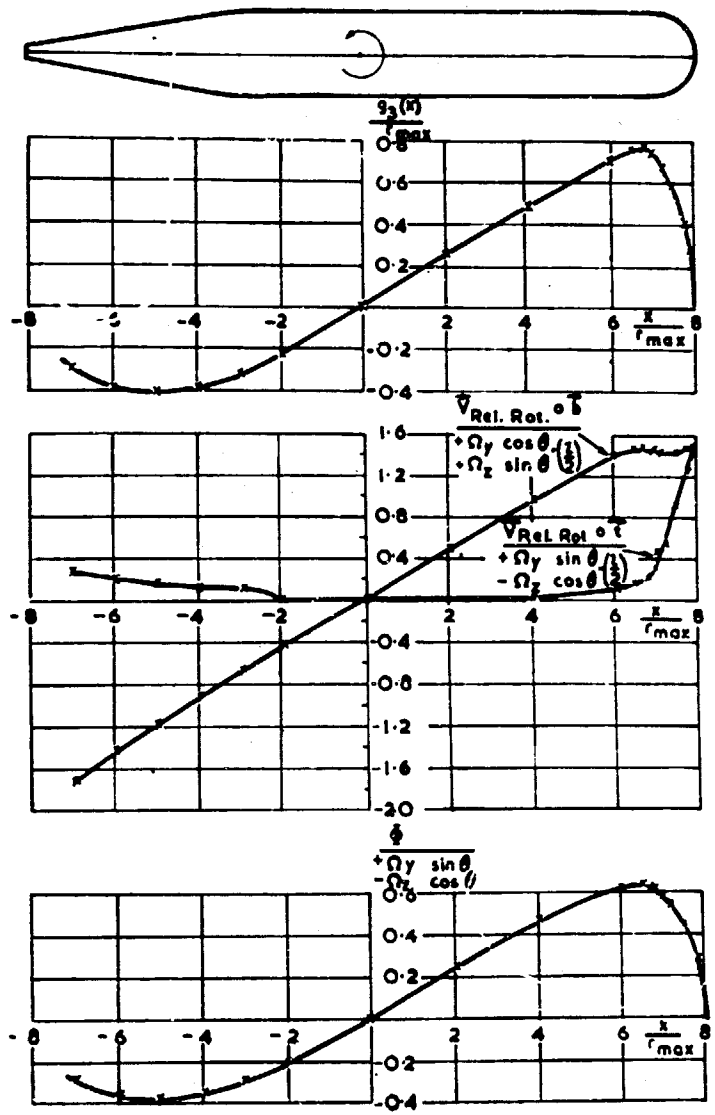


FIG. 9. Torpedo in rotation.—Source distribution, relative velocity and potential.