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MEASUREMENT OF RAPID TEMPERATURE FLUCTUATIONS IN PULSATING GAS FLOW

BY

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Introduction

The purpose of this paper is to reveal the work carried out in an attempt to find a method for measuring the time variation of temperature throughout the cycle of a pulse jet engine. The problem of measuring such temperature has not yet been satisfactorily solved, as far as we know; obviously, the knowledge of instantaneous temperatures is of vital importance in the investigation of pulse jet engines.

This paper offers a detailed exposition of techniques and work carried out in a study of this problem with particular reference to a method proposing the use of an ultrasonic beam for measuring gas temperatures as suggested by Professor I. Fankuchen of the Polytechnic Institute of Brooklyn.

The solution of the problem must meet two main requirements. It is imperative that the temperature measuring system be able to follow the instantaneous temperatures with negligible time lag as the repetition rate of the pulse jet engine under investigation is 200 c.p.s. Furthermore the system must be able to withstand the high temperatures occuring in the combustion chamber of a pulse jet engine, while at the same time not interfering with the gas flow. All of the conventional temperature measuring methods must therefore be discarded.

Since the transit time of sound passing through a medium is a function of the temperature of the medium among other parameters, it was suggested that an ultrasonic beam be used to measure gas temperatures.

For the short path-length involved, the time lag is negligible. The radiating and receiving transducers can be placed so that they do not interfere with the gas flow and so that at the same time they are exposed only to the relatively low temperatures at the combustion chamber walls. If necessary, additional external cooling of the transducers may be applied.

The practicability of the method was from the very beginning doubtful, and discussions of the problem with several experts in the field of ultrasonics were discouraging. Mr. S. Young White, consulting engineer of New York City and Mr. Edwin E. Turner, Assistant Chief Engineer of the Submarine Signal Company, Boston, Mass., as well as many others, pointed out that in order to solve the problem conflicting requirements had to be fulfilled.

The three major problems were the construction of a suitable ultrasonic generator, the practicability of transmission through highly turbulent flow, and the evaluation of results. Difficulties were further augmented by the fact that literature concerning the transmission of ultrasonic waves through gases is virtually non-existent.

Notwithstanding the discouraging outlook investigations were begun along the following lines: (1) the development of an experimental supersonic generator to facilitate final design of the transducer, (2) the investigation of energy transmission in turbulent flow, (3) the construction of the final transducer, and (4) measurement of temperatures in the combustion chamber of a pulse jet.

The results of the investigation under (2) were, as was to be expected, negative. Further consideration of (4) indicated that the variables involved in the measurements would not be separable, and hence that even if it were possible to transmit energy through the turbulent gases, the results would still be a function not only of the temperature but also of an undeterminate function of the velocity and density of the combustion gases.

Analysis of the Problem

The fundamental fact upon which the method is based is that the vleocity of sound varies according to the following equation:

$$c^2 = \frac{p}{\rho}$$

where

c = velocity of sound
p = pressure
ρ = density

further the equation of state of the gas is

$$p = \rho RT \qquad R = R(T)$$

where

R = universal gas constant
T = abs. temperature

It is readily seen that for uniform gas distribution and ideal steady flow where

$$\rho u A = \text{constant}$$

the transit time of ultrasonic signals passing through the chamber is a direct measure of the instantaneous temperature. However, the distribution of the various combustion gases in the combustion chamber during the operation of the pulse jet is irregular and is an unknown function of space and time. The gases involved are not ideal and furthermore the flow is pulsating ; thus the continuity equation

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho Au) = 0$$

must be solved for the unknown density and velocity distributions. It is doubtful that any assumed distribution of the above parameters would give sufficiently accurate results for the conversion of transit time records into instantaneous temperatures.

Choice of Operating Frequency:

The repetition rate of the pulse jet is 200 c.p.s. It is desired to have at least ten measurements per cycle. As will be shown later, a high mechanical Q is essential for high efficiency in the transducer. If we assume a comparatively low value of 40 for the mechanical Q, the dissipation is

$$\Theta = \frac{2\pi}{Q} = \frac{6.28}{40} = 0.157$$

and the original amplitude will fall off 63% after

$$n = \frac{1}{0.157} \cong 6 \text{ cycles}$$

Since the total signal should last at least twice the build up and decay time, the minimum pulse duration should be twenty-four cycles. Using 2000 pulses per second (each having a pulse width of 24 cycles) a carrier of 100 k.c. still leaves a convenient interval between pulses and is a reasonable frequency if the Q can be kept sufficiently low. This consideration shows that to maintain a reasonable carrier frequency one must sacrifice efficiency. For comparison, a typical value used in underwater sound detection is Q = 180.

The Supersonic Generator:

The main factor in determining the most suitable supersonic generator was the operating temperature of approximately 1500°F. Piezo-electric devices were discarded because of the low Curie points of the various crystals and the impracticability of cooling. Electro-dynamic transducers were also discarded because of extremely low efficiencies at the high frequencies involved. Because of high Curie points (1000°C for Vanadium Permendur and 360°C for Nickle A) and the possibility of cooling, magnetostriction transducers proved to be the only possible solution, although they do not lend themselves well to generation of sound waves in air because of poor impedance match between the transducer and the air.

Initial attempts to construct transducers with Permendur were unsuccessful because of fabrication difficulties. Later attempts with Nickle A, annealed for one hour at 900°C were more successful. Even though nickel has a lower Curie point, it is more desirable than Permendur because of its superior magnetostrictive and mechanical characteristics.

An experimental transducer was built to operate about 10 k.c. so that a standard commercial microphone could be used as a receiver. The magnetostrictive element consisted of a single nickel tube. Experimental evidence showed that the increased efficiency to be expected from slotting the tube to reduce eddy current losses were overwhelmed by losses due to parasitic vibrations. Slotting was therefore omitted. The radiating element was a light rigid aluminum disc. Polarization was maintained by external D.C. excitation.

The experimental transducer was used to obtain basic design data for the final transducer and to investigate the possibility of transmission in non-steady turbulent flow. To evaluate the experimental results, some theoretical aspects concerning the efficiency of magnetostrictive transducers were considered.

Analysis of Magnetostrictive Transducers:

The analysis of magnetostrictive transducers were carried out as follows:

The basic energy equation of magnetostriction is

$$W = \frac{1}{4\pi}\int_0^B H\,dB - \varepsilon B^2 s + \frac{Es^2}{2}$$

where

W = potential energy per unit volume

H = total magnetic field intensity

B = total magnetic induction

s = strain

ε = magnetostrictive coefficient

E = Young's modulus

Hence the stress and the electric part of the field are

$$\sigma = \left(\frac{\partial W}{\partial s}\right)_B = -\varepsilon B^2 + Es$$

$$H_e = 4\pi \left(\frac{\partial W}{\partial B}\right)_s = H - 8\pi\varepsilon Bs$$

if $\lambda = 2\varepsilon B_o$ $\qquad$ B_o = polarization

then

$$\sigma = -\lambda B + Es$$

$$H_e = H - 4\pi\lambda s$$

$$B = \mu \times H$$

where x is a complex factor accounting for the eddy current losses:

$$x = x_o e^{-j\delta} = x_R - jx_I$$

Values of x_R and x_I are tabulated in "The Design and Construction of Magnetostriction Transducer", (N.D.R.C. Summary Technical Report, 1946).

If forces F_1 and F_2 act on both ends of the magnetostrictive rod of length l, cross-sectional area a, and density ρ, then

$$\frac{d\xi}{dl} = \beta B^2 = \lambda B$$

$$H = H_e - 4\pi\lambda s$$

$$\sigma = -\lambda B + Es$$

$$\sigma = -\lambda B_e - \frac{4\pi\lambda^2\mu x(\xi_2-\xi_1)}{l} + \frac{\partial\xi}{\partial x}$$

where $\frac{\xi_2-\xi_1}{l}$ is the average strain, and further

$$\rho a \frac{d\xi^2}{dt^2} dx = \frac{dF}{dx} dx$$

$$\rho \frac{d^2\xi}{dt^2} = \frac{d\sigma}{dx}$$

$$\rho \frac{d^2\xi}{dt^2} = E\frac{d^2\xi}{dx^2}$$

$$\xi = e^{jwt}k(A \cos kx + B \sin kx)$$

$$k = w\sqrt{\frac{\rho}{E}} = \frac{w}{c}$$

$$\frac{d\xi}{dx} = e^{jwt}k(-A \sin kx + B \cos kx)$$

if

$$x = o \quad \frac{d\xi}{dt} = -v_1 \qquad \sigma = \frac{F_1}{a}$$

$$x = 1 \quad \frac{d\xi}{dt} = v_2 \qquad \sigma = \frac{F_2}{a}$$

where v stands for velocity. Hence

$$A = \frac{-v_1}{jwe^{jwt}}$$

$$B = \frac{v_2+v_1}{\sin kl} \cdot \frac{k\ l}{jwe^{jwt}}$$

$$F_1 = \lambda aB_e - \frac{4\pi\lambda^2 a\mu x}{jwl}(v_2+v_1) - j\rho ca(v_1 \cot an\ k\ l + v_2 \operatorname{cosec}\ kl)$$

$$F_2 = \lambda aB_e - \frac{4\pi\lambda^2 a\mu x}{jw\lambda}(v_2+v_1) - j\rho ca(v_1 \operatorname{cosec}\ k\ l + v_2 \cot an\ k\ l)$$

$$B_e = \frac{4\pi\mu xN}{l}\ I$$

The electric impedance may be considered to be made up of two arbitrary parts: the core impedance Z_c, resulting from the magnetic flux passing through the core assuming no air gap; and that part due to leakage, i.e. due to all of the flux passing through the air.

$$Z_e = Z_l + Z_c$$

$$Z_c = \frac{jw4\pi N^2 a\mu x}{l}$$

Letting

$$G = \frac{4\pi\lambda a N\mu x}{l}$$

then

$$E = (Z_l + Z_c)I + Gv_1 + Gv_2$$

$$F_1 = -GI - (j\rho ca \text{ cotan } kl + \frac{G^2}{Z_c})v_1 - (j\rho ca \text{ cosec } kl + \frac{G^2}{Z_c})v_2$$

$$F_2 = -GI - (j\rho ca \text{ cosec } kl + \frac{G^2}{Z_c})v_1 - (j\rho ca \text{ cotan } kl + \frac{G^2}{Z_c})v_2$$

If the following transformation is made

$$F_1 = je_1 \qquad F_2 = je_2$$

$$v_1 = ji_1 \qquad v_2 = ji_2$$

the mutual terms of different signs of the matrix, characteristic of magnetostriction, take on the same signs and hence the electromechanical system may be represented by means of an equivalent circuit.

Since

$$E = (Z_1 + Z_o)I + jGi_1 + jGi_2$$

$$e_1 = jGI - (j\rho\, ca \cot kl + \frac{G^2}{Z_c})\, i_1 - (j\rho\, ca \operatorname{cosec} kl + \frac{G^2}{Z_c})i_2$$

$$e_2 = jGI - (j\rho\, ca \operatorname{cosec} kl + \frac{G^2}{Z_c})\, i_2 - (j\rho\, ca \cot kl + \frac{G^2}{Z_c})i_2$$

One can show that the equivalent circuit is as in Figure 1.

$$Z_{12} = j\rho\, ca \operatorname{cosec} kl + \frac{G^2}{Z_c}$$

$$Z_{11} - Z_{12} = j\rho\, ca \tan\frac{kl}{2}$$

Clamping the rod at one end $v_1 = 0$, $i_1 = 0$. Thus if we open circuit the corresponding end of the equivalent circuit, we obtain the equivalent circuit for the rod clamped at one end ($l = \frac{1}{4}\lambda$). The transducer will be operated at resonance, circuits, and therefore the distributed parameters can be replaced by series circuits (Fig. 2).

In Figure 2:

$M_{tube} = \frac{la\rho}{2}$ ... equivalent lumped mass of the vibrating element

$K = \frac{Ea}{l}\frac{\pi^2}{8}$ ... equivalent lumped static stiffness

M_L ... mass of the radiating piston

R_L ... radiation resistance of air

The electric input impedance is given by

$$Z_i = R_i + jX_i = Z_e + \frac{Z^2_{em}}{Z_m + Z_l}$$

where

Z_{em} is the mutual impedance

Z_e is the electric impedance

Z_m is the mechanical impedance

Z_L is the impedance of the load

The efficiency can then be stated as

$$\eta = \frac{R_L}{R_i}\left[\frac{Z_{em}}{Z_m+Z_L}\right]^2$$

The motional impedance (if the electric side is open circuited) is

$$Z_{mot} = Z_i - Z_e = \frac{Z^2_{em}}{Z_m + Z_L}$$

$$Z_m + Z_L = (R_m + R_L) + j(wM - \frac{K}{w})$$

$$w_o = 2\pi f_o = \sqrt{\frac{K}{M}}$$

$$Q = \frac{\sqrt{MK}}{(R_m + R_L)}$$

where M and K are the equivalent lumped mass and stiffness respectively.

The efficiency at resonance is

$$\eta_{res} = \frac{|Z_i - Z_e|}{R_i} \cdot \frac{R_L}{|Z_m + Z_L|}$$

If $w = w_o$, then

$$Z_m + Z_L = R_m + R_L$$

Hence

$$\eta_{res} = \frac{1}{R_i} \frac{|Z^2_{em}|}{R+R_L} \left(1 - \frac{R_m}{R_m + R_L}\right)$$

where

$$Z_{em} = R_c + \frac{G^2}{R_m + R_L}$$

R_m .... mechanical dissipation

thus

$$\eta_{res} = \frac{1}{1+R_c\frac{R_m R_L}{|Z^2_{em}|}} \left(1 \frac{R_m}{R_m+R_L}\right)$$

$$R_c \frac{R_m+R_L}{|Z^2_{em}|} = \frac{R_c l^2 (R_m + R_L)}{(4\pi)^2 \lambda^2 a^2 N^2 \mu^2 x_o^2} = \frac{1}{k^2 Q_c Q \frac{x_o^2}{x_I}}$$

where

$$Q_c = \frac{X_c}{R_c}$$

and k, the magnetostrictive coupling, is

$$k = \sqrt{\frac{4\pi\lambda^2\mu}{E}}$$

The final expression for the efficiency at resonance is

$$\eta_{res} = \frac{1}{1 + \dfrac{1}{k^2 Q Q_c \dfrac{x_o^2}{x_I}}} \qquad \left[1 - \frac{R_m}{R_m + R_L}\right]$$

If we know the magnitude of the mechanical dissipation, the efficiency can be computed. The first term of the efficiency expression depends largely on the material constants and mechanical design, but the second factor can be easily altered by changing the area of the radiating face. The following is an investigation of the variation of η_{res} with respect to R_L.

$$\eta_{res} = \frac{Z_{em}^2}{R_I(R_m+R_L)} \; \frac{R_m}{R_L+R_m}$$

let

$$R_L = xR_m$$

$$\eta = \frac{x}{x+1} \cdot \frac{\alpha}{\alpha+1+x}$$

if

$$\alpha = k^2 \frac{x_Q^2}{x_I} Q_0 Q$$

$$\frac{d\eta}{dx} = \frac{1}{(x+1)^2} \frac{\alpha}{\alpha+1+x} - \frac{x}{x+1} \frac{\alpha}{(\alpha+1+x)^2} = 0$$

$$\frac{1}{x+1} - \frac{\alpha x}{\alpha+1+x} = 0$$

and hence

$$x = \frac{1}{2\alpha} [- \alpha - 1 + \sqrt{(\alpha-1)^2 + 4\alpha(\alpha+1)}]$$

If we measure the mechanical Q, R_m can be computed. Knowing the value of R_m, the optimum radiating surface for highest efficiency is determined. With the optimum dimensions of the radiating disc the final mechanical Q and the efficency at resonance can be computed. If the dimensions of the radiator are restricted by other requirements, the mechanical dissipation can be varied by the adjustment of the clamping. Experiments showed that maximum radiation was easily attained by adjusting the tightness of the clamping.

The mechanical Q can be measured indirectly by determining the decay factor θ. Mechanical parameters of the transducer must be measured with the electrical side open. The arrangement shown in Fig. 3 was used to determine θ.

The transducer was excited with a Hewlitt-Packard oscillator. A "Millisec" relay, driven by a second oscillator, was connected into the circuit so that the relay in series with the exciting oscillator acted as a square wave generator. Radiation of the transducer was received by a microphone connected to an oscilloscope. The sweep of the oscillo-scope was synchronized to the oscillator driving the relay. With the relay open, the electric side of the transducer is open circuited and the decay factor of the period corresponding to the open state of the transducer can be measured.

The experimental transducer was polarized with approximately $B_o = 3600$ Gauss at $H_o = 10$ Oersted. The wall thickness of the Nickel A tube was 0.001 in 0.0025 cm and the outside diameter was 0.95 cm. The driving frequency was in the vicinity of 10 k.c. at the resonant frequency of the transducer. The diameter of the aluminum disc radiator was large enough (3 cm) to keep the radiation impedance approximately ohmic. From the experimental data θ was calculated to be 0.164, and therefore

$$Q = \frac{2\pi}{.164} \cong 38$$

The other parameters of the transducer were:

cross-sectional area of the magnetostrictive tube $a = 0.00748 \text{ cm}^2$

length of magnetostrictive tube $l = 12.2$ cm

equivalent lumped static stiffness $K = \frac{\pi^2}{8} \frac{aE}{l} = 0.156 \times 10^{10}$ dyn/cm

where $E = 2.08 \times 10^{-12}$ at $H_o = 10$ oersted

equivalent lumped mass off the tube $M_{tube} = 0.396$g

equivalent lumped mass off the disc $M_{disc} = 0.85$g

equivalent lumped total mass $M = 1.246$g

radiating face $a_r = 7.1 \text{ cm}^2$

radiation resistance $R_L = \rho_{air} c_{air} a_r = 290$ ohm

at 20°C and 76 cm of mercury

$\lambda = 1.6 \times 10^4 \text{dynes}/_{g} \text{ cm}^2$ for $H_o = 10$ oersted

$\mu = 64$

$k = 0.31$

number of driving turns $N = 540$

$x_o = 0.75$

$x_I = 0.2$

$x_R = 0.7$

$$X_c = \frac{w 4\pi N^2 a \mu x_R}{l} = 116 \text{ ohm}$$

$$R_c = \frac{w 4\pi N^2 a \mu x_I}{l} = 33.2 \text{ ohm}$$

$$Q_c = \frac{X_c}{R_c} = 3.5$$

With these values the mechanical dissipation is

$$R_m = \frac{\sqrt{MK}}{Q} - R_L = 866 \text{ ohm}$$

It was assumed that for transducers of similar construction this value would remain approximately the same and could serve as a basis of design. For a given R_m one can calculate the optimum dimensions for the radiating surface with the given parameters as was shown previously.

With the given parameters

$$\alpha = k^2 \frac{x_o^2}{x_I} Q_c Q = 35.7$$

$$x = 0.64 = \frac{R_L}{R_m}$$

the optimum radiation resistance therefore is

$$R_L = 0.64 \, R_m = 554 \text{ ohm}$$

and the optimal area

$$a_r = \frac{R_L}{\rho_{air} \, c_{air}} = 13.9 \text{ cm}^2$$

and hence the optimum diameter of the disc is 4.2 cm. Using this diameter the new parameters are

$$M_{disc} = 1.36 \text{ g}$$

$$M = 1.756 \text{ g}$$

$$Q = 37.7$$

The efficiency with the optimal radiating surface is

$$\eta_{res} = 0.37$$

These computations indicate a rapid method for the design of magnetostriction transducers when high accuracy is not required. The first step in the design of a new transducer therefore would be the measurement of the mechanical dissipation of a similar type transducer. Then the proper relationship between the radiation resistance and mechanical dissipation for optimum efficiency is calculated. The area of the radiator thus determined, the other requirements for the dimensions of the piston must be considered, such as those imposed by the desired radiation pattern. If the discrepancy between the two values obtained is not too great, the latter diameter may be selected and the clamping adjusted empirically in order to bring the mechanical dissipation to the optimum value for maximum efficiency.

The efficiency, however, is not the only factor affecting the choice of the radiating surface. The ultrasonic beam dragged by the combustion gases will be deflected up and down the combustion chamber at velocities which may approach that of sound in the medium. Therefore the beam may be deflected as much as 45° from its initial position. Hence it is necessary to produce an ultrasonic beam having a divergence of approximately 90°. The divergence of the beam, which is a function of the diameter of the radiating surface, can be found by computing the radiation pattern for a piston radiating transversally into a tube.

Directivity Pattern.

Figure 4 shows the notation used in the calculation of the directivity pattern. The velocity potential obeys the wave equation

$$\nabla^2 \phi + k^2 \phi = 0$$

$$\phi(r) = \phi n(\varphi) e^{\pm j \varkappa_{mn} z}$$

$$\phi_n(\varphi) = A_{nm} J_n(k_{cnm} \varphi) \cos m\phi$$

where A_{nm} is a normalization factor.

The boundary condition is

$$J_n'(k_{cnm} R) = 0$$

The solution of this equation is obtained by application of a Green's function method. The aim is to find an operator such that

$$\int G(r,r')(\nabla^2+k^2)\phi(r)\,d\tau = 0$$

Using Green's theorem one obtains

$$\int \phi(r)[\nabla^2 G(r,r')+k^2 G(r,r')]\,d\tau = \int [\phi(r)\frac{\partial}{\partial n} G(r,r')-G(r,r')\frac{\partial}{\partial n}\phi(r)]\,ds$$

If

$$\nabla^2 G(r,r')+k^2 G(r,r') = -\delta(r-r')$$

and since

$$\int \delta(r-r')\,d\tau = 1$$

it follows that

$$\phi(r) = \int [G(r,r')\frac{\partial}{\partial n}\phi(r)]\,ds' - \int \phi(r')\frac{\partial}{\partial n} G(r,r')\,ds'$$

The boundary condition on G is

$$\frac{\partial G}{\partial n} = 0 \quad \text{if} \quad \rho = R$$

We assume that

$$G(r,r') = \sum_{n,m} \frac{\phi n(\rho)\,\phi n(\rho')^{-j\varkappa_{nm}|z-z'|}}{2j\varkappa_{nm}}$$

This expression satisfies the homogeneous wave equation. To assure that it satisfies the singularity at r', we investigate the following expression

$$\frac{\partial G}{\partial z}\Big]_{z'_-}^{z'_+} = -\delta(\varphi-\varphi') = -\frac{2j\varkappa_{nm}\sum_{nm}\phi_n(\varphi)\phi_n(\varphi')e^{-j\varkappa_{nm}|z'-z'|}}{2j\varkappa_{nm}}$$

This is valid because of the orthonormality of the eigen-functions, since

$$\sum_{nm} \phi_n(\varphi)\ \phi_n(\varphi') = \delta(\varphi-\varphi')$$

$\frac{\partial\phi}{\partial n} = 0$ everywhere on the bounding surface except on the piston and hence after normalization if $z > z'$ the velocity potential is

$$\phi(r) = -\frac{\partial\phi}{\partial n}\sum_{nm}\frac{\sqrt{\varepsilon_n}}{\sqrt{\pi}}\ \frac{J_n(k_{cnm}\varphi)e^{-j\varkappa_{nm}z}}{2j\varkappa_{nm}\sqrt{(k_{cnm}R)^2-n^2}} \cdots \cos n\phi \iint_{\text{on disc}} R\cos n\phi'\, e^{j\varkappa_{nm}z'}\, d\phi'\, dz'$$

If

$$z' = \sigma'\cos\delta'$$

$$\phi' = \frac{\sigma'\sin\delta}{R}$$

$$\phi(r) = -\frac{\partial\phi}{\partial n}\sum_{nm}\frac{\sqrt{\varepsilon_n}}{\sqrt{\pi}}\ \frac{J_n(k_{cnm}\varphi)e^{-j\varkappa_{nm}z}}{2j\varkappa_{nm}\sqrt{(k_{cnm}R)^2-n^2}}\cos n\phi \int_0^a\int_0^{2\pi}\cos\frac{n\sigma'\sin\delta'}{R}\, e^{j\varkappa_{mn}\sigma'\cos\delta'}\, \sigma'\, d\sigma'\, d\delta'$$

where

$$\varepsilon_n = 1 \quad \text{if} \quad n = 0$$

$$\mathcal{E}_n = 2 \quad \text{if} \quad n \neq 0$$

The numerical evaluation of this result appears to be so involved that only methods of approximation seem to be practical. In this case, however, it is reasonable to estimate the divergence of the beam for an infinite baffle as the dimensions of the radiator are small compared to the diameter of the combustion chamber ($a = 0.33$ cm for 100 k.c.)

In this case the approximate divergence of the ultrasonic beam is given by

$$\theta = \frac{\pi a^2}{\beta 4 \lambda^2} = \frac{\pi}{\beta^2}$$

if

$$a = \beta\lambda$$

Hence for 90° conical divergence

$$\beta = \sqrt{2} \cong 1.4$$

This value was checked by the following experiment. A tube of sufficiently large diameter was formed of sheet aluminum and the transducer mounted as on the combustion chamber. A microphone, which served as the receiving element, was adjusted 45° off the axis of the transducer on the opposite side of the tube. Intensity measurements were made with discs of successively decreasing diameters, starting with $a = 2.5\lambda$. The disc with a diameter $a = 1.8\lambda$ produced a sufficiently large intensity. Since the efficiency is also increased with the increase of diameter this value must be lowered somewhat to $\beta = 1.5$. This agrees reasonably with the experimental value because the tube evidently has a diverging effect. Hence all diameters lower than $1.5\,\lambda$ will satisfy the directivity requirements. We note therefore that for the experimental transducer the diameter for maximum efficiency is less than $1.5\,\lambda$ and requirements for maximum efficiency and optimum directivity are both met.

Investigation of Energy Transmission through Turbulent Flow.

In order to investigate the possibility of transmission through turbulent gases the following experiments were performed. The transducer and a microphone were fixed approximately 10 cm apart. An air flow was established between the microphone and the transducer perpendicular to their axis, by means of a blower. The intensity of radiation was measured without air flow, in approximately steady flow, and in flow scrambled with the help of various screens. The screens consisted of cardboard sheets with concentrically arranged holes of different size. The non-slotted nickel tube of the transducer was changed to a slotted one because the latter had resonant frequencies at 6 k.c., 10 k.c. and 15.5 k.c. In this way investigations could be made with three different frequencies. The intensity of radiation did not change appreciably with approximately steady air flow. The beam, however, was practically erased at all three frequencies when any of the screens were used. It can therefore be concluded that if the long waves of the experimental transducer show such sensitivity to scattering phenomena, transmission will not be practical for the much shorter and thus more sensitive waves of the 100 k.c. transducer.

Conclusions

At this stage of the development it became more and more evident that in the remaining time and with the available funds and equipment the ultimate aim of measuring instantaneous temperatures by ultrasonic waves would not be attainable. The power output of the transducer would need to be increased considerably above that used in the experiments in order to obtain appreciable transmission. (The order of magnitude of power fed into the experimental transducer was 1 Watt.) High power driving of the transducer, however, increases experimental and analytical difficulties tremendously. Analysis of high power under water transducers made by various other laboratories yielded negative results. The permeability and the magnetostrictive coefficient no longer remain constant. Cavitation phenomena also appear, and voltage and current wave forms greatly influence the power. At the same time the difficulty of the contradictory requirement of high Q and high frequency increases. Under these circumstances everything seems to suggest the use of piezo-electric crystals, but at the present time there seems no way to eliminate the problem introduced by high temperatures. Sensitivity of the receiving end must also be raised considerably, and this has severe limitations. Even if satisfactory transmission could be established there remains the problem of the evaluation of the experimental results. As pointed out earlier in this paper it is doubtful that sufficiently accurate separation of the variables could be made to obtain true temperature recordings.

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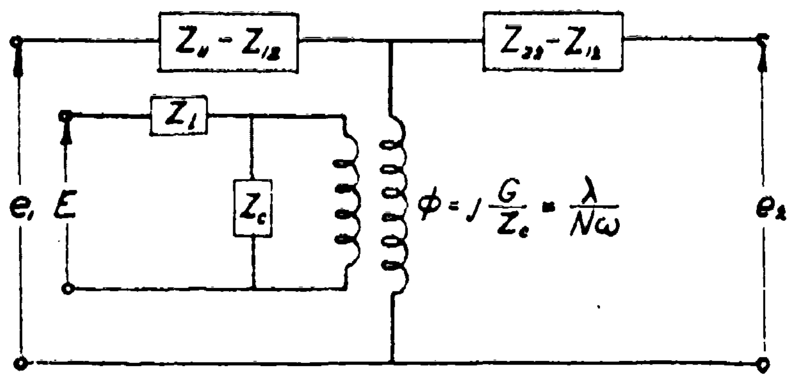
$Z_{11} - Z_{12}$
$Z_{22} - Z_{12}$
Z_l
Z_c
e_1
E
$\phi = j\frac{G}{Z_c} = \frac{\lambda}{N\omega}$
e_2

FIG. 1

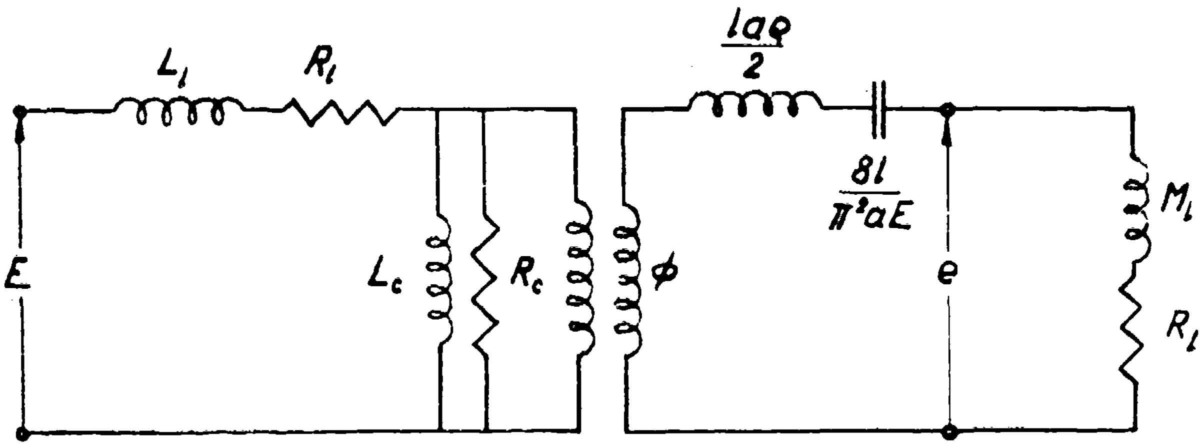
L_l
R_l
$\frac{laQ}{2}$
$\frac{8l}{\pi^2 aE}$
E
L_c
R_c
ϕ
e
M_L
R_L

FIG. 2

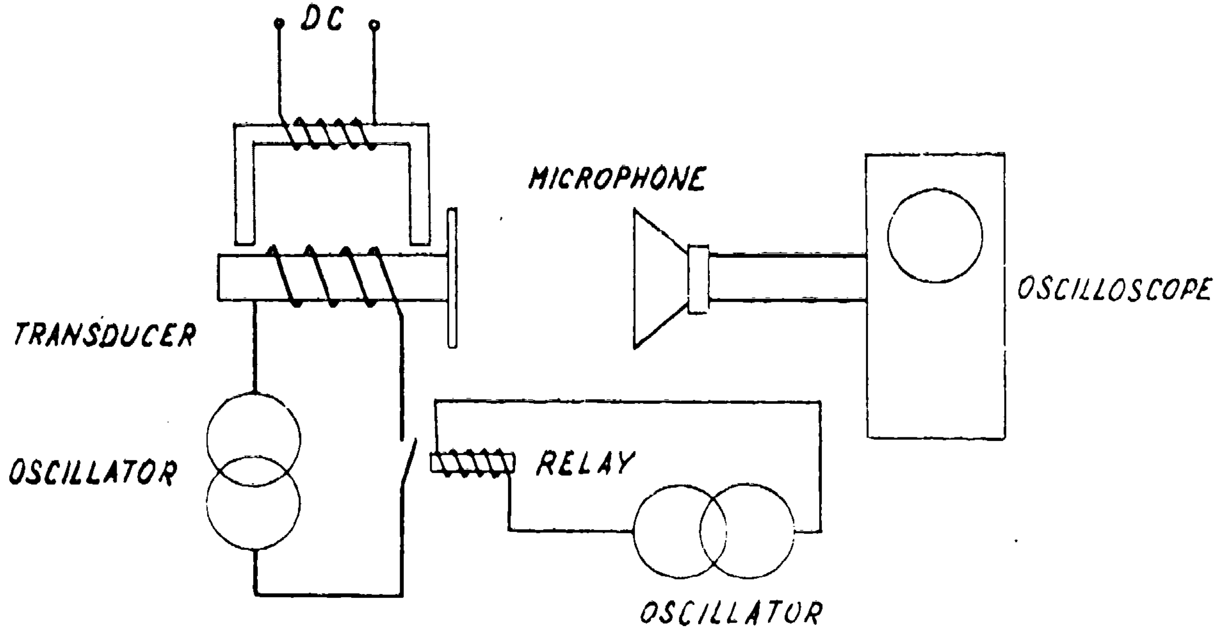
DC
MICROPHONE
OSCILLOSCOPE
TRANSDUCER
OSCILLATOR
RELAY
OSCILLATOR

FIG. 3.

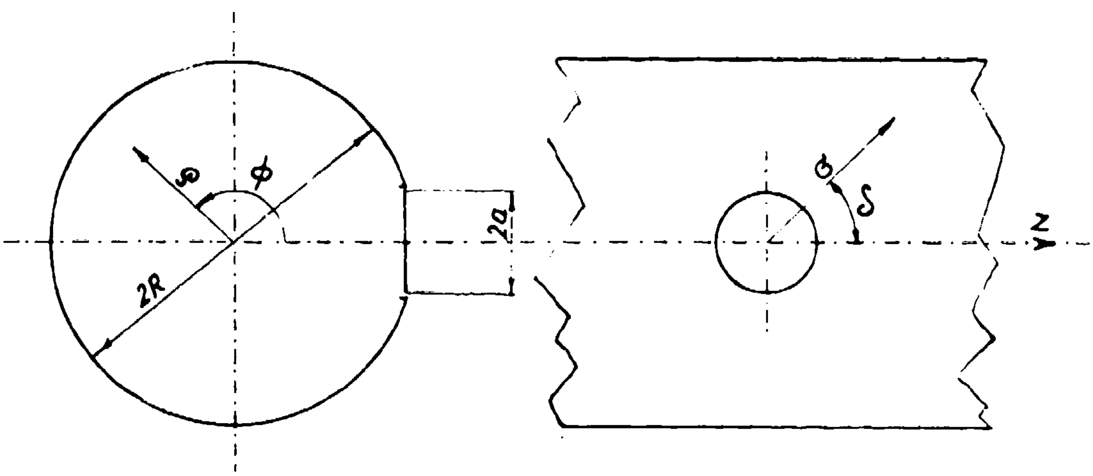
φ
Φ
2R
2a
σ
δ
Z
Y

FIG. 4.